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Statistical contextual explanation of quantum paradoxes

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This year we celebrate 100 years of quantum mechanics (QM). Incorrect interpretations of QM and incorrect mental models of the invisible details of quantum phenomena lead to paradoxes. To explain these, we advocate the statistical contextual interpretation (SCI) of quantum mechanics. State vectors (wave functions) and various operators are purely mathematical entities that permit quantitative probabilistic predictions. "State vector" describes an ensemble of identically prepared physical systems, and a specific "operator" represents a class of equivalent measurements of a physical observable. A collapse of wavefunction is not a mysterious and instantaneous physical process; a collapsed quantum state describes a new ensemble of physical systems prepared in a particular way. A value of a physical observable, such as a spin projection, associated with a pure quantum ensemble is a characteristic of this ensemble created by its interaction with measuring instruments. Probabilities are objective properties of random experiments in which empirical frequencies stabilize. Following Einstein, SCI rejects the claim that QM provides a complete description of individual physical systems, but it remains agnostic about whether a more detailed subquantum description can be found or is necessary. In conformity with Bohr contextuality, SCI rejects Bell-local and Bell-causal hidden variable models. Nevertheless, by incorporating into a probabilistic model contextual hidden variable measuring instruments, long distance quantum correlations studied in Bell tests can be explained without evoking quantum nonlocality or retro-causality. SCI allows the explanation of several quantum phenomena without evoking quantum magic. SCI does not claim to provide a complete description of quantum phenomena; in fact, it is unknown whether quantum probabilities even provide a complete description of existing experimental data. Time series of experimental data may contain much more information than is obtained using empirical frequencies and histograms. Therefore, predictable completeness of QM must be tested and not taken for granted.

KEYWORDS

EPR paradox, Bell-CHSH inequalities, Bell tests, entanglement, quantum nonlocality, contextuality, completeness of quantum mechanics

1 Introduction

In 1925, Werner Heisenberg, Max Born, and Pascual Jordan developed matrix mechanics (Heisenberg, 1925; Born and Jordan, 1925; Author anonymous, 2024a), the first consistent formulation of quantum mechanics (QM). To commemorate this achievement, 2025 has been declared the International Year of Quantum Science and Technology (IYQ) by the United Nations.

Despite the incredible advances made in quantum science and technology over the past century, there is still no consensus regarding its interpretation and limitations (Author anonymous, 2024b; Schlosshauer et al., 2013; Kupczynski, 2018a; Kupczynski, 2024a). Incorrect interpretations of QM and incorrect mental models of invisible details of quantum phenomena lead to paradoxes and speculations about quantum weirdness and quantum magic. Most of these paradoxes are due to the "individual" interpretation, according to which an instantaneous collapse of wave function describing individual physical system(s) is triggered by a single measurement performed on one of these systems.

We here review and advocate a statistical contextual interpretation (SCI) which is free of paradoxes (Einstein and Schilpp, 1949; Einstein, 1936; Ballentine, 1989; Ballentine, 1998; Kupczynski, 2007; Kupczynski, 1973; Kupczynski, 1987a; Kupczynski, 2005; Kupczynski, 2006; Kupczynski, 2015a; Kupczynski, 2016a; Kupczynski, 2017a; Khrennikov, 1999; Khrennikov, 2024; Khrennikov, 2009; Khrennikov, 2016; Allahverdyan et al., 2013). According to this interpretation, a quantum state is not an attribute of an individual physical system which can be changed instantaneously. The so-called collapse of the wavefunction is not a mysterious physical process. Quantum state/wavefunction is a mathematical entity representing an equivalence class of subsequent preparations of the physical systems. Quantum states together with specific operators representing physical observables are used to make probabilistic predictions for a statistical scatter of measured values of these observables in well-defined experimental contexts. A value of a physical observable, such as a spin projection, associated with a pure quantum ensemble is a characteristic of this ensemble created by its interaction with measuring instruments. Probabilities are objective properties of random experiments in which empirical frequencies stabilize. SCI rejects the claim that quantum mechanics provides a complete description of individual physical systems, but it remains agnostic on whether a more detailed subquantum description can be found or is necessary. In conformity with Bohr contextuality, SCI rejects Bell-local and Bell-causal hidden variable models. Nevertheless, by incorporation into probabilistic model contextual hidden variables measuring instruments, the quantum correlations studied in Bell tests can be explained without evoking quantum nonlocality. SCI does not claim to provide a complete description of quantum phenomena. In fact, it is not even known whether quantum probabilities provide a complete description of existing experimental data. Time series of experimental data may contain much more information than is obtained using empirical frequencies and histograms.

SCI (Kupczynski, 2006; Kupczynski, 2007; Kupczynski, 2016a; Kupczynski, 2017a) is similar but not identical to Ballentine's statistical (Ballentine, 1989; Ballentine, 1998) and Khrennikov's Växjö interpretation (Khrennikov, 1999; Khrennikov, 2024; Khrennikov, 2009; Khrennikov, 2016). In Ballentine's statistical interpretation, the quantum state also describes an ensemble of similarly prepared systems, not individual systems. This interpretation avoids the need for wave function collapse. It is compatible with hidden variable theories but contrary to SCI and Växjö interpretation it acknowledges that such theories must be non-local to comply with Bell's theorem (Ballentine, 1998). Växjö interpretation combines realism at the subquantum level with the contextuality of quantum observables. The value of an observable depends on the measurement context, in conformity with Bohr's complementarity and contextuality. The quantum probabilities are conditional probabilities. In contrast to SCI, it introduces the concept of a "prespace," suggesting that both classical and quantum spaces are reductions of a more fundamental reality.

A probability can have a different meaning (Khrennikov, 1999; Author anonymous, 2024c). In SCI, it is an objective property of a random experiment in which empirical frequencies stabilize. Thus, a probabilistic description of quantum phenomena can hardly be considered a complete description of individual physical systems (Einstein, 1936; Ballentine, 1989; Ballentine, 1998; Kupczynski, 2006).

Therefore, Einstein believed that QM is an emergent theory and that a more detailed description of quantum phenomena should be found (Einstein and Schilpp, 1949; Einstein, 1936; Ballentine, 1989). Bohr insisted that quantum probabilities were irreducible and that QM provided a complete description of quantum phenomena and experiments (Bohr, 1963; Bohr, 1987; Plotnitsky, 2009; Plotnitsky, 2012).

Heisenberg (1927) demonstrated the uncertainty principle according to which one may not measure simultaneously, with arbitrary accuracy, a linear momentum p and a position x of a sub-atomic particle, $\Delta x \Delta p \leq h$, where h is a Planck constant. The principle was generalized by Robertson (1929) and its precise statistical meaning was given by Kennard (1927). We have two experiments performed on two identically prepared beams/ ensembles of "particles". In one experiment, we measure their linear momenta and, in another, their positions. A statistical scatter of experimental data is described by respective standard deviations and $\sigma_x \sigma_p \leq \hbar\ /2$ where $\hbar = h/2\pi$. This interpretation only refers to a statistical scatter of measurement outcomes and not to positions and linear momenta of "particles" if no measurements are performed. According to the Copenhagen interpretation (CI), all speculations about the sharp unmeasured values of linear momenta and positions of sub-atomic particles are meaningless, and QM does not imply that an electron can be here and a meter away at the same time (Kupczynski, 2024a; Kupczynski, 2024b), as incorrectly claimed by several authors.

In 1935, Einstein, Podolsky and Rosen (Einstein et al., 1935) proposed a thought experiment—the "EPR paradox"—intended to demonstrate the incompleteness of quantum mechanics. They considered two entangled particles which interacted in the past moving away from each other in distant locations. According to the Copenhagen interpretation (CI), measuring the position or momentum of one particle would instantly give information about the position or momentum of its distant partner without disturbing it in any way. Thus, physical properties of objects exist independently of measurement, contrary to CI. Bohr (1935) explained that EPR inference requires different incompatible but complementary experiments and that it could not provide more information about an individual physical system than was allowed by QM.

