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The pn interaction and isospin symmetry

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A possible correlation between isospin symmetry/breaking and the average proton-neutron interaction of the last particles, δV_{pn}, is discussed. This correlation is tested for $T_z = \pm 1/2$ mirror nuclei in terms of a differential of δV_{pn}, $\Delta(\delta V_{pn})$, and their low-lying excited levels. Some nuclei, whose mass measurements will be useful for future studies, are suggested.

KEYWORDS

proton-neutron interaction, isospin, mirror nuclei, mass measurements, γ-ray spectroscopy

1 Introduction

The strong nuclear force is considered charge-independent and has charge symmetry. The latter means that the interaction strength between protons and neutrons is the same, and being independent of charge means that the sum of proton-proton (*pp*) and neutron-neutron (*nn*) interaction strengths is two times proton-neutron (*pn*) interaction strength. If the Coulomb interaction is ignored, charge independence and charge symmetry will have the same meaning for isobaric nuclei which have the same mass number with different proton and neutron numbers.

Mirror nuclei are pairs of atomic nuclei in which the number of protons in one nucleus equals the number of neutrons in the other, and *vice versa* (e.g., ^{25}Mg and ^{25}Al). In such mirror isobaric nucleus pairs, we expect similar nuclear structures [1]. We can easily see this from similar level schemes.

To understand this, it is useful to define the concept of isospin, T. Both protons and neutrons are assigned the same isospin value of $T = 1/2$, but differ in their isospin z-projection. Protons have $T_z = -1/2$, while neutrons have $T_z = 1/2$. Isospin symmetry is related to similar behavior of nucleons (protons and neutrons). Since some configurations such as *pp* and *nn* with $T = 0$ are forbidden, the Pauli principle should not be forgotten at this point. That is, the isospin symmetry only connects to $T = 1$ in the *pp* and *nn* interactions. For a given nucleus, the isospin projection is given by $T_z = (N\text{-}Z)/2$ where Z and N are, respectively, the number of protons and neutrons. While the low-lying states of a nucleus with given T_z, which we focus on here, generally have $T = |T_z|$, higher states can have higher T values, being part of more extended multi-isobar isospin multiplets.

Mirror nuclei have different T_z. The similar nuclear structure in such nuclei means that their excited states are (almost) identical, in terms of both their energies and spin-parity values. For example, the low-lying states of the $A = 23$ isobaric nuclei, ^{23}Na with $T_z = 1/2$ and ^{23}Mg with $T_z = -1/2$, are shown in Figure 1. As can be seen from the figure, the level schemes of the two nuclei are almost identical, so their nuclear structures are expected to be very similar. For these states, these nuclei exhibit good isospin symmetry. The assumption of perfect isospin symmetry implies that the difference between the binding energies of the mirror nuclei is zero if the differences in the Coulomb interaction in the two nuclei are ignored. Isospin symmetry breaking can occur due to increases in parts of the Coulomb

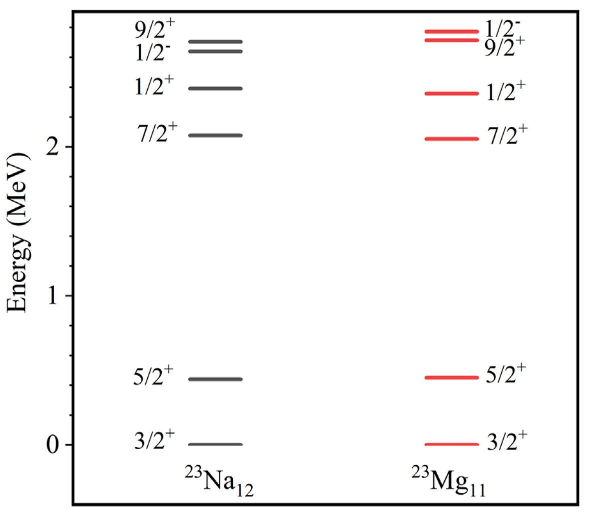

FIGURE 1
(Color online) Low-lying levels and spin-parity assignments for A = 23 [2], T = 1/2 mirror nuclei are shown.

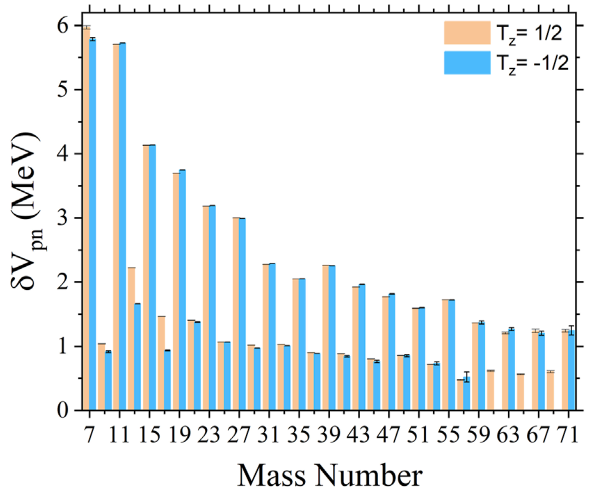

FIGURE 2
(Color online) Experimental δV_{pn} values as a function of mass number for mirror T_z = ±1/2 nuclei. For each mass, there are two δV_{pn} values shown with different colors, namely, T_z = 1/2 with orange and T_z = -1/2 with blue. There is no T_z = -1/2 data at A = 61, 65, 69 due to the lack of direct mass measurements. Masses are based on Refs. [22, 23].

interaction, especially as the mass number increases. Isospin breaking can also occur for other reasons beside the Coulomb interaction (e.g. [3, 4]). By taking these isospin symmetry breaking effects into account, the isospin concept can provide a tool for understanding the excitation energies and binding energies of exotic nuclei that are difficult to reach experimentally. In addition, the study of isospin symmetry breaking plays an important role not only in nuclear physics but also in particle physics, especially in testing the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix [5–9].

