ABSTRACT MATHEMATICAL COGNITION

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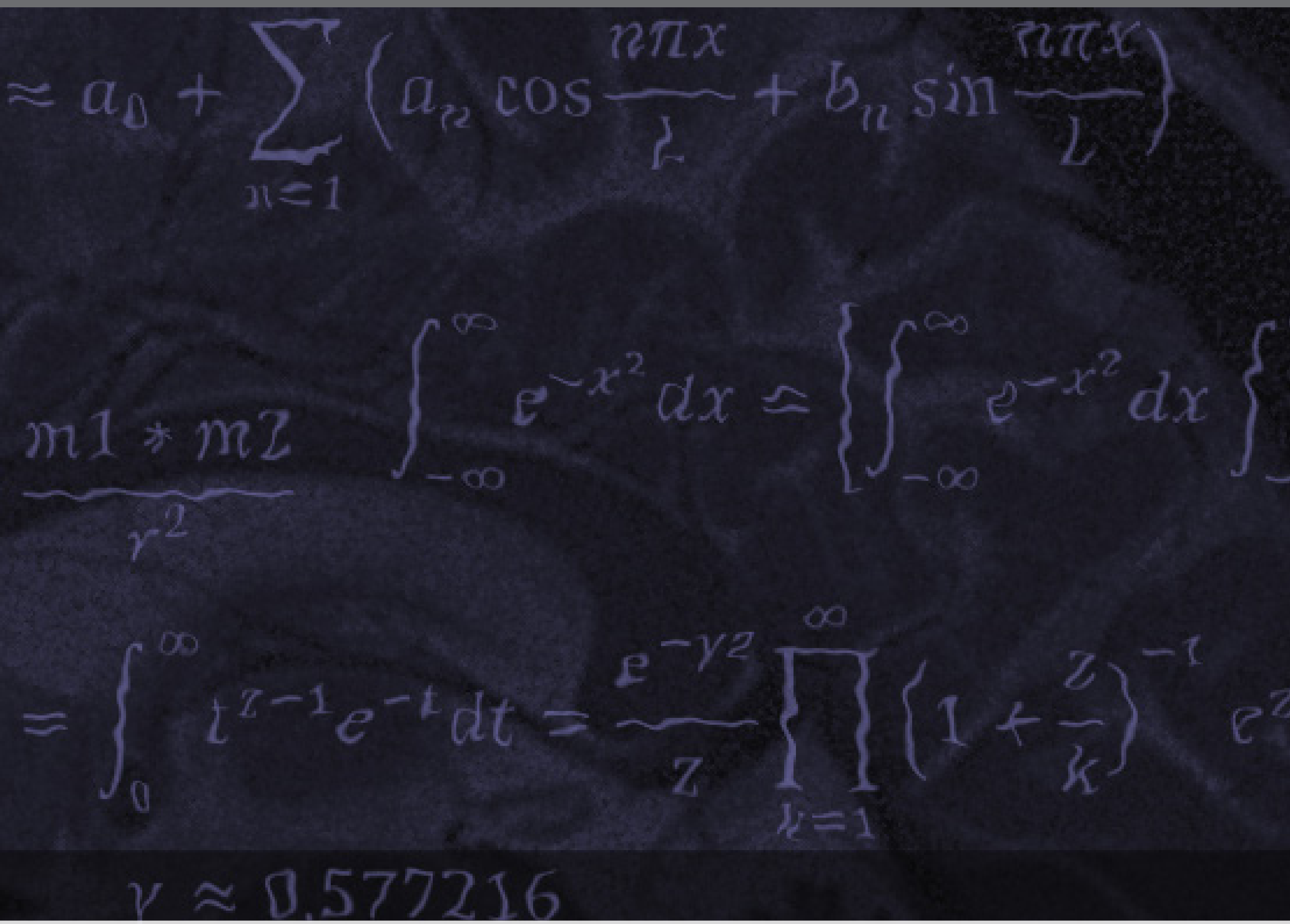

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ABSTRACT MATHEMATICAL COGNITION

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Despite the importance of mathematics in our educational systems little is known about how abstract mathematical thinking emerges. Under the uniting thread of mathematical development, we hope to connect researchers from various backgrounds to provide an integrated view of abstract mathematical cognition.

Much progress has been made in the last 20 years on how numeracy is acquired. Experimental psychology has brought to light the fact that numerical cognition stems from spatial cognition. The findings from neuroimaging and single cell recording experiments converge to show that numerical representations take place in the intraparietal sulcus. Further research has demonstrated that supplementary neural networks might be recruited to carry out subtasks; for example, the retrieval of arithmetic facts is done by the angular gyrus. Now that the neural networks in charge of basic mathematical cognition are identified, we can move onto the stage where we seek to understand how these basics skills are used to support the acquisition and use of abstract mathematical concepts.

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Editorial: Abstract Mathematical Cognition

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Keywords: mathematical cognition, abstract concepts, learning, developmental psychology, expertise development

The Editorial on the Research Topic

Abstract Mathematical Cognition

Despite the importance of mathematics in our educational systems, little is known about how abstract mathematical thinking emerges. Most research on mathematical cognition has been dedicated to understanding its more simple forms such as seriation and counting. Although these forms constitute the foundational plinth upon which all other maths skills develop, the gap between basic skills and the processing of complex mathematical concepts is poorly understood. What has come to be sufficiently well understood, however, is how numeracy is acquired. The 90s marked a change in our approach to human cognition in general and to mathematical cognition in particular. Neuroimaging technologies have enabled localization of neural activity, revealing that mathematical cognition, like other forms of cognition and skills, depends upon a network of activation. The key finding from neuroimaging and single cell recording is that numerical information is held in the intraparietal sulcus. Now that the core of mathematical cognition has been identified it is time to understand how basic skills are used to support the acquisition and use of abstract mathematical concepts. Chassy and Grodd (2012) opened the door for abstract mathematical cognition by examining for the first time the neural correlates of negative numbers, an abstract mathematical concept that emerges early on in mathematical curricula. The present issue reports crucial advances in our understanding of the neural underpinnings of abstract mathematical cognition.

For a general introduction to the topic the reader is referred to the article signed by Moeller et al. The article offers an excellent overview of the networks that are involved to some degree in processing quantities, the very basis of mathematical cognition. The authors' conclusion strengthens the view that a frontal parietal network constitutes the essence of our abilities in mathematics. The fronto-parietal network has been highlighted by a number of studies and is thought to underpin the learning of mathematical concepts. By increasing the complexity of the concepts stored in our memory, we improve the quality of our understanding of the physical world in the first stages of mathematical cognition. Abstract concepts are then able to emerge from concrete, physical quantities.

On the path of mathematical development, the first step toward an abstract representation of concepts is the shift from concrete, object-based cognition to the use of symbols. The symbols, though arbitrary, represent concrete quantities that help children quantify and thus understand the world around them. Roesch and Moeller support this view by suggesting that an internal representation of fingers contributes to the actual ability to represent quantities. In a similar vein, a cross cultural study authored by Bender and Beller compares the Western counting system to a Polynesian language of the Tonga island, offering a unique view of how concrete counting of different objects leads to an abstract representation of numbers; thus demonstrating that the roots

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of abstract mathematical cognition emerge from basic, sensory abilities (a long standing view that finds a new echo here). By highlighting the concrete roots of mathematical cognition, the authors of these studies open the debate on the inheritance of mathematical skill by pointing toward very concrete sensory performance.

The symbols in a later stage of mathematical development are used to represent concepts of an abstract nature. That is, once the notion of natural number is acquired, the next step toward expertise is to formalize operations as abstract entities. For example, the operation $5 + 4 = 9$ is concrete and can be taught by using objects. Dowker demonstrates that pupils tend to use the same problem-solving strategies to solve problems in subtraction and addition problems. Since the properties of the two operations differ the application of the same strategy leads the pupil to commit errors. Pupils have to learn a new set of properties to be able to solve subtraction. Similarly, Huber et al. argue that mental representations of fractions do not differ from natural numbers; what do differ are the strategies used to encode information. Dowker's and Hubet et al.'s views are in line with the study of Mihulowicz et al. who, by comparing left and right lesioned patients, showed that arithmetic operations are underpinned by different networks. The view of some educators, that subtraction and addition are mirror operations, is mistaken. It is interesting to note that teaching might be adapted so that different approaches could be used to teach different operations. The studies highlight the fact that learning arithmetic includes knowledge that is not purely numerical. This is our first hint indicating that educational strategies might have a huge influence on the ability of students to learn abstract concepts. The next stage in mathematical learning is the step consisting in moving from concrete (arithmetic) to abstract (algebraic) relationships. A study by Susac et al. looked at this move and showed that it requires about 4 years of training to master this new step toward abstract thinking in mathematics. It is crucial to note that these 4 years are in addition to the many years required for correctly mastering the basics. Mathematical learning is a long road. It calls for pedagogical approaches that are specific to each level.

Two main variables might modulate the acquisition of mathematical expertise: Educational system and inherited factors. The idea that teaching practices impact heavily on the ability of students to develop their skills in abstract mathematical cognition is demonstrated by Prado et al. The authors ran a cross cultural study comparing Chinese and American students on problem-size effects, and show that educational practices, which differ in the 2 countries, impact on the wiring of the network in charge of symbolic arithmetic. In line with this result, McLean and Rusconi attempt to bridge the gap between the findings of academic science and the practical problems faced by teaching institutions when dealing with students with mathematical difficulties. After revealing the cognitive factors underpinning the acquisition of mathematical knowledge, McLean and Rusconi discuss the types of interventions that may help students with mathematical difficulties. With respect to inherited factors, Zhang et al. have shown that gifted adolescents display a highly integrated fronto-parietal network, hence displaying a more efficient link between the representation of numbers in the parietal cortex and working memory in the prefrontal cortex.

The many findings of the articles in this special topic call for further research to see how specific neural networks serve various abstract mathematical concepts.

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A review on functional and structural brain connectivity in numerical cognition

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Only recently has the complex anatomo-functional system underlying numerical cognition become accessible to evaluation in the living brain. We identified 27 studies investigating brain connectivity in numerical cognition. Despite considerable heterogeneity regarding methodological approaches, populations investigated, and assessment procedures implemented, the results provided largely converging evidence regarding the underlying brain connectivity involved in numerical cognition. Analyses of both functional/effective as well as structural connectivity have consistently corroborated the assumption that numerical cognition is subserved by a fronto-parietal network including (intra)parietal as well as (pre)frontal cortex sites. Evaluation of structural connectivity has indicated the involvement of fronto-parietal association fibers encompassing the superior longitudinal fasciculus dorsally and the external capsule/extreme capsule system ventrally. Additionally, commissural fibers seem to connect the bilateral intraparietal sulci when number magnitude information is processed. Finally, the identification of projection fibers such as the superior corona radiata indicates connections between cortex and basal ganglia as well as the thalamus in numerical cognition. Studies on functional/effective connectivity further indicated a specific role of the hippocampus. These specifications of brain connectivity augment the triple-code model of number processing and calculation with respect to how gray matter areas associated with specific number-related representations may work together.

Keywords: brain connectivity, DTI, white matter pathways, fronto-parietal network, numerical cognition

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In the history of neurology, attempts to explain normal and impaired cognitive function following brain damage have alternated between two extreme perspectives; specifically, views based on localization of function and views based on functional connectivity. The localizationist view ascribes specific cognitive functions to gray matter (GM) brain areas with cognitive impairments attributed to lesions of these specific areas. Prominent historical examples of this view include the work of Broca (1861) and Wernicke (1874), who associated language production and perception, respectively, with specific cortical structures. Another prominent example of localization of function is the work of Brodmann (1909), who proposed a map of 46 cortical areas—so-called Brodmann areas (BA)—and their functionality. This work still influences neuro-scientific research today. In contrast, connectionist views of brain function take the connections of white matter (WM) pathways to be instrumental to cognitive functions, with disrupted connections also leading to impairments of the respective cognitive functions. Interestingly, such a connectionist view of brain function was proposed by Campbell (1905) at about the same time as Brodmann introduced his localizationist approach. Later, Reinvang (1985), amongst others, suggested "systemic localization" to be the overarching principle of brain organization, in which the functional role of a given brain area is not determined by its anatomical structure alone but also by its relationships to other areas—an argument, for which there is increasing empirical evidence (e.g., López-Barroso et al., 2013; see Catani et al., 2012, 2013 for reviews). Thus, it is the integrity and specific interplay of activated GM cortical areas connected by WM fiber tracts which underlie human cognitive functions.

Recently, brain hodology, the science of connectional anatomy (Catani and Ffytche, 2005), which characterizes the WM connections between brain regions, has become accessible to evaluation in the living brain by using diffusion tensor imaging (DTI). While functional magnetic resonance imaging (fMRI) identifies functionally defined cortical areas, tractography goes beyond this approach and indicates, by which WM tracts these areas are connected. This provides a powerful tool to study brain connectivity patterns underlying cognitive functions. By quantifying the diffusion characteristics of water molecules (Le Bihan and Breton, 1985), which diffuse more freely along than across myelinated tracts, it is possible to obtain *in vivo* estimates of WM fiber orientation at the voxel level (Basser et al., 1994). This information gives rise to diffusion tensor tractography (Conturo et al., 1999; Jones et al., 1999; Mori et al., 1999; Basser et al., 2000; Poupon et al., 2000), in which WM tracts are reconstructed in three dimensions by sequentially piecing together discrete voxel level estimates of fiber orientation to extrapolate continuous trajectories. Diffusion tensor tractography methodology has established the existence of neural networks associated with language processing (e.g., Saur et al., 2008) and also networks subserving attentional functions (e.g., Umarova et al., 2010). Accumulating such evidence has substantiated the functional role of WM connections in both language as well as attentional processing (e.g., Rijntjes et al., 2012). There have even been suggestions to conceptualize aphasia (e.g., Forkel et al., 2014) and neglect as disconnection syndromes (e.g., Bartolomeo et al., 2012; Thiebaut de Schotten et al., 2014) arising from disrupted neural connections between the involved cortex areas.

Numerical cognition and the syndrome of acalculia, a collection of impairments in processing numbers and mental calculation, have also witnessed a history of localisationist and connectionist views, although their study started later in history and they were less well investigated than language. Henschen (1920), who coined the term acalculia, also considered calculation mechanisms to rely on a complex anatomo-functional system, subserved by distinct cortical centers and their interconnections.

In the present paper we summarize and review the existing evidence on brain hodology underlying numerical cognition. Comparable to the cases of language and attention, considering WM connections may provide a more comprehensive understanding of human numerical cognition and its impairments (see also Matejko, 2014; Matejko and Ansari, 2015). First attempts were made to conceptualize acquired acalculia (Klein et al., 2013b) but also its developmental counterpart dyscalculia (DD) as disconnection syndromes (Kucian et al., 2014). Therefore, we will first give a brief overview regarding the neural GM correlates of numerical cognition before augmenting those neuro-functional data with recent evidence on WM connectivity made accessible by technical advances in DTI. In this review we use a broad definition of numerical cognition that encompasses tasks reflecting basic numerical competencies (e.g., magnitude comparison) but also mental arithmetic (e.g., addition, subtraction, multiplication, etc.), as also required in standardized tests of mathematical and/or intellectual abilities. Studies investigating higher mathematics (such as algebra, analysis or inferential procedures, etc.) and their neuro-structural correlates are not included in the current review.

Neural Correlates of Numerical Cognition

In the past two decades, significant progress has been made in uncovering the neural basis of numerical cognition (Menon, in press, for a review). The triple-code model (TCM) of Dehaene et al. (2003) reflects a unique integration of behavioral and neuro-functional aspects, proposing three different representational codes for numbers and their neural correlates. (i) A bi-hemispheric *numerical magnitude representation* associated with the intraparietal sulcus (IPS); (ii) A *verbal representation* of numbers associated with left perisylvian language areas and the left angular gyrus (AG) which is recruited in verbally mediated operations like number naming as well as arithmetic fact retrieval; and (iii) A *visual number form representation* specialized for recognizing Arabic digits and associated with bilateral fusiform regions. From its initial form the TCM assumed that number processing requires the close interplay of domain-specific number-related parietal as well as domain-general (pre)frontal processes involving working memory and executive control. This suggests that numerical cognition is subserved by a multi-modular and distributed system within the human brain.

So far, the TCM has not taken into account an explicit and detailed delineation of the connecting fiber pathways subserving this multi-modular organization, probably due to the non-availability of appropriate imaging methods at the time of its initial formulation. Nevertheless, in the first version of the anatomo-functional TCM (Dehaene and Cohen, 1995), and in a series of subsequent detailed single case studies, the involvement of intra-hemispheric (cortico-subcortical, fronto-parietal) as well as inter-hemispheric (commissural) pathways for number processing and calculation was highlighted. Moreover, observed patterns of impairment (e.g., pure alexia for numbers) were also explained by a disconnection account (Cohen and Dehaene, 1995; see also Klein et al., 2013a). Nevertheless, the vast majority of recent neuroimaging studies have focused on the localization of activated GM areas. WM connections underlying numerical cognition were not considered specifically in most cases. We identified 10 studies investigating functional connectivity (**Table 1**), and 17 studies investigating structural WM connections in numerical cognition (see **Table 2**) from the last ten years. The increasing number of publications in recent years may not only reflect increasing research interest but also progressive availability and validity of DTI sequences (e.g., Soman et al., 2015) and appropriate processing software.

Almost all studies aimed at specifying the fronto-parietal network underlying numerical cognition as suggested by the TCM. In this vein, intra-hemispheric fronto-parietal connections (e.g., Rykhlevskaia et al., 2009; Tsang et al., 2009; Matejko et al., 2013; Navas-Sánchez et al., 2014) and inter-hemispheric (intra)parietal to (intra)parietal connections (e.g., Cantlon et al., 2011; Krueger et al., 2011; Klein et al., 2013b; Park et al., 2013) were of primary interest in most studies. In the following we will summarize and review the existing evidence regarding brain connectivity in numerical cognition. First, functional and effective connectivity (reflecting correlations between activation in specific brain areas) will be considered. Subsequently, we will elaborate on studies addressing structural connectivity, which allow identification of anatomical WM fiber tracts involved in numerical cognition.

Brain Connectivity in Numerical Cognition

Correlations Between Activated Brain Areas—Functional and Effective Connectivity

A first way of evaluating the connectivity between specific brain regions involves computing functional connectivity; specifically, the correlation patterns between neural *GM* activation elicited in different brain regions, while performing a specific numerical task. Highly correlated activation in two different brain areas is assumed to indicate that these areas may work together (see **Table 1** for an overview of studies investigating functional connectivity). Emerson and Cantlon (2012) used a symbolic-nonsymbolic number matching task to localize number-specific activation in parietal and (pre)frontal cortex areas in four- to eleven-year-old children. They then correlated the time series of activated voxels within frontal regions of interest (ROIs) with parietal ROIs to obtain a measure of fronto-parietal

TABLE 1 | Overview of studies investigating functional/effective connectivity underlying numerical cognition.

Nr.	Authors	Year	Connectivity analysis	Task	Participants	Connections
1	Tang et al.	2006	Functional connectivity	Magnitude comparison, addition	Chinese: 23.8 ± 0.8 years; English-speaking: 26.8 ± 2.3 years	VFG – SMA, L SMA – L PMA, L PMA– Broca, Broca – Wernicke, VFG –L IPC, L IPC – Wernicke
2	Krueger et al.	2011	Effective connectivity (GCM)	Multiplication	26 ± 6.7 years	R IPS – L IPS; R IPS – R DLPFC; L precG,- L preSMA; L preSMA –L/R DLPFC; L IPS –L DLFPC
3	Rosenberg-Lee et al.	2011	Functional connectivity	WIAT, WMTB-C	7–9 years	L DLPFC –L AG, L SPL
4	Cho et al.	2012	Effective connectivity (PPI)	Addition	7–10 years	R Hippocampus –L DLPFC; L VLPFC
5	Emerson and Cantlon	2012	Functional connectivity	TEMA, Matching numbers, faces, words, and shapes	4–11 years	IPS –PFC, IFG, insula
6	Supekar et al.	2013	Functional connectivity	WASI; WIAT, WMTB-C, Reading, addition verification and production	8–9 years	R Hippocampus –R MTG, R SMA, L DLPFC, L VLPFC, L BG
7	Park et al.	2013	Effective connectivity (PPI)	non-symbolic Addition, number matching, shape matching	18–29 years	R IPS –L IPS, L sensorimotor cortex
8	Park et al.	2014	Effective connectivity (PPI)	Magnitude comparison on digits, dots, and line lengths	4–6 years	R SPL –L SMG, R preCG
9	Qin et al.	2014	Effective connectivity (PPI)	Addition	7–9, 14–17, & 19–22 years	Hippocamus –L/R DLPFC, L IPS
10	Rosenberg-Lee et al.	2015	Effective connectivity (PPI)	Addition, subtraction	7–9 years, 16 with dyscalculia	Hyperconnectivity IPS –AG, L SMG, R MFG, R IFG, VMPFC in dyscalculia

L: left; R: right; GCM: Granger causality mapping; PPI—Psycho-Physiological Interactions; TEMA: Test of Early Mathematics Ability (Ginsburg and Baroody, 2003); WMTB-C: Working Memory Test Battery for Children (Pickering and Gathercole, 2001); WASI: Wechsler Abbreviated Scale of Intelligence (Wechsler, 1999); WIAT: Wechsler Individual Achievement Test (Wechsler, 2005); VFG: visual fusiform gyrus; PMA: premotor association areas; Broca: Broca's area; Wernicke: Wernicke's area; IPC: intraparietal cortex; IPS: intraparietal sulcus; DLPFC: dorsolateral prefrontal cortex; (pre)SMA: (pre) supplementary motor area; preCG: pre central gyrus; SPL: superior parietal lobe; AG: angular gyrus; VLPFC: ventrolateral prefrontal cortex; IFG: inferior frontal gyrus; MTG: middle temporal gyrus; BG: basal ganglia; SMG: supramarginal gyrus; VMPFC: ventromedial prefrontal cortex.

TABLE 2 | Overview of studies investigating structural connectivity underlying numerical cognition.

Nr.	Authors	Year	Connectivity analysis	Task	Participants	White matter tracts
1	Barnea-Goraly et al.	2005a	DTI, ROI analysis (6 directions)	WISC number tasks	7–20 years, VCFS	–
2	van Eimeren et al.	2008	DTI, ROI analysis (32 directions)	WIAT number tasks	7–9 years	Atlas-based: SCR, ILF
3	Rykhlevskaia et al.	2009	DTI, fiber tractography (probabilistic and deterministic, ROI analyses), 23 directions	WASI, WIAT, WMTB-C	7–9 years, 23 with dyscalculia	Tractography-based: ILF, IFOF, thalamic radiation, caudal forceps major
4	Tsang et al.	2009	DTI, ROI analysis (12 directions)	Multiplication, exact and approximate addition, WISC, WRAT, Reading	10–15 years	Atlas-based: SLF
5	van Eimeren et al.	2010	DTI, ROI analysis (12 directions)	Four basic arithmetic operations	26.4 ± 3.0 years	Atlas-based: SCR
6	Cantlon et al.	2011	DTI, fiber tractography ROI analysis (deterministic, 15 directions)	Number comparison symbolic and non-symbolic	6 years	Tractography-based: Callosal isthmus
7	Hu et al.	2011	DTI, TBSS analysis (15 directions)	Digit/letter span, WAIS, 3 years of abacus training	10 years	Atlas-based: Internal capsule, thalamic radiation, corona radiata, SLF, ILF
8	Klein et al.	2013b	DTI, fiber tractography ROI analyses (probabilistic, 61 directions)	Mental addition	28 ± 5 years	Tractography-based: SLF, EC/EmC
9	Klein et al.	2013a	Fiber tractography (deterministic)	–	49 years, single case	Tractography-based: EC, SLF
10	Kucian et al.	2013	DTI, ROI analysis (21 directions)	ZAREKI, WISC, Corsi	10 years, 15 with dyscalculia	Atlas-based: SLF, adjacent to IPS
11	Navas-Sanchez et al.	2013	DTI, ROI analysis (16 directions)	Math Talent Program, Madrid, Spain	12–15 years	Atlas-based: Corpus callosum, internal capsule, SLF, SCR, EC, thalamic radiation
12	Matejko, et al.	2013	DTI, TBSS analysis (31 directions)	PSAT	17–18 years	Atlas-based: SLF, SCR, corticospinal tract
13	Li et al.	2013a	DTI, fiber tractography (probabilistic and TBSS, 30 directions)	WISC	10–11 years	Tractography-based: SLF, ILF, inferior fronto-occipital fasciculus
14	Li et al.	2013b	DTI, fiber tractography (probabilistic and TBSS, 15 directions)	Abacus training for 3 years	10 years	Tractography-based: Forceps major
15	Willmes et al.	2014	DTI, fiber tractography, ROI analysis (deterministic, 61 directions)	Parity judgment, magnitude comparison from Klein et al. (2010)	18–25 years	Tractography-based: EC/EmC, SLF
16	Van Beek et al.	2014	DTI (45 directions)	Addition, Subtraction; Multiplication, Division, WISC, WMTB-C, word and pseudoword reading	11–13 years	Anterior arcuate fasciculus
17	Klein et al.	2014	DTI, fiber tractography (deterministic, 61 directions)	Number bisection, exact/approximate addition	19–42 years	Tractography-based: MdLF, ILF, SLF, EC/EmC, cingulate bundle

PSAT: Preliminary Scholastic Aptitude Test (College Board USA, 2006); WASI: Wechsler Abbreviated Scale of Intelligence (Wechsler, 1999); WIAT: Wechsler Individual Achievement Test (Wechsler, 2005); WISC: Wechsler Intelligence Scale for Children (Wechsler, 2004); WMTB-C: Working Memory Test Battery for Children (Pickering and Gathercole, 2001; WRAT: Wide Range Achievement Test (Wilkinson and Robertson, 2006); ZAREKI-R: Testverfahren zur Dyskalkulie bei Kindern (von Aster et al., 2005).

connectivity. Interestingly, stronger fronto-parietal connectivity was associated with better math proficiency, emphasizing the importance of integrated fronto-parietal processing in numerical cognition. Tang et al. (2006) observed differential patterns of fronto-parietal functional connectivity for Chinese- and English-speaking participants in both a magnitude comparison task and a mental addition task. The authors argued that Chinese-speaking participants seemed to engage more strongly a visuo-premotor association network for solving these tasks (involving visual fusiform gyrus and premotor association areas). On the other hand, native English speakers seemed to largely employ language-based processes relying on left perisylvian cortices (including Broca's and Wernickes area) for the same tasks.

The important role of integrated fronto-parietal processing was further substantiated by Supekar et al. (2013), who investigated the neural predictors of arithmetic skill acquisition in 8–9-year-old children before an 8-week math tutoring program. The authors found that functional connectivity of the hippocampus with dorsolateral and ventrolateral prefrontal cortices as well as with the basal ganglia prior to tutoring predicted subsequent learning effects. This finding was interpreted to indicate that "individual differences in the connectivity of brain regions associated with learning and memory, and not regions typically involved in arithmetic processing, are strong predictors of responsiveness to math tutoring in children" (Supekar et al., 2013, p. 8230). In another study evaluating the manifestation of numerical learning in brain connectivity Rosenberg-Lee et al. (2011) investigated changes in the connectivity of prefrontal and more posterior brain areas between 2nd and 3rd grade using a cross-sectional approach. They observed differential functional connectivity between left DLPFC and posterior brain areas. In particular, changes in functional connectivity between 2nd and 3rd grade were stronger in what the authors termed dorsal (superior parietal lobe, AG) as compared to ventral stream areas (parahippocampal gyrus, lateral occipital cortex, lingual gyrus).

