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Evaluation of agricultural product distribution efficiency under the perspective of agricultural-commerce integration

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This study evaluates the efficiency of agricultural product distribution through the lens of agricultural-commerce integration, focusing on Fujian Province, China. By innovatively dividing the distribution process into two stages—production and sales—and employing a non-radial Slack-Based Measure (SBM) two-stage network Data Envelopment Analysis (DEA) model, the research provides a more comprehensive assessment of distribution efficiency. The results reveal that Xiamen maintained full efficiency in both stages, while Fuzhou improved from 0.681 in 2015 to full production efficiency by 2019. In contrast, Nanping's sales efficiency remained as low as 0.041. The results show that while overall distribution efficiency has improved, significant regional disparities persist, particularly in the sales stage. The paper highlights the central role of agricultural product distribution companies in optimizing resource allocation and enhancing efficiency through collaborations with emerging agricultural entities and strategies to reduce distribution costs. Additionally, the study introduces a novel approach by categorizing cities based on their efficiency levels and proposing tailored improvement strategies for each category. High-performing cities like Xiamen and Fuzhou should focus on value-added processing, while low-efficiency cities like Nanping require targeted interventions to improve sales performance. This research contributes to the literature by integrating agricultural-commerce integration into the evaluation of distribution efficiency and offers actionable insights for policymakers and stakeholders in the agricultural sector looking to improve agricultural product distribution systems.

agricultural product distribution, agricultural-commerce integration, efficiency evaluation, stage data envelopment analysis, Fujian

1 Introduction

To enhance the distribution, sales, and optimization of agricultural products while ensuring shared benefits for farmers, businesses, consumers, and other stakeholders, the international community increasingly emphasizes integrating agricultural and commercial systems. For instance, in 2018, China's Ministry of Commerce issued the "Notice on Promoting Agricultural-Commercial Integration to Support Rural Revitalization," aiming to establish a new distribution model for agricultural products. This model emphasizes the

development of interconnected, cooperative relationships between agricultural operators and distribution enterprises, such as through frameworks like "production bases + leading enterprises/wholesale markets/supermarkets" and "farmers + professional cooperatives + leading enterprises." These initiatives seek to align the interests of all stakeholders in the agricultural supply chain and foster sustainable, long-term partnerships.

Globally, agricultural operators often face challenges due to their small-scale operations and limited integration across the value chain. This lack of integration creates two major contradictions that hinder sustainable agricultural development. First, the relationships between agricultural producers and distribution enterprises are often characterized by short-term, loose, and unstable interactions, resulting in inefficiencies in resource allocation and power imbalances. Independent distribution enterprises frequently operate without effective horizontal or vertical information exchange within the supply chain, further exacerbating these issues (Sun et al., 2022; Yang et al., 2021). Second, the disconnect between producers and sellers undermines the potential for building stable, mutually beneficial partnerships. This fragmentation not only weakens trust and collaboration across the supply chain but also hampers efforts to promote sustainable agricultural practices (Liu and Zeng, 2022; Zhou et al., 2023). Addressing these contradictions is critical to fostering a more efficient and equitable agricultural system.

In response to these challenges, the introduction of policies focused on agricultural-commercial integration aims to increase the share of more stable and long-term agricultural product distribution models, such as contract farming, integrated production and sales, and cooperative ventures. These models support a more interconnected and resilient agricultural system that aligns the interests of producers, distributors, and consumers while also contributing to the goals of sustainable food systems. By creating new agricultural production and sales relationships that are more closely linked, these initiatives can enhance the overall efficiency of agricultural product distribution—a key indicator of policy success.

Previous research has explored agricultural product distribution efficiency from various perspectives, including rural supermarkets, agricultural e-commerce, and supply chain management. However, many of these studies have been limited by narrow analytical frameworks, often focusing on specific distribution models or sectors. Agricultural-commercial integration, by contrast, offers a broader and more nuanced approach, considering the full spectrum of stakeholders involved in the production, distribution, and consumption of agricultural products. This model emphasizes the sustainability of the food system, highlighting the need for efficient resource use, reduced environmental impact, and equitable economic outcomes for all stakeholders.

From the perspective of agricultural-commercial integration, the efficiency of agricultural product distribution can be evaluated through a more holistic lens, considering not only economic factors but also social and environmental dimensions. This approach aligns with the principles of sustainable food systems, which seek to ensure food security, promote fair trade, and minimize environmental harm.

Moreover, while previous studies have typically employed Data Envelopment Analysis (DEA) models, especially radial DEA, to assess distribution efficiency, these methods have limitations when evaluating complex, multi-stage agricultural systems (Liu et al., 2022; Su et al., 2019; Wen et al., 2021; Yuan et al., 2022). Conventional radial DEA models assume proportional changes in inputs and outputs, which oversimplifies the complexities of real-world agricultural systems (Gerami et al., 2022; Su et al., 2019). Existing studies on DEA in agriculture primarily focus on production efficiency. For example, Wagan et al. (2018) analyzed agricultural production efficiency in China and Pakistan, emphasizing the role of technical efficiency but neglecting the distribution process. Similarly, Zhou et al. (2023) investigated agricultural total factor productivity in China, revealing regional disparities but overlooking agricultural product distribution efficiency, especially from an integrated agricultural-commerce perspective. Further research has explored agricultural supply chains and ecological efficiency. Mu et al. (2025) used a network DEA model to assess agricultural supply chains but did not focus on the distinct stages of distribution, such as production and sales. Sun and Sui (2023) examined ecological efficiency using DEA combined with neural networks, emphasizing sustainability but not addressing the efficiency of agricultural product distribution. In addition, Zhuo et al. (2020) evaluated agricultural loan efficiency, focusing on financial supply chains rather than the distribution process.