Many isobaric nuclei with different isospin projections, such as T_z = ±1, T_z = ±2 have been investigated by experimental charge-exchange reactions [10, 11] and β-decay studies (e.g. [12]). In such studies, the $B(GT)$ values from isobaric T_z = ±1 nuclei to a T_z = 0 nucleus can be compared using both experimental techniques. If the experimental values of $B(GT)$ are similar, isospin symmetry between mirror, T_z = ±1, nuclei can be confirmed. If the values are different, the isospin symmetry may be broken.

2 Approach and methods

In this paper, we explore another observable as a possible indicator or signature of isospin symmetry or its breaking. Since the valence proton-neutron interaction plays an important role in the evolution of nuclear structure [13–15], we will investigate whether an empirical measure of those strengths correlates with isospin symmetry or its breaking. This measure is called δV_{pn} [16; 17; 18] and is the average interaction strength of the last proton(s) and neutron(s). It reflects the spatial overlap of their respective wave functions. We will examine values of δV_{pn} for nuclei near $Z = N$ and will also discuss a related quantity obtained from adjacent δV_{pn} values. We can extract the strengths of these interactions for the last valence proton(s) and neutron(s) from the following expressions in terms of binding energies [16, 17]:

$$\delta V_{pn}^{oe}(Z,N) = \frac{1}{2}\left[\left(B_{Z,N} - B_{Z,N-2}\right) - \left(B_{Z-1,N} - B_{Z-1,N-2}\right)\right] \quad (1)$$

$$\delta V_{pn}^{eo}(Z,N) = \frac{1}{2}\left[\left(B_{Z,N} - B_{Z,N-1}\right) - \left(B_{Z-2,N} - B_{Z-2,N-1}\right)\right] \quad (2)$$

where B is the nuclear binding energy

$$B_{Z,N} = \left(Zm_p + Nm_n - M\right)c^2 \quad (3)$$

and M in Equation 3 is the nuclear mass. Equations 1, 2 are given for odd-A. More detailed information can be found in Ref. [18]. Here we look at other applications of δV_{pn} to understand nuclear structure and its trends.

3 Results and discussion

In recent years, many light nuclei have been studied especially in such contexts as of the island of inversion, appearance, and disappearance of closed shells, etc. [19]. In addition, such nuclei have been studied in terms of δV_{pn}, in particular for the case where the values of δV_{pn} have obvious spikes at $Z = N$. This has been explained by Wigner's SU(4) symmetry [20, 21]. In these $Z = N$ nuclei, since protons and neutrons fill the same nuclear shell model orbitals, there can be a large spatial overlap between the proton and neutron wave functions and therefore we expect a large interaction between protons and neutrons, δV_{pn}. As the mass number increases, the values of δV_{pn} decrease presumably due to the Coulomb and spin-orbit interactions, and perhaps due to the greater average spacing of the last protons and neutrons.

Turning now to isobaric mirror nuclei, Figure 2 shows the experimental δV_{pn} values of odd-A T = ±1/2 mirror nuclei *versus* their mass numbers. There are two δV_{pn} values in each mass number

TABLE 1 A list of the nuclei discussed in this study with the T_z, δV_{pn} and $\Delta(\delta V_{pn})$ values. While the table has data up to A = 77, Figures 2, 3 have data up to A = 71, which is the largest mass number in which a pair of δV_{pn} is experimentally known. Bold face is used for high values of δV_{pn} with even-Z to draw attention to these nuclei; see the text for details.

Nucleus	Z	N	T_z	$\delta V_{pn}^{oe,eo}$ (keV)	$\Delta(\delta V_{pn})$ (keV)
^{7}Li	3	4	1/2	5970 (25)	
^{7}Be	4	3	−1/2	5785 (25)	−185 (35)
^{9}Be	**4**	5	1/2	**1037(4)**	
^{9}B	5	4	−1/2	914 (13)	−123 (13)
^{11}B	5	6	1/2	5706.8 (5)	
^{11}C	**6**	5	−1/2	**5727.6(5)**	21 (1)
^{13}C	**6**	7	1/2	**2222(1)**	
^{13}N	7	6	−1/2	1661 (3)	−562 (3)
^{15}N	7	8	1/2	4132.0 (1)	
^{15}O	**8**	7	−1/2	**4138.4(3)**	6.4 (1)
^{17}O	**8**	9	1/2	**1462.5(4)**	
^{17}F	9	8	−1/2	935 (7)	−527 (7)
^{19}F	9	10	1/2	3696.6 (1)	
^{19}Ne	**10**	9	−1/2	**3746.7(2)**	50.0 (3)
^{21}Ne	**10**	11	1/2	**1403(1)**	
^{21}Na	11	10	−1/2	1377 (5)	−26 (6)
^{23}Na	11	12	1/2	3181.40 (2)	
^{23}Mg	**12**	11	−1/2	**3192.0(1)**	10.6 (1)
^{25}Mg	**12**	13	1/2	**1065.0(1)**	
^{25}Al	13	12	−1/2	1065.0 (3)	0.3 (3)
^{27}Al	13	14	1/2	2999.7 (3)	
^{27}Si	14	13	−1/2	2992.0 (1)	−7.7 (3)
^{29}Si	**14**	15	1/2	**1015.10(3)**	
^{29}P	15	14	−1/2	971 (5)	−44 (5)
^{31}P	15	16	1/2	2274.1 (2)	
^{31}S	**16**	15	−1/2	**2290.5(2)**	16.4 (3)
^{33}S	**16**	17	1/2	**1027.10(3)**	
^{33}Cl	17	16	−1/2	1006 (2)	−21 (2)
^{35}Cl	17	18	1/2	2047.1 (3)	
^{35}Ar	**18**	17	−1/2	2049.4 (4)	2.3 (4)
^{37}Ar	**18**	19	1/2	**900.7(1)**	

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TABLE 1 (*Continued*) A list of the nuclei discussed in this study with the T_z, δV_{pn} and $\Delta(\delta V_{pn})$ values. While the table has data up to A = 77, Figures 2, 3 have data up to A = 71, which is the largest mass number in which a pair of δV_{pn} is experimentally known. Bold face is used for high values of δV_{pn} with even-Z to draw attention to these nuclei; see the text for details.