Krueger et al. (2011) used multivariate Granger causality to evaluate *effective* connectivity in adult numerical cognition. Multivariate Granger causality mapping not only quantifies the co-activation of two brain regions for a given task, but also allows one to assess the direction of the connections between the respective areas. The authors observed a fronto-parietal network for multiplication, involving a reciprocal parietal IPS-IPS circuit which subserves number magnitude information. This magnitude processing related network was also interlaced with a reciprocal fronto-parietal circuit from the dorsolateral prefrontal cortex and the IPS associated with the execution and updating of arithmetic operations. Importantly, the parietal cortex received more inputs from the frontal cortex than the other way around, indicating the central role of the parietal cortex in number processing.

Another method to evaluate effective connectivity is the approach of psychophysical interaction analysis (PPI), as used by Park et al. (2013, 2014, see also Cho et al., 2012; Qin et al., 2014). For adults, Park et al. (2013) used custom-made reaction time experimental tasks assessing (i) non-symbolic addition and subtraction, (ii) number matching as well as (iii) shape matching. They found increased effective connectivity within the right parietal cortex as well as between the right and left parietal cortices for arithmetic tasks in general and subtraction in particular. Importantly, the degree of effective connectivity was associated positively with behavioral performance in the subtraction task. Furthermore, Park et al. (2014) investigated effective connectivity of the right parietal cortex with the left supramarginal gyrus and the right precentral gyrus in 4–6-year-old children. The degree of connectivity from the right parietal cortex to the right precentral gyrus was predictive of performance on a standardized symbolic math test (see **Figure 1** for an overview of the connections suggested by functional/effective connectivity analyses). Using the same method, Rosenberg-Lee et al. (2015) investigated differences in functional connectivity of the IPS between typically developing 7–9-year-old children and a sample of children from the same age group with DD. Interestingly, the authors found what they called hyperconnectivity of the bilateral IPS in children with DD with ventro- and dorsolateral PFC as well as the SMG. The authors attributed this phenomenon to involvement of compensatory mechanisms. On the other hand, they also suggested that the "engagement of these circuits may result in the activation of problem-irrelevant information that in turn disrupts problems solving" (p. 18, see also below for findings on structural connectivity in children with DD).

The conclusion of Supekar et al. (2013), stating that hippocampal-prefrontal connectivity is specifically associated with numerical learning, was corroborated by an evaluation of effective connectivity of the hippocampus. Using PPI, Cho et al. (2012) revealed strong causal bidirectional connectivity between the right hippocampus and the left VLPFC and DLPFC to be associated with the application of retrieval-based solution strategies to an addition task in 7–10-year-old children. The influence of hippocampal-prefrontal connectivity on numerical learning was further specified in a paper by Qin et al. (2014), which so far represents the only longitudinal study investigating the influence of brain connectivity on numerical development. The authors observed that the connectivity of the hippocampus with prefrontal and parietal cortices was predictive of the gain in 7–9-year-old children's fact retrieval fluency in mental addition over a period of more than 1 year. Thus, numerical development seems to be associated with changes in hippocampal-neocortical connectivity.

Summarizing the results of studies evaluating functional and effective connectivity associated with numerical cognition clearly indicates that number processing involves a widespread network including (intra)parietal (e.g., IPS, SPL, SMG, AG) but also (pre)frontal cortex sites (e.g., DLPFC, VMPFC, preCG, SMA, IFG) as well as the hippocampus. The latter seems to be particularly involved in numerical learning and development because hippocampal-prefrontal as well as hippocampal-parietal connectivity is reliably associated with children's use of more sophisticated retrieval-based solution strategies in mental arithmetic. Additionally, the strength of fronto-parietal connectivity was associated with better math proficiency. In line with the TCM these analyses of functional and effective connectivity provide converging evidence for numerical cognition to be subserved by a fronto-parietal network also incorporating hippocampal structures. However, those analyses do not allow for the identification of anatomical WM fiber tracts connecting brain areas with correlated brain activations. This can only be achieved by considering structural connectivity. In the following, we describe another set of studies which evaluated structural connectivity in two different ways to pinpoint the WM tracts involved in numerical cognition.

Structural Connectivity

There are two different approaches to investigate structural connectivity (see **Table 2** for an overview of studies evaluating

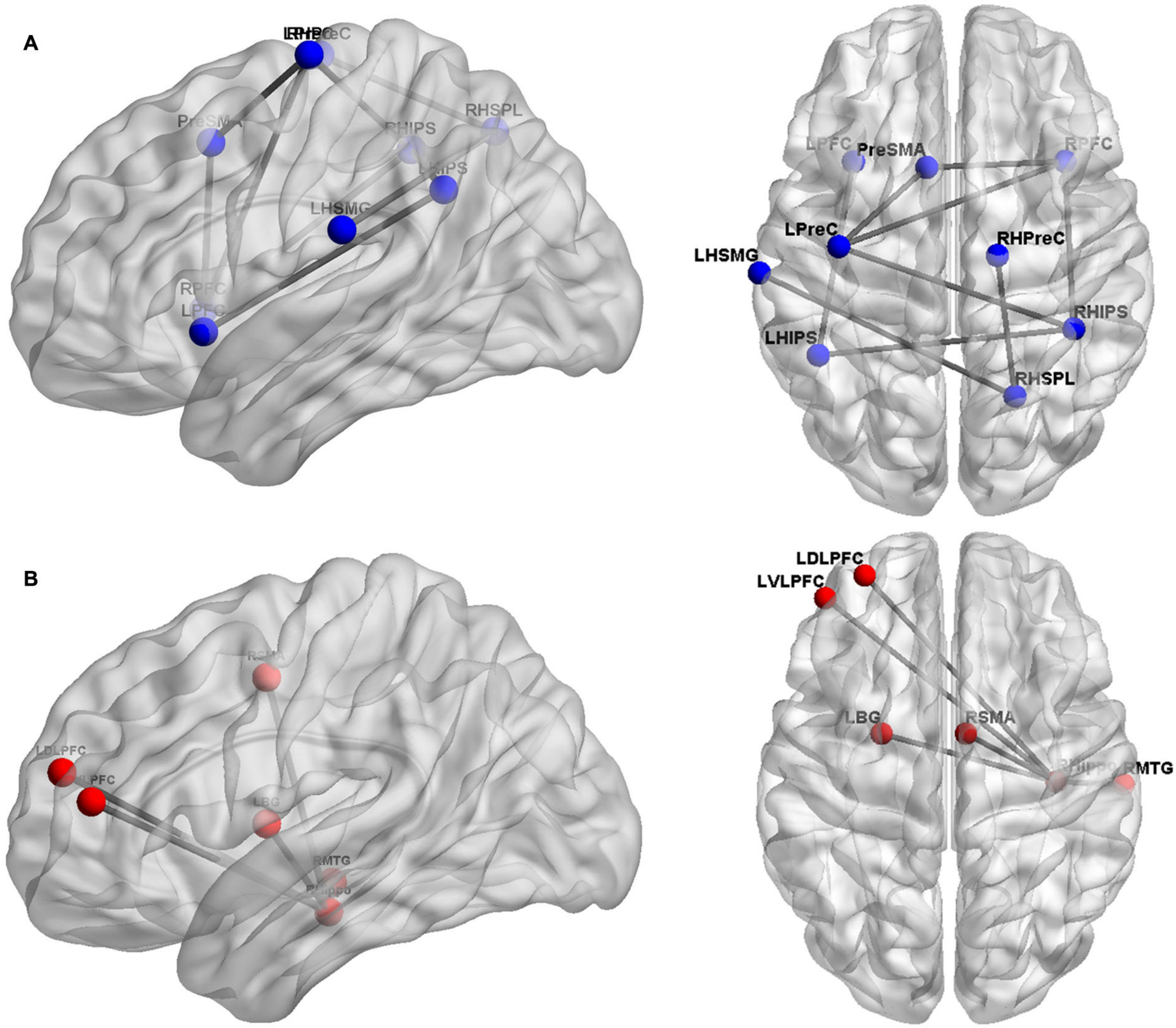

FIGURE 1 | Overview of cortical sites considered in the studies evaluating functional/effective brain connectivity. Panel **(A)** depicts the cortical sites (blue spheres) and their functional/effective connectivity patterns (gray lines), found by Krueger et al. (2011), Emerson and Cantlon (2012), and Park et al. (2013, 2014). These studies primarily focused on the fronto-parietal network of numerical cognition. Panel **(B)** shows the cortical regions (red spheres) and their functional/effective connectivity patterns (gray lines) identified by Supekar et al. (2013) found to be predictive of numerical learning.

structural connectivity). The first is to evaluate the correlation of diffusion parameters in predefined ROI with either behavioral performance or with fMRI activation peaks observed for numerical tasks. Fractional anisotropy (FA) and/or radial diffusion (RD; see Mukherjee et al., 2008 for an explanation of the physical principles) are often used diffusion parameters in these analyses. Additionally, ROIs are usually located to reflect a specific (mostly) atlas-identified WM pathway. The second approach is fiber tractography, which allows for the virtual reconstruction of entire WM pathways. Thereby, diffusion tensor tractography can characterize not only the orientation but also the integrity of WM fibers *in vivo* and noninvasively (Basser et al., 1994). The following section will discuss studies using atlas-based ROI analyses and fiber tractography in turn.

Atlas-based ROI Analyses of Diffusion Measures

Studies using ROI analyses provided evidence for the involvement of anterior to posterior association and projection fiber tracts in numerical cognition. With respect to *anterior to posterior association fiber tracts*, Rykhlevskaia et al. (2009) observed that increased FA in a temporo-parietal ROI incorporating parts of the superior longitudinal fasciculus (SLF), the inferior longitudinal fasciculus (ILF), and the inferior fronto-occipital fasciculus (IFOF) was associated with better performance of 7–9-year-old children in the arithmetic subtest of an IQ test. The importance of fronto-parietal connections was further corroborated by Tsang et al. (2009). These authors used a combination of tests administered outside and inside the scanner, to investigate the association between FA in a central part of the SLF and performance in a computerized approximate arithmetic task. In particular, the authors considered performance scores of 10–15-year-olds from the arithmetic subtest of a scholastic achievement test to control for the specificity of their results. They found an association of higher FA in the SLF and better performance in approximate arithmetic, indicating the importance of fronto-parietal connectivity for performance in mental arithmetic.

As regards projection fiber tracts, Rykhlevskaia et al. (2009) reported that increased FA in a temporo-parietal ROI incorporating parts of the anterior thalamic radiation and the cortico-spinal tract was associated with better performance of 7–9-year-old children in the arithmetic subtest of an IQ test. Comparably, van Eimeren et al. (2008) used ROI analyses to investigate the association of WM connectivity with arithmetic performance. They found that in 7–9-year-old children increasing FA in ROIs from the superior corona radiata (SCR) was associated with better performance in the arithmetic subtests of an IQ test administered outside the scanner. To a lesser degree this also held true for FA in ROIs from the ILF. van Eimeren et al. (2010) also found evidence for an involvement of the SCR in numerical cognition. They observed that higher FA in a ROI reflecting a central segment of the left SCR was associated reliably with a stronger BOLD response in the left AG, as recorded during retrieval-prone calculations in a sample of mostly university students.

Other studies reported a *combination of projection and association fiber tracts* to be recruited in numerical cognition. Matejko et al. (2013), using tract-based spatial statistics (TBSS), found higher FA in the left SLF, SCR, and cortico-spinal tract of 17 to 18-year-olds to be associated with better performance in the arithmetic subtest of a scholastic achievement test. TBSS (see also Hu et al., 2011; Li et al., 2013b) employs a voxel-wise statistic followed by the projection onto an alignment-invariant mean FA skeleton in order to derive clusters, in which FA correlates with a dependent variable. These clusters may be but do not necessarily need to be used for ROI analyses. Instead, most other studies reported in this paragraph extracted FA for ROIs in tracts of interest (after atlas-based identification) from aligned and spatially smoothed diffusion imaging data (Jones et al., 2005).

There is also another approach to investigate structural connectivity in numerical cognition. Instead of associating diffusion measures in specified ROIs with performance or GM activation, another subset of studies evaluated differences in ROI-based diffusion parameters between different populations. Rykhlevskaia et al. (2009) found reduced FA in ROIs located in the IFOF, ILF, SLF, amongst others, in children with developmental DD, as compared to typically developing children. Recent data by Kucian et al. (2014) substantiated impairments in WM connectivity as indicated by reduced FA and RD in a posterior part of the SLF, in particular in children with DD. These authors suggested that DD may be considered a disconnection syndrome (see also Klein et al., 2013a for the case of acquired acalculia). Navas-Sánchez et al. (2014), when studying math-gifted adolescents, observed higher WM integrity, as indicated by higher FA in ROIs located in the SLF adjacent to inferior parietal cortex areas. Barnea-Goraly et al. (2005a) found that reduced arithmetic competencies in velocardiofacial syndrome may be caused by structural WM aberrations in inferior parietal cortex (see also Lebel et al., 2010; Till et al., 2011, for WM differences associated with impaired numerical performance in children with fetal alcohol spectrum disorder and youths with multiple sclerosis, respectively).

Finally, there were another two studies specifically investigating the influence of the duration of abacus use on brain connectivity. Li et al. (2013b) employed TBSS and found increased FA in the left callosal forceps major only, whereas Hu et al. (2011) observed increased RD for abacus users after three years of abacus training in a variety of WM connections, including "the internal capsule (IC), corona radiata and posterior thalamic radiation" (p. 19) as well as the SLF.

In summary, ROI analyses of DTI data are commonly used to detect diffusion parameter alterations, as measured by FA values. However, it is worth noting that WM tracts specified by these analyses are simply those that pass through the respective ROI, as indicated by comparison with a brain atlas. Importantly, this means that the specified tracts were not identified directly to connect to cortex sites of interest. Fiber tractography based on task-related fMRI data, however, enables the virtual reconstruction of WM pathways connecting cortex areas found or assumed to be included in s processing model such as the TCM.

Fiber Tractography

A last set of studies interested in structural connectivity used either probabilistic or deterministic fiber tractography to identify WM connections. Fiber tractography aims at delineating WM pathways involved by virtually reconstructing the most probable WM tract pathways between user-defined seed points. Probabilistic tracking differs from deterministic tracking in that the probability for false negative reconstructions of specific tracts is taken into account. This probability is typically elevated in areas where fibers cross, merge or kiss. By employing both probabilistic and deterministic fiber tracking, Rykhlevskaia et al. (2009) compared the brain connectivity pattern in children with and without developmental DD. The authors found that typically developing children showed more inter-hemispheric (superior parietal) connectivity as well as stronger connectivity of the right temporal-parietal cortex. Probabilistic and deterministic fiber tracking analyses linking WM and GM alterations in children with developmental DD "point to tracts connecting the fusiform gyrus with temporal-parietal WM, most likely via the ILF, as a major locus of neuroanatomical abnormalities in DD" (Rykhlevskaia et al., 2009, p. 11).

Cantlon et al. (2011) employed deterministic fiber tracking to further investigate the influence of inter-hemispheric IPS to IPS connectivity on number processing in typically developing children. The authors observed that FA within tracked fibers of the left isthmus of the corpus callosum was correlated positively with performance in a number magnitude comparison task administered in the scanner in six-year-old children. On the other hand, Li et al. (2013a) used probabilistic fiber tracking to identify and differentiate the course of the different anterior to posterior association fiber tracts. Using TBSS, FA values of the WM tracts identified were correlated with children's performance in the arithmetic subtest(s) of an IQ test. Reliable positive associations between FA values and arithmetic performance were observed for the left SLF, ILF and bilateral IFOF.

More recent studies using probabilistic or deterministic fiber tracking have tried to integrate the identified pathways for numerical cognition into the broader architecture of dorsal and ventral processing streams, as previously done for other cognitive domains such as language (e.g., Hickok and Poeppel, 2007; Rauschecker and Scott, 2009; Weiller et al., 2011). Klein et al. (2013b) investigated WM connections between seed points observed to be activated in either more difficult (calculation-based) or more easy (retrieval-based) addition problems. For both conditions the authors reconstructed pathways encompassing the SLF and the external/extreme capsule (EC/EmC) system indicating that both magnitude- and fact retrieval-related processing were subserved by two largely distinct networks, both of them comprising dorsal and ventral connections. This distinction between magnitude- and fact retrieval-related processing on the level of structural brain connectivity was further substantiated by the results of Klein et al. (2014). These authors showed that the proposed differentiation generalizes to other numerical tasks (i.e., number bisection and exact/approximate addition). This indicates that magnitude- and fact retrieval-related processing may indeed rely on different neural networks, even though these networks operate in an integrated manner to solve numerical tasks most efficiently. This is also in line with the results of Van Beek et al. (2014). These authors found that higher FA in the left anterior portion of the arcuate fasciculus specifically predicted better addition and multiplication but not subtraction and division performance. As the arcuate fasciculus links frontal with temporo-parietal cortex sites the authors argue that "the association between the left arcuate fasciculus-anterior and addition/multiplication reflects involvement of phonological processing" (p. 117) related to arithmetic fact retrieval.

Finally, Willmes et al. (2014) observed a common ventral fronto-parietal connection encompassing the EC/EmC system for the general cognitive operation of *semantic classification* for the domains of language (e.g., word/non-word decisions) and number processing (e.g., odd/even judgments). Interestingly, this network appeared to be augmented by a dorsal connection to the IPS running along the SLF, when number magnitude was decision relevant, as is the case in number magnitude comparison.

In summary, analyses of structural connectivity underlying numerical cognition indicate crucial involvement of both association fiber tracts running from anterior to posterior (e.g., SFL, EC/EmC system, ILF, and IFOF) as well as projection fiber tracts (e.g., SCR, thalamic radiation) and transcallosal commissural fibers connecting the bilateral intraparietal sulci. Comparable to the results of studies investigating functional/effective connectivity, the findings from structural connectivity analyses corroborate the proposition of a fronto-parietal network subserving numerical cognition. In particular, the SLF and the EC/EmC system constitute important fronto-parietal pathways connecting number-specific areas in the parietal cortices (e.g., IPS, AG) with number unspecific areas in (pre)frontal cortex (e.g., DLPFC, IFG), as proposed by the TCM.

White Matter Pathways in Numerical Cognition

There is considerable convergence with respect to the fronto-parietal WM pathways connecting domain-specific number-related parietal brain areas (IPS, AG) to more domain-general (pre)frontal areas. The SLF and the EC/EmC system were identified repeatedly to be associated with fronto-parietal processing in numerical cognition (e.g., Rykhlevskaia et al., 2009; Tsang et al., 2009; van Eimeren et al., 2010; Klein et al., 2013a,b; Matejko et al., 2013; Kucian et al., 2014; Navas-Sánchez et al., 2014; see **Figure 2** for a schematic illustration). Importantly, the association of fronto-parietal connectivity with numerical performance encompassing these systems is not only in line with the results of the functional connectivity analyses described above, but also corroborates the propositions of the TCM. Furthermore, involvement of projection fibers such as the (superior) corona radiata, possibly connecting motor cortices and subcortical structures such as the thalamus, were frequently observed to be involved in numerical cognition (van Eimeren et al., 2008, 2010; Rykhlevskaia et al., 2009; Hu et al., 2011).

The importance of the SCR is hard to reconcile with the results of functional connectivity analyses, which usually did not consider cortico-subcortical connections or subcortical structures. Nevertheless, the involvement of these projection fibers is in line with propositions of earlier versions of the TCM by Dehaene and Cohen (1995, 1997), which have not been pursued systematically so far—possibly due to the cortex-centerd focus of recent (fMRI) research on numerical cognition.

With respect to the involvement of the SCR it is interesting that fiber tracts, which have been found to be involved in number processing less consistently, seem to be closely related neuro-anatomically. In particular, the cortico-spinal tract (Matejko et al., 2013; investigated in some cases at the level of the internal capsule (Hu et al., 2011; Navas-Sánchez et al., 2014) and the right thalamic radiation (Rykhlevskaia et al., 2009; Hu et al., 2011) are often aggregated to constitute the SCR. Neuro-anatomically the cortico-spinal tract, but also the superior peduncle of the thalamic radiation are part of the corona radiata—as is the SCR. Therefore, functional involvement of the SCR seems particularly reasonable from a theoretical point of view. The thalamic radiation directly connects the cortex with the ventrolateral thalamus, while the cortico-striatal tract connects the cortex indirectly with the ventrolateral thalamus via the striatum. Thus, these structures connect GM areas known to be involved in numerical cognition either directly or indirectly (via the basal ganglia) with the thalamus. Interestingly, early

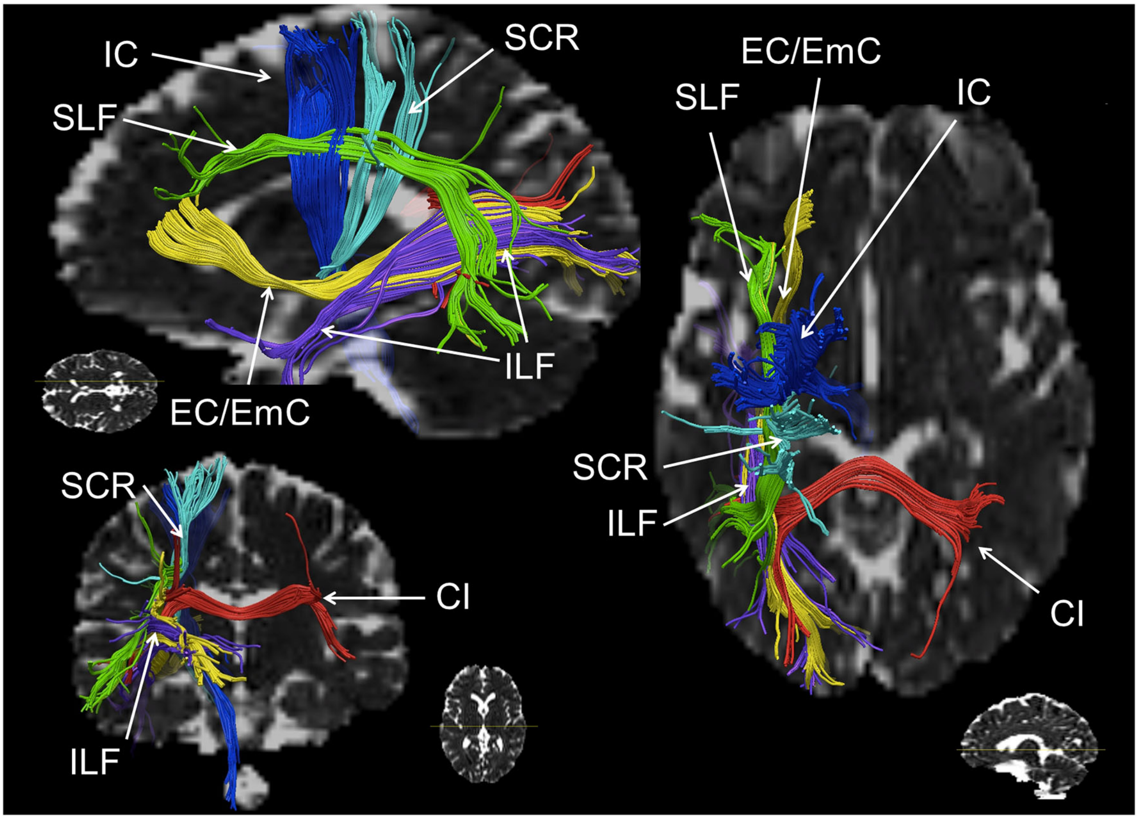

FIGURE 2 | Schematic reconstruction of association (green, yellow), projection (blue) and commissural (red) fiber tracts repeatedly observed in numerical cognition tasks (in axial, sagittal and coronal orientation). The superior longitudinal fasciculus (SLF) is displayed in green, the inferior longitudinal fasciculus (ILF) and the external/extreme capsule (EC/EmC) system are depicted in yellow, parts of the internal capsule (IC) in dark blue, the superior coronar radiata (SCR) is shown in light blue, and interhemispheric parietal to parietal connections encompassing the callosal isthmus (CI) are shown in red. Virtual dissections were performed for one individual with seed regions chosen deliberately for illustration purposes only, regarding white matter (WM) pathways involved in numerical cognition.

versions of the TCM (Dehaene and Cohen, 1995, 1997) suggested a vital role of the basal ganglia and the thalamus in numerical cognition. On the other hand, assuming the involvement of the cortico-spinal tract or the IC to be associated with processing of numerical information is less evident, because for both structures connections to the parietal lobes are strongly associated with motor and somatosensory processes (e.g., Newton et al., 2006; Lotze et al., 2011; see Catani et al., 2012 for an overview). However, all evidence for an involvement of these different parts of the corona radiata comes from studies specifying the involved WM in an atlas-based approach (either ROI analyses, e.g., Rykhlevskaia et al., 2009; Navas-Sánchez et al., 2014 or TBSS, e.g., Hu et al., 2011; Matejko et al., 2013). We suggest that any strong interpretation of the functional involvement of the SCR, the thalamic radiation, the cortico-spinal tract, the cortico-striatal tract and even commissural fibers should be made with great care, because the atlas-based identification of WM tracts highly depends on where exactly the respective ROI is placed (see **Figure 3** for a schematic illustration). Generally, ROI analyses do not provide virtual reconstructions of the WM tracts connecting two GM areas associated with number processing. Therefore, the WM tracts identified by ROI analyses reflect all tracts passing through the respective ROI, instead of considering only those tracts connecting the WM areas of interest as in fiber tractography. For the present WM tracts (i.e., SCR, cortico-spinal tract/IC, thalamic radiation, cortico-striatal tract, and commissural fibers) a respective ROI may incorporate fibers of more than one tract, and small variation in the location of the ROI may easily change the involved fiber tracts. It is even easily possible to capture *all* of the latter tracts in one atlas-based ROI. Importantly, this variability of results can be reduced by employing methods of fiber tractography, which makes them highly desirable for more studies evaluating brain connectivity in numerical cognition. While studies on functional and atlas-based structural connectivity paved the ground for a more general understanding of the interaction of WM with GM in numerical cognition, the actual connectivity within the fronto-parietal network of numerical cognition seems to be captured better using tractography-based analyses.