Additionally, many traditional DEA models fail to account for inefficiencies at specific stages of the supply chain, particularly in multi-stage processes such as production, processing, and retail. These limitations hinder the ability to accurately evaluate the performance of agricultural-commercial integration, which inherently involves interconnected and multi-dimensional processes.

To address these methodological gaps, this study adopts a non-radial SBM two-stage network DEA model, which represents a significant advancement in efficiency evaluation techniques. Unlike radial DEA models, the non-radial Slack-Based Measure (SBM) approach captures inefficiencies more comprehensively by considering slack variables in inputs and outputs, allowing for a more granular analysis of resource allocation and utilization. Furthermore, the two-stage network structure explicitly models the interdependencies between different stages of the agricultural supply chain, providing a clearer understanding of how inefficiencies propagate across production, processing, and retail stages. This integrated framework is particularly suited to agricultural-commercial systems, as it reflects the complex, interconnected nature of agricultural product distribution.

The innovative aspect of this study lies in its use of a non-radial SBM two-stage network DEA model to assess agricultural distribution efficiency within the context of agricultural-commercial integration. This methodological advancement offers a more robust tool for evaluating distribution efficiency across multiple stages of the agricultural supply chain, including production, processing, and retail. By integrating sustainability considerations into the analysis, the study contributes to the growing body of research on sustainable food systems and offers valuable insights for policymakers, agricultural enterprises, and

consumers seeking to optimize food distribution processes in an environmentally and socially responsible manner.

2 Materials and methods

2.1 Agricultural-commercial integration and agricultural product distribution

Agricultural-commercial integration refers to the deepening and expansion of collaborative models between agricultural product distribution enterprises and agricultural operators, aiming to foster a more cohesive and efficient agricultural supply chain. This model seeks to enhance the alignment of interests between agricultural producers and distribution enterprises, allowing the latter to penetrate the upstream production phase and establish a closely integrated agricultural product distribution system. In this context, agricultural-commercial integration is not merely about optimizing distribution efficiency but also about creating a more sustainable and resilient agricultural system by ensuring that the various actors within the supply chain work together in a more interconnected and coordinated manner.

One of the core objectives of agricultural-commercial integration is to optimize the agricultural product distribution process through integration and collaboration across multiple stages of the supply chain. The agricultural sector, particularly in developing countries, faces significant challenges in terms of distribution efficiency, which is often undermined by several persistent issues: the absence of intensive, alliance-based distribution channels, unstable relationships among supply chain actors, inadequate information exchange, poorly developed service systems, and deficiencies in quality and safety management. These challenges not only reduce the overall efficiency of the agricultural product distribution system but also impede efforts to build sustainable and resilient food systems that can meet the needs of both consumers and producers.

In the context of agricultural product distribution, enterprises involved in this sector play a critical role at both the production and sales stages. On the production side, they act as essential intermediaries, connecting farmers with markets and providing access to new technologies, technical support, and financial resources. By investing in upstream production processes, these companies ensure that farming practices align with market demand, quality standards, and sustainability goals. This involvement promotes the adoption of more sustainable agricultural methods, enhances resource efficiency, and contributes to better environmental outcomes, thereby supporting the broader objectives of sustainable food systems.

On the sales side, agricultural product distribution enterprises are essential for connecting products with consumers. They act as the bridge between producers and end consumers, ensuring that agricultural products reach markets efficiently while maintaining product quality and safety. In the context of agricultural-commercial integration, distribution enterprises can enhance their role by fostering long-term, stable relationships with farmers, thereby reducing the volatility and unpredictability that often characterize agricultural markets. This relationship is critical for improving the efficiency of agricultural product distribution and,

in turn, enhancing the sustainability of the agricultural system as a whole.

To address the challenges that persist in the agricultural distribution sector, methods such as organizational integration, information integration, and resource integration can be employed (Cai and Hua, 2021; Jiang et al., 2022; Liu and Tang, 2023; Wen et al., 2021; Xu et al., 2023). These strategies allow for a more collaborative and coordinated approach between agricultural product distribution enterprises and agricultural operators, enabling them to work together more effectively to optimize the flow of products, reduce waste, improve resource utilization, and enhance overall system efficiency. Organizational integration involves the alignment of processes and structures within agricultural distribution enterprises and their partners, such as establishing joint operational teams or shared logistics systems to streamline transportation and inventory management. For example, a distribution enterprise might collaborate with local farmers to create a centralized packaging and storage facility, reducing redundancy and improving efficiency. Information integration facilitates better data sharing and decision-making, such as implementing digital platforms that allow real-time monitoring of supply chain activities, including inventory levels, transportation schedules, and market demand. For instance, farmers can use mobile apps provided by distribution enterprises to update crop yields and receive instant feedback on delivery schedules. Resource integration ensures that both tangible (e.g., financial investment, infrastructure) and intangible (e.g., knowledge, expertise) resources are pooled effectively to achieve common goals, such as co-investing in cold chain logistics to minimize spoilage of perishable goods or organizing joint training programs to enhance technical skills in sustainable farming practices. For example, a distribution company might fund the construction of refrigerated trucks while agricultural operators contribute their expertise in optimizing harvest timing to ensure product quality.