Nucleus	Z	N	T_z	$\delta V_{pn}^{oe,eo}$ (keV)	$\Delta(\delta V_{pn})$ (keV)
^{37}K	19	18	−1/2	887.0 (3)	−13.7 (3)
^{39}K	19	20	1/2	2261.9 (2)	
^{39}Ca	20	19	−1/2	2253.9 (3)	−8.1 (4)
^{41}Ca	**20**	21	1/2	**882** [3]	
^{41}Sc	21	20	−1/2	841 (12)	−41 (12)
^{43}Sc	21	22	1/2	1922 (1)	
^{43}Ti	**22**	21	−1/2	**1954(4)**	40 (4)
^{45}Ti	**22**	23	1/2	**800.1(6)**	
^{45}V	23	22	−1/2	761 [20]	−39 (20)
^{47}V	23	24	1/2	1770.5 (6)	
^{47}Cr	**24**	23	−1/2	**1814(6)**	44 (6)
^{49}Cr	24	25	1/2	**854(4)**	
^{49}Mn	25	24	−1/2	851 (16)	−3 (17)
^{51}Mn	25	26	1/2	1592 (4)	
^{51}Fe	26	25	−1/2	**1601(6)**	9 (6)
^{53}Fe	26	27	1/2	714 (1)	
^{53}Co	27	26	−1/2	732 (25)	18 (25)
^{55}Co	27	28	1/2	1724 (1)	
^{55}Ni	28	27	−1/2	1721 (3)	−3 (3)
^{57}Ni	28	29	1/2	476 (1)	
^{57}Cu	29	28	−1/2	520 (80)	44 (80)
^{59}Cu	29	30	1/2	1364.1 (4)	
^{59}Zn	**30**	29	−1/2	**1370(25)**	6 (25)
^{61}Zn	30	31	1/2	615 (10)	
^{61}Ga	31	30	−1/2		
^{63}Ga	31	32	1/2	1206 (13)	
^{63}Ge	**32**	31	−1/2	**1266(22)**	60 (25)
^{65}Ge	32	33	1/2	561 (5)	
^{65}As	33	32	−1/2		-
^{67}As	33	34	1/2	1239 (26)	

(Continued on the following page)

TABLE 1 (*Continued*) A list of the nuclei discussed in this study with the T_z, δV_{pn} and $\Delta(\delta V_{pn})$ values. While the table has data up to A = 77, Figures 2, 3 have data up to A = 71, which is the largest mass number in which a pair of δV_{pn} is experimentally known. Bold face is used for high values of δV_{pn} with even-Z to draw attention to these nuclei; see the text for details.

Nucleus	Z	N	T_z	$\delta V_{pn}^{oe,eo}$ (keV)	$\Delta(\delta V_{pn})$ (keV)
^{67}Se	34	33	−1/2	1201 (32)	−37 (41)
^{69}Se	34	35	1/2	604 (15)	
^{69}Br	35	34	−1/2		
^{71}Br	35	36	1/2	1240 (22)	
^{71}Kr	**36**	35	−1/2	**1245(71)**	51 (74)
^{73}Kr	36	37	1/2	697 (68)	
^{73}Rb	37	36	−1/2		
^{75}Rb	37	38	1/2	1411 (21)	
^{75}Sr	38	37	−1/2		
^{77}Sr	38	39	1/2	786 (42)	
^{77}Y	39	38	−1/2		

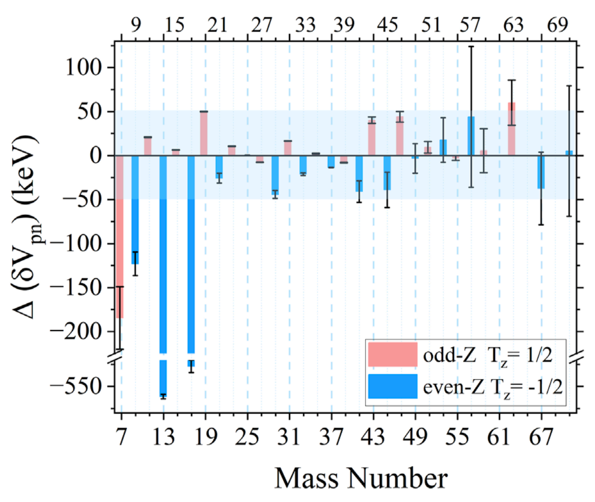

FIGURE 3
(Color online) Experimental $\Delta(\delta V_{pn})$ values as a function of mass number for mirror T_z = ±1/2 nuclei. Shadowing is used to point out a 50 keV band around zero. See also Table 1.

shown with vertical bars for T_z = 1/2 (orange) and T_z = -1/2 (blue). The A = 61, 65, and 69 nuclei have only T_z = 1/2 data due to missing experimental values for the masses of the involved nuclei.

Perhaps a simple way of stating the systematics in Figure 2 is that δV_{pn} is large for nuclei with $A = 4k$ - 1 and small for nuclei with $A = 4k + 1$. Interestingly, large and small δV_{pn} values involve different sets of T_z values (see Equations 1, 2), the large bars contain $|T_z|$ equals 0, 1/2 and 1; small bars contain $|T_z|$ values 0, 1/2, 1 and 3/2.