Generally, there seems to be notable agreement regarding the identified fronto-parietal WM tracts, but so far there

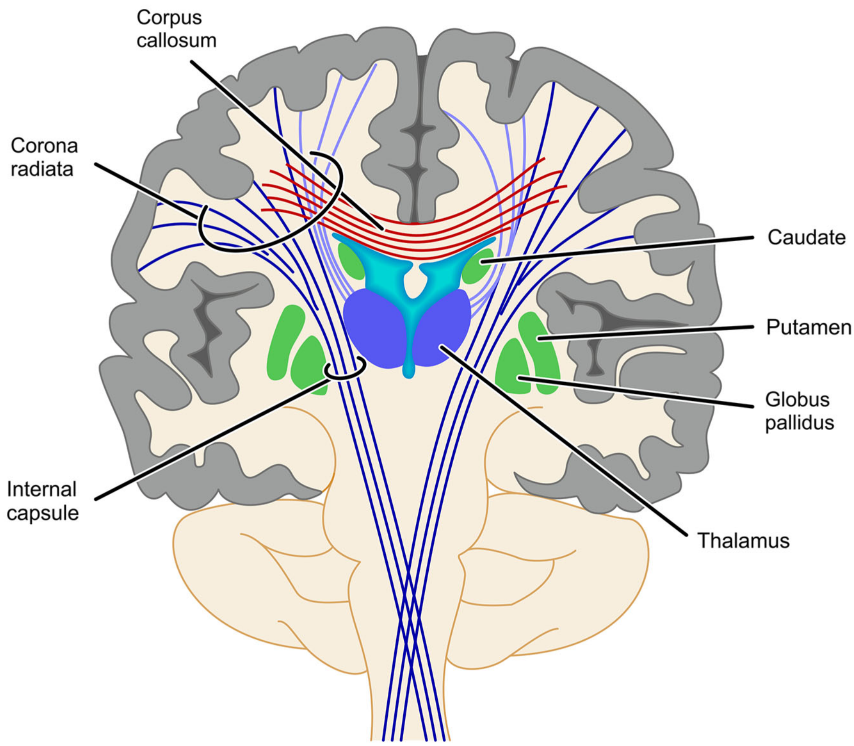

FIGURE 3 | Schematic illustration of problems with the identification of projection fibers. Fiber tracts identified by atlas-based ROI analyses depend strongly on where exactly along this bundle of tracts the respective ROI is placed. As most of the ROIs reviewed in this article placed their ROI somewhere between the basal ganglia and the cortex it is obvious that such a ROI might well involve fibers of the SCR, the cortico-spinal tract/IC, the thalamic radiation (light purple) or even commissural fibers (cortico-striatal tract not depicted for reasons of clarity).

is no coherent picture with respect to lateralization of the tracts involved. Some studies reported bilateral hemispheric connections associated with number processing (e.g., Rykhlevskaia et al., 2009; Hu et al., 2011; Klein et al., 2013b), whereas others only found significant results for left hemispheric fiber tracts (e.g., van Eimeren et al., 2008; Tsang et al., 2009; Kucian et al., 2014; Matejko et al., 2013) or deliberately chose to focus on the left hemisphere, because the authors were specifically interested in verbal numerical representations supposed to be left-lateralized (e.g., Klein et al., 2013a,b; Willmes et al., 2014). As regards interhemispheric connections, only Cantlon et al. (2011) directly compared inter-hemispheric (intra)parietal to (intra)parietal connections and found them to be reliable for the genu, isthmus and posterior splenium tracts of the corpus callosum. Additionally, WM tracts along the forceps major (e.g., Rykhlevskaia et al., 2009; Klein et al., 2013a; Li et al., 2013a,b) were observed to be involved in numerical cognition connecting (intra)parietal areas.

Finally, other WM pathways have been found to be involved in number processing less consistently. In particular, involvement of the IFOF (e.g., Rykhlevskaia et al., 2009; Li et al., 2013a), connecting frontal cortex sites with occipital sensory cortical areas was reported. Since all reported studies used visually presented stimuli this most probably reflects involvement of visual perceptual processes in numerical cognition. Additionally, the ILF was repeatedly found to be associated with numerical performance (e.g., van Eimeren et al., 2008; Rykhlevskaia et al., 2009; Li et al., 2013a). As the ILF represents a parieto-temporal connection, it may be most likely involved in connecting left-hemispheric perisylvian language areas, as proposed to be associated with the verbal representation of numbers in the TCM.

Other Methodological Limitations and Implications

Substantial convergence concerning the involvement of WM pathways in numerical cognition is notable, considering the variety of methodological approaches and tasks employed to investigate quite different populations. The different methods for investigating brain connectivity come with specific advantages and limitations. Functional connectivity analyses (e.g., Emerson and Cantlon, 2012) can only provide temporal correlations between activation in remote brain areas. Effective connectivity analyses (e.g., Krueger et al., 2011; Cho et al., 2012) additionally specify the direction in which one neuronal system exerts an influence over another. However, both attempts do not allow identification of WM tracts in a strict sense. As regards structural connectivity, ROI analyses of DTI data (e.g., van Eimeren et al., 2008, 2010; Matejko et al., 2013) require *a priori* hypotheses about the WM tracts involved. Additionally, intra- and inter-individual variability in the delineation of ROIs limits their reproducibility and reliability, as discussed above regarding involvement of the SCR. On the other hand, the interpretation of results from probabilistic and deterministic tractography (e.g., Rykhlevskaia et al., 2009; Willmes et al., 2014) depends on placement and size of the seed regions as well as on algorithm settings (Aoki et al., 2007). Additionally, tractography is particularly complex for regions where fibers cross, kiss, or merge, possibly leading to artefactual reconstructions (Basser et al., 2000). Furthermore, numerical performance was not only assessed inside (e.g., Tsang et al., 2009; Cantlon et al., 2011) but also outside (e.g., Rykhlevskaia et al., 2009; Matejko et al., 2013) the scanner. Moreover, assessment procedures also ranged from standardized diagnostic test instruments (e.g., TEMA, WIAT) over specific subtests selected from those standardized tests (e.g., subtest numerical operations from the WIAT; e.g., Barnea-Goraly et al., 2005a; Rosenberg-Lee et al., 2011; Emerson and Cantlon, 2012) to custom-made experimental reaction time tasks (e.g., Krueger et al., 2011; Park et al., 2013).

Finally, about half of the studies evaluated WM connectivity in children up to the age of 11 years (e.g., Emerson and Cantlon, 2012; Park et al., 2014), another three included adolescents between 11 and 18 years of age (e.g., Matejko et al., 2013; Navas-Sánchez et al., 2014), and only five studies investigated healthy adults (mostly students, e.g., Krueger et al., 2011; Park et al., 2013). Furthermore, there was one single-case stroke patient study (Klein et al., 2013a).

This shows that a considerable number of studies on brain connectivity in numerical cognition has been conducted with children and adolescents. Therefore, it seems important to evaluate whether these data shed new light on the potential development of WM connectivity within the fronto-parietal network underlying numerical cognition. However, no specific systematic trend was evident for age. Fronto-parietal associations (e.g., SLF) as well as projecting tracts (e.g., SCR) were found to be involved in numerical cognition from the youngest ages studied. This finding may be expected, because the identified fiber tracts (e.g., ILF, SLF, SCR, etc.) are important neuro-anatomical structures, which develop independently from specific cognitive functions during childhood and adolescence (e.g., Barnea-Goraly et al., 2005b; Huang et al., 2006; Asato et al., 2010; see Peters et al., 2012 for a recent meta-analysis).

Against this background, future studies are needed to evaluate how far the functional coupling of particular cortex areas or WM pathways associated with numerical cognition are specific to number processing and/or calculation. The study of Willmes et al. (2014) provides a first step in this direction. The authors investigated the WM connections associated with the general cognitive operation of *semantic classification* across the domains of language and number processing. They observed a common fronto-parietal connection encompassing the EC/EmC system for semantic classification, irrespective of content. This is in line with evidence suggesting that functional specificity of GM cortex areas (e.g., as suggested by Brodmann, 1909) is hard to reconcile with the differential involvement of cortical areas in a variety of tasks. For instance, Simon et al. (2002, 2004; see also Humphreys and Lambon Ralph, 2014 for a recent fMRI meta-analysis over 8 cognitive domains including number processing) found that the IPS was activated not only in number processing but also (partly overlapping) for the initiation of saccades, attention shifting, grasping, pointing, and language processing. This corroborates the notion of systemic localization, as suggested early on by Reinvang (1985). In this view, domain specificity may not be a question of localization (which cortex areas?) or connectivity (which fiber tracts?). Instead, it is important to concomitantly consider which cortex areas are connected to which other areas by which fiber tracts. The particular combination of cortex areas connected by specific fiber tracts may then be an indicator of domain-specificity.

Moreover, the question of how numerical learning and development manifest in WM connectivity is of particular interest. There is evidence for qualitative changes in GM activation patterns following numerical learning (e.g., Delazer et al., 2003; Kaufmann et al., 2011). There are now indications that learning may change WM connectivity parameters (Sagi et al., 2012; see Menon, 2013 for implications on cognitive development). For numerical cognition, Supekar et al. (2013) were able to show that numerical learning is specifically predicted by the connectivity of the hippocampus with prefrontal cortex sites (see **Figure 1**, see also Cho et al., 2012). Additionally, Qin et al. (2014) found longitudinal gains for fact retrieval fluency in 7–9-year-old children in mental addition, which were predicted by effective connectivity of the hippocampus with prefrontal and parietal cortices. Thus, numerical development as characterized by increasing use of retrieval-based solution strategies seems to be associated specifically with changes in hippocampal-neocortical connectivity. In the future, it might also be interesting to study the development of the different networks underlying arithmetic fact retrieval and number magnitude processing as identified by Klein et al. (2013a).

In addition to variation by age, there are five studies investigating brain connectivity in special populations: children with DD (Rykhlevskaia et al., 2009; Kucian et al., 2014; Rosenberg-Lee et al., 2015), individuals with velocardiofacial syndrome (Barnea-Goraly et al., 2005a), and math-gifted children (Navas-Sánchez et al., 2014). Molko et al. (2004) also provide first evidence regarding structural alterations in fiber orientation in children with Turner syndrome, who often present with numerical deficits.

Synthesis and Perspectives

Considering the variety of methodological approaches, types of assessment instruments, and populations investigated, the convergence of evidence regarding brain connectivity in numerical cognition is remarkable. Analyses of both functional/effective as well as structural connectivity consistently corroborate the propositions of the TCM: Numerical cognition seems to be subserved by a widespread network including (intra)parietal (e.g., IPS, SPL, SMG, AG) but also (pre)frontal cortex sites (e.g., DLPFC, VMPFC, preCG, SMA, IFG), as well as the hippocampus. Studies on functional/effective connectivity indicate a specific role of the hippocampus in numerical development. In children, hippocampal-prefrontal as well as hippocampal-parietal connectivity were found to be associated with the acquisition of retrieval-based solution strategies, while in adults hippocampal-parietal connectivity was associated with the retrieval of arithmetic facts. On the other hand, analyses of structural connectivity provide converging evidence for functional involvement of association fibers from the SLF dorsally as well as the EC/EmC system ventrally as the primary fronto-parietal connections in the numerical cognition network. Synced with commissural fibers such as inter-hemispheric IPS to IPS connections running transcallosally along the callosal isthmus and the forceps major, all these results further corroborate the proposition of the TCM of numerical cognition being subserved by fronto-parietal neural networks.

The specifications in this review of the WM pathways involved in numerical cognition also extend the TCM with respect to how GM areas associated with specific number-related representations (e.g., IPS: number magnitude vs. AG: arithmetic facts) may work together. The involvement of projection fibers such as the SCR (in particular the part of the thalamic radiation and the cortico-striatal tract) may revive the importance of the basal ganglia as well as the thalamus, which were already incorporated in early versions of the TCM by Dehaene and Cohen (1995, 1997). As a consequence, it is now possible to evaluate the idea that numerical impairments arise from WM disconnections between specific cortical areas in an individual brain (Kucian et al., 2014 for developmental DD; Klein et al., 2013a for acquired acalculia). This is in line with considerations for deficits in other domains (e.g., Schlaug et al., 2009 for aphasia and Rusconi et al., 2009 for a disconnection account of the Gerstmann syndrome).

These new insights into the WM correlates of numerical cognition also come with methodological as well as theoretical implications for future studies on brain hodology underlying numerical cognition. On the methodological side it would be interesting to combine different approaches to obtain a more comprehensive picture of the neural WM and GM correlates of numerical cognition. For instance, functional and structural connectivity analyses may be complemented by fMRI and voxel-based morphometry (see Rykhlevskaia et al., 2009, for a first attempt). Additionally, the ultrahigh-resolution 3D-model of the human brain ("BigBrain") provides unprecedented information on the interconnection of cortical regions (Amunts et al., 2013). Taken together, considering brain connectivity not only seems inevitably mandatory in order to understand human numerical cognition but it also opens up new avenues for future research.

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Mathematical difficulties as decoupling of expectation and developmental trajectories

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Recent years have seen an increase in research articles and reviews exploring mathematical difficulties (MD). Many of these articles have set out to explain the etiology of the problems, the possibility of different subtypes, and potential brain regions that underlie many of the observable behaviors. These articles are very valuable in a research field, which many have noted, falls behind that of reading and language disabilities. Here will provide a perspective on the current understanding of MD from a different angle, by outlining the school curriculum of England and the US and connecting these to the skills needed at different stages of mathematical understanding. We will extend this to explore the cognitive skills which most likely underpin these different stages and whose impairment may thus lead to mathematics difficulties at all stages of mathematics development. To conclude we will briefly explore interventions that are currently available, indicating whether these can be used to aid the different children at different stages of their mathematical development and what their current limitations may be. The principal aim of this review is to establish an explicit connection between the academic discourse, with its research base and concepts, and the developmental trajectory of abstract mathematical skills that is expected (and somewhat dictated) in formal education. This will possibly help to highlight and make sense of the gap between the complexity of the MD range in real life and the state of its academic science.

Keywords: mathematical difficulties, mathematical development, children, school curriculum, intervention

There has been increasing interest in *mathematical difficulties* (MD) particularly as government departments seek to understand why countries such as the US and the UK have low levels of functional numeracy. For example, Gross et al. (2009) found that around 25% of those able to work in the UK do not have essential mathematical skills, and Parsons and Bynner (2005) reported that those with poor numeracy were twice as likely to be unemployed as those with competent levels. These low levels of attainment have been linked with the developmental disability dyscalculia where low mathematics achievement stands against a background of otherwise normal skills (e.g., language, memory, visuo-spatial attention, etc.), and is characterized as a primary impairment of number skills (Butterworth, 2005, 2010). However it seems unlikely that dyscalculia alone can account for the findings as its prevalence rates range between 1.3 and 10.3% (Devine et al., 2013). It seems thus likely that a large proportion of those with poor numeracy would instead have MD, which we theorize encompass a range of mathematical learning shortcomings that may manifest at various developmental stages and originate in a variety of underlying causes. To explore at what point in time these diverse difficulties may impact on mathematic ability, this review will explore what children are expected to learn as they go through the mathematics curriculum. Clarity on the "expectation trajectory" should prove extremely useful in a research field where no comprehensive and consensus model of a "developmental trajectory" is yet available. We will then identify a recent model that has outlined the basic cognitive components involved in mathematical skill development. This model lends itself naturally to identifying individual causes of MD, especially when difficulties are conceptualized as a decoupling between developmental and expectation trajectory at different stages in formal education.

DEFINING MD

Although most people, when asked, will report having struggled with mathematics at some point in their lives, objective difficulties with the learning of mathematics are said to present when mathematical achievement is significantly lower than the average obtained by the appropriate age group. Official figures of attainment seem to suggest that in the UK 10% of children in formal education do not reach the required standards by ages 7 and 11 (DfES, 2012), and in the US 18% of children in formal education do not reach the required standards by ages 10 and 14 (National Center for Education Statistics, 2011; see also *Mathematics Curriculum* section). Individual achievement can be measured against consensus ideal standards (e.g., Common Core Standards for Mathematics, accessed August 9 2013, http://www.corestandards.org/Math/Practice), overall class achievement, or standardized tests. The latter are often based on academic and/or pedagogical models of mathematical cognition (e.g., Wide Range

Achievement Test-Revised (WRAT-R; Jastak and Wilkinson, 1984; Woodcock Johnson-Revised (WJ-R) Calculation and Applied Problems subtests; Woodcock and Johnson, 1989).

In developmental research children are typically selected as MD from a single assessment of their mathematical ability. One problem with selecting children this way is that there is little consensus about the method of selection. Some have included children who show a discrepancy between IQ and mathematics performance (e.g., Lindsay et al., 2001), but more commonly researchers have applied a cut-off criterion where children who perform below a given percentile on a standardized measure of mathematics achievement are defined as having MD (e.g., Geary et al., 2000; Butterworth, 2003; Szucs et al., 2013). Despite the popularity of a cut-off selection there is little consensus about at what level the cut-off criterion should be set and this can lead to differing cognitive profiles emerging from different research studies (Murphy et al., 2007). Another problem with selection based on a single assessment is that mathematics requires a range of different skills and these skills are different depending upon the child's expected stage of their mathematical development, so the same MD label may in principle indicate very different profiles. For example, MD at Grade 1 could indicate inability to use place value and perform simple additions and subtractions, whereas a classification of MD at Grade 5 could either indicate inability to translate numerical information into a Cartesian framework and solve geometrical problems or still indicate lack of more basic skills such as fluency in arithmetic operations (see e.g., **Figure 1**). In other words, inclusion in a MD group can be a reflection of these different stages of their understanding. The tests used for screening may differ for different age groups to reflect the expected stage of their development and so within-cohort differences are mostly meaningful in relation to age of the children and curriculum standards. Moreover, approximately 30% of individuals who are classified as having some sort of MD at any one time will not remain in the same category (i.e., they will not be classified as MD at further testing) over time (Silver et al., 1999; Mazzocco and Myers, 2003). Repeated testing however is extremely infrequent in practice and only a few targeted longitudinal studies have been conducted so far (e.g., Geary et al., 2000; Jordan et al., 2002, 2003; Vukovic and Siegel, 2010).

A host of persistent and or temporary factors have been proposed as the underlying cause of MD but no universal consensus risk assessment, prognostic and rehabilitative model is available as yet. Severe MD are known to be associated with psychological, neurological, and genetic conditions, such as epilepsy, Turner's syndrome, fragile X syndrome, phenylketonuria and ADHD (Shalev et al., 2000). Furthermore MD are often co-morbid with delayed language development and behavioral disorders (e.g., Manor et al., 2001). In cases where MD are co-morbid with other learning difficulties or pathological conditions, it is very difficult to discriminate whether the mathematics impairment is a primary deficit or whether it is instead due to a deficit in other functions. But even in cases where MD do not seem to be associated with any other conditions and a clear achievement-potential discrepancy is found in otherwise non-problematic individuals, several potential mechanisms may be at play. Despite this, no empirically-based normative developmental trajectory for mathematics learning has been yet established, and the non-problematic range of variation for age-appropriate levels is currently untested (see also Szucs and Goswami, 2013).

MATHEMATICS CURRICULUM

It is thus informative to take a closer look at template "expected developmental trajectories" of recent formulation, setting the standard against which individual or cohort performance will be contrasted within and between schools in a near future. Amount of tolerance toward performance deviance from the standards will probably depend on school-specific pedagogical and curriculum choices and on the average achievement levels of the cohort involved. Professional diagnoses of MD, often based on standardized tests will then typically follow in the most severe cases and dedicated support staff may be called in. It is however important to bear in mind that difficulties with math are unlikely to receive the same support as language difficulties, due to the only relatively recent awakening of public awareness (e.g., Bynner and Parsons, 1997). In this section we provide an overview of the guiding principles and targets behind the mathematics curriculum for England and the US and explore the impact that curricula can have on MD by setting age-appropriate targets.

We have chosen to provide descriptions of the English and US curriculum as both governments have developed very clear standards. For those interested in a comparative perspective, The Trends in International Mathematics and Science Study (TIMSS) has made some comparisons on curricula in many different countries but it is beyond the scope of this review to outline each of these and we thus redirect the reader to the latest report (TIMSS 2011 International Results in Mathematics, accessed 20 September 2013, http://timssandpirls.bc.edu/timss2011/downloads/T11_IR_Mathematics_FullBook.pdf). We will first describe two different curricula and compare their approach to learning this multi-layered skill. We look at what each child is expected to have achieved by the end of each level or grade, thus showing what cognitive components may impact at different stages of mathematics development. This discussion comes at a critical time as the national curriculum for mathematics in England has just completed its consultation period at the Department for Education and is due to be implemented in September 2014. The US is already in the stages of implementing a new curriculum and it is currently under adoption in 45 states.

ENGLAND

In England, all children in state funded schools are measured on their academic progress at 4 stages in their school career (approximately age 7, 11, 14, and 16). These are known as Key Stages 1–4 (KS1-4). Mathematics is a compulsory national curriculum subject at all 4 key stages (see **Table 1**; a full description of the curriculum can be found at http://www.education.gov.uk/schools/teachingandlearning/curriculum/primary/b00199044/mathematics). Assessment within the first three Key Stages is measured in levels. A child can reach one of 3 different levels of achievement (i.e., typically Level 1–3 for Key Stage 1, Level 4–6 for Key Stage 2 and Level 5–7 for Key Stage 3, although there may be overlaps such as children leaving KS 1

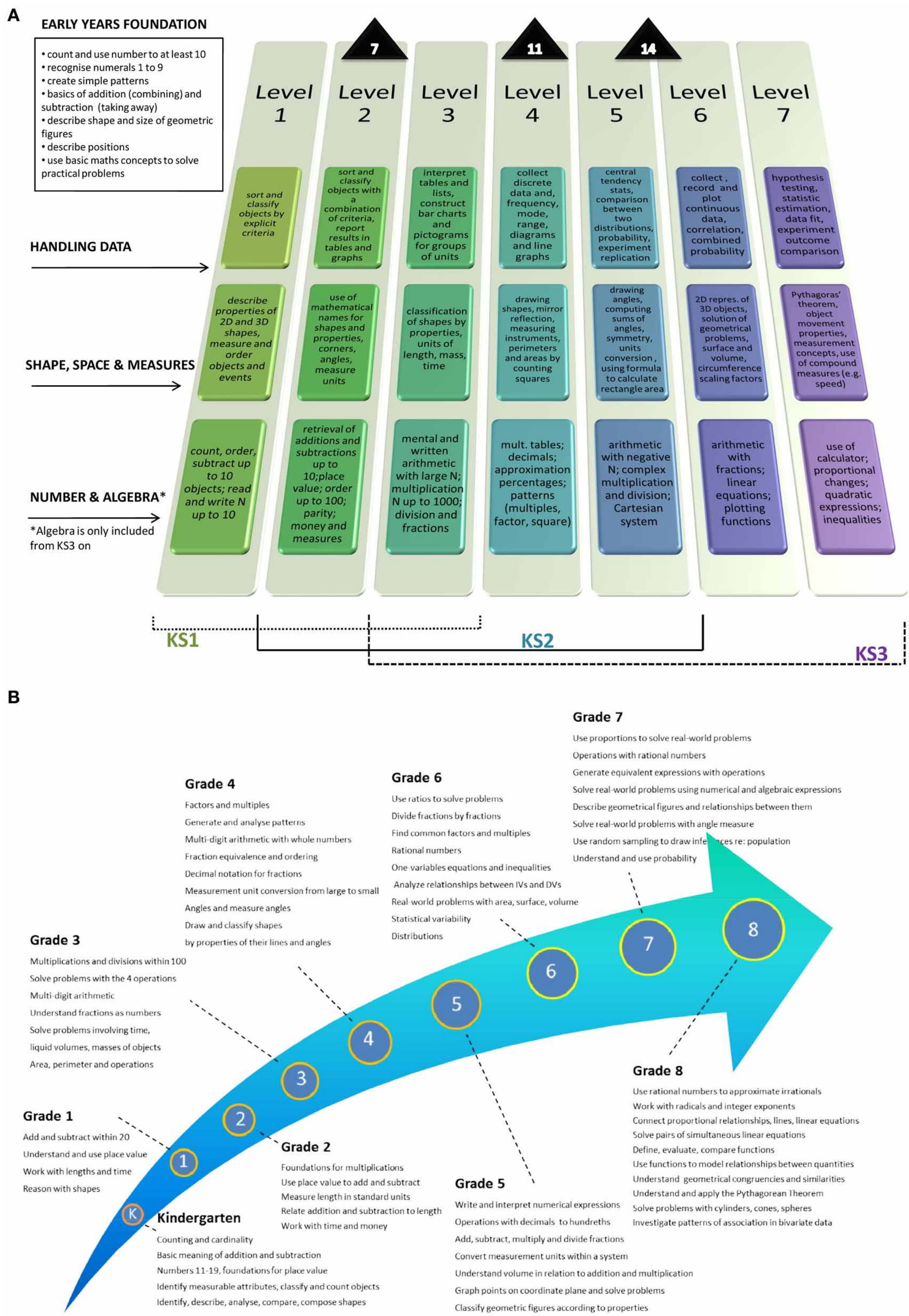

FIGURE 1 | (A) Expectation trajectory with attainment targets for Number and Algebra, Shape, Space, and Measures, and Handling Data from the UK curriculum. On average, children reach Level 2 at age 7, Level 4 at age 11 and Level 5–6 at age 14. **(B)** Expectation trajectory with targets by grade from the Core Standards.

Table 1 | Breakdown of Key Stages in the English curriculum per school year and chronological age.

Chronological ages	School years	Key stage
3–5	Pre-school/reception	EYFS
5–7	1–2	1
7–11	3–6	2
11–14	7–9	3
14–16	10–11	4

at Level 4 or only achieving Level 3 at the end of KS 2), ranked in ascending order of skill complexity (see **Figure 1A**). In addition, mathematics is included for 3–5 year old children within the Early Years Foundation Stage (EYFS). This review concentrates on this EYFS, as well as KS 1 and 2 as this encompasses the ages (3–11 years) where most research has focused on MD, however a brief description of KS3 is included as this includes math skills that some exceptional children in primary school can work toward.

At the EYFS, children are introduced to mathematics through guidelines set out in the Problem Solving, Reasoning, and Numeracy framework (DCSF, 2008). Within this framework, there are skills outlined for using numbers as labels and for counting (e.g., using number names accurately, counting up to four and beyond and recognizing numerals); calculating (e.g., using the vocabulary involved in adding and subtracting, understanding "more" and "less" to compare two numbers, relating addition to combining two groups and subtraction to "taking away"); and shapes, spaces, and measures (e.g., using language to compare quantities, talk about, recognize and recreate simple patterns, using words to describe position). Both formative and summative assessment of these skills is recorded in each child's Early Learning Profile. Early years practitioners are encouraged to use play as part of the child's learning activities and the focus is on providing the basic skills necessary to make the transitions into KS1.

At all stages there is a general attainment target for using and applying mathematics. However this does not have detailed standards and is included to ensure that teachers instruct students about the connections between different areas of mathematical knowledge (National Curriculum for England Mathematics, 1999, p. 6). The skills measured at KS1 come under two broad sections called number and shape, and space and measures. Under each section the curriculum outlines a number of standards that set out detailed targets. For example, within the numbers section, the target for counting states that "Pupils should be taught to count reliably up to 20 objects at first and recognize that if the objects are rearranged the number stays the same; be familiar with the numbers 11–20; gradually extend counting to 100 and beyond" (National Curriculum for England Mathematics, 1999, p. 16). The target for number patterns and sequences is to "create and describe number patterns" and use this knowledge to make predictions. This includes patterns of multiples of 2, 5, and 10, sequences of odd and even numbers and the relationship between halving and doubling (National Curriculum for England Mathematics, 1999, p. 16). At KS1, academic performance is assessed via individual teacher assessment against the National Curriculum Attainment Targets rather than by examination and pupils is expected to achieve KS1 level 2. The latest government figures were published in 2011 and they show that 90% of children were achieving the expected level.

At KS2 the skills measured are number; shape, space, and measures, handling data, and mental arithmetic. Again each section has standards with associated targets. For example, the target for counting states that "Pupils should be able to count on and back in tens or hundreds from any two- or three-digit number; recognize and continue number sequences formed by counting on or back in steps of constant size from any integer, extending to negative integers when counting back" (National Curriculum for England Mathematics, 1999, p. 21). Within the handling data, the targets for processing, representing and interpreting data include "interpreting tables, lists and charts used in everyday life; constructing and interpret frequency tables, representing and interpreting discrete data using graphs and diagrams (National Curriculum for England Mathematics, 1999, p. 27). Pupils at KS2 have formal assessments in the final year of primary school and this provides information about the children's math performance before they move onto secondary schooling and KS 3 and 4. Pupils are expected to reach a KS2 Level 4 standard in mathematics and schools were set a target to ensure 60 per cent of pupils achieve this standard (DfES, 2010). In 2011, the percentage of pupils attaining level 4 or above at KS2 was 84% (DfES, 2012). Whilst this may seem a high level, there were still a substantial number of schools with attainment below the 60% target that suggests that many children are not achieving the necessary skills in mathematics before they progress to secondary school education.