2.2 Model

Existing research on agricultural product distribution has largely concentrated on assessing the efficiency of distribution outcomes, with relatively limited attention given to the efficiency of processes. However, agricultural product distribution is a multistage, systematic process involving various stages from production to final sales, and the efficiency of each stage has a significant impact on overall performance. If we focus solely on outcome-oriented efficiency metrics while neglecting process efficiency, we may fail to capture the real bottlenecks within each stage of the distribution system, which could obscure areas where improvements are most needed. Therefore, in order to conduct a more comprehensive evaluation of agricultural product distribution efficiency, it is essential to adopt a phased approach, evaluating each stage separately to identify and optimize the factors that influence efficiency across the distribution process.

In the context of agricultural-commerce integration, the distribution model is characterized by closer alignment and connection between the production and sales stages. Unlike traditional distribution models, agricultural-commerce integration

extends beyond merely handling the sales end of agricultural products; it reaches upstream into the production stage, promoting deep coordination and aligning interests across the supply chain. This integrated approach overcomes the disconnect typically found between production and distribution, facilitating a seamless flow of information, resources, and products across all stages of the supply chain. Consequently, evaluating efficiency within the agriculturalcommerce integration framework requires a comprehensive view that accounts for efficiency at each stage, not just the end results. Traditional efficiency evaluations, if applied without considering the complete distribution process, may not fully capture the systemic improvements that agricultural-commerce integration can bring to distribution efficiency. Therefore, it is necessary to introduce a phased approach to efficiency evaluation, analyzing the distinct contributions of the production and sales stages to understand the full impact of integration on distribution performance.

To establish this, we reference the theoretical framework proposed by Aijun et al. (2017) and Yang et al. (2021) and categorize agricultural product distribution efficiency into two stages: production-stage efficiency and sales-stage efficiency. By breaking down the distribution process into these two stages, we can examine more precisely how agricultural-commerce integration influences distribution efficiency. Specifically, production-stage efficiency focuses on the effective use of agricultural inputs, such as seeds, fertilizers, and machinery, which are transformed into agricultural products during the production process. This stage also includes financial and technical support provided by distribution enterprises, which play a significant role in enhancing the quality and quantity of production, ensuring that products meet downstream market demands. Within the agricultural-commerce integration model, the involvement of distribution enterprises in the production stage helps to minimize information asymmetries between producers and the market, improving overall production efficiency and providing a strong foundation for subsequent distribution activities.

In the sales stage, efficiency evaluation shifts to how agricultural products are effectively channeled to the market and generate economic returns. The role of distribution enterprises is particularly critical in this stage, encompassing logistics, marketing, customer engagement, and other activities that help convert agricultural products into revenue. Efficient management of these functions not only enables agricultural products to reach the market in a timely and cost-effective manner but also directly impacts the satisfaction of end consumers and the financial success of all supply chain stakeholders. By constructing this two-stage network model, as illustrated in Figure 1, we gain a structured approach to measure and improve distribution efficiency across the agricultural supply chain within the agricultural-commerce integration framework.

2.3 Method

Data Envelopment Analysis (DEA) has proven to be a more suitable analytical method than traditional econometric approaches, particularly for efficiency measurement across comparable organizations or products. As a non-parametric method, DEA uses linear programming to assess efficiency by transforming inputs into outputs (Zhu, 2009).

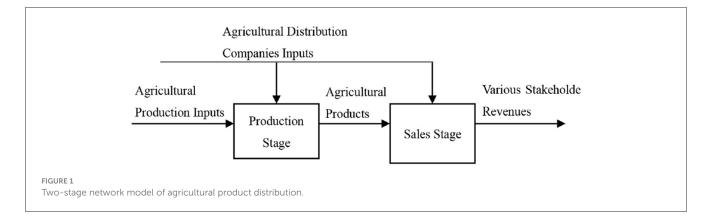
Agricultural-commerce integration covers agricultural product distribution process from production to sales, requiring evaluation of multi-stage processes. Unlike conventional DEA models that treat the entire process as a "black box," network DEA models allow for granular analysis of each stage (Tone and Tsutsui, 2010). Agricultural product distribution involves multiple interconnected stages, each with distinct inputs, outputs, and operational mechanisms. Traditional econometric methods often rely on predefined functional forms and assumptions, which may not adequately capture the complexity and interdependencies of multi-stage processes. In contrast, DEA provides a nonparametric approach that is well-suited for evaluating efficiency in such contexts, as it does not require prior assumptions about the functional relationships between inputs and outputs (Zhu, 2009).

Furthermore, the specific choice of the two-stage network DEA model is particularly appropriate for this study because it enables a detailed examination of efficiency across both the production and sales stages. Unlike conventional DEA models that treat the process as a single entity, the network DEA framework allows for disaggregation of the distribution process into distinct stages, facilitating a more granular analysis. This is critical for agricultural-commerce integration, where the coordination between production and sales stages plays a pivotal role in overall efficiency. By explicitly modeling the connections and dependencies between these stages, the network DEA approach provides insights into how inefficiencies in one stage may propagate and impact the overall system performance (Tone and Tsutsui, 2010).

The adoption of the non-radial Slack-Based Measure (SBM) model further strengthens the suitability of the chosen methodology. Traditional radial DEA models often assume proportional changes in inputs and outputs, which may lead to biased efficiency estimates when slack exists in certain inputs or outputs. The SBM model addresses this limitation by directly accounting for slack, providing a more accurate and realistic assessment of efficiency (Gerami et al., 2022). This is particularly relevant in agricultural product distribution, where slack-such as excess inventory, underutilized resources, or unmet market demands—can significantly distort efficiency measurements. By incorporating the SBM model within the network DEA framework, this study ensures that the evaluation captures inefficiencies at both the production and sales stages, offering a comprehensive and robust analysis of distribution efficiency under the agricultural-commerce integration model.