There is another systematic effect in Figure 2. For mass numbers where δV_{pn} is large (e.g., A = 7, 11, 15, 19, etc.), δV_{pn} is always higher for T_z = -1/2 for even-Z and odd-N except for A = 7, 27, 39, 55, 57, 59 and 67. For mass numbers where δV_{pn} is small (e.g., A = 9, 13, 17, 21, etc.), δV_{pn} is again always higher for cases of even-Z and odd-N but now for T_z = 1/2. That is, except for a few mass numbers and regardless of what T_z is, δV_{pn} is always higher in the case of even-Z and odd-N compared to odd-Z and even-N. This effect is even more visible in Table 1 which shows the data on which Figures 2, 3 are based on. Bold face is used for the cases of even-Z and high δV_{pn} values for each mirror pair.

When we look at the trends of the large values of δV_{pn} in Figure 2, we see a smooth decrease except at A = 39 and 55 in which δV_{pn} increases a little compared to the general downward trend. For A = 39, the δV_{pn} values of (Z,N) = (19, 20) and (20, 19) are very close to each other within their error bars. A small increase is seen because both Z and N contain the magic number 20. Similarly, in A = 55, the effect of the magic number 28 is observed in (Z,N) = (27, 28) and (28, 27). For the smaller pairs of bars, in the case of A = 17, the effect of the magic number eight should also be considered for (Z,N) = (8, 9) and (9, 8). After the decrease in A = 9, there is an increase in A = 13. The question here is whether A = 9 is exceptionally low or A = 13 high.

At this point, it is useful to introduce an empirical quantity related to δV_{pn}, but which is more sensitive to details of the p-n interactions. It is basically a differential of δV_{pn}. If we expect the nuclear structures of the mirror isobaric nuclei to be nearly identical, then we expect the δV_{pn} values of these nuclei to be quite close to each other. Although the δV_{pn} values of these mirror isobaric nuclei appear to be close to each other in Figure 2, the difference between two experimental δV_{pn} values of T_z = -1/2 and T_z = 1/2 is quite interesting. This quantity, $\Delta(\delta V_{pn})$, is defined as follows:

$$\Delta\left(\delta V_{pn}\right)(Z,N) = \delta V_{pn}^{T_z=-1/2} - \delta V_{pn}^{T_z=1/2}. \quad (4)$$

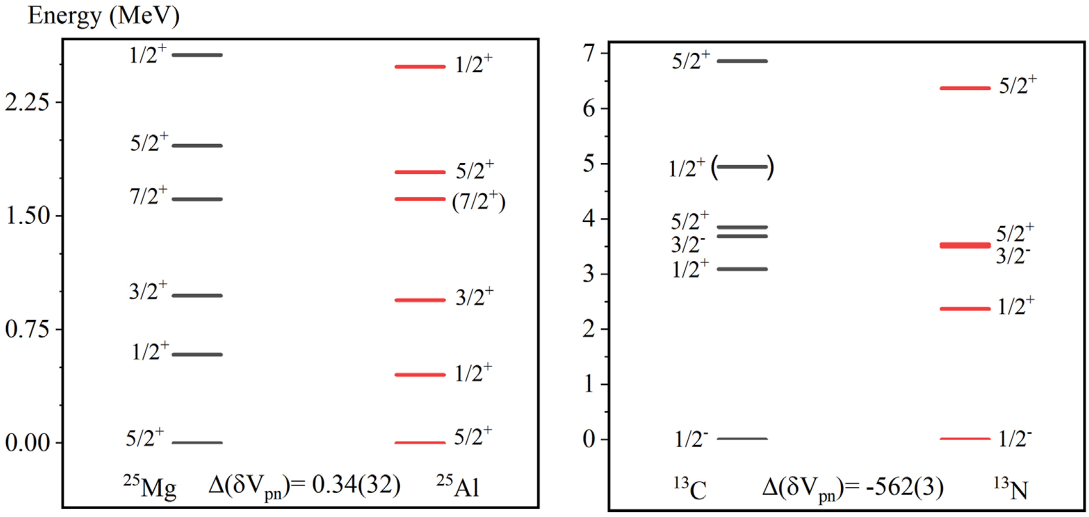

FIGURE 4
(Color online) Low-lying levels and spin-parity assignments [2] for A = 25 (left) and A = 13 (right), T_z = ±1/2 mirror nuclei are shown. $\Delta(\delta V_{pn})$ values are also given in keV.