The skills measured at KS3 come under the same broad topic headings as at KS2. As expected, the level of difficulty and range of skills required increases. For example, within the handing data topic children are expected to move up to a level of understanding where they can use statistical calculations and begin to use probability. Again each section has standards with associated targets. Similar to KS1, pupils are assessed through teacher assessment and pupils are expected to reach either KS3 Level 5 or 6 in mathematics (National Curriculum for England Mathematics, 1999, p. 7). In 2011, the percentage of pupils attaining level 5 or above at KS2 was 81% (DfES, 2012). A more detailed representation of the attainment targets at each level with Key stages 1–3 is presented in **Figure 1A**; using and applying mathematics is not included in this figures as there the attainment targets are not as detailed as the other sections.

UNITED STATES

In contrast to the English system, the mathematics curriculum in the US has been largely up to individual states (before introduction of the new curriculum); there was no common curriculum. Nevertheless performance on these diverse curricula has been assessed by the National Assessment of Educational Progress (NAEP) at Grades 4 (age 9–10 years). In 2011, a nationally representative sample of 209,000 children from 21 urban districts of the US were assessed on five content areas: number properties and operations (e.g., computation with or understanding of whole numbers and common fractions and decimals), measurement (e.g., knowledge of units of measurement for capacity, length, area, volume and time), geometry (e.g., knowledge and understanding of simple shapes, and relationships between shapes such as symmetry and transformations), data analysis, statistics, and probability (e.g., understanding data collection and organization, reading and interpreting representations of data, and basic concepts of probability), and algebra (e.g., understanding of algebraic representation, patterns, and rules; graphing points on a line or a grid; and using symbols to represent unknown quantities). This found that 82% of pupils were classed as at or above basic in mathematics, which suggests that by age 9 years, 18% of children experience some form of difficulty learning mathematics. Furthermore, a similar assessment conducted with over 175,000 eighth-graders (age 13–14 years) found that the percentage at or above basic levels drops to 73% (National Center for Education Statistics, 2011). Thus it appears that the number of children with difficulties increases as they progress through the curriculum. Although it is not possible to compare performance between the English and US children because the measures of assessment differ considerably, as well as the ages of the children, it is clear that a significant number of children in both countries are not achieving attainment targets in mathematics. Furthermore, statistics collected at later stages of schooling show that performance drops further as children progress through their schooling and gets decoupled from the expectation trajectory in about 20–30% of the children (National Center for Education Statistics, 2011).

Given that the evidence suggests that there is decoupling, it is worth describing here that a new Common Core State Standards Initiative (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010) has recently proposed a math curriculum that is to be adopted by the majority of states from 2014 (see **Figure 1B**). This curriculum lays out the mathematics content that should be learned at each grade level from kindergarten to Grade 8 (see http://www.corestandards.org/Math for a full description of the curriculum). Educators in the US and elsewhere have found it necessary to redefine what students should be able to understand and do when learning mathematics. They defined common core standards, while recognizing that the assumption that what is learnt before should determine what is learnt at a later stage is unwarranted, given the current state of the science. At the moment, indeed, only partial models of learning pathways to mathematical concepts and skills can be obtained from scientific and education research, with very few exceptions (see e.g., LeFevre et al., 2010). The criteria for the standards were developed from academic research; analyses of which skills are required of students entering college and workforce training programs and by looking at standards from high achieving nations and data from the TIMSS in collaboration with some of the teaching bodies within the US. For the purpose of this review, which concentrates on children up to age 11, we will report the four key domains: Operations and Algebraic Thinking; Number and Operations in Base 10; Measurement and Data; Geometry. In addition Counting and Cardinality is included for Kindergarten and Number and Operations is included for Grades 3 and 5. Within each domain, there are several standards, clustered into related standards. For example, during Kindergarten, within the domain of Counting and Cardinality, children are expected to acquire number names and the count sequence sufficiently to count up and determine the number of objects in a set and to compare numbers; within Operations and Algebraic thinking they should understand the concept of addition as putting together and adding to, and subtraction as taking apart and taking from; within Number and Operations in Base 10 they should be able to work with numbers up to 19 and begin to understand place value; within Measurement and Data, they should be able to describe and compare measurable attributes such as length or weight, and classify objects and count the number of objects in categories; and within Geometry, they should be able to identify and describe shapes such as squares, triangles and circles as well as analyze, compare, create, and compose shapes (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010; Common Core Standards for Mathematics, p. 10). The first assessments of this new curriculum are due to begin in the 2014–2015 school year.

One of the guiding teaching principles made explicit by the Core Standards is teachers' focus on "mathematical understanding" as the royal pathway, along with procedural learning, to meaningful achievement. "There is a world difference between a student who can summon a mnemonic device to expand a product such as $(a + b)(x + y)$ and a student who can explain where the mnemonic comes from. [The latter] may have a better chance to succeed to a less familiar task such as expanding $(a + b + c)(x + y)$." (Common Core State Standard Initiative for Mathematics, 2010, p. 4)

Note that the proposed assessment is tightly connected with this definition of "understanding." If assessment is only focused on the ability to reach and provide the correct solution to a given problem, it will often confound procedural or mechanic learning with mathematical understanding. Specifically assessing mathematical understanding means assessing: (1) the ability to generalize knowledge to novel situations, (2) the ability to explain the underlying meaning of procedures.

- Difficulty with math is not only defined by an inability to follow the procedure but may unveil a deeper problem (i.e., a lack of mathematical understanding)
- To assess learning, we needn't focus exclusively on achievement: apparently normal achievement at one stage, may still lead to later difficulties with numbers, if it is exclusively driven by procedural learning
- Difficulties may appear at a later stage due to lack of proper understanding at earlier stages; a child who shows MD at Grade 7 may not have understood concepts from Grade 5 and 6 despite normal achievement.

To illustrate how development is thought to develop, **Figure 1B** shows the trajectory of US children following the new Common Core State Standards curriculum.

COMPARING CURRICULA

Both the current English and new US curricula provide clear and detailed targets for arithmetic development and how children will build up an understanding of this complex discipline, and there are many commonalities between them. For example, they both have a strong focus on counting and place value within the early years and use this skill as a basis for progression onto calculations. Calculations are conducted first with single digit and then multi-digit numbers. They both also have a focus on shapes, space, and measurement that begins in early years and is included at all levels of the curriculum. However there are also differences that may impact on the selection of MD children. The US curriculum is much more strongly focused on number and operations within the early grades and purposely does not introduce additional topics until later schooling (The Hunt Institute, 2011). The idea behind this was to achieve an intricate grounding in these skills which can then be taken forward to new skills later in the trajectory. There is also more emphasis on conceptual understanding than rote procedural learning. The English curriculum does introduce these other topics. For example it has a strong emphasis on patterns. Even at KS1, children are expected to be able to "create and describe number patterns" whereas within the US curriculum patterns are not included as a standard until Grade 4 (approximately 9–10 years of age). Another contrast is that the English curriculum outlines the use of mental models for calculation. The target is to develop rapid recall of number facts and procedures; a target which calls directly on memory processes.

EXPECTATION TRAJECTORY AND IMPLIED COGNITIVE SKILLS

In the remainder of this article we will provide an overview of one of the mathematical development models that, in our opinion, looks promising for the identification of potential causes and areas of intervention in MD. These can occur at any stage of primary schooling when there is a decoupling between the general expectation and an individual's actual developmental trajectory.

Although our knowledge on mathematical learning and cognition has enormously expanded in the last few decades, there is no consensus or comprehensive developmental trajectory for mathematical skills, let alone a consensus model on MD. Grade placements for specific topics are therefore suggested on the basis of national and international comparisons, educators' collective experience, and researchers' and mathematicians' professional judgment. By establishing a standard set of principles and objectives on such basis, the initiative opens to the possibility of improving the process on a large scale as research on learning and effectiveness progresses.

The expected developmental trajectory, based on a consensus between education professionals (**Figures 1A,B**) rather than academic research output and theoretical models, will thus continue to set the main standard against which a given individual or cohort will contrast their performance and will be deemed as having MD or not in the years to come. The amount of tolerance toward deviance from the standards (and thus criteria for MD) is likely to be influenced by school-specific pedagogical and curriculum choices and also on the average achievement levels of the cohort involved. Once the most severe cases are identified as such, professional diagnoses of MD will then be typically undertaken and dedicated support staff may be called in—although difficulties with mathematics are unlikely to receive the same support as language difficulties (Butterworth et al., 2011).

The most straightforward type of MD (or MD risk) diagnosis is probably the one done at the earliest stages such as Kindergarten and Grade 1, where although multiple cognitive skills are already interacting to enable numerical understanding, such understanding is still very far removed from of the level of abstraction expected in later years. Typically diagnosis even at this early stage follows from a child's poor performance on a standardized test of mathematics especially in comparison to performance on measures of other abilities such as reading or IQ (e.g., Geary et al., 2000, 2007; Murphy et al., 2007; Chu et al., 2013), although other studies suggest that screening using experimental measures of number sense such as approximate number system acuity or skills in several counting tasks may be suitable (e.g., Jordan et al., 2006, 2009; Chu et al., 2013). At this stage abstraction is very much rooted in and inferred from concrete experiences such as those outlined above in the Kindergarten Common Core standards. These imply very basic and foundational skills, part of which may rest on a core number processing toolkit and basic cognitive abilities we share with animals (see e.g., Gallistel, 1989; Butterworth, 1999; Kawai and Matsuzawa, 2000; Dehaene, 2001). Part already rests on an interaction with symbolic processing skills and independent functions such as language and spatial processing (see e.g., Jordan et al., 2009; Cirino, 2011). It is apparently on these very concrete and relatively simple foundations that abstract mathematics starts being taught and learnt. In following years and up to the adult stage, with increasing abstraction the picture becomes much more complex and difficult to decipher.

LeFevre et al. (2010) have recently proposed a model including multiple cognitive factors that may contribute to the developmental trajectory and determine an individual's mathematical outcomes throughout developmental stages. Their model is based on the triple-code neuropsychological model of adult numerical processing (Dehaene et al., 2003), one that has collected the widest consensus and empirical support in recent years. LeFevre et al.'s (2010) model provides a simple and promising framework that could be especially well suited to identify cognitive precursors that may become important to fulfill expectations at different stages of the developmental trajectory up to adult age. No doubt, such framework would benefit from further refinements and the inclusion of possible additional cognitive precursors (see **Figure 2**). However the model does lend itself well

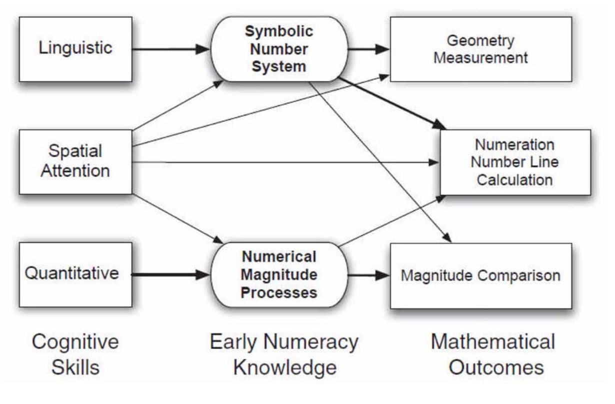

FIGURE 2 | Schematic of the Pathways model with predicted relations among cognitive precursors, early numeracy skills, and mathematical outcome measures. From LeFevre et al. (2010) Child Development. With permission. Note this is Canadian.

to the translation of academic concepts into educational targets by providing a theoretically-driven framework to evaluate and predict achievement targets. Moreover, by suggesting developmental pathways that are compatible with the guiding assumptions of much research in adult numeracy, knowledge will be easier to update and predictions about potential neural substrates could also be more easily derived from adult studies.

A USEFUL WORKING FRAMEWORK

LeFevre et al. (2010) have provided an initial test of their model in a longitudinal study with a large cohort of children from preschool and Kindergarten (aged between 4; 5 and 6; 6 years), following their progress in mathematics for 3 years. They hypothesized that linguistic, quantitative, and spatial attention pathways contribute independently to number skills and that they vary in their unique and relative contributions to mathematical outcomes, depending on task demands. Their test for basic quantitative knowledge (a cognitive precursor of more complex numeracy knowledge) was an object counting task with small sets of objects, and they used subitizing speed as a summary index (i.e., the speed in correctly recognizing numerosities from 1 to 3). Subitizing is generally considered a reasonable index of children's quantitative knowledge (see e.g., Landerl et al., 2004) although a visuo-spatial short-term memory component may also be at play (Feigenson et al., 2004). Linguistic skills were assessed via measures of vocabulary and phonological awareness (Dunn and Dunn, 1997; Wagner et al., 1999); and spatial attention skills were measured with an adaptation for children of the spatial span task (aka Corsi blocks test; see Passolunghi and Cornoldi, 2008). Each of these indexes, therefore, captured a complex of skills rather than a single element in relation with language, quantity (or numerosity) and space, while still maintaining some level of specificity.

As a measure of early numeracy skills, LeFevre et al., used the number of correct responses in single and multi-digit number naming from Arabic format and the percentage of correct trials in a non-linguistic arithmetic task on small quantities (e.g., mental operations between sets of objects; see Levine et al., 1992). These tasks are meant to maximally tap on either the linguistic or the quantity code (see **Figure 2**). Finally, as measures of mathematical outcomes at Grade 2, both standardized and more experimental tests were used: the Numeration, Geometry and Measurement subtests from the KeyMath Test-R (Connolly, 2000) and the Calculation tests from the WJ Tests of Achievement-R (Woodcock and Johnson, 1989) covering most of the skills required by Grade 1 and 2, a Number Line task (Laski and Siegler, 2007) requiring to place numbers in the appropriate position on a line whose extremes are labeled as 0 and 1000 and taken as a measure of coordination between symbolic and quantitative knowledge, and a comparison task between single digit numbers whose physical size was orthogonally varied with their numerical size, tapping on symbolic but especially quantitative knowledge (Landerl et al., 2004; Holloway and Ansari, 2009).

The thickness of the connectors in **Figure 2** between cognitive precursors (left-hand boxes) and numerical knowledge (central boxes) indicates the relative importance of the contribution of cognitive precursors to early numeracy skills measures as they emerged from a multiple regression analysis. Measures of linguistic skills predicted up to 30% of the variance in the symbolic number system task, whereas subitizing latency predicted up to 32% of the variance in the non-linguistic arithmetic task. Neither of them predicted performance in the alternative number task. Spatial attention was apparently involved in both the symbolic and the magnitude task (predicting 16 and 15% of the variance respectively). Three different factors, corresponding to the linguistic, spatial, and quantitative pathways, were entered in a further multiple regression analysis to assess their predictive power on the standardized and experimental mathematical outcomes (right-hand boxes). Overall they accounted for a substantial proportion of variability (26–56%) in both the conventional and the experimental outcome measures. As shown by connector thickness, the relative contribution of each pathway varies with the outcome considered.

The linguistic pathway (i.e., individual measures of vocabulary and phonological awareness and number naming) contributed to all mathematical outcomes, but especially those related with geometry and measurement, numeration and calculation (i.e., classical tests of school achievement) and the Number Line task. The spatial attention pathway resulted involved in all outcomes, except for the experimental magnitude comparison task, whereas the quantitative pathway was found to contribute to magnitude comparison, numeration, number line and calculation but not to geometry and measurement.

In summary, most mathematical outcome measures in the LeFevre et al. (2010) study, including standardized batteries, depend on the functioning of the symbolic number system, with a heavily linguistic component. Whether the symbolic number system may itself be related to the quantitative pathway was assessed with a more complex quantitative task, non-symbolic arithmetic by Gilmore et al. (2010). They suggested that "children's non-symbolic numerical abilities [. . .] appear to contribute to their achievement in mathematics primarily because they are associated with children's successful learning of number words and symbols, which figures prominently in [. . .] the kindergarten mathematics curriculum and the assessment of mathematical learning [. . .]" (Gilmore et al., 2010; p. 8). Furthermore, Gilmore et al. (2010), had partialled out the effect of literacy achievement and verbal intelligence. On the whole, these findings would suggest that primary impairments in the use of the symbolic system (Ansari, 2008) and/or linguistic deficits (Manor et al., 2001) will exert a pervasive negative effect on individual trajectories of mathematical achievement. For example, in the UK curriculum the rudiments of an abstract symbolic number system beyond 10 (i.e., beyond the number of fingers, typically used as concrete and intuitive representations for both cardinality and ordinality) constitute attainment targets at Level 2. Thus the developmental trajectory of pupils with impairment in the symbolic pathway could start diverging at about age 7 and become more and more decoupled from the expectation trajectory throughout schooling. The quantitative pathway alone, indeed, will become progressively inadequate to handle the abstraction of concepts and complexity of skills expected in later years (e.g., numbers of increasing size, mental arithmetic with two and three digit numbers, decimals, recognition of pattern in the number series, negative number arithmetic, linear and quadratic equations). Under the curriculum proposed by the Standards, difficulties would emerge even earlier, between Kindergarten and Grade 1 (i.e., ages 5 and 6), due to the early introduction of place value, with two-digit numbers and operations between them (see also **Figure 3**). MD from impairment in the symbolic system at age 5 in the US might therefore be diagnosed as MD at age 7 in the UK. Based on adult models of the role of language in number processing, specific impairments in the language system will particularly compromise the learning of rote memory arithmetic, and in particular multiplications and complex mental operations (Dehaene et al., 2003). Major difficulties will thus start to emerge when pupils reach US Grade 3 or UK attainment Level 3 (i.e., around the age of 8), particularly if children do not spontaneously discover alternative strategies to verbal representations. Teachers may also teach alternative strategies.

What could be considered as core numerosity processing (see e.g., Butterworth, 2010) or analogue magnitude processing (Dehaene et al., 2003), and whose selective impairment is thought to underlie developmental dyscalculia (e.g., Rubenstein and Henik, 2009) does not appear to contribute as importantly to tasks in the mathematics curriculum tapping geometry and measurement. In the presence of a problem in the quantitative system, achievement tests involving these tasks may therefore be relatively spared when compared to tasks of numeration, calculation, number comparison etc. from the earliest age. In a recent training study, Park and Brannon (2013) reported a relation between adult performance improvements in tasks tapping the approximate number system (a likely quantitative precursor), and performance improvements in corresponding symbolic arithmetic tasks. This connection between an approximate number system and mathematics proficiency would seem also to be domain-specific, and Dewind and Brannon (2012) showed that improvement in a comparison task involving the approximate number system did not generalize to a homologue visuo-spatial task. This calls the prediction of developmental dissociations between difficulties arising from impairments at the level of non-verbal, non-symbolic quantity processing and impairments in auxiliary domains—even if conceptually related with quantity, such as spatial processing—with the latter exerting more subtle and elusive effects on the developmental trajectory of number processing as opposed to measurement and geometry skills.

In LeFevre et al.'s (2010) study, acomplex measure of spatial attention and working memory was found to contribute to geometry and measurement, and all other outcome measures, except for symbolic magnitude comparison with single digits. Purely attentional deficits may therefore compromise most achievement tasks except for those primarily resting on the core quantitative pathway. MD will thus be more subtle than with those deriving from impairment in the symbolic and quantitative pathways, yet equally spread across attainment targets from UK Level 2 or US Grade 1, when place value and visuo-spatial geometric properties are taught. Additional strain may be put on the system when working toward UK Level 5 (or at US Grade 5), with the introduction of the Cartesian system and connections between number, geometry, and measurement. It is however also possible that this may in fact provide a novel and affordable method to parse space, thus improving these children's performance in

Cognitive Precursor*	K	G1	G2	G3	G4	G5
L	counting & ordinality; numbers 11-19 and place value; classify and count objects; describe and compose shapes	add and subtract within 20; understand and use place value; measure length; time & money	multiplications; multi-digit additions and subtractions; measure length in standard units; time & money	word problems; multi-digit arithmetic; fractions as numbers; basic physics and geometry problems	factors & multiples; patterns; decimal notation for fractions; measurement unit conversion; angles; shapes by properties	numerical expressions; operations with decimals and fractions; volume as addition and multiplication; Cartesian plane; geometric figures
S	counting & cardinality; basic meaning of addition and subtraction; numbers 11-19 and place value; classify and count objects; describe and compose shapes	add and subtract within 20; understand and use place value; measure length; time & money	multi-digit additions and subtractions; measure length in standard units; time & money	multi-digit arithmetic; fractions as numbers; word, physics and geometry problems	factors & multiples; patterns; fraction equivalence & ordering decimal notation for fractions; angles; shapes by properties	numerical expressions; operations with decimals and fractions; volume; Cartesian plane; geometric figures
Q	cardinality; basic meaning of addition and subtraction	time & money; measure length	measure length in standard units; time & money	understand fractions as numbers; physics problems	fraction equivalence & ordering	volume as addition and multiplication
B	counting & cardinality; basic meaning of addition and subtraction	add and subtract within 20	transition to more 'mature' calculation strategies			

*It is here assumed that all precursors remain available and may contribute at any stage, even when not indicated, as fall-back strategy, offload, route to deeper understanding and/or to achievement; however deficits in cognitive precursors may have different outcome at separate stages. L = linguistic pathway; S = spatial attention pathway; Q = quantitative pathway; B = body pathway

FIGURE 3 | Hypothetical coupling of expectation and developmental trajectory. Targets for Kindergarten and the first five grades have been associated with their most likely cognitive precursors on the basis of an expanded Pathways model (see LeFevre et al., 2010 and our section "Expectation Trajectory and Cognitive Skills").

mathematics. An interesting possibility that would need to be explored with *ad hoc* empirical studies.

It may be surprising that there is not an independent pathway for working memory within this framework as many researchers have found that performance on working memory measures can specifically predict mathematics performance (e.g., Holmes and Adams, 2006; Bull et al., 2008). Indeed research that has specifically explored the impact of working memory subsystems (phonological loop, visuo-spatial sketch pad (VSSP), and central executive) in the longitudinal development of mathematical learning has suggested that the VSSP may be important for younger children (e.g., Bull et al., 2008; Holmes et al., 2008; Simmons et al., 2008). One suggestion is that young children's mental representations of quantities rely heavily on visual-spatial representations, as they have not yet developed a spontaneous verbal rehearsal system. As children progress through school they increasingly use verbal representations of quantities such as number words and the role of the VSSP has less impact (see Rasmussen and Bisanz, 2005). One reason for apparent missing pathway is that LeFevre et al.'s description differs from others. For example, many studies have included variations of the Corsi Blocks task as their measure of visuo-spatial working memory. LeFevre et al., also used a version of this task but describe it as a spatial attention task due to problems distinguishing the nature of the task. Nevertheless LeFevre et al., note that a more detailed account about the role of working memory in mathematical learning may be necessary. In particular they suggest that working memory may play an important role in integrating knowledge from the linguistic and quantitative pathways.

More recently LeFevre et al.'s (2010) basic architecture was used as a working framework by Cirino (2011) who maintained the original conceptual distinctions but expanded the range of tasks (symbolic vs. non-symbolic) used to measure the effects of quantity precursors in Kindergarten on a single outcome measure (i.e., small written sums). Interestingly, the symbolic (with a strong linguistic element) vs. non-symbolic distinction between precursors of later mathematical outcomes was also highlighted by Jordan et al. (2006, 2009), who reported how children's socio-economic status defined by their family income level interacts with the symbolic/linguistic pathway (but also see Mejias and Schiltz, 2013). That is to say children from low-income families will enter primary school with an initial disadvantage due to poorer start-up symbolic/linguistic resources despite showing in most numerical tasks (e.g., verbal and non-verbal counting, verbal and non-verbal arithmetic, estimation, number patterns) similar growth trajectories as children from high-income families, with the notable exception of verbal story problems. Therefore, given the pervasive effects that the symbolic number processing pathway may exert on later mathematical outcome (see also Jordan et al., 2002), and their characterization of MD as difficulty in story problems and arithmetic combinations], children from low socio-economic backgrounds should be considered at higher risk for MD and intervention strategies could be specifically devised from Kindergarten. This point has also been corroborated more recently by Gilmore et al. (2010) who highlighted two factors contributing to mathematics achievement: a non-symbolic aptitude, which is essentially insensitive to differences in socio-economic status, and symbolic ability that may be responsible for the higher achievement levels found in association with higher socio-economic status. They point out that preschool exposure to conventional symbol systems is higher in higher for children of wealthy families (Jordan et al., 1992; Griffin and Case, 1996), therefore the achievement gap due to impoverished symbolic environment may be eliminated by targeted interventions at Kindergarten (Siegler and Ramani, 2008). Interestingly, such interventions may also provide useful evidence for the putative causal link between symbolic skills and the developmental trajectory of mathematics learning, which as of now can only be two co-varying variables due to the correlational character of the evidence reported above.

It has also been shown that reading difficulties predict lower number skills especially those implying verbal sequential factors, and should therefore be treated as risk factors too (Jordan et al., 2006). Manor et al. (2001) established a relation between developmental language disorders and measures of mathematical outcomes. In particular, both receptive and expressive language impairments were associated with low scores in reasoning principles and arithmetic operations. Only expressive deficits, however predicted poor performance in counting principles. Despite the sometimes different theoretical frameworks adopted by different research groups, especially regarding whether a single number core processing module or domain general skills are at the origin of difficulties with mathematics (see e.g., Bull et al., 2008; Locuniak and Jordan, 2008; Desoete et al., 2009; Geary et al., 2009), data suggest that mathematics difficulties at Kindergarten will persist and predict an atypical growth rate in the following years (Morgan et al., 2009). This is not to say that MD cannot appear after an otherwise normal developmental trajectory in successive years or a child who has experienced difficulties since Kindergarten cannot benefit from interventions at a later stage (the predictive power of early mathematics difficulties on later difficulties never reaches 100%, and MD can also appear at later stages).

ADDING NEW COMPONENTS TO THE ORIGINAL FRAMEWORK

As previously mentioned, the merit of LeFevre et al.'s (2010) model consists of bringing together in a simple but comprehensive framework the main cognitive modules that are expected to interact and inform mathematical outcomes across the normal developmental trajectory, rather than focusing on one single cognitive domain. It does so by paralleling a well-known neuropsychological model of adult mathematical cognition (Dehaene et al., 2005). Specifics about the component pathways and their operationalization's (e.g., whether approximate quantities and numerosity processing should be considered as partially independent subcomponents of the quantity pathway) can be improved by testing predictions and expanding the model's evidence base. Interesting connections with MD can be established by classifying groups of individuals (e.g., children with Williams syndrome vs. children with spina bifida vs. dyslexics) based on the pathway that may be most problematic. With the model, MDs characterized by different patterns of development-expectation decoupling and educational outcomes may be diagnosed and assessed (e.g., LeFevre et al., 2010). To this purpose, and based on the literature on both adult and developmental number processing, we would like to suggest that additional components may be useful to create a model to diagnose/assess/predict MD.