The EPR paradox was rephrased by Bohm (1951) in terms of measurements of a particle's spin. If you measure the spin of one particle, you instantly know the spin of the other. According to QM, outcomes are produced in irreducibly random ways, but in an ideal EPR-B experiment they are perfectly correlated or anti-correlated in specific randomly chosen experimental settings. This is called the

"EPR-B paradox", since a pair of fair dice cannot always produce correlated outcomes (Mermin, 1985; Mermim, 1993; Kupczynski, 2017b; Kupczynski, 2020).

(Bell, 1965, 2004) abandoned irreducible randomness and proposed the Local Realistic Hidden Variable Model (LRHVM) in which outcomes are predetermined at a source. Clauser and Horne (1974) abandoned predetermination and proposed the Stochastic Hidden Variable Model (SHVM). LRHVM describes entangled pairs/qubits as pairs of socks and SHVM as pairs of dice. In these models, correlations between distant outcomes coded ±1 must obey Bell–CHSH inequalities (Clauser et al., 1969).

Later, hidden variables were assumed to represent all common causes of events in distant laboratories, and the Local Hidden Variable Model (LHVM) (Mermin, 1993; Bell, 2004; Valdenebro, 2002; Wiseman, 2014) could be rejected in several Bell tests (Hensen et al., 2015; Giustina et al., 2015; Shalm et al., 2015; Handsteiner et al., 2017; The BIG Bell Test Collaboration, 2018; Rosenfeld et al., 2017; Zhang et al., 2022; Storz et al., 2023).

Since Bell–CHSH inequalities are violated by some quantum predictions and by experimental data, the majority of the physics community believes that no other locally causal explanation of quantum correlation is possible. Therefore, nature does exhibit non-locality, and entangled particles can influence each other instantaneously across huge distances. This is a source of extraordinary metaphysical speculation about experimenters' freedom of choice, retro-causality, quantum nonlocality, and quantum magic.

It has been widely explained that such speculations are unfounded (Kupczynski, 2006; Kupczynski, 2007; Kupczynski, 2016a; Kupczynski, 2017a; Kupczynski, 2018a; Khrennikov, 1999; Khrennikov, 2024; Khrennikov, 2009; Khrennikov, 2016; Allahverdyan et al., 2013; Kupczynski, 1973; Kupczynski, 1987a; Kupczynski, 2005; Kupczynski, 2015a; Accardi, 1981; Accardi et al., 2002; Accardi, 2005; Accardi and Uchiyama, 2007; Aerts, 1982; Aerts, 1986; Aerts et al., 2000; Boughn, 2022; Czahor, 1988; Dzhafarov, 2021; Fine, 1982; Hance and Hossenfelder, 2022; Hess and Philipp, 2005; Hess, 2014; Hess et al., 2009; Hess et al., 2016; Hess, 2022; Jaynes and Skilling, 1989; Jung, 2017; Khrennikov, 2007; Khrennikov, 2008; Khrennikov, 2019; Khrennikov, 2020a; Khrennikov, 2022; Kupczynski, 1987b; Khrennikov, 1986; Khrennikov, 2012; Khrennikov, 2014; Khrennikov, 2018b; Khrennikov, 2021; Khrennikov, 2023a; Khrennikov, 2024a; Khrennikov, 2024b; De Muynck et al., 1994; De Muynck, 2002; Nieuwenhuizen, 2009; Nieuwenhuizen, 2011; Nieuwenhuizen and Kupczynski, 2017; Peres, 1978; Pitovsky, 1983; Pitovsky, 1994; De la Peña et al., 1972; Zhao et al., 2008).

- In spin polarization correlation experiments (SPCE) and other Bell tests, we have four incompatible random experiments for different pairs of settings. LRHVM use a unique probability space and a joint probability distribution to describe these experiments, what is only possible in rare circumstances, and what is clearly incompatible with experimental protocols in Bell Tests (Kupczynski, 2007; Kupczynski, 2016a; Kupczynski, 2017a; Khrennikov, 1999; Kupczynski, 1987a; Kupczynski, 2005; Kupczynski, 2015a; Kupczynski, 2017b; Accardi et al., 2002; Accardi, 2005; Accardi et al., 2007; Accardi and Uchiyama, 2007; Aerts, 1982; Aerts, 1986; Czahor, 1988; Fine, 1982; Hess and Philipp, 2005; Hess, 2014; Hess et al., 2009; Hess et al., 2016; Hess, 2022; Khrennikov, 2007; Khrennikov, 2008; Khrennikov, 2019; Khrennikov, 2020a; Khrennikov, 2022; Kupczynski, 2024c; Pitovsky, 1994; De la Peña et al., 1972).
- In 1982, Arthur Fine was the first to clearly demonstrate that the following statements are mutually equivalent (Fine, 1982). 1) There is a deterministic hidden-variables model for the experiment. 2) There is a factorizable, stochastic model. 3) There is one joint distribution for all observables of the experiment, returning the experimental probabilities. 4) There are well-defined, compatible joint distributions for all pairs and triples of commuting and non-commuting observables. 5) The Bell inequalities hold.
- Bell and CHSH inequalities are trivial algebraic properties of experimental spreadsheets (Kupczynski, 2020; Hess and Philipp, 2005; Kupczynski, 2018b; De Raedt et al., 2017; De Raedt et al., 2023; De Raedt et al., 2024) containing quadruplets of ±1 which are, in fact, samples drawn from a statistical population described by some joint probability distribution of four compatible random variables. The outcomes of Bell tests are displayed using four spreadsheets each containing only couples ±1. The violation of Bell–CHSH inequalities only provides the evidence that the data in these four spreadsheets cannot be reshuffled to form quadruples (De Raedt et al., 2023; De Raedt et al., 2024).
- In QM, interactions of instruments with physical systems during the measurement process may not be neglected, and outcomes are not passively registered pre-existing values of the physical observables. Therefore, the Bell-causal hidden variable model suffers from a theoretical "contextuality loophole" (Kupczynski, 2015a; Kupczynski, 2017b; Kupczynski, 2020; Kupczynski, 2021; Kupczynski, 2023a; Kupczynski, 2024e; Kupczynski, 2024a; Nieuwenhuizen, 2009; Nieuwenhuizen, 2011; Nieuwenhuizen and Kupczynski, 2017) because it fails to correctly include setting-dependent variables that describe measuring instruments at the moment of measurement.

A detailed discussion of EPR-type paradoxes and Bell Tests in the spirit of SCI may be found in Kupczynski (2006), Kupczynski et al. (2007), Kupczynski (2016a), and in a dedicated section of this study. As we conclude in Kupczynski (2024b) and Kupczynski (2024c), Bell tests allow only the rejection of probabilistic couplings provided by Bell-local and Bell-causal hidden variable models. If contextual variables, describing varying experimental contexts, are correctly incorporated into a probabilistic model, then Bell–CHSH inequalities cannot be proven, and *nonlocal quantum correlations* may be explained intuitively.

This study is organized as follows. Section 2 recalls different definitions of probability and Bertrand`s paradox. We explain that in physics, probabilities are objective properties of random experiments in which empirical frequencies stabilize. Section 3 compares classical and quantum observables and filters. Section 4 recalls EPR-B paradoxes and explains them using SCI. In Section 5, quantum predictions for an ideal EPR-B experiment are derived. Section 6 gives an explanation of how Bell–CHSH inequalities are trivial arithmetic properties of N×4 spreadsheets containing ±1 entries and can be rigorously derived only for random experiments described by four binary jointly-distributed random variables. Section 7 discusses hidden variable models proposed to explain EPR-B experiments. Section 8 is about loophole free Bell Tests, their interpretation, and their implications. Section 9 presents a contextual hidden variable model, which allows an explanation of long-range correlations observed in Bell tests. A more detailed analysis of existing time-series of data in order to elucidate the problem of completeness of quantum mechanics is advocated in Section 10. Additional conclusions are presented in Section 11.

2 Probability and Bertrand paradox

Probability and randomness are subtle notions long debated by mathematicians and philosophers. There are several definitions of probability (Khrennikov, 1999; Author anonymous, 2024a; Author anonymous, 2024b).