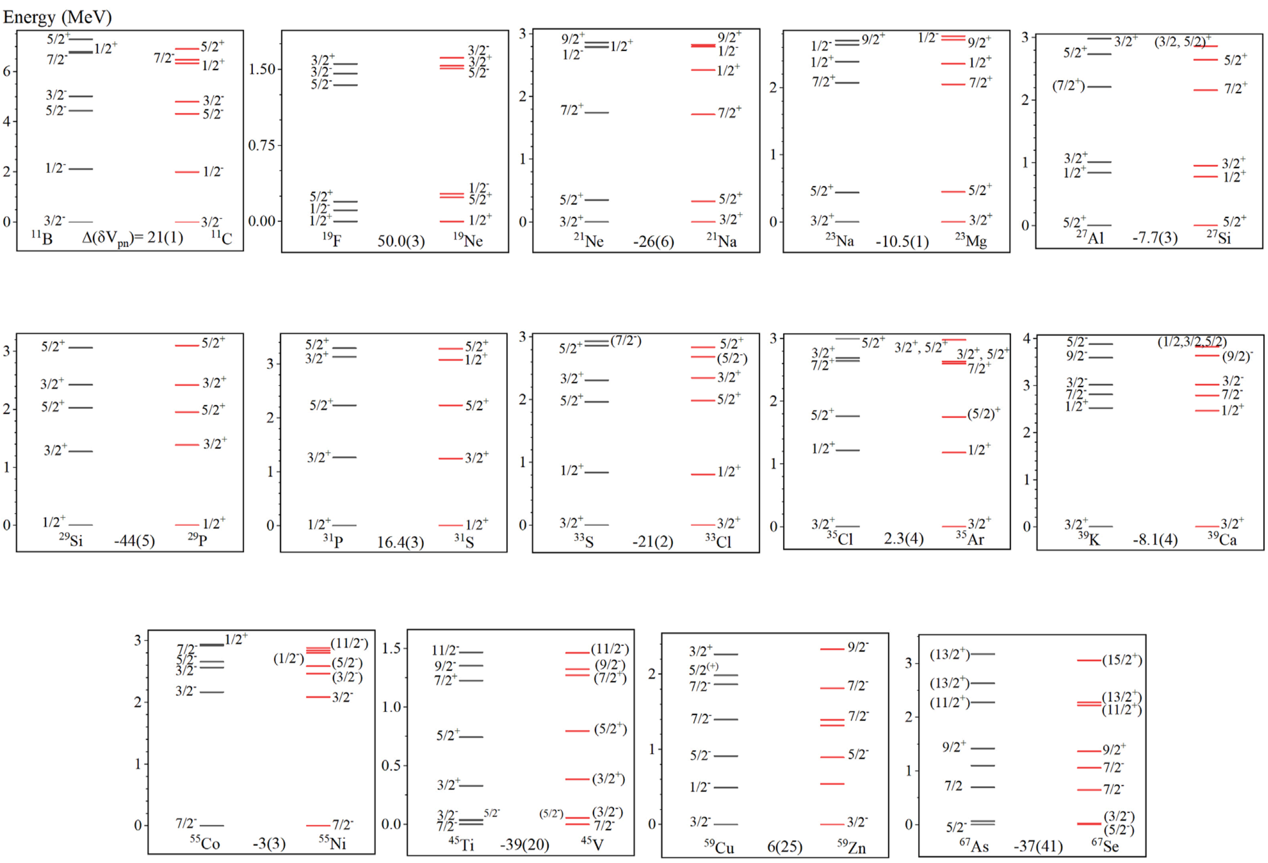

FIGURE 5
(Color online) Similar to Figure 4 for more cases of consistency between small values of $\Delta\delta V_{pn}$ and level schemes that are very similar.

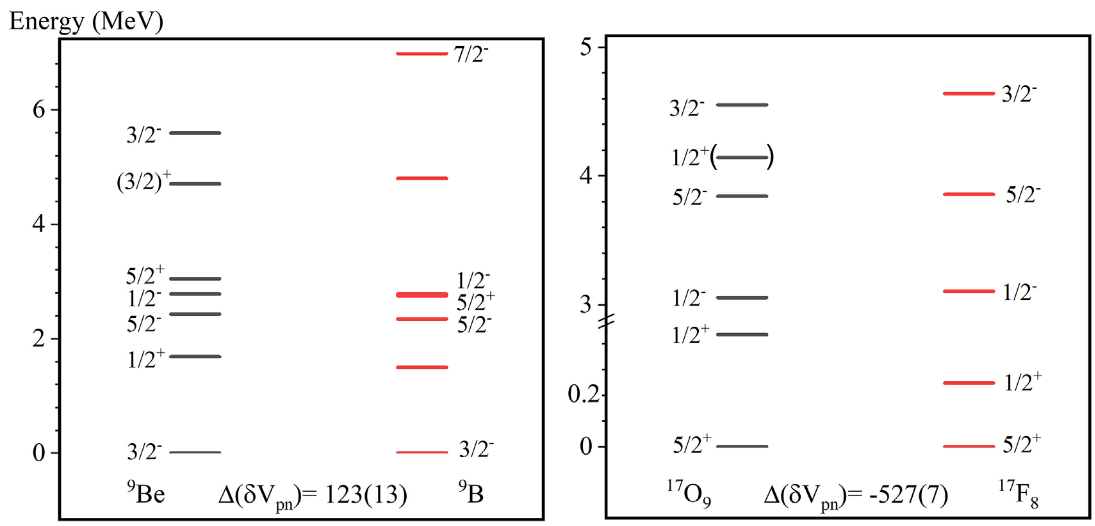

FIGURE 6
(Color online) Similar to Figure 4 but with cases of consistency between dissimilar excitation spectra and a large $\Delta(\delta V_{pn})$ value.