For example in addition to spatial and linguistic precursors, body representations may be related to numeracy skills, as suggested by the interactions between finger gnosis and number processing in both adults and children. In a few studies, MD was reportedly associated with impairments of finger gnosis, left-right orientation and writing, also known under the name of developmental Gerstmann syndrome (see e.g., Miller and Hynd, 2004). In the past, this was typically taken as evidence for a functional connection between all of these abilities and between finger gnosis and the development of number skills in particular (see e.g., Butterworth, 1999). The connection seems to be corroborated by the fact that acquired brain lesions localized to the left posterior hemisphere often produce the adult version of Gerstmann's syndrome (a cluster of neuropsychological symptoms characterized by left–right confusion, agraphia, acalculia, and finger agnosia; Gerstmann, 1940). Likewise, TMS studies have identified contiguous neural substrates in adult participants with causal effects on numerical processing, finger gnosis, and categorical left–right discrimination (Rusconi et al., 2005; Hirnstein et al., 2011). Without undermining the significance of these associations, Kleinschmidt and Rusconi (2011) have recently suggested that the Gerstmann functions, including finger gnosis and calculation may indeed be supported by a network of cortical regions in the left posterior parietal lobe whose intraparietal projections converge toward a common subcortical bottleneck location. A small and localized lesion to the bottleneck location will cause systematic association of symptoms. The adult version of Gerstmann's syndrome would thus be characterized as an anatomical syndrome, meaning that the four symptoms may not be functionally interdependent, and yet still subjected to the very same local neural efficiency parameters and maturation constraints. This may also suggest an anatomical mechanism for the somewhat elusive developmental version of Gerstmann syndrome and provide an additional cluster of non-numerical predictors—although not necessarily cognitive precursors - of mathematics achievement, that could help identify a neurofunctional locus for certain patterns of MD.

At least a transient phase of finger counting and finger calculation almost invariably precedes the mature mathematical cognition in the developmental trajectory, although educators may have different views on its utility (e.g., Moeller et al., 2011). In fact, the use of fingers to represent number is ubiquitous across ages and cultures (Dantzig, 1954; Butterworth, 1999). Children use finger counting as an initial strategy to understand and keep track of counting and calculate, even if this is often seen as just a very primitive strategy (Geary et al., 2007). Amputees and children with congenital agenesia of hands and fingers use phantom fingers as quantifiers (Poeck, 1964). Finger counting strategies also tend to be used by older children and adults with MD, to make up for deficient mental number representations. Furthermore, performance in tests of finger gnosis before formal schooling selectively predict mathematical outcomes at a later age (e.g., Fayol et al., 1998; Noël, 2005) and it has been reported that early finger training may improve numerical abilities at a later stage (Gracia-Bafalluy and Noël, 2008; but see Fischer, 2010). According to a very popular idea, these latter findings are consistent with numerical knowledge being represented together with the same sensory and motor features that are engaged during learning (see e.g., Fischer, 2012). There is indeed empirical evidence that traces of finger counting habits influences—not necessarily always in a beneficial way—symbolic number representations and calculation processes (Domahs et al., 2008, 2010; also see Fischer and Brugger, 2011) for a review on other relevant interactions). Another possibility is that the crosstalk between numerical and body representations is not integral to numerical representations but provides a means to offload and free working memory resources while processing numerical information in a task-dependent way (e.g., Fischer, 2006). Of relevance to this context, however, are not the exact mechanisms underlying the cross-talk between fingers and numbers and whether traces of finger processing are indeed integral to the numerical representation. The consensus and empirical evidence that finger counting does play a role in the development of numerical skills could thus suggest a useful expansion and improve the predictive power of the LeFevre et al. (2010) model by including a dedicated body representation component amongst the cognitive precursors.

In addition, as noted above, although working memory is included in the LeFevre et al. (2010) framework as part of spatial attention pathway, an additional or more detailed component may be required to address the more complex aspects of mathematical development. For example, LeFevre at al. note that working memory may be involved in the coordination of information from the world and from memory. The central executive is usually considered the working memory subsystem responsible for coordinating information, including controlling attentional resources (Baddeley, 2003). There have been many studies which have shown that children with MD show impaired performance compared to typically developing children on tasks which are designed to tap into central executive processes (e.g., Bull et al., 1999; Geary et al., 2004; McLean and Hitch, 1999). However the role of the central executive within mathematical development is less well understood. Recently, LeFevre et al. (2013) examined executive attention, which they suggest encompasses executive functioning and the central executive in working memory, in children's development of mathematics. Children completed executive attentional tasks and mathematical tasks (specifically tasks on knowledge of the number system and arithmetic fluency) at either 8 or 9 years of age. They repeated the mathematical tasks 1 year later. Using structural equational modeling, LeFevre et al., showed that executive attention was concurrently predictive of both knowledge and fluency but predicted growth in performance only for fluency. LeFevre et al., conclude that the executive functioning may be particularly important in the early years of mathematical development when new tasks are being taught and learned. We would also expect that executive functioning, rather than being a cognitive precursor, may play a crucial role in integrating knowledge from the linguistic and quantitative pathways.

In **Figure 3** we show how the model could be used to draw predictions on the cognitive abilities that are necessary at each developmental stage as specified in the Core Standards. This in turn will suggest what cognitive problems may subtend MD at different stages.

INTERVENTIONS

One may assume that precise knowledge of the mechanism(s) underlying an individual's difficulties with mathematics may be a prerequisite for devising tailored teaching, remediation and intervention strategies. Therefore it is important to have a well-defined model that can encapsulate where difficulties may occur and how remediation can pinpoint these difficulties.

All researchers and educators agree that mathematic competence is not a single well-defined skill but encompasses a range of skills. What is clear from the expectation trajectory is that low attainment, particularly measured at single assessment, can also reflect a single or multitude of difficulties with mathematical concepts. The evidence suggests that although there may be around 2–10% of the population with the severe specific difficulty dyscalculia, it is also likely that the 15–20% described in the Parsons and Bynner (2005) report have difficulties with only certain aspects of mathematics. These difficulties may be sufficient to hinder their education and employment prospects. Of course there may be other reasons for low achieving population such as math anxiety (e.g., Ashcraft, 2002) or poor teaching. However it is often difficult to disentangle these from poor attainment. In addition, as noted above, some problems may be due to co-morbid developmental disorders such as dyslexia or ADHD (Rubenstein and Henik, 2009). Nevertheless even those with co-morbid conditions will have difficulties which impact at different stages of mathematic development.

Thus the question remains on the best way to assist those who do have underlying difficulties. In a series of well designed empirical studies, Fuchs and colleagues (see Fuchs et al., 2009, 2013; Powell et al., 2009; Powell and Fuchs, 2010) developed and tested the effects of extensive training of children with MD on targeted foundational skills, for example counting or retrieval, on typical math achievement tasks (e.g., Number Combinations). They derive the rationale for their interventions (which are also available commercially as the software Pirate Math) from existing empirical evidence linking specific foundational skills with more complex math and curriculum targets. Dowker (2009) noted that there has been an increase in the number of intervention programs as the government and charities highlight problems in numeracy. Dowker is very clear in her recommendations that any intervention should be individualized to reflect the fact that math is a multi-layered skill and difficulties can occur at different stages. Recent reviews of the efficacy of interventions for students who are showing signs of struggling with numeracy (e.g., Kroeger et al., 2012), have tried to assess a selection of the current range of interventions available and suggest future directions. Kroeger et al. (2012) evaluated 20 commercially available programs (mostly available in the US) by exploring whether each program was developed from neuroscientific research, what cognitive processes were targeted by the program, and the kind of research that supported the program. They explicitly implemented this approach because they believe that the most effective intervention practices would integrate research from neuroscientists and cognitive developmental psychologists as well as math educators. In particular it has been shown that the impact of neuroscientific data can influence the general public perception of research, including interventions, as brain research appears more compelling than behavioral data (Weisberg et al., 2008). However, for an intervention to be deemed successful it must build on evidence from all three fields.

Kroeger et al. (2012) found that only three programs included publisher-reported use of neuroscience research in their development, and here they focused on the triple code model (Dehaene et al., 2003). These were Fluency and Automaticity through Systematic Teaching with Technology (FASTT Math), Number Worlds (NW), and The Number Race (NR). In addition only FASTT Math, NW and NR plus two others (Accelerated Math (AM), Corrective Mathematics (CM) were supported by empirical, peer-reviewed research on their efficacy. Their review concluded that although 4 of these 5 intervention programs showed improvements on test scores, the programs emphasize representation of number sense, akin to the quantitative pathway, math facts and working memory. For example, in the NR, quantitative pathways and math facts are trained. Children play a computer game that requires them to first carry out a numerical comparison task; they must choose the larger of two quantities of treasure faster than a competitor. The competitor is essentially the computer program represented by a character on the screen, and the difference in magnitude between the two quantities can be large or small to manipulate difficulty. Furthermore the quantity can be represented in a non-symbolic format, sets of gold pieces, in symbolic Arabic numerals or symbolic number words. Presenting the numerical information in different ways is designed to strengthen links between representations of number (Wilson et al., 2006). At a higher level of difficulty, the quantities can only be worked out on completion of arithmetic problems (e.g., is 6–2 bigger than $4 + 0$?). On completion of the comparison task, the game moves counting task. The set of treasure they chose in the comparison task is placed next to treasure from a competitor. The child then races against their competitor by moving the same number of squares in grid as they have pieces of treasure. This is done by counting each piece of treasure one at a time and hence loading on one-to-one correspondence and cardinality. Kroeger et al.,note that few intervention programs have focused on problem solving or executive function although the CM program may load onto both of these as it attempts to teach students rules and strategies to help solve arithmetic problems. Executive function is potentially one cognitive process that underlies mathematical understanding and, in general, it appears that apart from number sense there is little intervention targeting the underlying cognitive processes. To mesh with our expectation trajectory—intervention could be targeted where a difficulty is found.

Another commercially available intervention, which has been developed in the UK and not included in Kroeger et al., is Catch Up Numeracy. In this intervention program children are individually assessed and provided with targeted sessions building on their strengths and weaknesses. This is low intensity intervention program and children complete just two 15 min remediation sessions a week. However it has shown some good improvements with children who have weaknesses in their math performance but are not necessarily dyscalculic. For example, Holmes and

Dowker (2013) showed that children identified as MD who followed the Catch Up program for 30 weeks showed significant gains in their numeracy. These gains were twice as large as other children with MD who had received no intervention and more than the gains expected from typically developing children.

One of the interventions mentioned here, the Number Race (http://www.thenumberrace.com), is freely available and is very explicitly connected with the adult neuropsychological model that also shaped LeFevre et al.'s (2010) framework. More recently the researchers have added another game to develop fluency in arithmetic, the Number Catcher (http://www.thenumbercatcher.com/). Peer reviewed research is not available for the Number Catcher but research in the Number Race suggests effects on core numerical processing. For example, Räsänen et al. (2009) tested 30 preschoolers who had been identified as having poor numeracy skills and compared them to 30 typically developing children. Half the children followed a software program called Graphogame-Math that trains children to compare small numerical differences; and half the children played the Number Race games. After 3 weeks of playing the games 10–15 min a day, both experimental groups demonstrated improved performance in number comparison but did not improve in other number skills such as verbal and object counting. The developers thus suggest that their programs should be used in conjunction with other techniques. Both the Number Race and Number Catcher do make use of game software to engage children with mathematics. This application of games for an educational purpose, or gamification, is becoming an increasingly more popular way to motivate learners (e.g., Deterding, 2012).

In summary, there is growing demand and concurrent development of interventions for math difficulties. There is some evidence to suggest that intervention can improve scores on mathematics tests but also a warning that the intervention is not targeted sufficiently at the underlying cognitive skills of mathematics nor designed for individuals who may show differing profiles of difficulties. Future interventions should draw upon the growing body of evidence that mathematic difficulties can occur at different stages and for different underlying reasons. Some existing intervention programs might eventually lead to significant improvements in mathematical understanding if the program attempts to pinpoint specific skills that are required for mathematical competence. That should be attuned to different cognitive pathways and combination of skills at every developmental stage (e.g., **Figure 3**) and should be conducted within a theoretically-driven framework. In this way, applied can also be used to feed back into theory and contribute with new knowledge toward the delineation of an empirically-based developmental trajectory.

AUTHOR CONTRIBUTIONS

The authors contributed equally to the writing of this paper. The order of authorship is arbitrary.

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Considering digits in a current model of numerical development

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Numerical cognition has long been considered the perfect example of abstract information processing. Nevertheless, there is accumulating evidence in recent years suggesting that the representation of number magnitude may not be entirely abstract but may present a specific case of embodied cognition rooted in the sensory and bodily experiences of early finger counting and calculating. However, so far none of the existing models of numerical development considers the influence of finger-based representations. Therefore, we make first suggestions on (i) how finger-based representations may be integrated into a current model of numerical development; and (ii) how they might corroborate the acquisition of basic numerical competencies at different development levels.

Keywords: mathematical cognition, numerical development, embodied cognition, finger-based representation, finger counting

INTRODUCTION

The mental representation of number magnitude is often seen as the perfect example of what is called abstract. This seems reasonable as the quantity information conveyed by any number is independent of the characteristics of the objects in the set denoted, such as size, color, weight, etc. In line with this, Gauss wrote in a letter to Bessel in 1830 that “we must admit with humility that [. . .] number is purely a product of our minds”—a claim which seems to be corroborated by recent data (e.g., Condry and Spelke, 2008; Cantlon et al., 2009). Nevertheless, there is also accumulating evidence in recent years suggesting that the representation of number magnitude may not be entirely abstract. Instead, it seems to not only depend on input format (see Cohen-Kadosh and Walsh, 2009 for a review and discussion) but might even represent a specific instance of embodied cognition, rooted in sensory and bodily experiences (Lakoff and Núñez, 2000; Núñez, 2004). Importantly, early finger counting has been suggested to play a vital role for the development of a representation of embodied numerosity (e.g., Domahs et al., 2010; Fischer and Brugger, 2011; Moeller et al., 2012).

On a very basic level this assumption is corroborated by the observation that the majority of children use their fingers when learning to count or calculate at some point in their numerical development (e.g., Fuson and Hall, 1983; Fuson, 1988). Additionally, it has repeatedly been found that finger-based numerical representations still influence number processing in adults. Di Luca et al. (2006), for instance, observed that there seems to be an association between specific fingers and numbers. When asked to respond to a presented number (1–10) by pressing a corresponding key, adults’ responses were faster when the key had to be pressed by a finger which was associated to the respective number in their prototypical finger counting strategy (e.g., 1 corresponding to thumb, 2 corresponding to index finger, etc.). Another line of research evaluated in how far symbolic number processing is influenced by the respective finger counting system used. In this context, Domahs et al. (2010) contrasted possible influences of the German and Chinese finger counting system. In the German finger counting system each number from 1 to 10 is not only assigned to a specific finger but also associated with a specific finger pattern following 1-to-1 correspondence (i.e., the thumb stretched out for 1, thumb and index finger for 2, thumb, index, and middle finger for 3, etc.). This means that for numbers >5 both hands are required for the respective finger pattern (i.e., one full hand plus the thumb of the other hand for 6). In contrast, in the Chinese system this 1-to-1 correspondence only applies to numbers up to 5 whereas numbers larger than 5 are indicated symbolically using one hand only. The authors found that in a symbolic magnitude comparison task, reaction times of German-speaking participants were significantly longer when at least one of the to-be-compared numbers was associated with a finger counting pattern requiring both hands (e.g., 4 vs. 6 or 6 vs. 8). Domahs et al. (2010) argue that Chinese participants did not show this increase in reaction times because their finger counting pattern only required one hand for all numbers up to 10.

Based on these and other results, Moeller et al. (2012) suggested that finger-based representations should be considered a distinct representation of number magnitude that is automatically activated whenever we encounter a number. However, so far none of the existing models of numerical development (e.g., von Aster and Shalev, 2007; Krajewski and Schneider, 2009) considers the influence of finger-based representations. Therefore, the aim of the current article is to discuss how finger-based representations might corroborate the acquisition of basic numerical competencies and to derive first suggestions on how finger-based representations may be integrated into a current model of numerical development.

DIGITS IN A CURRENT MODEL OF NUMERICAL DEVELOPMENT

The currently most sophisticated model of early numerical development was proposed by Krajewski and Schneider (2009). The authors assume numerical competencies to develop on three consecutive levels through an association between non-numerical abilities such as quantity discrimination, the understanding of part-whole relations, etc. and more specifically numerical skills such as counting. The following line of argument will focus primarily on the numerical skills of the model because those should be accessible and promotable by finger-based numerical representations. In the following section, we will first argue how finger-based representations will add to each of the three levels of the developmental model of Krajewski and Schneider (2009) before we will discuss possible constraints and limitations of our propositions.

LEVEL I: BASIC NUMERICAL SKILLS

Krajewski and Schneider (2009) propose that on the first level of development, children learn to recite the exact number word sequence and become skilled at counting. At this stage, many of them start to use their fingers by adopting the finger counting system of the respective cultural area, even without any specific instruction to do so. In the German finger counting system, which we will refer to as the working example throughout this article, the finger counting sequence starts with the thumb for 1 and then goes on with the index finger for 2, the middle finger for 3, the ring finger for 4, the pinkie for 5, restarting the same sequence at the thumb of the other hand for 6 and so on to 10 (see **Figure 1**; Level I). Thus, each number word is linked to one specific finger as it is the case in most finger counting systems of Western cultures (Bender and Beller, 2011, 2012; see below for a discussion on cultural differences in finger counting). Due to this link between fingers and numbers, the counting principle of one-to-one-correspondence is easily understandable (Brissaud, 1992). On a very basic level, even the acquisition of the number words themselves might be corroborated by making use of fingers, as the finger-number association may help perceive the number words as phonological discrete items (Beller and Bender, 2011) and contributes, as some kind of marker, to memorizing them (Brissaud, 1992; Fayol and Seron, 2005; De La Cruz et al., 2014). Additionally, the counting principle of stable order and the ordinal concept of numbers might be conveyed as well (Brissaud, 1992; Fayol and Seron, 2005; Crollen et al., 2011), because the involved motor sequence during finger counting (e.g., stretching out thumb, stretching out index finger, etc.) is just as stable as the number word sequence. This might help understand that for instance "ten", which is assigned to the ultimate finger counted, comes after "nine", which is associated with the penultimate finger counted.

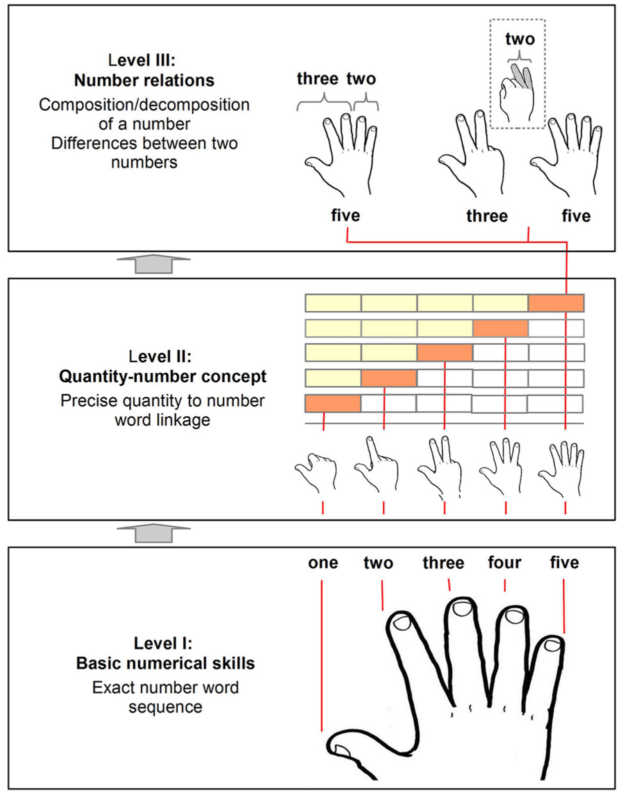

FIGURE 1 | Schematic illustration in what way finger-based representations may be integrated into the model of early mathematical development by Krajewski and Schneider (2009). On level I the acquisition of the number word sequence and basic counting principles can be supported by finger-based representations as each finger may be associated with a specific number word. On level II the development of quantity-number associations and of spatial-numerical representations can be corroborated because each finger pattern reflects the cardinality of the counted set. Finally, on level III the acquisition of initial calculation abilities such as number composition, decomposition, and comparison can be fostered by finger-based representations as fingers allow for grouping, regrouping, and comparing.

LEVEL II: QUANTITY-NUMBER CONCEPT

On level II of the developmental model by Krajewski and Schneider (2009), children are suggested to become aware of the quantitative meaning conveyed by each number word. The acquisition of this so called cardinal number concept can also be supported by fingers (Brissaud, 1992). During finger counting, following the German finger-counting routine, digits are stretched out one after another whereas outstretched ones are not pulled in again (see **Figure 1**; Level II). This procedure allows for linking each number word to the corresponding quantity and for perceiving this quantity-number association both visually as well as through tactile and even proprioceptive sensations. By nature, finger counting is thus "cardinalized counting" (Brissaud, 1992) because quantities increase steadily (one by one) with every additional finger added during counting. This visualizes not only the respective cardinal values but also their progressive summation (see **Figure 1**; Level II). As this summation usually occurs in a specific spatial direction, it was suggested recently that finger counting might even modulate the spatial representation of magnitude, also known as the mental number line (Fischer, 2008). For instance, Pitt and Casasanto (2014) observed that just 15 min of training finger counting in leftward direction, this means (in palm up position) from the right thumb for 1 over the left pinkie for 6 to the left thumb for 10, extinguished and partially reversed the usual association of small numbers with left and larger numbers with right which was observed in Western participants (e.g., Dehaene et al., 1993). Taken together, these considerations indicate that fingers might not only contribute to the acquisition of ordinal counting but may also corroborate the understanding of cardinal quantity-number associations and the development of spatial-numerical associations (Fischer, 2008; Tschentscher et al., 2012).

LEVEL III: NUMBER RELATIONSHIPS

On level III of their development model, Krajewski and Schneider (2009) argue that children start to learn that numbers not only convey quantities but also allow for describing relationships between quantities. This insight enables them to compose (e.g., 2 and 3 equals 5) and de-compose numbers (e.g., 5 is decomposable into 3 and 2) as well as to quantify the precise difference between two numbers (e.g., 3 is distinct from 5 by 2). Although these abilities already reflect initial calculations, they may nevertheless be corroborated by finger-based representations. Many children use their fingers when they start to calculate but not only, as often criticized, to keep track of items while counting up (e.g., Fuson, 1988) but also to visualize and combine the involved quantities as a whole (Siegler and Shrager, 1984). To combine, for example, 3 and 2 in this way, children might first stretch out thumb, index finger, and middle finger simultaneously; then add ring finger and pinkie, just to conclude finally that the result is 5, as all fingers of one hand are stretched out in the end. The other way round, finger-based strategies can also be used to visualize number decomposition. For this purpose, children might first display the initial quantity with their fingers (e.g., all fingers of one hand for 5) and then separate two subgroups of digits from each other (e.g., thumb, index, and middle finger for 3 vs. ring finger and pinkie for 2) by putting some digits closer together (see **Figure 1**; Level III; left side). In addition to compositions and decompositions, fingers might even exemplify differences between numbers. When children compare, for instance, the finger magnitudes 3 and 5 with each other, they can easily see them differ by 2, namely by ring finger and pinkie (see **Figure 1**; Level III; right side). This indicates that even number relationships, which are required for initial calculations, can be conveyed by finger-based numerical representations.

Taken together, these considerations strongly suggest that finger-based numerical representations are well-suited to corroborate the acquisition of basic numerical concepts such as counting as well as the understanding of cardinality and number relations.

POSSIBLE CONSTRAINTS AND LIMITATIONS

While above considerations clearly argue for a specific role of finger-based representations in children's numerical development, there also seem to be constraints and limitations to this account (e.g., Bender and Beller, 2011; Previtali et al., 2011 for discussions of this point).

A first point to consider is whether finger-based representations are a necessary step in children's numerical development. This would be a very strong claim and hard to proof. Actually, Butterworth et al. (2011) observed that some indigene Australian cultures do not at all use their fingers in numerical contexts but are nevertheless able to perform simple calculations. On the other hand, Poeck (1964) reported the case of a girl born without forearms, who counted the fingers of her phantom hands to solve simple arithmetic problems. Additionally, Crollen et al. (2011) found that even blind children use their fingers to count and calculate, yet less often and systematically. Against this background, we are confident that finger-based numerical representations can corroborate the acquisition of basic numerical concepts, although we do not wish to claim that they are mandatory or even necessary to develop basic numerical concepts.

A second point to consider is whether above mentioned advantages of finger-based numerical representations may be generalizable across culturally differing finger counting routines. Between cultures, finger counting routines vary, for instance, (i) in the finger on which counting is started (i.e., thumb for 1 in Germany, index finger for 1 in the US, little finger for 1 in Iran); (ii) how the finger counting sequence continues from 6 to 10 (i.e., with the same finger sequence on the second hand in most Western cultures or with the first hand using finger symbolic gestures in China). In our opinion, it seems reasonable to assume that the acquisition of basic numerical concepts (i.e., counting, understanding of cardinality and number relations) may be corroborated best, when fingers and numbers are associated in 1-to-1-correspondence and a stable finger sequence is used. Thus, it should not matter which finger (on which hand) is used to begin and how the finger counting sequence continues because these two preconditions allow for both, associating (i) a specific finger with a specific number; and (ii) a specific finger pattern with a specific cardinality. In case of the Chinese finger counting system, for instance, this is only fulfilled for numbers up to 5. For numbers exceeding 5, which are represented symbolically (see above), the order of the counting sequence is still stable but there is no 1-to-1-correspondence between fingers and numbers. Therefore no finger is specifically associated with number 7, nor is the cardinality of 7 reflected in its associated finger pattern, which makes decompositions and compositions impossible (see also Domahs et al., 2012 for the case of the Korean finger counting system).

Finally, recent evidence indicates that finger counting habits vary not only across cultures but also on the individual level. Wasner et al. (2014a) observed, for instance, that whether German participants started counting on their left or right hand was influenced reliably by whether both of their hands were equally available. Additionally, Wasner et al. (2014b) found that finger patterns differed—at least for specific numbers (e.g., 4)—when participants were asked to either count to that number (i.e., by thumb, index, middle, and ring finger) or to show the respective number as a spontaneous finger pattern (i.e., index, middle, ring finger, and pinky). Irrespective of the fact that there tends to be some flexibility in adults' finger counting habits, it seems reasonable that children benefit most, if they stick to a stable motor sequence when learning to count on fingers (for similar results on object counting see Kamawar et al., 2010). Importantly, this is corroborated by a recent study on cognitive robotics, in which De La Cruz et al. (2014) observed "that learning the number words in sequence along with [stable] finger configurations helps the fast building of the initial representation of number in the robot" and "the internal representations of the finger configurations themselves [. . .] sustain the execution of basic arithmetic operations" (p. 1).

Nevertheless, it should be noted that all studies on the differences of finger-based representations across individuals as well as cultures were conducted primarily with adult participants. To the best of our knowledge, there is currently no study investigating the flexibility of children's finger counting habits. Thus, further research is needed to clarify how children develop finger-counting habits and whether different finger counting routines impact on numerical development differentially.

CONCLUSION

In summary, above considerations clearly suggest that finger-based representations can corroborate the acquisition of basic numerical concepts at all three levels of the developmental model proposed by Krajewski and Schneider (2009): They seem to be helpful for (I) learning the number word sequence and basic counting principles because numbers and fingers are associated in 1-to-1 correspondence during finger counting; (II) understanding the quantity-number association and developing a spatial-numerical representation because during finger counting numbers are not only associated with specific fingers but also with finger patterns that indicate the progression of magnitudes; and (III) acquiring initial calculation abilities such as composition and decomposition or number comparison because fingers allow for grouping, regrouping and comparing. Notwithstanding these reasonable benefits, we do not intend to claim that finger-based representations are a mandatory or necessary prerequisite of successful numerical development. Nevertheless, they can positively influence the acquisition of basic numerical concepts. Against this background, it seems plausible that numbers may not be "purely a product of our minds", as suggested by Gauss in his letter to Bessel but in fact reflect a specific case of embodied cognition that roots in the bodily experiences of early finger counting and calculating. Therefore, we strongly suggest that digits should be considered in current models of numerical development.