Classical probability is the ratio of the number of favorable outcomes to the total number of possible outcomes. For example, the probability of drawing a black king from a deck of 52 cards is 2/52 = 1/26. *Geometric probability* is the probability that a point chosen at random within a certain geometric figure will satisfy a given condition, and it is calculated as the ratio of the area (or length, volume, etc.) of the favorable region to the area of the entire region. For example, the probability of hitting a specific region on the dartboard can be calculated by dividing the area of that region by the total area of the dartboard.

Frequentist probability is the relative frequency of occurrence of an experiment's *outcome* "in the long run" of outcomes (theoretically if the experiment could be repeated an infinite number of times). It is an objective property of a random experiment. Another objective probability is *propensity*, which is defined as the tendency of some experiments to yield a certain outcome, even if they are performed only once. A *subjective probability* is based on the personal judgment of an agent and quantifies her degree of belief of how likely an event is to occur.

The limitations of the classical and geometric probabilities became evident due to Bertrand's paradox. This demonstrates how different methods of defining "randomness" can lead to different probabilities for the same event. In 1889, Bertrand posed the following problem. Consider an equilateral triangle inscribed in a circle. What is the probability that a randomly chosen chord of the circle is longer than a side of the triangle? He provided three different methods to choose a random chord, each yielding a different probability (Bertrand, 1889; Author anonymous, 2024c).

Bertrand's paradox can be rephrased in a more intuitive way (Kupczynski, 1987a). If we consider two concentric circles on a plane with radii R and R/2 respectively, we can ask what the probability P is that a chord of the larger circle chosen at random cuts the smaller one at least one point? The various answers seem to be equally reasonable. If we divide the ensemble of all chords into sub-ensembles of parallel chords, we find that P = ½. If we consider sub-ensembles of chords having the same beginning, we find that P = ⅓. Finally, if we choose midpoints of chords lying in small circle, we find that P = ¼.

The solution of Bertrand's paradox is simple. Different probabilistic models leading to different answers correspond to random experiments performed using different specific experimental protocols. It proves the contextual character of probabilities and their intimate relation to specific random experiments (Kupczynski, 2015a). Therefore, the probability of obtaining "heads" in a coin flipping experiment using a specific coin and a specific flipping device is neither a property of the coin nor of the flipping device. It is only a property of the whole experiment: "flipping this particular coin with that particular flipping device." This is why in physics, probabilities are objective properties of phenomena and random experiments in which empirical frequencies stabilize.

3 Classical *versus* quantum: properties, filters, and observables

In classical physics, measurement outcomes may contain experimental errors, but measurements are assumed to be non-invasive, meaning that they do not change the properties they measure. Therefore, macroscopic physical systems are described by properties p_i (i = 1, . . . ,n) quantified by the values of classical compatible observables which can be measured in any order.

If we have a mixed statistical ensemble (a beam) B of macroscopic systems, we can choose systems having particular properties using classical filters. *A classical filter F_i or a macro selector is a device which passes only through systems having a property p_i. Classical filters operate according to Boolean yes-or-no logic. If we have n different properties, we have n filters corresponding to them. A lattice of classical filters have simple properties: $F_i\ F_j = F_j\ F_i\ F_j = F_j\ F_i$. There also exists a maximal filter $F = F_1\ F_2 \ldots F_n$ which transforms a mixed statistical ensemble into a pure statistical ensemble in which all the systems have exactly the same properties* (Kupczynski, 2015a). Mixed statistical ensembles of physical systems can be described by a joint probability distribution of random variables associated with measured physical observables.

In quantum experiments, the information obtained about invisible physical systems is indirect and obtained from their interactions with macroscopic measuring instruments. As Bohr correctly insisted, the atomic phenomena are characterized by "...the impossibility of any sharp separation between the behaviour of atomic objects and the interaction with the measuring instruments which serve to define the conditions under which the phenomena appear" (Bohr, 1987, v. 2, pp. 40–41). Quantum observables have the following properties of Bohr-contextuality (Khrennikov, 2020b; Kupczynski, 2021): *the output of any quantum observable is indivisibly composed of the contributions of the system and the measurement apparatus.*

The formalism of QM was inspired by optical experiments with polarized light. Linearly polarized light passes without noticeable attenuation by a subsequent identical polarizer. The intensity of linearly polarized light after a passage through another polarizer is reduced according to Malus law = $I_0 \cos^2 \theta$, where I_0 is the initial intensity and θ is the angle between the light's initial polarization direction and the axis of the polarizer.

Discrete atomic spectral lines and the photoelectric effect proved that exchanges of energy between electromagnetic field and matter are quantized, and "carriers" of quantized exchanged energy are called "photons." Therefore, linearly polarized monochromatic light is usually represented as a beam of linearly polarized photons carrying energy *hν*. This mental picture is misleading because we cannot see photons—they are not point-like objects. When a sophisticated photon detector, after several steps of signal enhancement, produces a click, we conclude that a photon was detected. The intensity of light is now measured by counting clicks on detectors. We say that each linearly polarized photon has a probability (propensity) = $\cos^2\theta$ to pass through a polarizer if θ is the angle between the direction of the photon's initial polarization and the axis of the polarizer.

After passing through a quantum filter, the linear polarization of light becomes a contextual property of photons. *A quantum filter F_i is a device which creates a contextual property "i": passing by F_i. A physical system having a property " i" has a probability (propensity) p_{ij} to pass through another filter F_j, acquiring after the passage a new property" j." Quantum filters are idempotent, $F_i F_i = F_i$, but in general they do not commute Fi Fj ≠Fj Fi, and the lattice of quantum filters is isomorphic to the lattice of projectors on subspaces of a Hilbert space. Quantum filters are not selectors of pre-existing attributes of physical systems but are creators of the contextual properties defined above* (Kupczynski, 2015a).

Incompatible filters, such as polarizers with non-parallel axes, create incompatible contextual properties which cannot be measured simultaneously and, if measured in a sequence a previous contextual property, is destroyed in a new measurement. As explained in the preceding section, the probabilities are objective properties of phenomena and random experiments, and thus considering propensity as the property of individual physical systems (here, invisible photons) is in fact unfounded. This is why vectors in SCI quantum state are not considered to be properties of the individual physical systems. Treating a wavefunction as an attribute of the individual physical system leads to the EPR paradox, which is discussed in the next section.

4 EPR paradox and statistical contextual interpretation

Resumed here is the discussion of the EPR paradox in Kupczynski (2016a). Before the publication of the EPR paper, it was believed that:

A1: Any pure state of a physical system is described by a specific unique wavefunction Ψ.

A2: Any measurement causes a physical system to jump instantaneously into an eigenstate of the dynamical variable being measured. This eigenstate becomes a new wavefunction describing a state of the system.

A3: A wave function Ψ provides a complete description of a pure state of an individual physical system.

EPR considers two particular individual systems, I + II, in a pure quantum state; they interacted in the past, separated, and evolved freely afterward (Einstein et al., 1935). Using A2, they concluded that

- A single measurement performed on one of the systems—for example, on system I—gives instantaneous knowledge of the wave function of system II moving freely far away.
- By choosing two different incompatible observables to be measured on system I, it is possible to assign two different wave functions to system II (the same physical reality: the second system after the interaction with the first).

Since a measurement performed in a distant location on system I does not disturb system II in any way, according to A1 and A3 system II should be described by a unique wavefunction and not by two different wave functions. Moreover, these wave functions are eigenstates of two non-commuting operators that represent incompatible physical observables which allow indirect deduction of the values of these incompatible physical observables for the same system II without disturbing it in any way which contradicts Heisenberg uncertainty relations and CI.