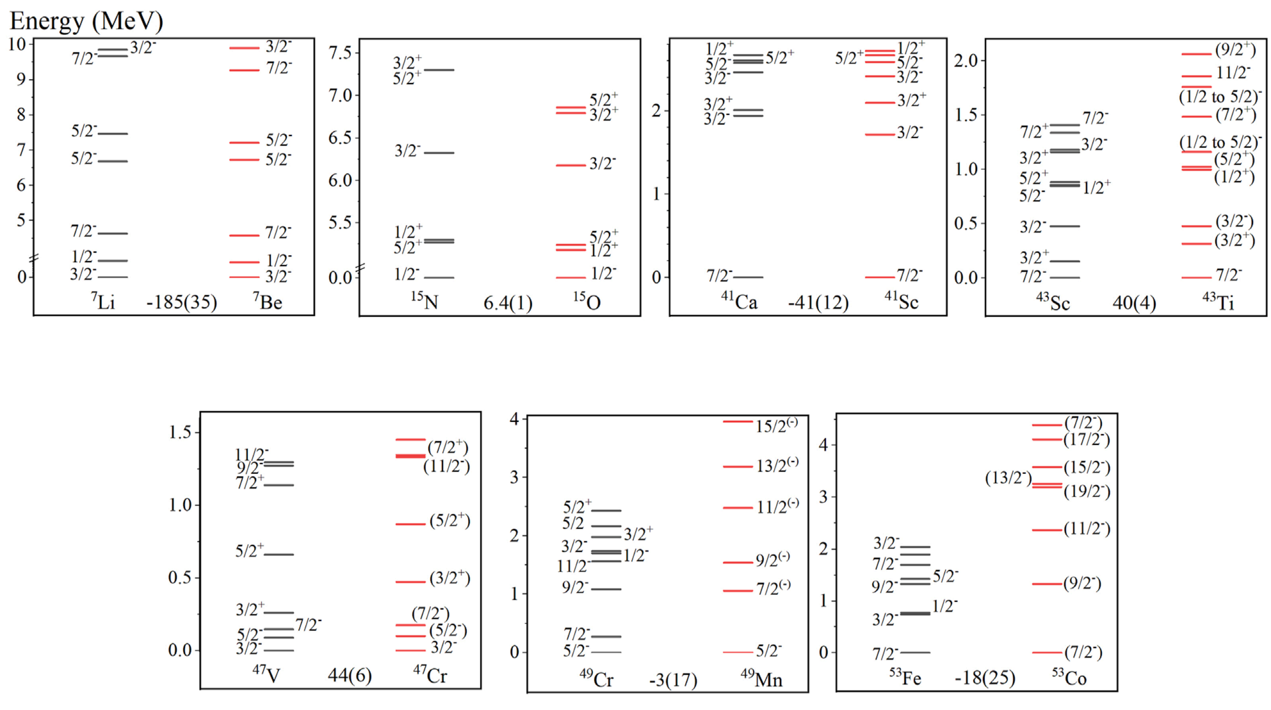

FIGURE 7
(Color online) Similar to Figure 4 but with cases of disagreement between the value of $\Delta(\delta V_{pn})$ and the excitation spectra. A = 7 shows similar level schemes and a large $\Delta(\delta V_{pn})$ value. All the other panels show small (< 50 keV) $\Delta(\delta V_{pn})$ values and spectra of mirror nuclei that either disagree with each other or where further data on levels and J^{π} assignments are needed to evaluate the level of agreement.

Equation 4 and a similar approach as presented here have recently been discussed in Refs. [24–26]. Here, however, we investigate which nuclei have isospin symmetry by looking at both the $\Delta(\delta V_{pn})$ values and some of the lowest excited states in mirror pairs.

The $\Delta(\delta V_{pn})$ results are shown in Figure 3. There are some clear trends in the results. Except for A = 7, which seems highly anomalous, the pink bars (for odd-Z, T_z = 1/2) are always positive (in some cases the values are very close to zero where uncertainties generally overlap with zero). The blue bars (for even-Z T_z = -1/2) are always negative, except in a few cases above A = 50 where the data has large uncertainties that again overlap with zero. There are also some other interesting features. As can be seen in the figure, there are quite high negative $\Delta(\delta V_{pn})$ values for a few mass numbers such