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Of adding oranges and apples: how non-abstract representations may foster abstract numerical cognition

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INTRODUCTION

The realm of numbers constitutes just one of many fields of mathematical cognition, but arguably a pivotal one. It is also among those core domains of knowledge that—while being prepared for unfolding in the human species (Feigenson et al., 2004; Hyde, 2011)—nonetheless requires cultural mediation to unfold to its full potential: Not only is the availability of a conventionalized counting sequence essential for accurate counting and calculating (Gordon, 2004; Pica et al., 2004; Frank et al., 2008; Spaepen et al., 2011), acquiring a counting sequence in the first place is also crucial in more fundamental ways: for grasping the concept of precise quantities, for comprehending the ordinal and cardinal nature of numbers, or for learning the algorithms of basic arithmetics that then pave the way for higher levels of mathematics.

Learning to count also promotes acquaintance with some of the more general principles that characterize mathematics such as abstractness. In fact, one of the first principles to be learned in this process is that numbers are abstract—all kinds of entities can be counted with the same number words (Gelman and Gallistel, 1978; but see also Cohen Kadosh and Walsh, 2009). But not all counting sequences seem to reflect this principle. A substantial number of Oceanic languages, for instance, have counting sequences whose usage is restricted to specific objects, while other objects are counted otherwise (Bender and Beller, 2006a,b).

This pattern of counting different things differently seems to directly contradict the abstractness principle and has thus been taken as an earlier stage in the evolution of numerical thinking (e.g., Klix, 1993). While the latter assumption was refuted elsewhere (Beller and Bender, 2008), the question remains open of how (if at all) such apparently non-abstract counting sequences may foster abstract numerical cognition. Here, we defend the position that the Oceanic counting sequences are not only *compatible* with an abstract understanding of numbers, but may even *promote* such an understanding. To this end, we propose to conceive of these sequences as the verbal components of the mathematical code, which provide the symbols that people use to represent and manipulate abstract mathematical concepts. Analyzing how the specific properties of these symbol systems affect the processing of numerical information will help us to understand better how abstract mathematical thinking emerges.

COUNTING SEQUENCES AND THEIR COGNITIVE IMPLICATIONS

In general, each counting sequence consists of a limited set of symbols for basic numbers and (optionally) some composition rules for representing larger numbers. These symbols and composition rules constitute a distinct numeration system, the properties of which may differ substantially across languages (Chrisomalis, 2004; Bender and Beller, 2012; Widom and Schlimm, 2012).

The system's internal structure, for instance, depends on its dimensionality (Zhang and Norman, 1995). One-dimensional systems are unstructured; they either use the same symbol in a cumulative manner to indicate increasing set size (as in tallies), or employ specific symbols distinctively to indicate distinct set sizes (as with the Arabic digits from 1 through 9). In contrast, two-dimensional systems like the English number words make use of a base (in this case: "ten"), which is raised to various powers ("hundred," "thousand," etc.). Number words in between are composed according to the addition and multiplication principle, as in "two hundred and three."

Counting sequences are cultural tools, whose properties may give rise to "representational effects" (Zhang and Norman, 1995), that is, they affect how numerical information is represented and processed (Nickerson, 1988; Fuson, 1990; Miller et al., 1995; Zhang and Wang, 2005; Schlimm and Neth, 2008; Domahs et al., 2010; Beller and Bender, 2011; Krajcsi and Szabo, 2012). An analysis of such representational effects will help us to illuminate the cognitive implications of specific counting systems.

SPECIFIC COUNTING SYSTEMS IN OCEANIC LANGUAGES

In a large number of Oceanic languages, two types of verbal systems co-exist: regular systems for general counting and systems restricted to counting specific objects in a particular manner (Bender and Beller, 2006a,b). For illustration, take the Polynesian language spoken on Tonga, an island group in the Western Pacific.

Tongan employs five numeration systems: a general and four specific ones. All of them contain primary numerals for the numbers 1 through 10 and for the powers of the base up to 10^5 (Bender and Beller, 2007). The specific systems deviate from each other and from the general system in three ways: They make use of diverging counting units; they employ distinct lexemes for some of the powers; and they are applied to only one kind of object each. Accordingly, sugar-cane is counted in pairs, whereas coconuts, pieces of yam for planting, and fish are counted in pairs when few, but in scores when numerous.

It was especially the object-specificity of counting that arrested researchers' interest early-on and nurtured the assumption that speakers of languages like Tongan may lack an abstract concept of number (Klix, 1993). However, viewing this feature in the context of the other two peculiarities allows for a more accurate assessment. It reveals that number word composition in the specific systems remains highly systematic. In fact, the rules for composing number words in the general system require only marginal modifications (namely acknowledgement of the counting unit and the specific power numerals) to generate number words in the specific systems. This structural alignment, together with the older age of the general system, also suggests that the specific systems were deliberately developed out of the general one (Bender and Beller, 2006a,b, 2007).

While the structure of the general counting sequence was retained, the counting unit to which its constituents referred (and hence the value of the counted set) was increased. This transformation of values follows the same principle that is inherently instantiated in two-dimensional systems, namely the multiplication principle for composing larger number words. In these systems, the base and its powers are counted as if they were objects: "three hundreds" is similar to "three baskets." Specific systems carry this abstraction one step further by implicating that the "three hundreds" may refer in fact to "three hundreds of pairs or scores."

Adopting the multiplication principle inherent in power term constructions (like "three hundreds") for the creation of specific counting systems is far from being trivial. It requires a sophisticated understanding of counting insofar, as number words are used now to count not just objects, but other numbers and even abstract counting units. To this end, countability is defined recursively, and in doing so, also paves the way for conceiving of multiplication as an algorithm for mental arithmetics.

What at first glance may look laborious—the recursive extraction of numerical values—can, thus, in fact be cognitively advantageous: It allows for more compact representations, which, in the absence of notation, not only reduces cognitive load (Beller and Bender, 2008), but also increases the speed and correctness of mental arithmetic (Lordahl et al., 1970; Bender and Beller, 2013).

CONCLUSION

With our analysis we hope to have demonstrated, that the apparently non-abstract representations in Oceanic counting systems have indeed fostered abstract numerical cognition. But beyond this rehabilitation of the specific systems and their users, this "exotic" phenomenon is of more general relevance to the cognitive sciences. It also serves as an instance of the recursive process in which cultural tools and cognitive achievements advance each other and thus as an instance of the "ratchet effect" (Tomasello, 1999; and see Wiese, 2003) of culture more generally, which also highlights the importance of anthropological insights for cognitive science theorizing (Beller et al., 2012). By their mere existence and usage, cultural tools may promote cognitive advancement. Designed to serve one purpose, tools generally have more properties than only those relevant to the task at hand, and these properties may then afford new ways of usage or reasoning (see also Miller and Paredes, 1996; Coolidge and Overmann, 2012). It is this extra value of cultural tools that, in the domain of mathematical cognition, promotes abstract thinking.

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The neural bases of the multiplication problem-size effect across countries

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Multiplication problems involving large numbers (e.g., 9 × 8) are more difficult to solve than problems involving small numbers (e.g., 2 × 3). Behavioral research indicates that this problem-size effect might be due to different factors across countries and educational systems. However, there is no neuroimaging evidence supporting this hypothesis. Here, we compared the neural correlates of the multiplication problem-size effect in adults educated in China and the United States. We found a greater neural problem-size effect in Chinese than American participants in bilateral superior temporal regions associated with phonological processing. However, we found a greater neural problem-size effect in American than Chinese participants in right intra-parietal sulcus (IPS) associated with calculation procedures. Therefore, while the multiplication problem-size effect might be a verbal retrieval effect in Chinese as compared to American participants, it may instead stem from the use of calculation procedures in American as compared to Chinese participants. Our results indicate that differences in educational practices might affect the neural bases of symbolic arithmetic.

Keywords: education, arithmetic, problem-size, fMRI, multiplication

INTRODUCTION

In mental arithmetic, the problem-size effect describes a well-known phenomenon whereby problem difficulty increases with the numerical size of the operands (Ashcraft and Guillaume, 2009). For example, single-digit multiplication problems involving relatively large numbers (e.g., 9 × 8) take longer to solve (and are more error prone) than problems involving smaller numbers (e.g., 2 × 3). Although the problem-size effect is one of the most widely observed phenomena in the cognitive arithmetic literature, its sources remain debated (Ashcraft and Guillaume, 2009).

On the one hand, the multiplication problem-size effect might occur because answers of large problems are more difficult to retrieve from long-term memory than answers of small problems (Campbell and Graham, 1985; Siegler, 1988; Ashcraft, 1992). This may be because large problems are not frequently encountered and practiced during arithmetic learning in school (Hamann and Ashcraft, 1986). Such problems tend to be associated with several candidate answers (Campbell and Graham, 1985). For example, 8 × 6 might be associated with the correct answer (i.e., 48) but also with incorrect neighboring answers from the multiplication table (e.g., 56). Because small problems are more practiced and less likely to be associated with interfering answers, their representations may be more differentiated in memory and answers of small problems should be more easily retrieved from long-term memory than large problems.

On the other hand, the multiplication problem-size effect might result from differences in strategy choices for solving small vs. large problems (Lefevre et al., 1996; Penner-Wilger et al., 2002). Specifically, LeFevre and colleagues have argued that large problems are less frequently solved by retrieval than small problems. Rather, answers of large problems may be derived from procedural calculation algorithms, such as decomposition (e.g., $8 \times 9 = 8 \times 10 - 8$; $6 \times 4 = 5 \times 4 + 4$) and transformation (e.g., $3 \times 8 = 8 + 8 + 8$) (Dowker, 2005). Procedural strategies are typically thought to be slower and more error prone than direct retrieval. Thus, the greater use of such strategies in large vs. small problems would explain the problem-size effect.

It has been proposed that the sources of the problem-size effect might depend upon differing educational backgrounds across countries (Lefevre and Liu, 1997; Campbell and Xue, 2001; Penner-Wilger et al., 2002). This hypothesis is supported by behavioral studies showing that the effect indeed differs across countries. First, although the multiplication problem-size effect can be observed on response times (RTs) and error rates in individuals educated in China and in North America, it is significantly smaller in Chinese than North American participants (Campbell and Xue, 2001). Second, whereas adults educated in North America report using both retrieval and calculation strategies when solving single-digit multiplication problems (Lefevre et al., 1996), adults educated in China report relying almost exclusively on retrieval when solving single-digit multiplication (Campbell and Xue, 2001). Third, the distributions of response times associated with small and large multiplication problems significantly differ between North American and Chinese adults, suggesting differences in strategy choices between groups (i.e., mixture of retrieval and calculation for North Americans and exclusive retrieval for Chinese) (Penner-Wilger et al., 2002). Overall, these behavioral results suggest that the source of the problem-size effect might depend upon factors associated with educational background across countries. Specifically, while the effect might be explained by differences in retrieval effort in Chinese adults, it might arise from differences in the use of calculation strategies in North American participants (Penner-Wilger et al., 2002).

Such behavioral results based on self-report and analyses of response times have been challenged, however. Specifically, it has been argued that verbal reports might be misleading because they are likely to be influenced by instructions (Kirk and Ashcraft, 2001) and might not accurately distinguish between calculation and retrieval (Fayol and Thevenot, 2012). Furthermore, because calculation procedures can be highly practiced and automatized, these might be implemented as fast as retrieval (Fayol and Thevenot, 2012). Therefore, analyzes of response times might not necessarily give meaningful insight into the strategies used in arithmetic problem-solving.

The goal of the present study was to provide additional evidence for the dependency of the problem-size effect on educational background by comparing its neural correlates in Chinese and American adults. Specifically, we used functional magnetic resonance imaging (fMRI) to measure the brain activity of adults educated in China and the United States while they evaluated small and large single-digit multiplication problems. Previous neuroimaging studies suggest that arithmetic processing relies on a heterogeneous brain network. On the one hand, left temporo-parietal regions are typically activated when problems are more likely to rely on fact retrieval, as is the case for single-digit multiplication (Lee, 2000; Andres et al., 2011, 2012; Prado et al., 2011), small problems (Stanescu-Cosson et al., 2000; Zhou et al., 2007; Jost et al., 2009; De Smedt et al., 2011), extensively trained problems (Zamarian et al., 2009), problems self-reported to be retrieved (Grabner et al., 2009), and exact arithmetic (Dehaene et al., 1999; Venkatraman et al., 2006). These regions are thought to support the verbal representation of math facts and include the left middle/superior temporal gyrus (Sandrini et al., 2003; Ischebeck et al., 2007; Zhou et al., 2007; Andres et al., 2011, 2012; Prado et al., 2011) and the left angular gyrus (Grabner et al., 2009, 2013; Zamarian et al., 2009). On the other hand, a dorsal fronto-parietal network is typically engaged when problems are more likely to involve the manipulation of numerical quantities, as is the case for single-digit subtraction (Lee, 2000; Piazza et al., 2007; Prado et al., 2011), large problems (Stanescu-Cosson et al., 2000; Zhou et al., 2007; Jost et al., 2009; De Smedt et al., 2011), untrained problems (Zamarian et al., 2009), problems self-reported to be calculated (Grabner et al., 2009), and approximate arithmetic (Dehaene et al., 1999; Venkatraman et al., 2006). This network involves the IPS, a region thought to be involved in the representation of numerical magnitudes (Nieder and Dehaene, 2009). It also involves the lateral and medial frontal cortices, which are thought to reflect the demands in working-memory and executive control associated with the manipulation of numbers (Ansari, 2008; Jost et al., 2009). Recently, these findings have been confirmed by a quantitative meta-analysis of the neuroimaging literature (Arsalidou and Taylor, 2011). This meta-analysis indicated that there are substantive overlap between the neural bases of numerical processing and arithmetic in the parietal and frontal cortices, suggesting that procedural strategies relying on numerical manipulation are likely to be used during arithmetic calculation. However, this meta-analysis also indicated that left temporal regions are specifically engaged during operations that are likely to rely on verbal fact retrieval, such as multiplication.

Overall, neuroimaging studies conducted on western adults (Stanescu-Cosson et al., 2000; Jost et al., 2009) and children (De Smedt et al., 2011) have observed a neural problem-size effect (i.e., greater activity for large than small problems) in the dorsal fronto-parietal regions typically involved in numerical calculation. While these findings suggest that the effect might stem from the greater use of calculation procedures in large than small problems, it is possible that this result might depend upon differences in cultural and educational background. To our knowledge, only two previous studies have investigated the neural correlates of arithmetic across Chinese and English languages (Tang et al., 2006; Venkatraman et al., 2006). First, by studying English–Chinese bilinguals, Venkatraman et al. (2006) found that solving arithmetic problems in a language different from the one used to learn them is associated with enhanced activity in several brain regions. These increases are observed in regions associated with verbal retrieval for exact arithmetic and regions associated with numerical manipulation for approximate arithmetic. Although this study supports the idea that regions involved in verbal retrieval and numerical processing are differentially engaged in arithmetic, it could not evaluate the effect of cultural and educational background on the neural bases of arithmetic as it focused on the same group of bilingual individuals. Second, Tang et al. (2006) recently compared the neural correlates of simple arithmetic processing in participants educated in China and Western countries. However, this study only did so with single-digit addition, and did not further dissociate between small and large problems. Because single-digit addition and multiplication diverge in terms of learning methods (Dehaene et al., 2003) and problem-solving strategies (Fayol and Thevenot, 2012), differences in the neural bases of the multiplication problem-size effect between Chinese and Western individuals remain unknown.

In the present study, we expected the neural bases of the multiplication problem-size effect to specifically differ between Chinese and Americans. Behavioral studies suggest that the problem-size effect preferentially may result from differences in retrieval effort in Chinese, whereas it preferentially may rely on differences in the use of calculation procedures in North Americans (Penner-Wilger et al., 2002). Therefore, we expected that the problem-size effect would be more strongly associated with activity in brain regions involved in the verbal representation of math facts (i.e., left mid-superior temporal gyrus and/or left angular gyrus) in Chinese as compared to American participants. Conversely, we hypothesized that the problem-size effect would be more strongly associated with activity in brain regions involved in numerical manipulation and arithmetic calculation (i.e., IPS and frontal regions) in American as compared to Chinese participants. As is common in the neuroimaging literature (Poldrack, 2006), most studies have indirectly inferred the role of temporal and parietal brain regions involved in arithmetic based on anatomical landmarks and prior research. It is increasingly believed, however, that such "reverse" inferences can be greatly strengthened by systematically localizing the cognitive processes of interest in each participant (Saxe et al., 2006). In the present study, we used independent localizer scans to identify the parietal and temporal cortices involved in verbal and numerical processing. This enabled us to improve the specificity and selectivity of our analyses.

MATERIALS AND METHODS

PARTICIPANTS

Thirty-two Chinese participants were recruited from the Beijing community in China, and 33 American participants were recruited from the Chicago community in the United States. Data from three Chinese and four American participants were excluded due to excessive movement in the scanner (i.e., greater than 3 mm). Two Chinese and three American participants were further excluded because their error rates were above 30%. Therefore, 27 Chinese participants [13 males; mean age = 24.2 years; standard deviation = 2.12; age range: 20–28 years] and 26 American participants [10 males; mean age = 25.2 years; standard deviation = 3.07; age range: 19–30 years] were included in the analyses. Chinese participants were native Chinese speakers, while American participants were native English speakers. All participants had a minimum of 13 years of education, which they completed in their respective countries (i.e., China or the United States). Although all participants were graduates from high-school, they varied regarding the number of years of post-secondary education they received. However, as emphasized by Campbell and Xue (2001) and Lefevre and Liu (1997), basic arithmetic skills such as single-digit multiplication are acquired and consolidated primarily during elementary education. Therefore, with respect to single-digit multiplication skill, it is unlikely that this variability in the number of years of post-secondary education might have affected our results. Nonetheless, to ensure that any fMRI differences between the Chinese and American groups were not driven by differences in math proficiency, we performed control analyses in which relevant effects were controlled for differences in multiplication skill (see below).

None of the Chinese participants were of Western descent and none of the American participants were of Asian descent. All subjects were right-handed and had no history of neurological or psychiatric disorders. Experimental protocols were approved by the local Institutional Review Boards, and informed consent was obtained from each participant. Chinese and American participants were compensated 75 RMB and 20 USD per hour for their time, respectively. Groups were comparable in terms of age ($t_{(51)} = 1.34$, $p = 0.19$) and gender (Fisher's Exact test: $p = 0.58$). The exact same individuals participated in the localizer and the arithmetic tasks.

TASK

In each trial of the multiplication task, participants evaluated the answer of a single-digit multiplication problem involving Arabic numerals (see **Figure 1A**). The exact same stimuli were employed for the Chinese and American groups. Following a previous study (Prado et al., 2011), we included 12 small and 12 large multiplication problems. In small multiplication problems, the two operands were smaller than or equal to 5 (e.g., 3×4). In large multiplication problems, both operands were larger than 5 (e.g., 6×7). Each problem was repeated twice with a true answer (e.g., $3 \times 4 = 12$) and once with a false answer, yielding 72 trials total in each task (36 small and 36 large problems). False answers were table-related. They corresponded to the answer that would be obtained by adding or subtracting 1 to the first operand (e.g., $3 \times 5 = 20$ or $3 \times 5 = 10$). Problems involving 0 (e.g., 3×0), 1 as second operand (e.g., 3×1) and ties (e.g., 3×3) were not included in the main experiment but were used in the practice session. Twelve problems with a correct answer and twelve problems with a false answer were included in the practice session for each task.

LOCALIZER SCANS

Our hypotheses involved regions of the parietal and temporal cortices involved in verbal and numerical processing (see Introduction). To identify those regions and improve the sensitivity and specificity of our analyses (Saxe et al., 2006), localizer scans were included in the experiment. In the verbal processing localizer (see **Figure 1C**), participants decided whether two visually presented words rhymed or not. Single character (monosyllables) Chinese words were used for the Chinese group and monosyllabic English words were used for the American group. To ensure that judgments were not based solely on orthographic similarities between words, orthography and phonology were manipulated independently. That is, the two words could have similar orthography and similar phonology (e.g., dime–lime; 漆-膝; 12 trials), similar orthography but different phonology (e.g., pint–mint; 萎-矮; 12 trials), different orthography but similar phonology (e.g., jazz–has; 敞-烫; 12 trials) or different orthography and different phonology (e.g., press–list; 倍-粉; 12 trials). Similar orthography in Chinese was operationalized as sharing the same phonetic radical (right part of the character). We also included a perceptual control condition in which two symbol strings were presented on the screen instead of word pairs (12 trials). In American participants, the two symbol strings consisted of rearranged parts of lower case Courier letters. In Chinese participants, the two symbol strings were single Tibetan characters. Tibetan characters were chosen because they are similar to Chinese characters in terms of visual complexity and configuration. The perceptual condition was designed to control for visual stimulation and response selection in both groups. All participants had to determine whether the symbol strings matched (the symbols matched in half of the trials). Twelve trials of each condition were presented in the practice session. Different sets of stimuli were used in the practice and in the scanning sessions.

In the numerical processing localizer (see **Figure 1B**), participants decided which of two visually presented dot arrays were composed of the larger number of dots (i.e., the larger

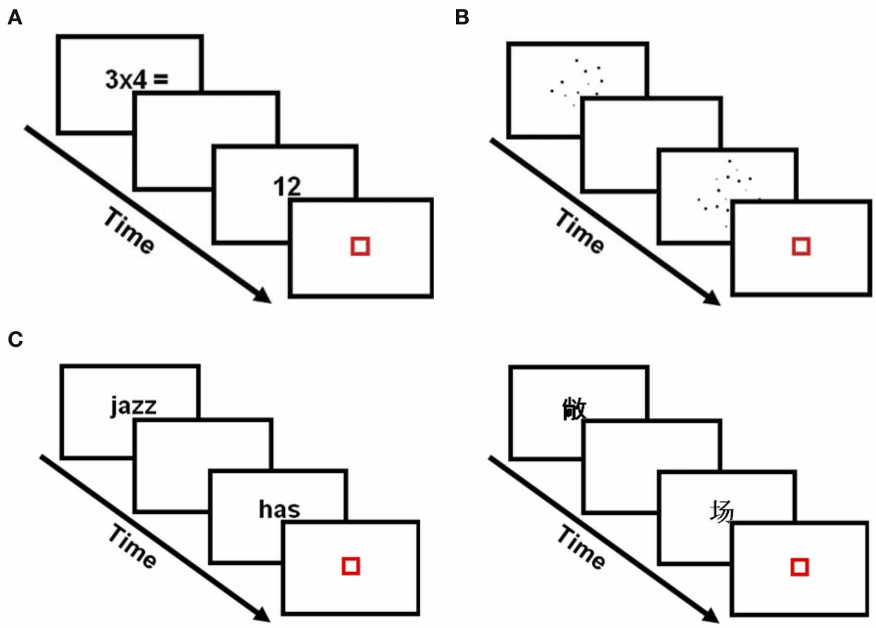

FIGURE 1 | Experimental and localizer tasks. (A) In the multiplication task, participants were asked to evaluate the answer of single-digit multiplication problems. Problem-sizes were either small (e.g., 3 × 4) or large (e.g., 6 × 7). **(B)** In the numerical processing localizer, participants decided which of two dot arrays were composed of the larger number of dots. **(C)** In the verbal processing localizer, American participants decided whether two visually presented English words rhymed or not (left) and Chinese participants decided whether two visually presented Chinese words rhymed or not (right). In all tasks, the first stimulus was presented for 800 ms, followed by a blank screen for 200 ms. A second stimulus was then presented for 800 ms, followed by a red fixation square for 200 ms.

numerosity). The exact same stimuli were employed for the Chinese and American groups. The numerical comparisons were "easy" (i.e., 12 dots vs. 36 dots; 24 trials), "intermediate" (i.e., 18 dots vs. 36 dots; 24 trials), or "hard" (i.e., 24 dots vs. 36 dots; 24 trials). Six different dot sizes were used and stimuli were controlled for differences in cumulative surface areas and distribution of dot sizes (Prado et al., 2011). Twelve trials of each condition were presented in the practice session. Different stimuli were used in the practice and in the scanning sessions.

EXPERIMENTAL PROTOCOL

Participants practiced the experimental and localizer tasks before entering into the scanner. In the scanner, the multiplication task and the numerosity processing localizer were decomposed in 2 functional runs of about 4 min each. The verbal processing localizer was administered in one single run lasting approximately 7 min. Participants also performed an additional subtraction evaluation task in the scanner. The data from this task will not be considered in this report. The order of the tasks was fully counterbalanced across participants. The timing and order of trial presentation within each run was optimized for estimation efficiency using optseq2 (http://surfer.nmr.mgh.harvard.edu/optseq/). Specifically, although trials appeared to be presented in a random order to participants, the timing and order of trials in each condition was calculated by optseq2 in order to remove the overlap from the estimate of the hemodynamic response (by introducing variable periods of fixation, or jitters). Behavioral responses were recorded using an MR-compatible keypad placed below the right hand. Visual stimuli were generated using E-prime software (Psychology Software Tools, Pittsburgh, PA) and projected onto a translucent screen that was viewed by the participants through a mirror attached to the head-coil.

Stimulus timing was identical in all tasks. A trial started with the presentation of a first stimulus (multiplication, dot array or word) for 800 ms, followed by a blank screen for 200 ms. A second stimulus (multiplication answer, dot array or word) was then presented for 800 ms. This second stimulus was followed by a red fixation square (duration: 200 ms) that indicated the need to make a response during an interval ranging from 2800 to 3600 ms. Twenty-four null trials were included in the multiplication task and the numerical localizer scan. Twelve null trials were included in the verbal localizer scan. In the null trials, a black square was presented for the same stimulus duration as in the experimental trials and participants were asked to press a button when the black square turned red.

IMAGING PROCEDURES

Data from the Chinese participants were collected at the State Key Lab of Cognitive Neuroscience and Learning at Beijing Normal University in China. Data from the American participants were collected at the Northwestern University's Center for Advanced MRI (CAMRI) in the United States. At both sites, the exact same scanner model (Siemens 3T TIM Trio MRI scanner; Siemens Healthcare, Erlangen, Germany) and exact same scanning parameters were used. The fMRI blood oxygenation level dependent (BOLD) signal was measured with a susceptibility weighted single-shot echo planar imaging (EPI) sequence. The following parameters were used: TE = 20 ms, flip angle = 80°, matrix size = 128 × 120, field of view = 220 × 206.25 mm, slice thickness = 3 mm (0.48 mm gap), number of slices = 32, TR = 2000 ms. Before functional image acquisition, a high resolution T1 weighted 3D structural image was acquired for each subject (TR = 1570 ms, TE = 3.36 ms, matrix size = 256 × 256, field of view = 240 mm, slice thickness = 1 mm, number of slices = 160).