Consider a set of n DMUs, each with K stages. Let m_k represent input quantities and r_k the output quantities for stage k. Under variable returns to scale assumption, the production possibility set includes constraints on inputs, outputs, and connecting variables between stages. The connecting variables are handled using the linking-free (LF) approach to maintain continuity between production and sales stages.

The SBM network DEA model for DMU_o is formulated as an optimization problem that minimizes the ratio of



weighted input efficiency to weighted output efficiency across all stages:

$$\rho_o^* = \min \frac{\sum_{k=1}^K w^k \left[1 - \frac{1}{m_k} \left(\sum_{i=1}^{m_k} \frac{s_i^k}{x_{io}^k} \right) \right]}{\sum_{k=1}^K w^k \left[1 + \frac{1}{r_k} \left(\sum_{r=1}^{r_k} \frac{s_r^{k+}}{y_{eo}^k} \right) \right]}$$
(1)

The conceptual model divides agricultural product distribution into two stages: production-stage efficiency and sales-stage efficiency, linked by connecting variables that represent the flow of products, information, and resources. The mathematical model operationalizes this by treating each stage as a node (k) in a network, with inputs (x^k) , outputs (y^k) , and connecting variables $(z^{(k,h)})$ linking adjacent stages. Efficiency at each stage is evaluated using the network SBM model, where stage-specific efficiencies (ρ_k) and overall efficiency (ρ_o^*) are calculated. This ensures alignment between the conceptual framework and its quantitative representation, allowing for a comprehensive evaluation of distribution efficiency.

The efficiency score $\rho_o^*=1$ indicates that DMU_o is overall efficient. Stage-specific efficiency can be calculated using the optimal slack values obtained from the solution. This model provides comprehensive efficiency measurement for both production and sales stages within the agricultural-commerce integration framework, enabling identification of improvement opportunities at each stage. The complete mathematical derivation and constraint specifications are provided in Appendix A.

2.4 Data

To ensure the alignment of the efficiency evaluation results with real-world conditions, we based our analysis on municipal-level statistical data from Fujian Province, utilizing data from nine municipalities collected between 2016 and 2020. Fujian Province serves as an ideal sample for this study because it has a robust agricultural industry and a well-developed distribution network, representing a region with active agricultural production and dynamic distribution channels. Furthermore, as a coastal province with strong trade connections, Fujian has significant agricultural exports, which adds complexity and relevance to the evaluation of agricultural product distribution efficiency. Studying Fujian

Province allows us to analyze the interplay between production and distribution efficiency within a region that exemplifies both traditional agricultural practices and modern distribution systems.

In constructing the two-stage network model for agricultural product distribution, we carefully selected input and output indicators for each stage to accurately represent the resources and outputs involved in the distribution process. The first stage focuses on agricultural production inputs, incorporating factors directly related to the production capacity and resource investment in agriculture. Specifically, we selected the agricultural labor force and the cultivated area of major crops as the primary input factors. From a theoretical perspective, the agricultural labor force represents human capital that directly influences production quality and coordination with distribution partners, which are crucial for distribution efficiency. Skilled agricultural workers enhance product quality consistency, reduce distribution losses, and improve supply chain coordination (Jouanjean et al., 2015). The agricultural labor force represents the human capital involved in production, reflecting the workforce's capacity to influence productivity, while the cultivated area of major crops indicates the physical resources allocated to agriculture, showing the extent of land investment. The cultivated area reflects scale economies in agricultural production, where larger areas enable bulk handling, reduce per-unit collection costs, and improve logistics efficiency by allowing distributors to collect larger volumes from fewer locations (Rada and Fuglie, 2019; Swinnen and Kuijpers, 2019). These indicators were chosen to capture the essential inputs that drive agricultural productivity and ensure a consistent supply of agricultural products.

Additionally, we included total assets of enterprises engaged in the wholesale and retail of agricultural products (fruits, vegetables, meat, poultry, eggs, dairy, and aquatic products) as a representation of the input provided by agricultural product distribution enterprises in the first stage. This indicator reflects the resource-based view where physical and technological assets (cold storage, transportation, processing equipment) directly determine distribution efficiency by enabling better quality preservation, faster delivery, and expanded market reach (Ali et al., 2018). This indicator reflects the capital resources and infrastructural investment from distribution enterprises, which significantly impact the efficiency of the distribution network by supporting logistics, storage, and market reach. The total assets of these enterprises are crucial in bridging production and distribution, as

they facilitate the movement of agricultural goods from producers to the market.

For the output of the first stage, we selected the total output value of agriculture to represent the volume of agricultural products generated. Output value captures both quantity and quality dimensions, reflecting market acceptance and product differentiation essential for distribution efficiency. Higher-value products indicate better quality and market positioning, leading to more efficient distribution channels and reduced waste (Barrett et al., 2017). This output serves as the connecting variable between the first and second stages, linking agricultural production with the subsequent distribution and sales processes. The total agricultural output value provides a comprehensive measure of the goods available for distribution and sets the foundation for evaluating how effectively these products reach the market and generate economic returns in the second stage.