EPR discussed particle positions and momenta, and Bohm discussed an experiment in which a source produces pairs of particles prepared in a spin singlet state (Bohm, 1951). One of a pair (photon or electron) is sent to Alice and another to Bob in distant laboratories. According to A1, each pair of photons is described by a state vector:

$$\Psi = \left(|+\rangle_P|-\rangle_P - |-\rangle_P|+\rangle_P\right)\big/\sqrt{2}. \tag{1}$$

—where $|+\rangle_P$ and $|-\rangle_P$ are state vectors corresponding to photon states in which their spin is "up" or "down" in direction **P**, respectively**.** If we measure a spin projection of a photon I on direction **P**, we have an equal probability of obtaining result "1" or "−1". If we obtain "1," a reduced state vector of the photon II is $|-\rangle_P$; if we obtain "-1," a reduced state vector of the photon II is $|+\rangle_P$. By choosing direction **P** for the measurement to be performed on photon I, when "photons are in flight and far apart" we can assign different incompatible reduced state vectors to the same photon II. In other words, we can predict with certainty and without in any way disturbing the second photon that the **P**-component of the spin of photon II must have the opposite value to the value of the measured **P**- component of the spin of photon I (Ballentine, 1998). Therefore, for any direction **P**, the **P**-component of the spin of photon II has unknown but predetermined value which contradicts QM and is called the "EPR-B paradox".

Bohr (1935) promptly replied to the EPR paper and explained that two different wave functions could be assigned to system II only in two different incompatible experiments in which both systems were exposed to different influences before the measurement on system I was performed. In order to make predictions concerning the individual physical systems in EPR scenario 1, much more detailed knowledge of how a particular pair was prepared in each of these incompatible experiments is necessary (Kupczynski, 2006).

In 1936, Einstein advocated a purely statistical interpretation of QM and explained that the EPR paradox disappears because "…Ψ function does not, in any sense, describe the state of one single physical system and reduced wave functions describe different sub-ensembles of systems" (Einstein, 1936). This statistical interpretation has been generalized and promoted with success by Ballentine 1989 and Ballentine 1998: "…the habit of considering an individual particle to have its own wave function is hard to break … though it has been demonstrated strictly incorrect".

According to the statistical contextual interpretation of QM (SCI) (Ballentine, 1998; Kupczynski, 2006; Kupczynski, 2007; Kupczynski, 2016a; Khrennikov, 2009; Allahverdyan et al., 2013):

1. A state vector Ψ is not an attribute of a single electron, photon, trapped ion, quantum dot, *etc.* A state vector Ψ or a density matrix ρ describe only an ensemble of identical state preparations of some physical systems.
2. A wave function reduction is neither instantaneous nor non-local. In an EPR experiment, a state vector describing system II obtained by reduction of an entangled state (Equation 1) of two physical systems I + II describes only a sub-ensemble of systems II being partners of those systems I for which a measurement of some observable gave the same specific outcome. Different sub-ensembles are described by different reduced state vectors.
3. A value of a physical observable, such as a spin projection, is not a predetermined attribute of a system but is a property of a pure ensemble of identically prepared physical systems created in the interaction with a measuring instrument (Kupczynski, 1987b, 2015a).

The solution of the EPR-B paradox given by SCI is simple: the wave function reduction is not instantaneous, and a reduced one-particle state $|+\rangle_P$ "describes" only an ensemble of partners of the particles I which were detected to have "spin down" in the direction **P.** For different directions **P,** we perform specific experiments, and we obtain a different sub-ensemble of particles II. Strong correlations between distant outcomes in EPR experiments are due to contextuality and various conservation laws. More detailed discussion of EPR and EPR-B paradoxes may be found, for example, in (Kupczynski, 2009).

5 Kolmogorov and quantum probabilistic models

Outcomes of any random experiment are described by a specific probability space Ω, σ-algebra of its all sub-ensembles F, and a probabilistic measure μ. A sub-ensemble $E \in F$ is an event corresponding to a subset of possible outcomes of a random experiment. A probability of observing this event is given by $0 \le \mu(E) \le 1$. In statistics, instead of Ω we use a sample space S which contains only the possible outcomes of a studied random experiment.

Every random experiment is defined by its experimental context C (Kupczynski, 2017a; Kupczynski, 2015a; Khrennikov, 1999; Khrennikov, 2024; Khrennikov, 2009; Khrennikov, 2016; Khrennikov, 2022). If its outcomes are discrete, it may be described by a random variable A and a probability distribution

$$P(a|C) = P(A = a\,|\,C) \tag{2}$$

and its expectation value

$$E(A|C) = \sum_a aP(a|C). \tag{3}$$

In quantum experiments, the context of an experiment is determined by a preparation of an ensemble of physical systems represented by a density operator ρ (or a state vector ψ) and by a Hermitian operator $\hat{A}$ representing the experimental set-up used to measure a physical observable A. Instead of (Equations 2, 3), we have

$$P\left(a\middle|\psi,\hat{A}\right) = |\langle a|\psi\rangle|^2, \tag{4}$$

—where $|a\rangle$ is a corresponding eigenvector of the operator $\hat{A}$ and

$$E\left(A|\psi,\hat{A}\right) = \langle\psi|\hat{A}|\psi\rangle. \tag{5}$$

If a density matrix ρ is used to describe a pure or mixed prepared ensemble, then

$$E\left(A|\rho,\hat{A}\right) = Tr\left(\rho\hat{A}\right). \tag{6}$$

In an idealized EPR-B experiment (Equation 1), which is impossible to implement, a source sends two correlated signals which arrive to distant laboratories, pass by polarization analyzers, and produce coincident counts on detectors. The experimental situation is much more complicated since clicks are not registered at the same time and one has to decide which clicks are correlated by introducing specific time windows and deciding how to use them in order to define coincident clicks (Kupczynski, 2017b; 2021).

An idealized EPR-B experiment is described by the following probabilistic model (Kupczynski, 2020, 2023a, 2024a; Cetto et al., 2020). Randomly chosen polarization measurement settings are (x, y), prepared ensemble E is described by $\rho = |\psi\rangle\langle\psi|$, $\hat{A}_x = \vec{\sigma}\cdot\vec{n}_x$ and $\hat{B}_y = \vec{\sigma}\cdot\vec{n}_y$ represent spin projections on the corresponding unit vectors, and

$$\begin{aligned} E\left(A_xB_y\right) &= Tr\left(\rho\hat{A}_x\otimes\hat{B}_y\right) = \langle\psi\,|\,\hat{A}_x\otimes\hat{B}_y|\psi\rangle = \sum_{\alpha\beta}\alpha\beta p_{xy}(\alpha,\beta) \\ &= -\vec{n}_x\cdot\vec{n}_y = -\cos\left(\theta_{xy}\right), \end{aligned} \tag{7}$$

—where $\hat{A}_x\otimes\hat{B}_y|\alpha\beta\rangle_{xy} = \alpha\beta|\alpha\beta\rangle_{xy}$, $p_{xy}(\alpha,\beta) = |\langle\psi|\alpha\beta\rangle_{xy}|^2$ and $\alpha = \pm1$ and $\beta = \pm1$ (Kupczynski, 2024b; Cetto et al., 2020).

The model is contextual because a triplet $\{\rho,\hat{A}_x,\hat{B}_y\}$ changes if a preparation or defined by Equations 4-7 experimental settings change. For each choice of settings (x, y), QM provides a specific Kolmogorov model.

Since $E(A_xB_y) = -1$ for $\theta_{xy} = (\theta_x - \theta_y) = 0$, it has been incorrectly claimed that QM predicts strict anti-correlations of two space-like events produced in an irreducibly random way. Since two space-like events produced randomly cannot be correlated $(E(A_xB_y) = 0)$, irreducible randomness was abandoned, and several hidden variable models were proposed to explain the correlations predicted by QM. In fact, QM does not predict strict correlations for EPRB-type experiments. Directions can only be defined by some small intervals I_x and I_y containing angles close to θ_x and θ_y respectively. Therefore, the correct quantum prediction for expectation values is (Kupczynski, 2016a, 1987b)

$$E\left(A_xB_y\right) = -\iint_{I_xI_y}\cos(\theta_1-\theta_2)\,d\rho_x(\theta_1)\,d\rho_y(\theta_2). \tag{8}$$

After defining in the next section Bell–CHSH inequalities, we will discuss several hidden variable models proposed to explain quantum correlations Equations 7, 8.