BEHAVIORAL DATA ANALYSIS

Behavioral studies have found that large multiplication problems were associated with both longer RT and higher error rates than small problems (Ashcraft and Guillaume, 2009). Errors, however, are known to elicit specific activity in brain regions and this may bias fMRI analyses (Holroyd and Coles, 2002). Therefore, only correct trials are analyzed in the present study and the behavioral multiplication problem-size effect is measured in terms of a difference in RT rather than error rate. Specifically, the behavioral multiplication problem-size effect was investigated by analyzing RT data on correct trials as a function of problem-size and group. This was done using a 2 × 2 ANOVA with the within-subject factor Problem-size (small, large) and the between-subject factor Group (Chinese, American).

fMRI DATA ANALYSIS

Data analysis was performed using SPM5 (Statistical Parametric Mapping) (www.fil.ion.ucl.ac.uk/spm). The first six images of each run were discarded to allow for T1 equilibration effects. The remaining functional images were corrected for slice acquisition delays, spatially realigned to the first image of the first run to correct for head movements, co-registered with the segmented anatomical image, normalized to the standard T1 Montreal Neurological Institute (MNI) template volume (normalized voxel size, $2 \times 2 \times 4\,mm^3$), and spatially smoothed with a Gaussian filter equal to twice the voxel size ($4 \times 4 \times 8\,mm^3$ full width at half maximum). The quality of the normalization was verified in each participant by visually checking the registration and ensuring an adequate correspondence between each individual's brain and the MNI template. Event-related statistical analysis was performed according to the general linear model. Trials in which an incorrect response was recorded were excluded from the analyses. Activation was modeled as epochs with onsets time-locked to the presentation of the first stimulus and with a duration of 2 s. Only hits (i.e., correct responses in problems with a true answer) were considered of interest in the behavioral and fMRI analyses of the multiplication task. All epochs were convolved with a canonical hemodynamic response function. The time series data were high-pass filtered (1/128 Hz), and serial correlations were corrected using an autoregressive AR (1) model.

Previous behavioral studies have found that the multiplication problem-size effect is larger for American than Chinese participants (Campbell and Xue, 2001). Such a difference in task performance might introduce a potential confound in the fMRI analysis because any group differences in activity could be potentially explained by this discrepancy (Church et al., 2010). To minimize this confound, we matched the Chinese and American groups in terms of their behavioral problem-size effect. Specifically, we iteratively removed from the fMRI analyses the Chinese participants with the smallest multiplication problem-size effect and the American participants with the largest multiplication problem-size effect until no significant difference was observed between groups. This procedure yielded two groups with 22 participants in each for the fMRI analyses (i.e., 44 participants total). To determine the neural correlates of the multiplication problem-size effect in these remaining participants, we calculated for each subject the contrast of large vs. small problems (i.e., the neural problem-size effect). The resulting individual contrast images were entered into two random effect (RFX) analyses: a one-sample t-test across all participants (Chinese and Americans) and a two-sample t-test coding each group separately. In both analyses, the mean-centered individual behavioral problem-size-effects were included as covariates to control for any remaining behavioral differences between groups. These analyses allowed us to identify (1) the voxels showing a significant neural problem-size effect across groups and between groups, as well as (2) the voxels whose difference in activity between large and small problems (i.e., the neural problem-size effect) co-varied with the difference in RT between large and small problems (i.e., the behavioral problem-size effect) across groups and between groups. In the localizer scans, we calculated for each participant the contrasts of (1) word pairs vs. symbol strings in the verbal processing localizer (word pairs > strings) and (2) hard vs. easy numerical comparisons in the numerical processing localizer (hard > easy). The resulting individual contrast images were subsequently entered into RFX one-sample t-tests.

Unless otherwise noted, group-level statistical tests were controlled for a family-wise error (FWE) rate of $p < 0.05$ across the whole brain, via a combination of individual voxel threshold of $p < 0.005$ and cluster extent threshold of $880\,mm^3$ (i.e., 55 voxels). The cluster extent threshold was determined by Monte Carlo simulations (5000 iterations) conducted using the "AlphaSim" program (http://afni.nimh.nih.gov/pub/dist/doc/manual/AlphaSim.pdf) using an estimate of the smoothness of the data provided by SPM. Additionally, when no significant effect was present at this threshold, activations were examined with a FWE rate of $p < 0.1$ (across the whole brain). This was achieved by using a more lenient individual voxel threshold of $p < 0.01$ and a cluster extent threshold of $1120\,mm^3$ (i.e., 70 voxels) (estimated by AlphaSim). Such a more lenient threshold allows for an examination of more diffuse activations (Hasson et al., 2007) and indicates a statistical tendency. It is thus more informative than uncorrected thresholds because it allows for an interpretation of the results while giving a precise idea about the rate of false positive (Bennett et al., 2009).

In addition, small volume corrections were applied to *a priori* regions of interest of the parietal and temporal cortex identified in the localizer scans. These were the right IPS identified in the numerical processing localizer ($x = 36$, $y = -48$, $z = 47$) and the left Middle Temporal Gyrus (MTG) identified in the verbal processing localizer ($x = -28$, $y = -58$, $z = 21$). For these two regions, activation was controlled for a FWE rate of $p < 0.05$ within a 12-mm radius sphere around each set of coordinates, via a combination of individual voxel threshold of $p < 0.005$ and cluster extent threshold of 128 mm^3 (i.e., 8 voxels) (using AlphaSim and the procedure detailed above). All coordinates are reported in MNI space. For anatomical localization, we performed a non-linear transformation from MNI to Talairach coordinates (Talairach and Tournoux, 1988) and identified the regions activated via the Talairach Daemon software (http://ric.uthscsa.edu/resources). Cross validations were performed by overlaying each map on anatomical reference images from the Brodmann and AAL (automatic anatomic labeling) maps included in the Mricron software (www.sph.sc.edu/comd/rorden/MRIcron/).

Brain activity in activated clusters was extracted for visualization using the SPM toolbox Marsbar (http://marsbar.sourceforge.net/). Regions of Interests (ROIs) included all voxels within the activated cluster. For each participant, we calculated the average activity for each trial type within an ROI by averaging the fMRI signal across all voxels within that ROI.

RESULTS

BEHAVIOR

Multiplication task

A main effect of Problem-size revealed that participants were slower at evaluating large than small multiplication problems [$F_{(1, 51)} = 29.23$, MSE $= 8, 554$, $p < 0.00001$]. Therefore, a significant behavioral problem-size effect (i.e., difference in RT between large and small problems) was observed across all participants. However, this effect interacted with Group [$F_{(1, 51)} = 6.67$, MSE $= 8, 554$, $p = 0.013$], such that the problem-size effect was greater in American than Chinese participants (144 ms for American participants, 52 ms for Chinese participants). This was the case despite the fact that the problem-size effect was significant in each group separately [Chinese: $t_{(26)} = 4.02$, $p = 0.0004$; American: $t_{(25)} = 4.23$, $p = 0.0003$]. Finally, the ANOVA revealed a main effect of Group [$F_{(1, 51)} = 51.97$, MSE $= 63, 026$, $p < 0.00001$], indicating that Chinese participants were faster than American participants.

In line with previous findings (Campbell and Xue, 2001), our results indicate that the behavioral multiplication problem-size effect was larger in American than Chinese participants. To minimize this behavioral confound in fMRI analyses, we attempted to match the Chinese and American groups in terms of their behavioral problem-size effect (see Materials and Methods). After the matching procedure, the multiplication problem-size effect was still significant across participants [Chinese: $F_{(1, 21)} = 16.24$, MSE $= 2, 482$, $p = 0.0006$; American: $F_{(1, 21)} = 14.42$, MSE $= 15, 109$, $p = 0.002$] (see **Figure 2**). However, the interaction between Group and multiplication Problem-size effect was no longer significant [$F_{(1, 42)} = 4.05$, MSE $= 8796$, $p = 0.051$], indicating that the problem-size effect was more comparable across groups (although there remained a numerical difference between the problem-size effect between the groups).

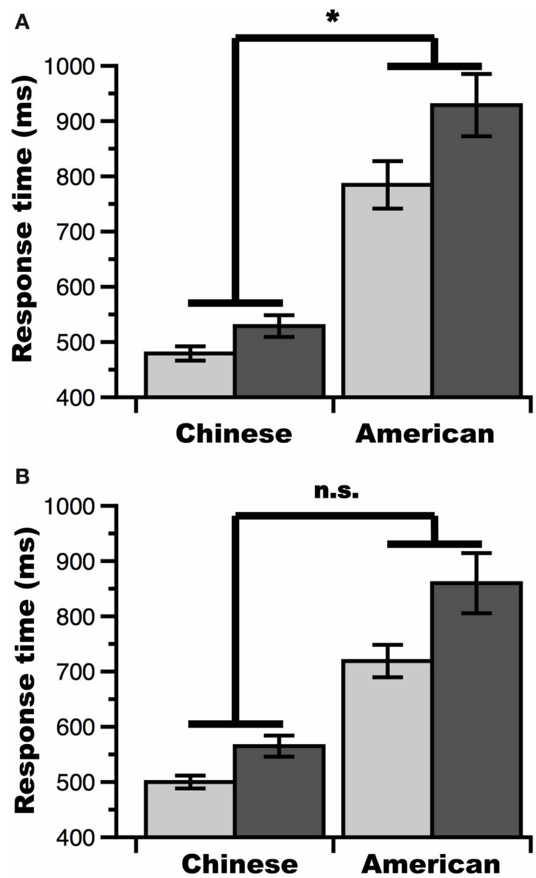

FIGURE 2 | Behavioral performance on the multiplication task. (A) Before equating for the magnitude of the problem-size effect between groups, the multiplication problem-size effect was significantly larger for American than Chinese participants. **(B)** After equating for the magnitude of the problem-size effect between groups, the multiplication problem-size effect was no longer significantly larger for American than Chinese participants. light gray, small problems; dark gray, large problems; *$p < 0.05$; n.s., non-significant; error bars, standard error of the mean.

Localizer scans

In the verbal processing localizer, mean RT for correct responses was submitted to a 2 × 2 ANOVA with the within-subject factor Stimulus type (word pairs, symbol strings) and the between-subject factor Group (Chinese, American). The ANOVA revealed a main effect of Stimulus type [$F_{(1, 51)} = 47.87$, MSE $=$ 8142, $p < 0.00001$], a main effect of Group [$F_{(1, 51)} = 24.03$, MSE $= 114, 762$, $p = 0.00001$] and an interaction of Stimulus type and Group [$F_{(1, 51)} = 44.81$, MSE $= 8142$, $p < 0.00001$]. Therefore, although participants took overall longer to evaluate word pairs than symbol strings, the effect was greater in Chinese (926 ms vs. 689 ms) than in American adults (1143 ms vs. 1137 ms).

In the numerical processing localizer, mean RT for correct responses was submitted to a 2 × 2 ANOVA with the within-subject factor Comparison difficulty (easy, intermediate, hard) and the between-subject factor Group (Chinese, American). We found a main effect of Comparison difficulty [$F_{(2, 102)} = 42.82$, MSE $= 2410$, $p < 0.00001$], indicating that RT increased as comparison difficulty increased (easy: 734 ms, intermediate: 760 ms, hard: 820 ms). Although the ANOVA revealed faster RT for Chinese than American participants [$F_{(1, 51)} = 28.13$, MSE = 246, 022, $p < 0.00001$], there was no interaction between Group and Comparison difficulty [$F_{(2, 102)} = 0.79$, MSE = 2410, $p = 0.46$]. Thus, consistent with previous research (Pinel et al., 2001; Prado et al., 2011), there was an inverse relationship between RT and the numerical distance between numerosities of the dot patterns (i.e., a distance effect). This effect, however, was not modulated by group.

Finally, the size of the main effect of Comparison difficulty in the numerical processing task was comparable to the size of the main effect of Stimulus type in the verbal processing localizer [86 ms vs. 121 ms, $t_{(52)} = 1.41$, $p = 0.16$]. Therefore, both localizer contrasts were comparable in terms of difficulty.

fMRI RESULTS

Localizer scans

As described in the Materials and Methods, localizer scans served to identify *a priori* regions of interest of the parietal and temporal cortices (i.e., IPS and MTG) involved in verbal and numerical processing for small volume correction of the main analyses. Across all participants, the verbal processing localizer identified a region of the left mid-superior temporal cortex more active for words than symbol strings ($x = -28$, $y = -53$, $z = 21$). Additional activation was observed in dorsal and ventral parts of the left Inferior Frontal Gyrus (IFG), left Middle Frontal Gyrus (MFG) and left Precentral Gyrus (PG) (see **Figure 3** and **Table 1** for a full list of activated regions). In the numerical processing localizer, enhanced activity was observed for hard than easy comparisons in the right IPS ($x = 36$, $y = -48$, $z = 47$). Additional activation was observed in a fronto-parietal network encompassing the left Precuneus, left ventral IFG, and Anterior Cingulate Cortex (ACC) (see **Figure 3** and **Table 1** for a full list of activated regions).

Multiplication problem-size effect

Across Chinese and American participants, a significant neural problem-size effect (i.e., greater activity for large than small multiplication problems) was observed in several fronto-parietal regions, including the left IPS, bilateral IFG, left MFG, and

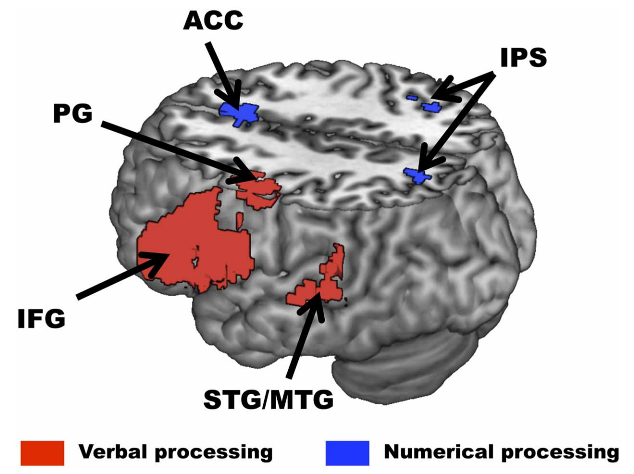

FIGURE 3 | Brain regions identified by the localizer scans across all participants. In the verbal processing localizer (red), greater activity for words than symbols was observed in the left Superior and Middle Temporal Gyri (STG/MTG), left Inferior Frontal Gyrus (IFG), and left Precentral Gyrus (PG). In the numerical processing localizer (blue), greater activity for difficult than easy comparisons was observed in the right Intraparietal Sulcus (IPS) and Anterior Cingulate Cortex (ACC). All activations are overlaid on a 3D rendering of the MNI-normalized anatomical brain. The upper part of the brain is cut out ($Z = 46$) to show activations in deeper sulci and along the medial wall of the cortex.

Table 1 | Clusters activated in the localizer scans across all participants.

Anatomical regions	~BA	Cluster size (mm^3)	MNI coordinates			*Z* scores
			X	*Y*	*Z*	
VERBAL PROCESSING LOCALIZER (WORDS > SYMBOLS)						
L. Inferior/Middle frontal gyrus	46/47	21200	−44	20	21	5.44
L. Middle temporal gyrus	21	1648	−28	−53	21	3.46
R. Caudate	–	7696	36	−43	2	4.86
L. Precentral gyrus	6	2336	−48	−3	48	4.62
L. Caudate	–	1600	−4	1	18	4.09
L. Middle/superior temporal gyrus	21/22	1952	−55	−37	6	3.91
L. Parahippocampal gyrus	28	976	−24	−18	−13	3.88
L. Anterior cingulate gyrus	25	880	−4	19	−4	3.85
L. Cuneus	18	2528	0	−73	15	3.24
R. Middle frontal gyrus	9	21200	4	50	23	3.11
NUMERICAL PROCESSING LOCALIZER (HARD > EASY COMPARISONS)						
R. Inferior frontal gyrus	9	4704	50	9	25	5.18
R. Anterior cingulate gyrus	32	896	10	21	39	3.83
L. Precuneus/superior parietal lobule	7	920	−18	−56	51	3.9
R. Intra-parietal sulcus	40	904	36	−48	47	3.78

Notes. All clusters survive a threshold of $p < 0.05$ FWE corrected for multiple comparisons.

L, left; R, right; ~BA, approximate Brodmann Area; MNI, Montreal Neurological Institute.

ACC (see **Table 2**). Critically, however, the effect differed between groups.

First, we found a greater neural problem-size effect for Chinese than American participants in the bilateral Superior Temporal Gyrus (STG), as well as in the left precentral/postcentral gyri and precuneus (see **Figure 4A** and **Table 2**). A visualization of the pattern of brain activity in the left and right STG revealed that the group difference was driven by a positive neural problem-size effect in Chinese participants, and a negative effect for American participants (see **Figure 4B** for a plot in the left STG). The left MTG/STG cluster identified in the verbal processing localizer did not overlap with the left STG cluster exhibiting the group difference in neural problem-size effect. However, overlap was observed in the left STG at a FWE corrected threshold of $p < 0.1$, indicating a statistical tendency.

Second, we found a greater neural problem-size effect for American than Chinese participants in the right IPS and ACC (activation in the right IPS was found after small volume correction based on the peak activity obtained in the numerical localizer task, see Materials and Methods) (see **Figure 4A** and **Table 2**). The peak activity of this right IPS cluster was less than 12 mm away from the peak coordinates of the right IPS region identified in the numerical processing localizer. In the IPS, an examination of the pattern of brain activity revealed that the group difference was driven by a larger positive neural problem-size effect for American than Chinese participants (see **Figure 4C**).

Simple effect analyses were then conducted to assess the significance of the neural problem-size effect in each group separately. First, in both the right IPS ($x = 30$, $y = -60$, $z = 36$; $Z = 3.77$) and the ACC ($x = -2$, $y = 24$, $z = 36$; $Z = 4.09$), we found a significant neural problem-size effect across American participants. Importantly, the effect in the IPS was absent in Chinese participants. Second, we did not find a significant neural problem-size effect across Chinese participants in either the left or right STG at our stringent FWE corrected threshold of $p < 0.05$. However, this effect tended to be significant in both of these regions, as revealed by further analyses conducted at a threshold of $p < 0.1$ (FWE corrected across the whole-brain). Furthermore, small multiplication problems tended to be associated with more activity than large multiplication problems (i.e., a reverse neural problem-size effect) in American participants ($p < 0.1$ FWE corrected) in both of these regions.

Individual differences in the multiplication problem-size effect

Overall, the results above suggest that the neural sources of the multiplication problem-size effect differ in Chinese and American participants. To test whether activity in the brain regions found above was related to behavioral performance, we then identified the voxels in which there was a reliable between-subject relationship between the behavioral and neural problem-size effects in Chinese and American participants across the whole-brain. Although we did not find any regions showing such a relationship across Chinese participants, we found that a larger neural problem-size effect was associated with a larger behavioral problem-size effect across American participants in the right IPS, ACC and right IFG (see **Figure 5A**). Critically, this relationship was more positive across American than Chinese participants in all of these regions (see **Figure 5B** and **Figure 5C** for a plot in the right IPS). Therefore, the neural bases of inter-individual

Table 2 | Clusters showing a multiplication neural problem-size effect.

Anatomical regions	~BA	Cluster size (mm^3)	MNI coordinates			*Z* scores
			X	*Y*	*Z*	
ACROSS ALL PARTICIPANTS (LARGE > SMALL)						
R. Insula	13	4592	34	20	4	5.62
L. Inferior frontal gyrus	47	13504	−40	18	−12	5.56
L. Precuneus/intraparietal sulcus	7/40	7328	−30	−46	44	4.75
R. Medial frontal gyrus	6	1936	0	14	52	4.42
L. Cuneus	18	2880	0	−80	24	4.22
CHINESE (LARGE > SMALL) > AMERICAN (LARGE > SMALL)						
L. Paracentral lobule/precuneus	5/7	14112	−10	−44	60	5.14
R. Superior temporal gyrus	42	1616	58	−24	4	4.68
R. Medial frontal gyrus	11	3360	6	48	−12	4.31
L. Superior temporal gyrus	22	1488	−64	−16	4	3.96
L. Precentral/postcentral gyrus	4/3	944	−54	−16	44	3.72
L. Insula	13	896	−38	−24	20	3.6
R. Fusiform gyrus	19	896	24	−60	−8	3.06
AMERICAN (LARGE > SMALL) > CHINESE (LARGE > SMALL)						
L. Medial frontal gyrus	8	1968	−4	20	52	3.59
R. Intraparietal sulcus*	40	128	32	−60	44	2.98

Notes. All clusters survive a threshold of p < 0.05 FWE corrected for multiple comparisons.

L, left; R, right; ~BA, approximate Brodmann Area; MNI, Montreal Neurological Institute.

**Region significantly activated after small volume correction.*

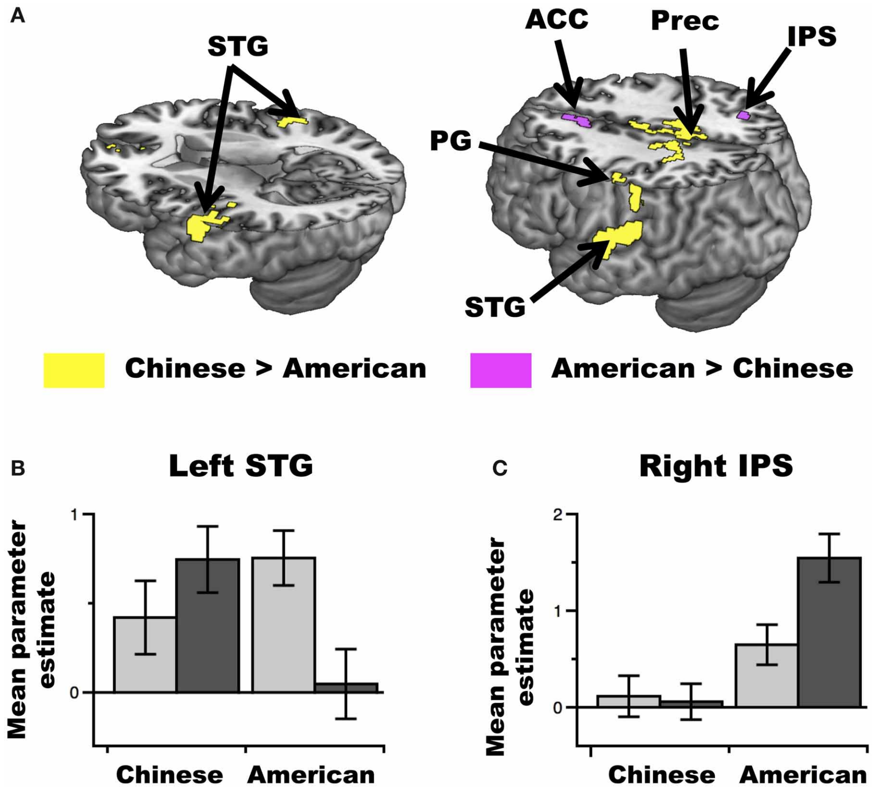

FIGURE 4 | Group differences in the neural problem-size effect (i.e., difference in activity between large and small multiplication problems). **(A)** A greater neural problem-size effect was observed in Chinese than American participants in the bilateral Superior Temporal Gyrus (STG), left Postcentral Gysrus (PG), and Precuneus (Prec) (yellow). A greater neural problem-size effect was observed in American than Chinese participants in the right Intraparietal Sulcus (IPS) and Anterior Cingulate Cortex (ACC) (purple). All activations are overlaid on 3D renderings of the MNI-normalized anatomical brain. The upper part of the brain is cut out at two different heights (left panel: $Z = 8$; right panel: $Z = 46$) to show activations in deeper sulci and along the medial wall of the cortex. **(B)** Plot of the brain activity observed for large (dark gray) and small (light gray) problems as a function of group in the left STG cluster (for visualization only). **(C)** Right: Plot of the brain activity observed for large (dark gray) and small (light gray) problems as a function of group in the right IPS cluster (for visualization only). Note that the scale is different than that in **(B)**.

variations in the problem-size effect also differed between the Chinese and American group.

Control analyses

The group differences in the neural problem-size reported above are consistent with our hypotheses. However, it is important to ensure that such effects are not driven by other factors.

First, consistent with previous reports (Lefevre and Liu, 1997; Campbell and Xue, 2001), American participants were less proficient in single-digit multiplication than Chinese participants: they displayed poorer overall performance and a larger problem-size effect. We controlled for group differences in behavioral problem-size effect by including this factor as covariate in our main analyses and by matching the groups in terms of this effect. To further rule out the possibility that overall group differences in multiplication performance were driving our results, we performed an additional set of analyses in which we included both problem-size effect and overall response time as nuisance covariates. The results obtained with this model were similar to the results obtained in our initial analysis. Specifically, all the clusters in which we observed differences in the neural problem-size effect in Chinese vs. Americans remained significant with these covariates. This suggests that none of our results were due to differences in overall performance between groups.

Second, as can be seen on **Figure 5C**, inter-individual differences in the behavioral problem-size effect were larger in the American than in the Chinese group. Although none of the American participants can be considered outliers (defined as >3 standard deviations from the mean), it remains possible that the more positive relationship between the behavioral and neural problem-size effects in the IPS for American compared to Chinese participants might be driven by a greater inter-individual variability. Therefore, we conducted another set of analyses in which we removed the two American participants with the largest behavioral problem-size effects, thereby equating inter-individual variability between groups. Again, the results obtained with these analyses were similar to the results obtained in our initial analyses. Specifically, there was still a reliable positive relationship across American participants in the right IPS and ACC (but not in the right IFG). This relationship was also greater in Americans than Chinese in both of these regions.

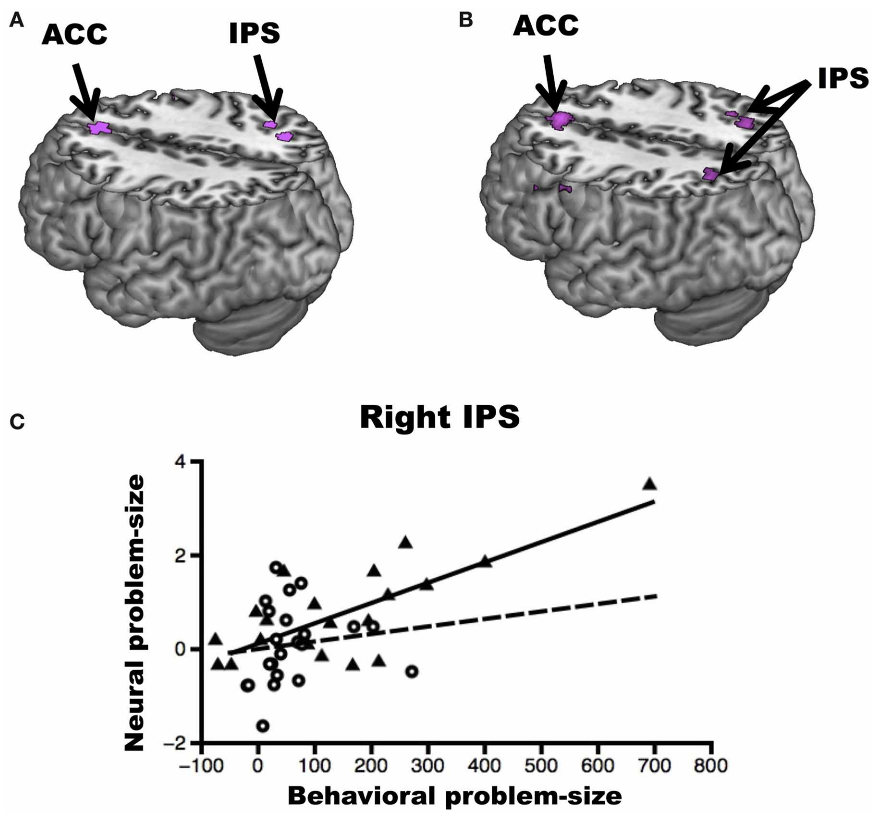

FIGURE 5 | Inter-individual variability in the multiplication problem-size effect. (A) Across American participants, a larger behavioral problem-size effect (difference in response time between large and small problems) was associated with a larger neural problem-size effect (difference in brain activity between large and small problems) in the right Intraparietal Sulcus (IPS) and the Anterior Cingulate Cortex (ACC). **(B)** A stronger positive relationship between behavioral and neural problem-size effects was observed in American than Chinese participants in the bilateral IPS and ACC. **(C)** Plot of the relationship between the behavioral and neural problem-size effect in the right IPS for American (triangles, solid line) and Chinese (circles, dotted line) participants (for visualization only). Brain activity was extracted in the cluster showing a stronger positive relationship between behavioral and neural problem-size effects in American than Chinese participants for multiplication. The relationships between behavioral and neural problem-size effects remained significant in the right IPS and ACC when the two subjects with the largest behavioral effects were removed from the analysis. All activations are overlaid on a 3D rendering of the MNI-normalized anatomical brain. The upper part of the brain is cut out ($Z = 46$) to show activations in deeper sulci and along the medial wall of the cortex.