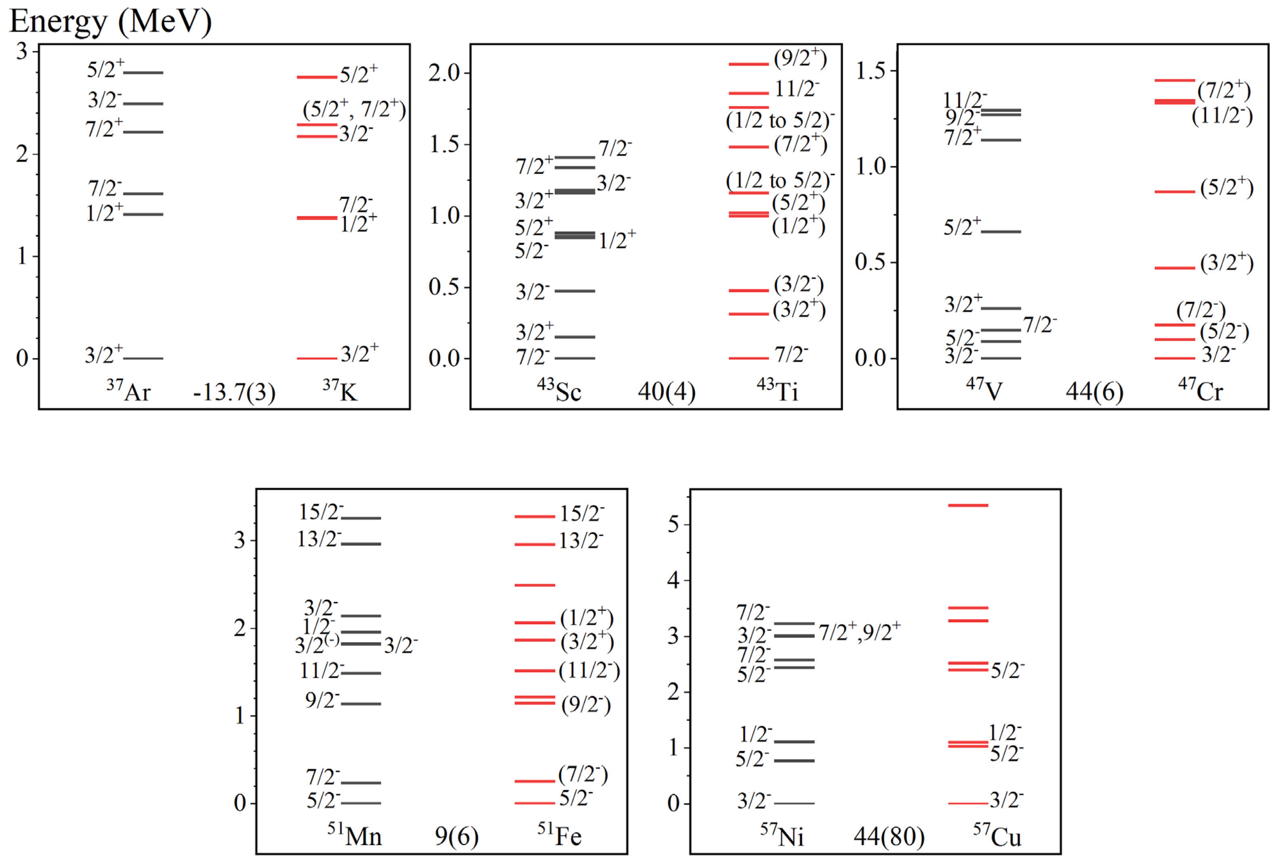

FIGURE 8
(Color online) Cases in which mirror level schemes and $\Delta(\delta V_{pn})$ values cannot be compared due to the need for more fully known level schemes (both levels and J^{π}) values.

as ~-600 keV for $A = 13$. The largest differences are seen at $A = 7$, 9, 13 and 17. A 50 keV band around zero is shaded as a reference to guide the eye. Most of the bars are within this 50 keV band. Note that the largest errors are at $A = 7$, 57, 63, 67 and 71. We see results for $\Delta(\delta V_{pn})$ closest to zero in many cases such as $A = 15$, 25, 37, 59, etc. Due to the lack of experimental mass values, there are no $\Delta(\delta V_{pn})$ values at $A = 61$, 65 and 69 (see also Figure 2).

What can we learn about the nuclear structure of mirror nuclei from these $\Delta(\delta V_{pn})$ values? Does a small value hint to a similar structure between mirror pairs? In other words, can $\Delta(\delta V_{pn})$ be used as a measure of isospin symmetry and/or its breaking? For example, in Figure 3, the $\Delta(\delta V_{pn})$ value of the mirror nuclei $A = 25$, ^{25}Mg and ^{25}Al, is approximately zero, while the $\Delta(\delta V_{pn})$ value of mirror nuclei $A = 13$, ^{13}C and ^{13}N, is approximately −600 keV. In this case, is the nuclear structure of $A = 25$ $T_z = \pm 1/2$ mirror nuclei more similar to each other compared to the nuclear structure of $A = 13$ $T_z = \pm 1/2$ mirror nuclei? The rest of this paper looks at this possibility in greater detail.

Each panel of Figure 4 shows some low-lying excited levels of a pair of mirror nuclei $A = 25$ (left) and $A = 13$ (right). As can be clearly seen, there is almost perfect similarity between the level schemes of ^{25}Mg and ^{25}Al, while there is very little similarity between ^{13}C and ^{13}N. In fact, the isospin symmetry between ^{25}Mg and ^{25}Al has been experimentally demonstrated [27]. This correlates very well with the $\Delta(\delta V_{pn})$ result. On the other hand, Ref. [3] shows isospin breaking in ^{13}C using pion inelastic scattering. The $A = 13$ spectra are very dissimilar and $\Delta(\delta V_{pn})$ is large. This pair of examples suggests that $\Delta(\delta V_{pn})$ may be useful as a filter or signature for the goodness of isospin, or its breaking. To study if this approach is accidental or not, one should look at each example of $\Delta(\delta V_{pn})$ shown above in Figure 3. Of course, the absolute binding energies of the two mirror nuclei are different because of the Coulomb interaction. But, this does not play a role in the figure since we normalize the ground state energies to zero.

Figure 5 shows all pairs of mirror nuclei with level schemes that are very similar, including $A = 23$ from Figure 1 but not $A = 25$ just shown in Figure 4. Here, similarity in the level schemes, the energy difference between the excited states (level spacing) and the fact that these similar states have the same spin-parity are used as criteria. Besides the fact that the level schemes of these nuclei are very similar, their $\Delta(\delta V_{pn})$ values are quite small. The nuclei with the largest $\Delta(\delta V_{pn})$ in Figure 5 are the $A = 19$ mirror nuclei with 50.0 (3) keV and the $A = 29$ mirror nuclei with 44 (5) keV. The others have maximum $\Delta(\delta V_{pn})$ values of ~25 keV.

These results confirm that small $\Delta(\delta V_{pn})$ values might be a useful filter for mirror nuclei with small isospin symmetry breaking. We will see below that there are some exceptions to this that need to be

TABLE 2 The successive columns of the table show δV_{pn} values for experimentally known nuclei with large errors, the mass excess errors for those nuclei contributing the largest uncertainties to δV_{pn}, and half-lives. The nuclei with unknown δV_{pn} are also listed in *Unknown* δV_{pn} column. Experimental masses are taken from Refs. [22, 23].

Z	N	δV_{pn}	Unknown δV_{pn}	Z	N	Needed Mass	Mass excess Error (keV)	$T_{1/2}$
29	28	^{57}Cu		29	26	^{55}Cu	160	55.9 (15) ms
30	29	^{59}Zn		30	28	^{58}Zn	50	86.0 (20) ms
31	32	^{63}Ga		31	30	^{61}Ga	21	166.0 (20) ms
32	31	^{63}Ge		32	30	^{62}Ge	37	73.5 (1) ms
33	34	^{67}As		32	34	^{66}Ge	30	2.26 (4) h
				33	32	^{65}As	42	130.3 (6) ms
34	33	^{67}Se		34	32	^{66}Se	61	51.0 (40) ms
36	35	^{71}Kr		36	34	^{70}Kr	140	45.19 (14) ms
				36	35	^{71}Kr	24	95.0 (4) ms
38	37		^{75}Sr	38	37	^{75}Sr	150	85.2 (22) ms
				38	36	^{74}Sr	–	27.6 (26) ms
39	38		^{77}Y	39	38	^{77}Y	–	–
				39	36	^{75}Y	–	–
				38	36	^{74}Sr	–	27.6 (26) ms
				38	34	^{72}Sr (unknown)	–	–

studied further. In some cases, like $A = 55$ and 59, further study of experimental spectra would be useful.