In the second stage, the output indicators reflect the income and value generated through the distribution of agricultural products. We chose agricultural value-added and revenue of wholesale enterprises dealing with agricultural products above specific quotas as the key outputs in this stage. Agricultural value-added represents economic value created through processing, packaging, and distribution activities beyond basic production, indicating efficient utilization of agricultural products and better market integration (Swinnen and Kuijpers, 2019). Agricultural value-added represents the economic value generated by agricultural activities after accounting for intermediate inputs, providing a measure of the economic contribution from agricultural activities. Wholesale enterprise revenue reflects distribution network effectiveness in reaching markets and satisfying consumer demands, serving as a proxy for market acceptance and distribution efficiency (Kumar and Reinartz, 2016). The revenue of wholesale enterprises reflects the income of various stakeholders in the distribution network, capturing the financial returns to distribution enterprises and indirectly representing the success of distribution in meeting market demands. Together, these indicators allow us to assess how efficiently agricultural products are distributed and how effectively they generate economic benefits for all stakeholders.

The Table 1 summarizes the input and output indicators used in each stage of the two-stage network model:

TABLE 1 Indicators for the two-stage network model of agricultural product distribution efficiency.

Stage	Indicator type	Indicator description	
First stage: production	Input	Agricultural labor force (number of people)	
	Input	Cultivated area of major crops (hectares)	
	Input	Total assets of agricultural distribution enterprises (million CNY)	
	Output	Total agricultural output value (million CNY)	
Second stage: distribution and sales	Input	Total agricultural output value (million CNY) (connecting variable)	
	Output	Agricultural value-added (million CNY)	
	Output	Revenue of wholesale enterprises (million CNY)	

By utilizing these carefully chosen indicators, we aim to capture a comprehensive picture of the agricultural product distribution process, from resource investment in production to economic outcomes in the distribution phase. This approach allows for a nuanced analysis of distribution efficiency, highlighting both production efficiency and sales performance, and ensuring that the evaluation results align closely with the real-world conditions in Fujian Province. The selection of Fujian Province, combined with the two-stage model and these targeted indicators, enables us to provide a realistic and reliable assessment of agricultural distribution efficiency within an integrated agricultural-commerce framework.

3 Results

We evaluated the agricultural product distribution efficiency of nine cities in Fujian Province using a non-radial SBM two-stage network DEA model, with results summarized in Table 2. In the production phase, Xiamen and Putian consistently achieved full efficiency (1.000), while Fuzhou improved from 0.681 in 2015 to 1.000 in 2019. Nanping, initially inefficient, reached full efficiency by 2019. Sales phase efficiency was generally lower, reflecting challenges in distribution. Xiamen remained fully efficient, Fuzhou improved steadily, but Nanping's performance was consistently weak, with a score as low as 0.041 in 2019. Overall, Xiamen excelled in both phases, Fuzhou showed significant progress, and Nanping lagged behind, requiring targeted improvements. These findings highlight the need for focused efforts to improve sales efficiency in underperforming cities.

The Table 3 presents the overall efficiency of agricultural product distribution in nine cities in Fujian Province during the period from 2015 to 2019, as well as the average efficiency at different stages. It can be observed that the difference in production stage efficiency among these cities is much smaller than the difference in sales stage efficiency, indicating that the variation in agricultural product distribution efficiency primarily stems from differences in the sales efficiency stage. When the nine cities in Fujian Province are ranked based on their 5-year average agricultural product distribution efficiency, from highest to lowest, the ranking is as follows: Xiamen, Fuzhou, Zhangzhou, Putian, Longyan, Quanzhou, Sanming, Ningde, and Nanping.

Fuzhou and Xiamen are the central cities in the northeastern and southwestern regions of Fujian, respectively, serving as the agricultural product hub markets in their respective areas. Therefore, the agricultural product distribution efficiency in these two cities is relatively high. Additionally, due to its advantageous geographical location compared to other cities and its lower distribution costs to Xiamen, Zhangzhou's agricultural products can flow into Xiamen to meet its demands, resulting in agricultural product distribution efficiency ranking just below that of Fuzhou and Xiamen.

For cities like Sanming, Ningde, and Nanping, although they possess abundant agricultural resources, their relatively inconvenient transportation infrastructure leads to higher distribution costs for many agricultural products. Consequently, their sales stage efficiency is relatively low, resulting in these cities

TABLE 2 Evaluation results of agricultural product distribution efficiency under the perspective of agricultural-commerce integration.

	Fuzhou	Xiamen	Putian	Zhangzhou	Sanming	Nanping	Ningde	Longyan	Quanzhou
First stage	First stage								
2015	0.681	1.000	1.000	0.679	0.865	0.745	0.988	0.976	0.844
2016	0.709	0.921	1.000	0.865	0.920	0.788	0.983	1.000	0.862
2017	0.822	1.000	1.000	0.973	0.934	0.726	0.988	1.000	0.885
2018	0.914	1.000	1.000	0.974	0.956	0.716	0.955	1.000	0.877
2019	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.910
Second sta	Second stage								
2015	0.663	1.000	0.224	0.586	0.196	0.126	0.200	0.239	0.284
2016	0.746	0.841	0.281	0.660	0.239	0.114	0.244	0.286	0.355
2017	0.782	1.000	0.394	0.518	0.275	0.066	0.221	0.367	0.416
2018	0.846	1.000	0.529	0.746	0.324	0.076	0.163	0.441	0.471
2019	1.000	1.000	0.586	0.739	0.681	0.041	0.173	0.531	0.282
Overall sc	Overall score								
2015	0.670	1.000	0.365	0.621	0.306	0.195	0.332	0.382	0.408
2016	0.730	0.878	0.439	0.742	0.370	0.183	0.389	0.445	0.488
2017	0.800	1.000	0.565	0.673	0.417	0.107	0.360	0.537	0.554
2018	0.877	1.000	0.692	0.844	0.478	0.121	0.274	0.612	0.601
2019	1.000	1.000	0.739	0.850	0.810	0.078	0.295	0.694	0.420

TABLE 3 Average value of staged efficiency of agricultural product distribution in nine cities in Fujian Province, 2015–2019.