6 Experimental spreadsheets and Bell–CHSH inequalities

Let us consider a random experiment described by four jointly distributed binary random variables (A, A’, B, B’) taking the values ± 1. In each trial of this experiment, four outcomes (a. a, b, b’) are obtained and displayed in an N×4 experimental spreadsheet (Kupczynski, 2020). Since b = b’ or b = -b’ thus

$$|s| = |ab - ab' + a'b + a'b'| = |a(b - b')| + |a'(b + b')| \le 2. \quad (9)$$

From Equation 9 we obtain CHSH inequality:

$$|S| \le \sum_{a,a',b,b'} |ab - ab' + a'b + a'b'| p(a, a', b, b') \le |E(AB)\text{-}E(AB')| + |E(A'B) + E(A'B')| \le 2, \quad (10)$$

—where p(a, a′, b, b’) is a joint probability distribution of (A, A′, B, B’), and $E(AB) = \sum_{a,b} ab p(a, b)$ is a pairwise expectation of A and B obtained using a marginal probability distribution $p(a, b) = \sum_{a',b'} p(a, a', b, b')$ (Kupczynski, 2020).

If all pair-wise expectation values in Equation 10 are estimated using the same N×4 experimental spreadsheet, then the inequality (Equation 10) is strictly obeyed by all finite samples. The inequalities (Equation 10) are in fact necessary and sufficient conditions for the existence of a joint probability distribution of only pairwise measurable ±1-valued random variables (Fine, 1982). The inequalities (Equation 10) are also valid if |A|≤1, |A’|≤1|, |B|≤1, and |B’|≤1. It is now well known that cyclic combinations of pairwise marginal expectations of jointly distributed binary random variables must obey non-contextuality inequalities (NCI) (Araujo et al., 2013). Bell–CHSH inequalities are a special case of NCI.

If we have four N×4 spreadsheets containing outcomes from four runs of the same random experiment, as discussed above, but we use each of these spreadsheets to estimate only one pairwise expectation E (A, B), E(A,B′), E (A’. B), and E(A’. B’) respectively, then 50% of the time, these estimates violate the inequality (Equation 10) (Kupczynski, 2016a; Kupczynski, 2023a; Gill, 2014), Only if N increases to infinity the probability of the violation of the inequality (Equation 10) tends to 0. Therefore, the violation of CHSH-inequality by experimental data in EPR-type experiments allows only the evaluation of the plausibility of particular probabilistic models (Kupczynski, 2024c). The next section will discuss such models.

7 Local realistic models for the EPR–Bohm experiment

7.1 Local realistic hidden variable model (LRHVM)

In an attempt to explain correlations in an ideal EPR-B experiment, (Bell, 1965, 2004; Kupczynski, 2015a, 2024d) proposed a probabilistic model in which outcomes registered in distant laboratories are predetermined at a source:

$$E(A_x B_y) = \sum_{\lambda\in\Lambda} A_x(\lambda) B_y(\lambda) P(\lambda), \quad (11)$$

—where $A_x(\lambda) = \pm 1$ and. $B_y(\lambda) = \pm 1$. In LRHVM, we have four jointly distributed random variables (A_x(L), B_y(L), $A_{x'}$(L), $B_{y'}$ (L)) being functions of the same random variable L. The random variable L describes a classical random experiment in which λ is sampled with replacement from a probability space Λ. For each value of λ, all outcomes can be calculated. LRHVM describes *entangled pairs* as pairs of socks, which can have different sizes and colors; for example, Harry draws a pair of socks, sends one sock to Alice and another to Bob, who in function of (x, y) record corresponding properties of color or size.

Since (A_x(L), B_y(L), $A_{x'}$(L), $B_{y'}$ (L)) are jointly distributed, they thus obey CHSH inequality:

$$|S| = \left|E(A_x B_y) + E(A_x B_{y'}) + E(A_{x'} B_y) - E(A_{x'} B_{y'})\right| \le 2. \quad (12)$$

Bell knew that in the EPR-B experiment, (A_x, B_y, $A_{x'}$, $B_{y'}$) are not jointly measurable and that their joint probability distribution does not exist. He did not notice that to prove his inequalities, he was tacitly using the existence of a joint probability distribution of (A_x(L), B_y(L), $A_{x'}$(L), $B_{y'}$ (L)). As explained in the preceding section, the inequalities (Equations 10 and 12) can be rigorously proven for a random experiment outputting in each trial four ±1 outcomes.

7.2 Stochastic hidden variable model (SHVM)

(Clauser and Horne, 1974; Kupczynski, 2024e) proposed a stochastic hidden variable model (SHVM) in which λ does not determine outcomes in a given trial but only their probability.

Using the notation of Big Bell Test collaboration (The BIG Bell Test Collaboration, 2018):

$$P(a, b|x, y) = \sum_{\lambda} P(a|x, \lambda) P(b|y, \lambda) P(\lambda), \quad (13)$$

—where P(-|-) denotes a conditional probability. Equation 13 for a fixed setting (x, y) describes a family of independent random experiments labelled by λ and

$$E(A_x B_y) = \sum_{\lambda} E(A|x, \lambda) E(B|y, \lambda) P(\lambda). \quad (14)$$

Pair-wise expectations defined by Equation 13 are also constrained by CHSH inequalities Equation 12. In SHVM, entangled photon pairs are described as pairs of dice, and the correlations which can be created in this model are quite limited.

7.3 Local causal hidden variable model (LHVM)

LHVM is a generalization of the preceding two models, where λ represents all possible common causes of events happening in distant laboratories, and “. . .they may include the usual quantum state; they may also include all the information about the past of both Alice and Bob. Actually, the λ′s may even include the state of the entire universe” (The BIG Bell Test Collaboration, 2018; Kupczynski, 2024a) —except that inputs (x, y) cannot depend on them.

$$P(a, b, x, y) = \sum_{\lambda} P(a|x, \lambda) P(b|y, \lambda) P(x, y|\lambda) P(\lambda) \quad (15)$$

and

$$P(x, y|\lambda) = P(x, y). \quad (16)$$

The condition (Equation 16) is called "measurement independence," experimenters' "freedom-of-choice" (FoC) or "no conspiracy" (The BIG Bell Test Collaboration, 2018; Hall, 2010; Myrvold et al., 2020; Blasiak et al., 2021; Kupczynski, 2024b, 2022). Since correlation does not mean causation, this terminology is based on the incorrect causal interpretation of conditional probabilities (Kupczynski, 2017a, 2021, 2023a, 2024a, 2024b, 2024c, 2022). In a probabilistic model, $P(x, y|\lambda) \neq P(x, y)$ does not imply that FoC is constrained by causal influences.

If λ represents ontic properties of entangled pairs or common causes, it thus cannot not depend in any sense on chosen settings:

$$P(\lambda, x, y) = P(\lambda) P(x, y) \Rightarrow P(\lambda|x, y) = P(\lambda). \quad (17)$$

However, hidden variables can also describe measuring instruments, so they can depend on the chosen settings (Kupczynski, 2006; Kupczynski, 2016a; Kupczynski et al., 2007). As Theo Nieuwenhuizen explained, the model (Equations 13–16) suffers from a theoretical *contextuality loophole* because the hidden variables describing measuring instruments had not been included (Nieuwenhuizen, 2009; Nieuwenhuizen, 2011; Nieuwenhuizen and Kupczynski, 2017).

There is no doubt that experimenters can freely choose binary random labels of their setting (x, y), and this is what they do (Hensen et al., 2015; Giustina et al., 2015; Shalm et al., 2015; Handsteiner et al., 2017; The BIG Bell Test Collaboration, 2018; Rosenfeld et al., 2017; Zhang et al., 2022; Storz et al., 2023). However, this random choice of labels (x, y) is followed by a choice of corresponding specific instruments and setting-dependent measuring procedures. Since measuring instruments play an active role in quantum experiments, it is reasonable to assume that outcomes depend not only on setting-independent hidden variables that describe prepared physical systems but also on setting-dependent hidden variables that describe local instruments and measuring procedures; and thus *statistical independence* (Equation 17) is violated:

$$P(\lambda|x, y) \neq P(\lambda). \quad (18)$$

Bell was the first to notice that if hidden variables depend on settings; then Bell–CHSH inequalities could not be derived. However, since Equation 18 implied the violation of Equation 16, this option was rejected as violating FoC (Kupczynski, 2017a; The BIG Bell Test Collaboration, 2018; Myrvold et al., 2020; Kupczynski, 2023a, 2024a, 2024b). As explained above, the violation of Equation 16 does not constraint FoC.