This idea can be tested in an inverse way. The $A = 9$ and 17 cases are shown in Figure 6 and have both incompatible level schemes and $\Delta(\delta V_{pn})$ values that are rather large. At first glance, there seems to be no serious difference between the two level schemes in each pair but, for example, if we look at the level spacing in ^{17}O and ^{17}F carefully, there is about a factor of two difference in the energies of their first excited levels. The large $\Delta(\delta V_{pn})$, ~-500 keV, also points to this disagreement. Indeed, in Ref. [4] $A = 17$ isospin breaking has been discussed on the basis of quark-meson coupling. Thus, we again see the use of $\Delta(\delta V_{pn})$ values as a signature, in this case of symmetry breaking. Note that ^{9}B has an unbound proton, therefore a large $\Delta(\delta V_{pn})$ may be expected. However, the mass ^{9}B is used not only for $\delta V_{pn}(^{9}\text{B})$ but also for $\delta V_{pn}(^{11}\text{B})$. In Figure 5, a small $\Delta(\delta V_{pn})$ value is given together with nice agreement on the level schemes of ^{11}B and ^{11}C. Clearly, the effects of extended proton radial distributions in proton unbound nuclei need further study.

While this correlation of $\Delta(\delta V_{pn})$ and the degree of similarity in mirror pair level schemes is suggestive of a new tool to assess isospin symmetry, however, there are also a few counter examples that may hint to its limitations. Figure 7 shows one case of similar level schemes but a large $\Delta(\delta V_{pn})$ for $A = 7$, −185 (35) keV, and a number of $T_z = \pm 1/2$ mirror nuclei with dissimilar level schemes but low $\Delta(\delta V_{pn})$ values. There is no noticeable anomaly in the δV_{pn} results for the ^{7}Li and ^{7}Be nuclei, except for mass error of about 50 keV for both ^{5}Li and ^{5}He. If $\Delta(\delta V_{pn})$ is a reliable filter for isospin breaking, one would expect more consistency of spectra and $\Delta(\delta V_{pn})$ values. This needs further investigation.

The rest of Figure 7 shows cases of dissimilar level schemes. Most of these are in heavier nuclei compared to the nuclei in Figure 5. As the mass number increases, the $\Delta(\delta V_{pn})$ filter may simply break down. As mentioned in the beginning of this paper, isospin breaking occurs when the mass number increases due to Coulomb force among protons. Also, especially in heavier nuclei, there can be states of higher $T(>1/2)$ at higher energies, which are part of extended isospin multiplets, and there can also be isospin mixing in complex states. This could lead to some differences in spectra.

Finally, there are a number of nuclei with insufficient data to assess the correlations. In these cases, either further spectroscopic or mass data would be highly useful. We first consider cases of insufficient level scheme information. In some nuclei, spin-parity of the excited levels is unknown or not fully known, and their $\Delta(\delta V_{pn})$ values are small. Such nuclei are shown in Figure 8. These nuclei should be studied by γ-ray spectroscopy. If the experimental data of these nuclei are clarified, further tests of the usefulness of $\Delta(\delta V_{pn})$ as a signature of isospin symmetry may emerge.

A recent γ-ray spectroscopic study focusing on isospin symmetry breaking is Ref. [28]. The study finds evidence for the breaking of isospin symmetry in the mirror system ^{71}Kr and ^{71}Br by β-decay. As seen in Figure 3, $\Delta(\delta V_{pn})$ of A = 71 has a large error. Therefore, in order to test our approach here with $\Delta(\delta V_{pn})$, ^{70}Kr and even maybe ^{71}Kr mass excess values should be improved. There are a number of other cases where additional mass measurements would be helpful to further study the use of δV_{pn} to assess the degree of isospin symmetry breaking. These are listed in Table 2 and provide motivation for further experimental mass measurements.

As seen in Figure 2, there are no δV_{pn} values at A = 61 T_z = -1/2, ^{61}Ga, A = 65 T_z = -1/2, ^{65}As, and A = 69 T_z = -1/2, ^{69}Br. Since the half-lives of ^{59}Ga, ^{63}As and ^{67}Br are in the order of nanoseconds, it is impossible to measure the masses of these nuclei today. Finally, the δV_{pn} values for ^{75}Sr and ^{77}Y are experimentally not known due to missing masses, as seen in Table 2. They are the heaviest nuclei suggested here where we can possibly test isospin symmetry/breaking with $\Delta(\delta V_{pn})$. The other nuclei in the table have δV_{pn} values but their errors can be improved. The masses needed for this purpose are also listed. The A = 79 T = 1/2 mirror nuclei do not have any δV_{pn} value for either T_z = 1/2 or T_z = -1/2 nuclei.

4 Conclusion

We have discussed a possible correlation between isospin symmetry in mirror nuclei and its breaking and empirical measures of the average proton-neutron interaction. The correlation is suggestive but not perfect, and breakdowns in it need to be further investigated by both γ-ray spectroscopy and mass spectrometry. For the latter, possible nuclides of interest are listed in Table 2.

Data availability statement

The original contributions presented in the study are included in the article/supplementary material, further inquiries can be directed to the corresponding author.

Author contributions

RBC: Writing – original draft, Writing – review and editing. KB: Writing – review and editing. RC: Writing – review and editing.

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Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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