City name	Overall efficiency	Production stage efficiency	Sales stage efficiency
Fuzhou	0.815	0.825	0.807
Xiamen	0.976	0.984	0.968
Putian	0.560	1.000	0.403
Zhangzhou	0.746	0.898	0.650
Sanming	0.477	0.935	0.343
Nanping	0.137	0.795	0.085
Ningde	0.330	0.983	0.200
Longyan	0.534	0.995	0.373
Quanzhou	0.494	0.876	0.362

ranking at the bottom in terms of agricultural product distribution efficiency within Fujian Province.

Figure 2 illustrates the changing trends in agricultural product distribution efficiency among the nine cities in Fujian Province from 2015 to 2019, showing a consistent overall improvement. This improvement coincides with the rapid expansion of agricultural product distribution enterprises in the province. Several nationally renowned companies, such as Yonghui Superstores, Missfresh, and Yuanchu Food, have emerged during this period. Concurrently, Fujian's geographical indication agricultural products, including Fuding White Tea, Gutian Tremella, and Ningde Yellow

Croaker, have gained significant recognition both within the province and across the country, contributing to the enhanced distribution efficiency.

During this time, as agricultural product distribution enterprises have expanded and geographical indication agricultural products have gained wider recognition, their collaboration with agricultural producers has grown stronger. This has resulted in substantial progress in various aspects, including technology diffusion, contract farming, and integrated production and marketing. As depicted by the trend line in Figure 2, the agricultural product distribution efficiency in Fujian Province has shown a positive trajectory over the past five years. This signifies that the joint efforts of agricultural product distribution enterprises and agricultural operators have contributed to the continuous and steady improvement of agricultural product distribution efficiency in Fujian.

According to the average values of different stages shown in Figure 2, the overall efficiency of agricultural product distribution in Fujian Province lies between production stage efficiency and sales stage efficiency. Notably, production stage efficiency surpasses sales stage efficiency by a significant margin, indicating that there is still considerable room for improvement in sales stage efficiency. Despite the decreasing annual input in terms of production factors in Fujian Province from 2015 to 2019, agricultural total output has steadily increased. This suggests that the efficiency of agricultural production has improved in recent years. This improvement can be attributed, on one hand, to advancements in agricultural production technology and, on the other hand, to the enhancement of management efficiency brought about by improvements in the production and marketing model.

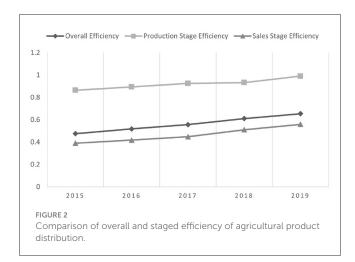
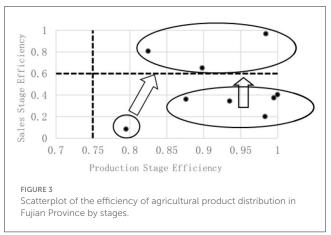


TABLE 4 Cluster grouping of agricultural product distribution efficiency in nine cities in Fujian Province.

Category	City names	Cluster center	Within-cluster variance
Category 1	Xiamen, Fuzhou, Zhangzhou	0.8457	0.0279
Category 2	Putian, Longyan, Quanzhou, Sanming, Ningde	0.2335	0.0186
Category 3	Nanping	0.5163	0.0043

To gain a deeper understanding of the efficiency of agricultural product distribution in the nine cities of Fujian Province, this study employs clustering analysis using SPSS to assess both the overall efficiency and stage-specific efficiencies of agricultural product distribution. The K-means clustering method is applied to group the nine cities into three categories, as detailed in Table 4. The clustering results demonstrate clear differentiation among the three categories, with cluster centers of 0.8457, 0.2335, and 0.5163 for Categories 1, 2, and 3, respectively (Figure 3). The within-cluster variances are relatively low (0.0279, 0.0186, and 0.0043), indicating good internal homogeneity within each cluster and validating the effectiveness of the clustering approach.

Category 1 cities, which include Xiamen, Fuzhou, and Zhangzhou, exhibit relatively high overall and two-stage efficiency in agricultural product distribution. Although Xiamen's production stage efficiency does not stand out significantly compared to other cities in Fujian Province, and both Fuzhou and Zhangzhou's production stage efficiency ranks in the middle to lower range among the nine cities, Xiamen, Fuzhou, and Zhangzhou demonstrate high levels of sales stage efficiency, with scores of 0.968, 0.807, and 0.650, respectively, contributing to their overall high agricultural product distribution efficiency. It is evident that the level of agricultural product distribution efficiency is primarily determined by sales stage efficiency. Therefore, these Category 1 cities should focus on fostering the leading distribution of agricultural products, continuously increasing the added value of agricultural products, and investing in agricultural and agrifood processing and brand building to further enhance sales



stage efficiency and maintain robust agricultural production and marketing relationships.