Bell clearly demonstrated that LRHVM is inconsistent with QM because there exist four particular experimental settings for which, using Equation 7, one obtains $|S| \leq 2\sqrt{2}$, which significantly violates Equation 12. Various Bell Tests (Hensen et al., 2015; Giustina et al., 2015; Shalm et al., 2015; Handsteiner et al., 2017; The BIG Bell Test Collaboration, 2018; Rosenfeld et al., 2017; Zhang et al., 2022; Storz et al., 2023) were performed in order to check the plausibility of local hidden variable models. Before explaining a contextual hidden variable model in which hidden variables depend on settings, the next section discusses recent Bell tests and their implications.

8 Bell tests and what they have proven

Bell tests are inspired by an ideal EPR experiment. Entangled pairs are created at a source and sent to distant locations or are created directly in distant laboratories using specific synchronized preparations/treatments such as entanglement swapping or entanglement transfer protocols (Kupczynski, 2024a). Despite differences, experimental protocols are subdivided into three steps:

1) Preparation of an ensemble E of pairs of entangled physical systems.
2) Random local choice of labels/inputs (x, y) using random number generators (RNG), and signals coming from distant stars (Handsteiner et al., 2017; The BIG Bell Test Collaboration, 2018) or/and human choices (The BIG Bell Test Collaboration, 2018; Rosenfeld et al., 2017; Zhang et al., 2022; Storz et al., 2023). This study uses four pairs of labels/inputs—(x, y), (x, y'), (x', y), and (x', y') —which denote four incompatible experimental settings/contexts.
3) Implementation of correlated and synchronized measurements in distant locations and readout of binary outcomes (a, b) (called outputs), which are the coded information corresponding to clicks on different distant detectors, *etc.*

In Bell Tests to each randomly chosen input (x,y) corresponds a specific pair of correlated distant experiments. Outcomes of these experiments are described by four pairs of binary random variables: (A_{xy}, B_{xy}), $(A_{xy'}, B_{xy'})$, $(A_{x'y}, B_{x'y})$, and $(A_{x'y'}, B_{x'y'})$ (Kupczynski, 2024a). Our notation is inspired by the contextuality-by-default approach (CbD) (Kupczynski, 2021; Dzhafarov and Kujala, 2014; Dzhafarov et al., 2015; Kujala et al., 2015) in which random variables measuring the same content in a different context are *a priori* stochastically unrelated, such as A_{xy} and $A_{x'y}$. It is evident that in Bell tests, a joint probability distribution of these eight random variables does not exist, and Bell– CHSH inequalities cannot be derived without additional assumptions (Khrennikov, 2022).

A pair of random empirical variables (A_{xy}, B_{xy}) describes a scatter of outputs in the experiment using settings (x, y). We have four random experiments described by specific empirical probability distributions. Using these distributions, we may test the plausibility of quantum and local hidden variable models proposed to explain a statistical scatter of outcomes in an ideal EPR-B experiment. If random variables in probabilistic models are denoted (A'_{xy}, B'_{xy}) in order to not be confounded with empirical random variables (A_{xy}, B_{xy}), then we say that a probabilistic model provides a *probabilistic coupling* if:

$$E(A_{xy}) = E(A'_{xy}), E(B_{xy}) = E(B'_{xy}), E(A_{xy}B_{xy}) = E(A'_{xy}B'_{xy}). \quad (19)$$

Therefore, in Bell tests, we are testing the plausibility of different probabilistic couplings, in particular for LRHVM:

$$E\left(A'_{xy}\right) = E\left(A'_{xy'}\right) = E\left(A_x\right), E\left(B'_{xy}\right) = E\left(B'_{x'y}\right)$$
$$= E\left(B_y\right), E\left(A'_{xy}B'_{xy}\right) = E\left(A_xB_y\right), \quad (20)$$

where (A_x, B_y, $A_{x'}$, $B_{y'}$) are jointly distributed Equation 11. More detailed discussion may be found in Kupczynski (2024a).

There is still much confusion in journals, books, and in social media concerning the metaphysical implications of the results of Bell tests (The BIG Bell Test Collaboration, 2018; Rosenfeld et al., 2017; Zhang et al., 2022; Storz et al., 2023), so it is beneficial to explain it here. Using LHVM Equations 15–17, one derives inequalities which must be satisfied by specific combinations of probabilities of events to be observed in the experiments performed using different experimental settings. These combinations are denoted "S," "J," or "T," which are called in brief "Bell parameters". If the observed parameter violates inequality, one can conclude that measured systems were not governed by any LHVM. It should be noted that this conclusion is always statistical and typically takes a form of a hypothesis test, leading to a conclusion of the form: "…assuming nature is governed by local realism, the probability to produce the observed Bell inequality violation … is P(observed or stronger | local realism)≤ p. This p-value is a key indicator of statistical significance in Bell Tests" (The BIG Bell Test Collaboration, 2018)

Since p-values in several experiments are very small, one concludes: *Local realism, i.e., realism plus relativistic limits on causation, was debated by Einstein and Bohr using metaphysical arguments, and recently has been rejected by Bell tests.* Such a conclusion is imprecise and misleading. As correctly observed by Wiseman (2014), "*the usual* philosophical meaning of 'realism' is the belief that entities exist independent of the mind, a worldview one might expect to be foundational for scientists." It is also claimed that Bell tests allow the rejection of *local causality*, where *Bell-local causality* is defined: *Alice's output* a *depends only on her input* x *and on* λ *describing all possible common causes included in the intersection of the of the backward light cones of* a *and* b *and independent of inputs* x *and* y.

It is true that tested probabilistic models have been motivated by various metaphysical assumptions. Nevertheless, Bell tests allow only the rejection of a statistical hypothesis that says that LHVM Equations 15–17 provides a probabilistic coupling (Equation 20) consistent with experimental data. Therefore, the violation of Bell–CHSH inequalities does not allow for far reaching metaphysical speculations. We agree also with De Raedt et al., (2023):

…all EPRB experiments which have been performed and may be performed in the future and which only focus on demonstrating a violation BI-CHSH merely provide evidence that not all contributions to the correlations can be reshuffled to form quadruples … These violations do not provide a clue about the nature of the physical processes that produce the data….

Similar conclusions have been drawn (Kupczynski, 1987a, 2018b, 2020; Dzhafarov, 2021; Hess and Philipp, 2005; Khrennikov, 2007, 2008, 2019, 2020a, 2022; De Raedt et al., 2024).

Bell tests confirm the existence of long range correlations between outcomes of experiments performed in space-like locations. If additional context-dependent variables that describe measuring instruments and procedures are correctly incorporated into a probabilistic model (Equation 11), then Bell–CHSH inequalities cannot be derived and "nonlocal "correlations can be explained without evoking quantum magic. Such a model is discussed in the next section.

9 Contextual hidden variable model and the violation of statistical independence

We incorporate into the model (Equation 11) additional variables that describe distant measuring contexts (Kupczynski, 2024a).

- $\lambda_1 \in \Lambda_1$ and $\lambda_2 \in \Lambda_2$ describe correlated physical systems and do not depend on measurement settings (x, y).
- $\mu_x \in M_x$ and $\mu_y \in M_y$ describe measurement procedures and instruments at the moment of measurement when settings (x, y) were chosen.
- Inputs/labels (x, y) are randomly chosen in separate random experiments.
- Outputs are created locally: $a = A'_x(\lambda_1, \mu_x) = \pm 1$ and $b = B'_y(\lambda_2, \mu_y) = \pm 1$

The resulting contextual model (CHVM) is defined by three equations

$$E\left(A_xB_y\right) = \sum_{\lambda\in\Lambda_{xy}} A_x(\lambda_1,\mu_x)B_y\left(\lambda_2,\mu_y\right)\mathrm{P}(\lambda_1,\lambda_2)\mathrm{P}_{xy}\left(\mu_x,\mu_y\right), \quad (21)$$

—where $\Lambda_{xy} = \Lambda_1 \times \Lambda_2 \times M_x \times M_y$,

$$\mathrm{P}(\mathrm{a,b,x,y}) = \sum_{\lambda\in\Lambda_{xy}} \mathrm{P}(\mathrm{a}\,|\,\lambda_1,\mu_x)\mathrm{P}(\mathrm{b}|\lambda_2,\mu_y)\mathrm{P}(\mu_x,\mu_y|\mathrm{x,y})\mathrm{P}(\mathrm{x,y})\mathrm{P}(\lambda_1,\lambda_2) \quad (22)$$

and

$$\mathrm{P}\left(\mu_x,\mu_y\,|\,\mathrm{x,y}\right) = \mathrm{P}_{xy}\left(\mu_x,\mu_y\right) \neq P\left(\mu_x,\mu_y\right) \quad (23)$$

In Bell tests, P(x, y) = P(x) P(y), but in the contextual model Equation 21 and in QM, it does not matter how labels (x, y) are chosen. In general, spaces Λ_{xy} for different settings (x, y) do not overlap and, as Larsson and Gill (2004) demonstrated, Bell–CHSH inequalities cannot be derived and $|S| \leq 4$.