Category 2 cities, including Putian, Longyan, Quanzhou, Sanming, and Ningde, demonstrate high levels of production stage efficiency but have a certain gap in sales stage efficiency compared to Category 1 cities. While there is no significant difference in agricultural production efficiency between these cities and Category 1 cities, with Putian even achieving the highest production stage efficiency in Fujian Province from 2015 to 2019, the inefficiency in connecting production to sales results in relatively lower sales stage efficiency, impacting overall agricultural product distribution efficiency. Therefore, Category 2 cities should consider expanding the scale of new agricultural business entities and increasing the proportion of contract farming to promote the stability of the agricultural supply chain.

Category 3 city, Nanping, exhibits the lowest levels of both production stage and sales stage efficiency. Nanping's input-output ratio in both production and sales stages is relatively lower compared to other cities in Fujian Province, with a production stage efficiency of 0.795 and a sales stage efficiency of 0.085. For cities in this category, active exploration of the application of new agricultural technologies, improving agricultural production efficiency, and attracting agricultural distribution enterprises to work closely with rural producers are essential. Leveraging methods such as "One Village, One Product" and rural e-commerce that can reduce agricultural product distribution costs will help unlock the potential value of agricultural products in these cities.

4 Discussion

4.1 Conclusions and recommendations

This study highlights several key insights into improving agricultural product distribution efficiency, especially within the context of agricultural-commerce integration. In line with the empirical data and the efficiency classification of cities, the following insights and tailored recommendations are provided for each category of cities.

Firstly, the research emphasizes the need to harness the integrative role of agricultural product distribution companies. These companies play a central role in addressing the inherent

conflict between large markets and small-scale production. By optimizing resource allocation, they can collaborate with emerging agricultural entities through contract farming or technology dissemination, thereby enhancing production-stage efficiency. However, cities with high efficiency (Category 1: Xiamen, Fuzhou, and Zhangzhou) have already demonstrated strong sales-stage efficiency, so for these cities, the focus should shift to enhancing the added value of agricultural products, optimizing marketing strategies, and developing branding. These measures will help further improve sales-stage efficiency and solidify their leadership in agricultural distribution.

Secondly, regional economic synergy is identified as a crucial factor in improving agricultural product distribution efficiency. The study highlights imbalances in efficiency across different cities in Fujian Province and proposes leveraging regional drivers to address these disparities. Cities in Category 1, such as Fuzhou and Xiamen, can act as central hubs to drive integration between peripheral and core regions. These cities have the infrastructure and market access to foster collaboration with smaller cities, particularly those in Category 2 (Putian, Longyan, Quanzhou, Sanming, and Ningde), which exhibit strong production-stage efficiency but struggle with sales-stage efficiency. By developing stronger supply chain links and expanding contract farming, these cities can boost their sales efficiency and improve overall agricultural distribution efficiency.

For Category 2 cities, while their production-stage efficiency is high (with cities like Putian even ranking at the top for production efficiency), they face challenges in linking production to sales. In these cities, the primary recommendation is to enhance coordination between producers and distribution companies. This can be achieved through scaling up agricultural business entities, increasing the proportion of contract farming, and leveraging technologies for better logistics and inventory management. By addressing inefficiencies in sales-stage distribution, these cities can move closer to the efficiency levels seen in Category 1 cities.

Category 3 cities, including Nanping, exhibit the lowest levels of both production-stage and sales-stage efficiency. For these cities, the key is to boost both production and sales efficiencies. In the production stage, there is a need to introduce more advanced agricultural technologies and improve infrastructure to reduce costs and increase output. In the sales stage, improving logistics networks and exploring e-commerce platforms can help lower distribution costs. Additionally, local governments and businesses should explore initiatives like "One Village, One Product" to better connect rural producers with markets, increasing the economic value of agricultural products and fostering more efficient distribution networks.

Lastly, tailored improvement strategies are essential given the varying levels of distribution efficiency observed across the cities. Based on the classification of cities into three efficiency levels, these recommendations are made:

For Category 1 cities (Xiamen, Fuzhou, Zhangzhou): Focus on enhancing the value-added potential of agricultural products, improving marketing strategies, and building strong agricultural brands. These cities should maintain their leadership in salesstage efficiency by continuing to invest in agri-food processing and product branding.

For Category 2 cities (Putian, Longyan, Quanzhou, Sanming, Ningde): Focus on scaling up operations of new agricultural entities, increasing production capacity, and improving supply chain coordination. This will help address the inefficiencies in linking production to sales, thereby improving sales-stage efficiency and overall agricultural distribution performance.

For Category 3 city (Nanping): Focus on technological innovation in agricultural production, infrastructure development, and logistics optimization. Additionally, strategies like rural ecommerce and improved producer-distributor collaboration will help unlock the potential value of agricultural products, enabling better distribution efficiency.

By aligning the improvement strategies with the empirical data from the clustering analysis and taking into account the specific needs of each city, this study provides a clear roadmap for optimizing agricultural product distribution efficiency in Fujian Province. These strategies can be broadly applied to other regions facing similar challenges in agricultural supply chains, leading to more efficient, sustainable, and profitable outcomes for all stakeholders.

4.2 Limitations and future research

While this study offers significant insights into the efficiency of agricultural product distribution in Fujian Province through the application of a two-stage SBM network DEA model, there are certain areas that future research could explore further. The focus on Fujian Province provides a valuable case study, but as agricultural and economic conditions vary across regions, future studies could examine whether the findings hold in other contexts or conduct comparative analyses to uncover broader patterns. One promising direction for future research lies in extending the current understanding of agricultural mergers to more integrated systems. Previous studies have highlighted the potential of mergers and acquisitions (M&As) to improve efficiency and sustainability, particularly through the optimization of resource allocation and energy use (Oukil et al., 2023, 2024). Investigating how these benefits can be realized within interconnected agricultural networks or systems could provide deeper insights into achieving long-term sustainability goals.