The model (Equations 21–23) violates statistical independence, and $P(x, y|\mu_x, \mu_y) \neq P(x, y)$:

$$\mathrm{P}\left(\mu_x,\mu_y,\mathrm{x,y}\right) = P_{xy}\left(\mu_x,\mu_y\right)\mathrm{P}(\mathrm{x,y}) = P\left(\mu_x,\mu_y\right) \rightarrow P\left(\mathrm{x,y}\,|\,\mu_x,\mu_y\right) = \frac{P\left(\mu_x,\mu_y\right)}{P\left(\mu_x,\mu_y\right)} = 1. \quad (24)$$

The Equation 24: $P(\mathrm{x,y}\,|\,\mu_x,\mu_y) = 1$ only "says" that if a hidden event $\{\mu_x,\mu_y\}$ "happened", then the settings (x,y) were used (Kupczynski, 2017a, 2021, 2023a, 2024a, 2022). It has nothing to do with *conspiracy* or FoC.

Since inputs (x,y) were chosen using signals from distant stars (Handsteiner et al., 2017), random number generators or random human choices were made during online computer games (The BIG Bell Test Collaboration, 2018), and thus the *freedom-of-choice loophole* was successfully closed, but it did not prove *statistical independence*. As I proposed in preceding papers, a violation of *statistical independence* should be called "Bohr-contextuality"—not to be cofounded with *CbD contextuality* (Dzhafarov and Kujala, 2014; Dzhafarov et al., 2015; Kujala et al., 2015) or simply *contextuality*.

CHVM violates Bell-locality and Bell-causality, but outputs are created in a locally causal way. Hidden variables describing physical systems and measuring contexts in space-like separated laboratories can be statistically correlated, but the violation of statistical independence and apparently non-local correlations may be explained without evoking spooky influences. It may be the effect of setting dependent post-selection of data (Kupczynski, 2017b, 2021, 2024a), or it may be due to the global space-time symmetries (Kupczynski, 2023a, 2024a, 2023b).

The model (Equation 21) can be further simplified. For example, μ_x can be a fixed set of variables describing experimental procedures labeled by x. If in a distant laboratory, a setting labeled by y is used, then a measuring instrument and/or laser beam are rotated by angle $\theta_{xy} = \theta_x - \theta_y$. Therefore, due to global rotational symmetry, $\mu_y = f(\mu_x, \cos(\theta_{xy}))$, and:

$$E\left(A_x B_y\right) = \sum_{\lambda \in \Lambda_{xy}} A_x(\lambda_1, \mu_x) B_y\left(\lambda_2, \mathrm{f}\left(\mu_x, \cos\left(\theta_{xy}\right)\right)\mathrm{P}(\lambda_1, \lambda_2)\right. \quad (25)$$

The model (Equation 25) seems to have enough flexibility in order to explain long range correlations in Bell tests depending on $\theta_{xy} = \theta_x - \theta_y$. The model (Equation 25) does not allow the derivation of any Bell-type inequalities.

10 Can a quantum-mechanical description of physical reality be considered complete?

This question asked by Einstein, Podolsky, and Rosen (EPR) (Einstein et al., 1935) and answered by Bohr (1935) has been debated for 90 years. Many incorrectly believe that the results of recent Bell tests prove that if we reconcile QM with general relativity, we will obtain a complete description of physical reality. In fact, we should be much more humble (Kupczynski, 2024c) because we even do not know whether QM is predictably complete.

QM gives probabilistic predictions for distributions of the results obtained in long runs of one experiment or in several repetitions of the same experiment on a single physical system. It is unclear how and in what sense a claim can be made that QM provides a complete description of individual physical systems. This is why (Einstein, 1936; Einstein and Schilpp, 1949) never accepted that a statistical theory may provide a complete description of individual physical systems and believed that QM should be completed by some microscopic theory of sub-phenomena that enable the reproduction of quantum probabilistic predictions.

According to Bohr, quantum probabilities describe completely quantum phenomena and experiments, and no more detailed sub-quantum description is possible or necessary. Quantum probabilities are thus irreducible, and QM is not an emergent theory. In statistical mechanics, probabilities reflect a lack of knowledge about the properties of physical systems. In SCI, quantum probabilities reflect a lack of knowledge about the interactions of physical systems with measuring instruments in well-defined experimental contexts. The Bertrand paradox teaches that probabilities are not properties of individual physical systems but are only properties of random phenomena and experiments as a whole. In this sense, they do not provide a complete description of individual physical systems.

Whether a more detailed description of quantum phenomena does exist is an open question, and several hidden variable models have been proposed and discussed. Bell tests permit the rejection of several hidden variable models but neither prove the completeness nor non-locality of QM. Several years ago, we pointed out that the question about the completeness of QM cannot be answered by constructing *ad hoc* sub-quantum hidden variable models. It can only be answered by a different and a more detailed analysis of experimental data (Kupczynski, 2006; Kupczynski et al., 2007; Kupczynski, 2016a, 1986, 1984).

In quantum experiments, outcomes are registered by online computers as finite time series of data. It can be a laser beam which, after passing by a PBS (polarization beam splitter), produces clicks on detectors coded ±1. It can be a physical system in a trap, a physical observable is measured, an outcome is recorded, and initial conditions in the trap are reset.

No single result is predictable in all these experiments. Empirical frequency distributions are obtained from long-term series of counts and compared with probabilistic predictions of QT. In this way, *predictable completeness* of QT is taken for granted, and any fine structure of time-series, if it existed, would be averaged.

Let us consider two experiments repeated N times each. In the first experiment, we obtain a time series of the results, 1,-1,1,-1, . . . 1,-1 . . ., and in the second, 1,-1,-1,1,1,1,-1,-1, 1,-1,-1,1,1,1,-1 . . . By increasing the value of N, the relative frequency of achieving 1 can approach ½ as close as we wish. However, it is not a complete description of these time series. By searching for reproducible fine structures in experimental time series, we can investigate whether QM is emergent without constructing specific hidden variable models.

In any more detailed description of quantum phenomena, pure quantum ensembles become mixed statistical ensembles with respect to additional uncontrollable parameters that describe physical systems and measuring instruments. There is a principal difference between a pure statistical ensemble and a mixed one. For a pure ensemble, any sub-ensemble has the same properties. Sub-ensembles of a mixed statistical ensemble may differ from one to another if mixing is not perfect. These differences can be, in principle, detected by using so called purity tests (Kupczynski, 2006, 1986, 1984; Kupczynski et al., 2007), which I introduced in a different context (Kupczynski, 1974, 1977).

Let us consider time series of outcomes T(S, E, i) obtained in an i^{th} run of an experiment E performed on physical system(s) S. Since we do not control the distribution of hidden variables, time-series T(S, E, i) may differ from run to run of the same experiment. Using the language of mathematical statistics, T(S, E, i) represents a random sample drawn from some statistical population. A pure ensemble is one characterized by such empirical distributions of various counting rates, which remain approximately unchanged for any rich sub-ensembles drawn from this ensemble in a random way

(Kupczynski et al., 2007; Kupczynski, 1973, 1986). Therefore, we must test the null hypothesis H_0:

Samples T(S, E, i) for different values of i are drawn from the same statistical population.