Additionally, While the study emphasizes production and sales efficiency, other factors, such as technological innovation, environmental considerations, or consumer behavior, were beyond the scope of this research. Exploring these aspects in future studies could complement the current findings and provide a more comprehensive understanding of distribution systems. Moreover, the DEA model employed assumes static conditions, which, while effective for the study's objectives, may not fully capture the dynamic nature of agricultural-commerce integration. Incorporating dynamic models or longitudinal data, alongside a deeper examination of how mergers influence system-wide efficiency over time, could further enrich the analysis and reveal evolving trends (Oukil, 2023, 2024). Despite these considerations, the study lays a strong foundation for evaluating agricultural product distribution efficiency and offers actionable insights that can guide both policymakers and stakeholders in optimizing distribution systems.

Author contributions

QZ: Conceptualization, Supervision, Validation, Writing – review & editing. DT: Conceptualization, Formal analysis, Methodology, Writing – original draft. XS: Methodology, Supervision, Writing – review & editing. JL: Formal analysis, Writing – review & editing.

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Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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Supplementary material

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Appendix A: Complete mathematical derivation of the SBM network DEA model

Consider a set of n Decision-Making Units (DMUs), each with K stages or nodes $(k=1,\ldots,K)$. Let m_k represent the input quantities and r_k the output quantities for each stage k. The interactions between different nodes, denoted as (k,h), form a network structure. The observed data include $\{x_j^k \in R_+^{m_k}\}$ (input quantities), $\{y_j^k \in R_+^{r_k}\}$ (output quantities), and $\{z_j^{(k,h)} \in R_+^{t_{(k,h)}}\}$ (connecting variables between nodes).

Under the assumption of variable returns to scale, the production possibility set is defined as $\{(\mathbf{x}^k, \mathbf{y}^k, \mathbf{z}^{(k,h)})\}$, where:

$$x^{k} \ge \sum_{j=1}^{n} x_{j}^{k} \lambda_{j}^{k}, \quad y^{k} \le \sum_{j=1}^{n} y_{j}^{k} \lambda_{j}^{k} \quad (k = 1, ..., K)$$
 (2)

$$\mathbf{z}^{(k,h)} = \sum_{j=1}^{n} \mathbf{z}_{j}^{(k,h)} \lambda_{j}^{k} = \sum_{j=1}^{n} \mathbf{z}_{j}^{(k,h)} \lambda_{j}^{h} \quad \forall (k,h)$$
 (3)

$$\sum_{j=1}^{n} \lambda_{j}^{k} = 1, \quad \lambda_{j}^{k} \ge 0 \quad \forall j, k$$
 (4)

For DMU_{o} , the input and output constraints with slack variables are:

$$\mathbf{x}_{o}^{k} = \mathbf{X}^{k} \lambda^{k} + \mathbf{s}^{k-}, \quad \mathbf{y}_{o}^{k} = \mathbf{Y}^{k} \lambda^{k} - \mathbf{s}^{k+} \quad (k = 1, ..., K)$$
 (5)

$$\mathbf{e}\lambda^k = 1, \quad \lambda^k \ge 0, \quad s_k^- \ge 0, \quad s_k^+ \ge 0 \quad \forall k$$
 (6)

where $\mathbf{X}^k = (\mathbf{x}_1^k, \dots, \mathbf{x}_n^k) \in R^{m_k \times n}, \mathbf{Y}^k = (\mathbf{y}_1^k, \dots, \mathbf{y}_n^k) \in R^{r_k \times n}$, and $\mathbf{s}^{k-}(\mathbf{s}^{k+})$ represents the slack variables for inputs (outputs).

For connecting variables, we employ the linking-free (LF) approach:

$$\mathbf{Z}^{(k,h)}\boldsymbol{\lambda}^h = \mathbf{Z}^{(k,h)}\boldsymbol{\lambda}^k \quad \forall (k,h)$$
 (7)

where $\mathbf{Z}^{(k,h)} = (\mathbf{z}_1^{(k,h)}, \dots, \mathbf{z}_n^{(k,h)}) \in R^{t_{(k,h)} \times n}$.

The complete undirected network SBM model is formulated as:

$$\rho_o^* = \min_{\lambda^k, s_k^-, s^{k+}} \frac{\sum_{k=1}^K w^k \left[1 - \frac{1}{m_k} \left(\sum_{i=1}^{m_k} \frac{s_i^{k-}}{x_{io}^k} \right) \right]}{\sum_{k=1}^K w^k \left[1 + \frac{1}{r_k} \left(\sum_{r=1}^{r_k} \frac{s_r^{k+}}{y_{ro}^k} \right) \right]}$$
(8)

subject to constraints (2)-(7), where $\sum_{k=1}^{K} w^k = 1$ with $w^k \ge 0$, and w^k represents the relative weight of node k.

If $\rho_o^*=1$, DMU $_o$ is overall efficient. The stage-specific efficiency is calculated as:

$$\rho_k = \frac{1 - \frac{1}{m_k} \left(\sum_{i=1}^{m_k} \frac{\sum_{i=1}^{k-*}}{x_{io}^k} \right)}{1 + \frac{1}{r_k} \left(\sum_{r=1}^{r_k} \frac{\sum_{i=1}^{k+*}}{y_{io}^k} \right)} \quad (k = 1, \dots, K)$$
 (9)

where \mathbf{s}^{k-*} and \mathbf{s}^{k+*} represent the optimal slack variables obtained from solving Equation (8).