Various statistical non-parametric compatibility tests can be used to test H_0.

Purity tests are not sufficient. To prove that QM is not *predictably complete*, it is necessary to study in more detail time series of data, detect some temporal fine structure, and find a stochastic model to explain it. Several methods are used to study and compare empirical time-series: frequency or harmonic analysis, periodograms, autocorrelation and partial autocorrelation functions, *etc.* (Kupczynski, 2009, 2011). The aim of most physical experiments is to compare empirical probability distributions with quantum probabilistic predictions. Therefore, all fine structures in time-series of data, if they exist, are averaged out and are not discovered.

Completeness of QM has been discussed for nearly 100 years, but a detailed study of experimental time series of existing experimental data is still to be done. As demonstrated recently with Hans de Raedt, sample inhomogeneity invalidates dramatically significance tests (Kupczynski and De Raedt, 2016); therefore, if sample homogeneity is not tested carefully enough, then the *sample homogeneity loophole* is not closed and statistical inference cannot be trusted (Kupczynski and De Raedt, 2016; Kupczynski, 2015b, 2016b).

11 Conclusion

This review article has explained why speculations about quantum nonlocality and quantum magic are rooted in incorrect interpretations of QM and/or in incorrect "mental pictures" and models that try to explain invisible details of quantum phenomena. In particular, it is not true that in Bell tests, entangled qubits behave as "a pair of dice showing always perfectly correlated outcomes."

We advocate an abstract statistical contextual interpretation (SCI) of QM which is free of paradoxes. SCI rejects the existence of a universal wave function. Quantum probabilities are objective properties of quantum phenomena. Whether these probabilities can be explained as emergent is an open question which cannot be settled by philosophical discussions and no-go theorems; it can be only elucidated by more detailed study of experimental time series of data than is usual.

Bell tests are subtle experiments that are imperfect implementations of an ideal EPRB experiment. It is often claimed that the violation of Bell–CHSH inequalities in these tests allow the rejection with great confidence of *local realism* and *local causality*. Such conclusions, though, are misleading.

Bell–CHSH are trivial properties of N×4 spreadsheets on which the outcomes of measurements of four jointly distributed random variables (e.g., A_x, B_y, $A_{x'}$, $B_{y'}$) are displayed. In Bell tests, such experimental spreadsheets do not exist because there are four pairs of distant random experiments performed using four incompatible experimental settings (x, y). These experiments are described by empirical probability distributions of four pairs of random variables (A_{xy}, B_{xy}). Bell–CHSH inequalities cannot be derived, and estimated pairwise expectations $E(A_{xy}\, B_{xy})$ are not constrained by these inequalities.

Probabilistic couplings can be postulated in order to explain statistical regularities in experimental data, such as $E(A_{xy}B_{xy}) = E(A_xB_y)$. The quantum probabilistic model and Bell-causal hidden variable model can only be tested as plausible probabilistic couplings (Kupczynski, 2024a). Quantum coupling (Equation 7) is constrained by quantum–CHSH inequalities: $|S| \le 2\sqrt{2}$ (Kupczynski, 2020; Kupczynski, 2024a; Khrennikov, 2019; Cirel'son, 1980; Landau, 1987). Local hidden variable couplings (Equations 11,13, 15–17) are constrained by Bell–CHSH inequalities: $|S| \le 2$.

It was incorrectly believed that if the *freedom-of-choice* loophole was closed then hidden variables could not statistically depend on randomly chosen binary inputs (settings' labels). This is untrue because variables describing distant measuring instruments used in different settings can depend on inputs and may be correlated due to global rotational symmetry. Therefore, closing the *freedom-of-choice* loophole does not close the *contextuality loophole*.

In contextual hidden variable models (Equations 21–23) and (Equation 25), which are neither Bell-local nor Bell-causal, distant outcomes are locally determined by setting independent hidden variables that describe prepared qubits and setting dependent hidden variables that describe distant measuring instruments and procedures. This model is only constrained by $|S| \le 4$. Due to global rotational symmetry, the pairwise expectation values of distant random variables (describing Alice's and Bob's outcomes) have to depend on angle $\theta_{xy} = \theta_x - \theta_y$, where (θ_x, θ_y) are the respective angles by which distant qubits are rotated before local read-outs.

We can intuitively explain how parameters describing measuring devices in space-like locations may obey the equation $\mu_y = f(\mu_x, \cos(\theta_{xy}))$, even if (θ_x, θ_y) are chosen perfectly randomly. We imagine two observers in front of two screens on which two identical triangles are projected. They record their observations by six coordinates $\mu=(x_1,\ y_1;\ x_2,\ y_2;\ x_3,\ y_3)$. Next, (θ_x, θ_y) are chosen randomly, and rotated triangles are projected onto respective screens. Now the observers' recordings differ: $\mu_x = R(\theta_x)\mu$, $\mu_y = R(\theta_y)\mu$ and $\mu_x = R(\theta_{xy})\mu_y$. Variables describing distant measuring devices and procedures can be strongly correlated without any spooky influences. We used a shortened notation according to which the rotation 2 x 2 matrices are applied at the same time to coordinates of three triangle's vertices.

Therefore, Bell tests prove only that the probabilistic coupling LHVM is inconsistent with the experimental data. They allow the rejection of Bell-locality and Bell-causality assumptions but have little to say about the completeness of QM or *local causality* in nature. As has been observed, *quantum nonlocality* is a misleading notion (Boughn, 2022; Czahor, 1988; Dzhafarov, 2021; Fine, 1982; Hance and Hossenfelder, 2022; Hess and Philipp, 2005; Hess, 2022; Jaynes and Skilling, 1989; Jung, 2017; Khrennikov, 2007; Khrennikov, 2008; Khrennikov, 2019; Khrennikov, 2020a; Khrennikov, 2022; Kupczynski, 2018b; Kupczynski, 2023a; Kupczynski, 2024a; Kupczynski, 2024b; Peres, 1978; Pitovsky, 1983; Pitovsky, 1994; Żukowski and Brukner, 2014; De Raedt et al., 2017; De Raedt et al., 2023; De Raedt et al., 2024; Żukowski and Brukner, 2014; Jung, 2020; Boughn, 2017), and extraordinary metaphysical speculations based on the results of Bell tests are unfounded.

Correlation does not mean causation. Alice's and Bob's experimental outcomes may be correlated, but a probabilistic scatter of Alice's outcomes cannot depend on what Bob is measuring in his distant laboratory. This is called "no-signaling."

No-signaling was verified and confirmed for raw experimental data in all Bell tests. Nevertheless, to study correlations in some experiments involves rejecting single clicks and combining coincident clicks in pairs on Alice's and Bob's detectors. This has created an apparent signaling in some experiments (Hensen et al., 2015; Weihs et al., 2024; Adenier and Khrennikov, 2007; Adenier and Khrennikov, 2017; Bednorz, 2017) which could be explained without evoking spooky influences (Kupczynski, 2017b; Kupczynski, 2021; Kupczynski, 2024a; Khrennikov, 2022). The presence of signaling patterns in the experimental data means that these data have to be described by random variables labelled by both the content and context of the experiment, and of course a joint probability distribution of such variables does not exist.

An external world certainly does exist and it does not depend on whether it is observed or not. Our mathematical models describe only imperfectly its different layers (Kupczynski, 2024c). Quantum phenomena under investigation depend on the detailed contexts of our experiments. The information obtained is contextual and complementary, but quantum probabilities are objective properties of quantum phenomena.

Questions about the completeness of quantum mechanics can only be answered by a search of reproducible fine structures in time series of experimental data which were not predicted by QM. It would not only demonstrate that QM may not provide the most complete description of the individual physical systems but also that QM is not *predictably complete* (Kupczynski, 2006; Kupczynski, 2009; Kupczynski, 2011).

We finish this article with words of Einstein (1936):

Is there really any physicist who believes that we shall never get any insight into these important changes in the single systems, in their structure and their causal connections … To believe this is logically possible without contradiction; but it is so very contrary to my scientific instinct that I cannot forego the search for a more complete description.

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Conflict of interest

The author declares that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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