Third, Chinese and American participants were scanned on two different MRI scanners. To minimize this factor, the exact same experimental protocol, scanner model, and scanning protocol were used. However, it remains possible that some of the between-group differences might still be affected by scanner-related factors (e.g., in shim or magnetic susceptibility). Importantly, such scanner-related biases should affect all contrasts to the same degree. This includes low-level contrasts in which one would not expect any differences between groups. To test for this possibility, we contrasted the brain activity associated with null trials in Chinese vs. American participants. We did not find any brain regions differentially activated between groups, even at a lenient threshold of $p < 0.01$ uncorrected. Therefore, it seems unlikely that systematic differences between scanners might have biased our results.

DISCUSSION

The problem-size effect is one of the most robust and consistent phenomena in the cognitive arithmetic literature (Ashcraft and Guillaume, 2009). Yet, there is a debate as to whether the effect reflects differences in retrieval effort or in the use of calculation procedures between large and small problems (Ashcraft and Guillaume, 2009). Several behavioral studies suggest that the sources of the problem-size effect might in fact depend upon differing educational backgrounds across countries (Lefevre and Liu, 1997; Campbell and Xue, 2001; Penner-Wilger et al., 2002). Specifically, while the effect might result from differences in retrieval effort in Chinese participants, it might stem from differences in the use of calculation strategies in North American participants (Penner-Wilger et al., 2002). The present fMRI study sought to test this hypothesis by investigating the neural bases of the multiplication problem-size effect in Chinese and American adults.

NEURAL MULTIPLICATION PROBLEM-SIZE EFFECT ACROSS ALL PARTICIPANTS

Across all Chinese and American participants, we found greater overall activity for large than small multiplication problems in a network of dorso-parietal brain regions encompassing the IPS as well as the lateral and medial prefrontal cortex. This finding is consistent with several previous neuroimaging studies that have shown that these regions are more active for large than small problems (Stanescu-Cosson et al., 2000; Zhou et al., 2007; Jost et al., 2009; De Smedt et al., 2011). Although this finding might be interpreted as reflecting greater use of calculation procedures in large than small problems (De Smedt et al., 2011), follow-up analyses revealed that it was mostly driven by the American group. As discussed below, our study suggests that the neural bases of the multiplication problem-size effect are affected by country and educational background.

NEURAL MULTIPLICATION PROBLEM-SIZE EFFECT IN CHINESE PARTICIPANTS

Our results revealed a larger neural problem-size effect in Chinese than American participants in the bilateral STG. The left STG cluster tended to overlap with the mid-superior temporal region identified in the rhyming task that was used as verbal processing localizer. Several neuropsychological (Sandrini et al., 2003; Van Harskamp et al., 2005) and neuroimaging (Zhou et al., 2007; Andres et al., 2011, 2012; Prado et al., 2011) studies have suggested that regions of the mid-superior temporal cortex (especially in the left hemisphere) are involved in the representation of math facts in verbal memory. For example, studies have found that lesions of the left mid-superior temporal regions are associated with impaired retrieval of multiplication facts (Lampl et al., 1994; Sandrini et al., 2003; Van Harskamp et al., 2005; Delazer et al., 2006). In a previous study, we have suggested that the left MTG might be involved in the storage of the semantic association between a multiplication problem and its answer (Prado et al., 2011). In the present study, the greater involvement of the left STG in Chinese than American participants is broadly consistent with a general role of the temporal cortex in lexical-semantic processing (Vandenberghe et al., 1996; Price et al., 1997; Rissman et al., 2003). However, it is interesting to note that the left STG is typically associated with phonological (rather than semantic) processing in the literature (Friederici, 2012; Wu et al., 2012) and that both the left and right STG are believed to play an important role in letter to speech sound mapping (Suzuki and Sakai, 2003; Hickok and Poeppel, 2007; Blau et al., 2009). Activation of the bilateral STG might thus also reflect the greater involvement of phonological representations during the processing of large vs. small multiplication problems in Chinese as compared to American participants. This might be due to the fact that, unlike Americans, Chinese memorize multiplication facts as rhyming formulas in school, thanks to the single-syllable structure of Chinese number words (this strategy is reflected in the name of the Chinese multiplication table, i.e., Nine Nine song).

It is interesting to note that a larger neural problem-size effect in Chinese than American participants was also observed in other brain regions, such as the precentral and postcentral gyrus and the precuneus. Such activations were not *a priori* predicted and, in the absence of any relevant localizers, must be interpreted with caution. However, the involvement of these regions may indicate that factors other than verbal retrieval might differentiate arithmetic processing in Chinese and American participants. For example, although only Arabic numerals were used in this task, reading experience may affect arithmetic processing (Tang et al., 2006). Because the Chinese writing system places greater demands on visuo-spatial processing than the English writing system, these activations might thus reflect enhanced visuo-spatial processing in Chinese participants (Tang et al., 2006; Cantlon and Brannon, 2007). Such activations might also reflect the use of alternative visuo-spatial strategies in Chinese participants, such as abacus imagery (Cantlon and Brannon, 2007).

NEURAL MULTIPLICATION PROBLEM-SIZE EFFECT IN AMERICAN PARTICIPANTS

We also found a larger neural problem-size effect in American than Chinese participants in the right IPS and the ACC. The IPS is believed to house neuronal populations sensitive to numerical magnitudes (Nieder and Dehaene, 2009) and to be a critical region for numerical processing in general (Ansari, 2008). This region is consistently found activated in tasks involving numerical comparison (Ansari, 2008) and arithmetic problem-solving (Dehaene and Cohen, 2007). Critically, enhanced activity in the IPS has been observed when problems are solved with calculation procedures rather than retrieved from memory (Grabner et al., 2009). Such enhanced activation of the IPS is typically accompanied with greater recruitment of frontal regions, including the ACC (Grabner et al., 2009). Recruitment of such frontal regions has been attributed to the greater demands in working-memory and executive control associated with calculation strategies (Delazer et al., 2003). Therefore, our findings suggest that the multiplication problem-size effect might result from a greater use of calculation procedures in large vs. small problems in American as compared to Chinese participants.

MULTIPLICATION AND VERBAL RETRIEVAL IN CHINA

Why would the problem-size effect be more associated with differences in verbal representations in Chinese than American participants? One possibility is that the Chinese education system places greater emphasis on verbal memorization methods than American education (Zhang and Zhou, 2003). To some extent, rote verbal teaching methods are employed in both China and the United States. Multiplication tables are used to teach multiplication in Chinese and American elementary schools, but those methods tend to be used earlier and more extensively in China than in the United States (Zhang and Zhou, 2003). The result is that Chinese children spend more time practicing multiplication facts than American children, both in school and at home. Rote verbal memorization of multiplication facts in Chinese children might also be facilitated by cultural specificities. For example, the Chinese multiplication table is shorter and easier to memorize than the tables typically used in American schools (Zhou et al., 2007). Rote verbal learning is further made easier by the relative transparency and conciseness of Chinese words for numbers, as compared to English words (Miller et al., 1995). Overall, these educational divergences might explain why a greater proportion of Chinese than North American adults rely on direct retrieval strategies to solve both small and large multiplication problems (Campbell and Xue, 2001), and why the multiplication problem-size effect might be more strongly related to differences in verbal representations in Chinese as compared to American participants.

MULTIPLICATION AND CALCULATION PROCEDURES IN AMERICANS

The greater reliance on calculation procedures in American than Chinese participants might be explained by the greater emphasis that American education tends to place on the comprehension of mathematical concepts during childhood (such as numerical magnitude or numerical order) than on verbal memorization methods *per se* (Graham and Fennell, 2001). Overall, less extensive reliance on rote verbal learning in the United States than in China is likely to lead to weaker associations between multiplication problems and their solutions in American as compared to Chinese adults, which might lead to a greater use of indirect calculation procedures. This may be especially true for problems involving large problem-sizes, which are typically less drilled in school than problems involving smaller operands (Hamann and Ashcraft, 1986). Indirect strategies used by American participants could involve decomposing a relatively large problem-size item (e.g., 9×8) into a multiplication problem that is easier to retrieve from memory (e.g., $10 \times 8 = 80$) and using a different operation to calculate the results (e.g., $80 - 8 = 72$). It might also involve transforming a multiplication problem (e.g., 3×8) into easier addition problems (e.g., $8 + 8 + 8$). These indirect strategies involve a manipulation of numerical magnitudes through addition or subtraction and are more likely to engage numerical processing mechanisms in the IPS (as well as control processes in the ACC) than verbal retrieval mechanisms in the mid-superior temporal gyrus. In keeping with these observations, we found more activity for large than small multiplication problems in American participants in the IPS and ACC, but not in any regions of the temporal cortex. Instead, we found that large problems tended to be associated with less activity than small problems in the left STG (see **Figure 4B**). Therefore, large problems might be more likely to be solved by backup strategies and calculation procedures than verbal retrieval in American participants. Overall, our results are consistent with the view that a failure to retrieve the answer of large problems and a more extensive use of calculation procedures in large vs. small problems might give rise to the problem-size effect in American adults (Lefevre and Liu, 1997; Penner-Wilger et al., 2002).

Interestingly, a previous study found greater activity in the left STG (as well as in the left IFG) for single-digit addition in English-speakers as compared to Chinese-speakers (Tang et al., 2006). Because this study did not categorize problems as a function of their sizes, it is impossible to know whether the effect was driven by small or large problems (or both). Nonetheless, an examination of the pattern of activity in the left STG in the present study (see **Figure 4B**) indicates that small multiplication problems also elicited numerically higher activity in American than Chinese participants. Therefore, the higher left STG activity for English than Chinese speakers observed by Tang et al. (2006) might have been primarily driven by small addition problems and may reflect greater retrieval effort for these problems in English speakers. Critically, although Tang et al. (2006) did not find any reliable group differences in regions associated with numerical calculation, English-speakers tended to engage more extensive activity in the right parietal cortex than Chinese-speakers (see Figure 1 in Tang et al., 2006). Our findings suggest that this effect might have been driven by large problems and might reflect a greater use of calculation procedures in English than Chinese-speakers.

INDIVIDUAL DIFFERENCES IN THE MULTIPLICATION PROBLEM-SIZE EFFECT

Further support for the greater use of calculation procedures in American than Chinese participants is given by an analysis of the inter-individual variability in the multiplication problem-size effect. We found that a larger behavioral problem-size effect was associated with a larger neural problem-size effect in both the IPS and ACC across American participants, but not across Chinese participants. Furthermore, this relationship was significantly stronger for American than Chinese participants. Therefore, even if the problem-size effect is likely to result from the use of calculation procedures in most American participants, this is especially true for participants exhibiting the largest problem-size effects. This was, however, not the case for Chinese participants. Surprisingly, we did not find any relationship between behavioral and neural multiplication problem-size effects in the left mid-superior temporal gyrus across Chinese individuals. Although it is always difficult to interpret a null effect, it is possible that this lack of relationship might be due to the smaller inter-individual variability observed in the Chinese sample than in the American sample. Future studies with a larger number of subjects and greater inter-individual variability might examine the relationship between behavioral and neural problem-size effects in Chinese participants.

OTHER FACTORS THAT MAY HAVE INFLUENCED THE BETWEEN-GROUP COMPARISON

Although we argue that the between-group differences observed here result from divergences in educational backgrounds across countries, other potential factors should be considered. For example, between-group differences might result from differences in MRI scanners (Costafreda et al., 2007; Gountouna et al., 2010; Yendiki et al., 2010), language processing (Bolger et al., 2005) and/or performance levels (Church et al., 2010). However, none of these factors appear to provide a better explanation of our findings than differences in educational methods across countries. First, although the present data were acquired at two different sites, the exact same experimental protocol, scanner type and scanning protocol were used in both sites. Several studies have shown that, when these precautions are taken, activation variability due to scanner site is small compared to inter-individual variability in the cognitive task (Costafreda et al., 2007; Gountouna et al., 2010; Yendiki et al., 2010). Those studies all conclude that multi-site studies are reliable. Second, arithmetic problems were presented in the same Arabic numeral form to both Chinese and American participants, thus controlling for linguistic differences between groups. Third, a limitation of our study is that we did not acquire measures of intellectual and arithmetic abilities for each participant. Therefore, the differences between the Chinese and American groups might be attributable to overall differences in arithmetic skill, rather than differences in educational background. This may be problematic because differences in proficiency have been found to affect the neural bases of arithmetic processing (Grabner et al., 2007; Matejko et al., 2012; Price et al., 2013). However, this possibility is unlikely for two reasons. Firstly, behavioral research has shown that equating groups of Chinese and North-American adults for overall (multi-digit) arithmetic performance does not remove differences in single-digit multiplication performance: Chinese are still faster overall and exhibit a smaller problem-size effect than North-Americans (Lefevre and Liu, 1997). Therefore, the smaller multiplication problem-size effect observed in Chinese than American participants is more likely to be due to cultural and/or educational factors than proficiency *per se* (Lefevre and Liu, 1997). Secondly, we controlled for overall group differences in skill by (1) matching groups in terms of size of the problem-size effect and (2) including in our fMRI analyses the behavioral problem-size effect and the overall response time as nuisance covariates. Therefore, although we cannot definitely rule out the hypothesis that some of our results might be attributable to differences in proficiency, we think that the differences observed in the present study are more likely to stem from differences in educational backgrounds.

CONCLUSION

In sum, our findings support the idea that the source of the multiplication problem-size effect may vary across countries (Penner-Wilger et al., 2002). Specifically, the neural dissociation observed between STG and IPS for large and small problems indicates that the effect is more likely to be due to reliance on verbal representations in Chinese than American individuals, while it might more likely result from the use of calculation procedures in American than Chinese individuals. Our direct demonstration of differences in the reliance on these underlying mechanisms in Chinese and American adults is in keeping with prior behavioral research based on self-report and analyses of reaction time (Lefevre and Liu, 1997; Campbell and Xue, 2001; Penner-Wilger et al., 2002). Together with Tang et al. (2006), our study indicates that the neural bases of elementary arithmetic are modulated by educational differences across countries. Such findings might be important for understanding the effects of different teaching methods on the neural representations of arithmetic (Dowker, 2005). They might also improve our knowledge of the neural bases of math learning disabilities across countries, as those are likely to stem from different sources (Geary, 2010) depending on education and cultural background.

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Single-digit arithmetic processing—anatomical evidence from statistical voxel-based lesion analysis

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Different specific mechanisms have been suggested for solving single-digit arithmetic operations. However, the neural correlates underlying basic arithmetic (multiplication, addition, subtraction) are still under debate. In the present study, we systematically assessed single-digit arithmetic in a group of acute stroke patients ($n = 45$) with circumscribed left- or right-hemispheric brain lesions. Lesion sites significantly related to impaired performance were found only in the left-hemisphere damaged (LHD) group. Deficits in multiplication and addition were related to subcortical/white matter brain regions differing from those for subtraction tasks, corroborating the notion of distinct processing pathways for different arithmetic tasks. Additionally, our results further point to the importance of investigating fiber pathways in numerical cognition.

Keywords: arithmetic, arithmetic facts, number processing, lesion analysis, stroke patients, fiber pathways

INTRODUCTION

Despite numerous fMRI studies reporting the neural correlates of number processing (e.g., see Dehaene, 2009; Nieder and Dehaene, 2009 for reviews; see Arsalidou and Taylor, 2011 for a meta-analysis), there is still no agreement about the cognitive mechanisms involved in basic arithmetic, nor have its neural bases been delineated sufficiently. There is no consensus even with regard to single-digit operations, such as "2 × 3," which are encountered frequently in every-day life and are assumed to be solved by retrieval from long-term memory without additional computation. These problems are commonly referred to as arithmetic facts, however a precise definition is rarely given (Domahs and Delazer, 2005).

Neuropsychological observations of double dissociations suggest that arithmetic facts are stored separately from other numerical information such as arithmetic concepts or procedures (Warrington, 1982; McCloskey et al., 1991b; McCloskey, 1992; Hittmair-Delazer et al., 1995; Delazer and Benke, 1997). The currently most influential model of numerical cognition, the Triple Code Model (TCM) by Dehaene et al. (2003) refers only to multiplication table facts, whereas addition is seen as a mixed operation, in which both direct and indirect processing pathways can be involved. In contrast, subtraction is assumed to rely essentially on magnitude processing, and it does not involve language-based processes (Lee and Kang, 2002). However, there is evidence that different arithmetic operations can be solved via diverse strategies (Lee, 2000) and considerable individual differences in strategy use have been reported (LeFevre et al., 1996a; Campbell and Xue, 2001; Thevenot et al., 2007). Self-reports suggest that the fact-retrieval strategy can be applied for easy items from all arithmetic operations, whereas it is much more often applied for multiplication (82%) and addition (75%) than for division and subtraction (cf. Campbell and Xue, 2001; Grabner et al., 2009). Thus, it has been suggested that single-digit addition tasks with sums smaller than ten, which do not involve "carrying," can also be retrieved from memory directly (LeFevre et al., 1996b; Stanescu-Cosson et al., 2000; Klein et al., 2013). Findings by Thevenot et al. (2013) suggest that the retrieval strategy for addition tasks is more common in older as compared to younger subjects. Moreover, the bimodal distribution of reaction time (RT) data (Campbell, 2008) indicates that basic subtraction problems can also be solved by the retrieval strategy.

Another question concerns the interrelation of arithmetic operations. If different operations can be solved by the same strategy—do they share common neural representations? Results of neuroimaging studies provide evidence for separate representations (Arsalidou and Taylor, 2011; Rosenberg-Lee et al., 2011), however there is no comprehensive data from a group study in patients so far. The TCM (Dehaene et al., 2003) posits that arithmetic fact retrieval is subserved by left-hemispheric perisylvian and language areas. So far the TCM neither specifies in detail all language areas involved nor how they are connected for processing numerical information.

Therefore, the current study aimed to investigate single-digit arithmetic tasks in different arithmetic operations (i.e., addition, subtraction, multiplication) in a sample of 45 acute stroke patients in order to identify the brain structures crucial for their execution.

The variability of diagnostic tests and methodologies used in previous neuropsychological studies on single-digit arithmetic does not allow for direct comparisons. Single case studies have included patients with very divergent etiologies, such as closed head injury (e.g., McCloskey et al., 1991a), Fahr's disease (Delazer et al., 2004), dementia (e.g., Pesenti et al., 1994), or radiotherapy following leukemia (Hittmair-Delazer et al., 1995). These studies are informative as to the nature of the putative cognitive processes involved in relation to some processing model, yet they are not suited for determining critical brain areas involved in particular functions. Voxel-wise lesion-behavior mapping (VLBM) methods applied in an unselected stroke sample are considered a powerful approach to identify not only those brain structures which are "involved" in arithmetic fact retrieval but rather which are critically required for normal functioning. These methods implement inferential statistical analyses irrespective of clinical diagnoses or specified regions of interest. Moreover, these methods also allow to identify potential new brain areas in the network explored (Bates et al., 2003; Rorden et al., 2007). However, for valid VLBM-findings the examined sample should be unselected and the lesions should possibly cover the entire brain or at least large portions of it. Furthermore, because of neural plasticity, reorganization processes, and spontaneous recovery changes in shape, location, and functional integrity of brain tissue depend on the time post-stroke, potentially affecting the results of a VLBM analysis (Karnath and Rorden, 2012). To minimize these intervening sources, we decided to investigate patients only in the acute phase.

The aim of the current study was two-fold. First, we aimed at identifying brain structures critical for the execution of single-digit calculation. Based on the assumptions of the TCM and results of further neuroimaging studies, as summarized in a recent meta-analysis (Arsalidou and Taylor, 2011), we expected involvement of left perisylvian regions in arithmetic fact retrieval. Second, the study set off to systematically examine whether single-digit problems from different arithmetic operations are dependent on the same or different neural circuits.

METHODS

PATIENTS

Forty-five acute stroke patients, 21 left-hemisphere damaged (LHD), and 24 right-hemisphere damaged (RHD) participated in the study. This unselected sample comprised all patients consecutively admitted to the Center of Neurology at Tübingen University Clinic during 33 months, who met the inclusion criteria: MR or CT-documented cerebral stroke with cortical involvement, max. 14 days post-stroke, no previous lesions, no other neurological or psychiatric diseases, no substantial micro-angiopathy or white matter alterations, right-handedness, and German language as their mother tongue. Demographic and clinical data of all patients is presented in **Table 1** and **Supplementary Table 1**. LHD patients were tested for language comprehension with the "Color-Figure" subtest items from the German adaptation of the Aphasia Check-List (ACL; Kalbe et al., 2005), and language production with the "Picture naming task" from the Aachener Aphasia-Bedside Test for acute patients (AABT; Biniek et al., 1992). Right-hemisphere patients underwent hemispatial neglect testing consisting of two cancellation tasks: "Letter Cancellation Task" (Weintraub and Mesulam, 1985) and "Bells test" (Gauthier et al., 1989), a copying task (Johannsen and Karnath, 2004), and a line bisection task (Heilman and Valenstein, 1979). Visual field deficits were assessed in all patients with a confrontation visual field test.

All patients gave their informed consent. The study was conducted in accordance with the ethical standards laid down in the 1964 Declaration of Helsinki and was approved by the ethics committee of the University Clinic Tübingen.

STIMULI AND PROCEDURE

Participants performed single-digit multiplication, addition, and subtraction tasks as part of a standardized neuropsychological battery examining number processing performance (Number Processing and Calculation (NPC) Battery; Delazer et al., 2003), also providing cut-off scores for impaired performance. In the NPC battery the three different arithmetic tasks constitute separate subtests. The standardized procedure requires that the testing per subtest is aborted after five consecutive incorrect or missing responses. Like for most cognitive neuropsychological assessment procedures, there is no time limit for answering the individual items. Self-corrections were allowed. Each calculation task was presented on a separate A4-sheet of paper (black digits printed on white paper, digit height: 7 mm). Sheets were aligned centrally on a table in front of the patient. Participants responded orally.

For the subsequent correlation of addition and subtraction tasks with lesion information we considered only those items of a subtest, which did not involve a carry or borrow operation (i.e., the sum was smaller than 10). Also, none of the items involved "0" or "1" as operands, because they are assumed to represent a distinct class of arithmetic problems implying rule-based processing (McCloskey et al., 1991a). With these item restriction criteria, the multiplication task comprised 36, the addition task 10, and the subtraction task 15 items.

Table 1 | Demographic and clinical data of the left- and right-hemisphere damaged patients.

		LHD	RHD
n		21	24
Sex (f/m)		15/6	11/13
Age (years)	Mean (SD)	61.6 (16.1)	61.0 (14.0)
Stroke type	Ischemic stroke	17	21
	Hemorrhagic stroke	4	3
Interval lesion onset to examination (days)	Mean (SD)	4.3 (1.9)	5.5 (2.8)
Interval lesion onset to imaging (days)	Mean (SD)	2.1 (2.0)	3.7 (3.5)
Education (years)	Mean (SD)	14.1 (4.4)	12.3 (4.5)
Contralateral paresis	% present	28.6	66.7
Visual field deficit	% present	23.8	20.8
Aphasia	% present	38.1	–
Neglect	% present	–	16.7

**Except for "Contralateral paresis" ($\chi^2 = 6.5$, $p = 0.011$) there are no significant differences between the groups.*

Table 2 | Raw scores (number of items solved correctly) observed for the two patient groups in each arithmetic task.

		LHD ($n = 21$)				RHD ($n = 24$)			
	Items	**Mean**	**Median**	***SD***	**Range**	**Mean**	**Median**	***SD***	**Range**
Addition	10	9.1	10.0	2.3	0–10	10.0	10.0	0.0	10–10
Subtraction	15	13.4	15.0	3.4	1–15	14.8	15.0	0.4	14–15
Multiplication	36	32.0	33.0	7.5	2–36	34.0	35.5	3.1	24–36

To operationally determine impaired performance in the arithmetic tasks, we used cut-off criteria. For multiplication the cut-offs provided with the NPC-battery were used (cf. Delazer et al., 2003). The NPC-battery provides no separate cut-offs for addition items with sums below 10, or for subtraction items with minuends below 10. However, ceiling performance is expected in a healthy population. Thus, patients were considered to be showing a deficit in a given fact-retrieval task if their performance was below a mastery criterion computed by means of a procedure derived from criterion-referenced measurement. Using exact binomial (95%, i.e., $1 - \alpha$) confidence intervals computed for the relative frequency of items solved correctly, performance is considered to be in the mastery range if the upper bound of that interval is higher than some (high) criterion probability, e.g., $p_c = 0.95$ or $p_c = 0.99$. In case of very easy tasks like addition under 10, $p_c = 0.99$ was considered to be appropriate. For the somewhat more difficult subtraction facts $p_c = 0.95$ was employed.

In addition, to detect possible dissociations for individual patients' performance on different tasks, we performed specific, freely available single-case statistical tests for differences in level of performance (http://homepages.abdn.ac.uk/j.crawford/pages/dept/SingleCaseMethodology.htm; Deloche and Willmes, 2000; cf. Crawford and Garthwaite, 2005; Willmes, 2010) implementing operational definitions for different types of performance dissociations (classical and strong) as conceptually proposed by Shallice (1988).

LESION ANALYSIS

We used diffusion-weighted images for patients, who underwent MR-imaging within the first 48 h after stroke-onset (Weber et al., 2000), or T2-weighted fluid-attenuated inverse-recovery (FLAIR) contrast MR-imaging, if images were acquired later than that (Brant-Zawadzki et al., 1996; Noguchi et al., 1997; Ricci et al., 1999; Schaefer et al., 2002). If MR-images were not available, we employed CT-images. If several subsequent imaging data sets were available for the same patient, we chose the session acquired closest to the time of behavioral testing and providing the best imaging contrast for lesion demarcation.

Lesion borders were marked directly in the individual MR- or CT-scan using MRIcron software (www.mricro.com/mricron). Subsequently, both the anatomical scan and the lesion shape were mapped onto stereotaxic space using the "Clinical Toolbox" for normalization (Rorden et al., 2012; www.mccauslandcenter.sc.edu/CRNL/clinical-toolbox) implemented in SMP8 (www.fil.ion.ucl.ac.uk/spm). Some of the normalized lesion images had to be adjusted manually to the standard template by validating specific anatomical landmarks such as the basal ganglia. This was particularly the case in patients with extended hemorrhage, in which the normalization algorithm may lead to an unrealistic specification.

To investigate the relationship between lesion location and performance in the calculation tasks, we carried out separate VLBM analyses for each arithmetic operation and patient group (LHD or RHD, respectively) using the non-parametric Liebermeister test of the NPM software (Rorden et al., 2007) provided by the MRIcron package. For each voxel, the two subgroups of patients with resp. without a lesion in a given voxel were compared with regard to showing resp. not showing a deficit in the particular task. Because of the heavily left-skewed distributions of total scores correct per arithmetic operation item set, only this dichotomous performance measure was employed. Voxels damaged in at least one patient were included in the analysis. The results were corrected for multiple comparisons using a permutation-based family-wise error-correction approach with $p < 0.05$. Cortical and subcortical areas corresponding to voxels with a significant lesion-performance link were identified in the MNI-single subject space according to the Anatomical Automatic Labeling atlas (Tzourio-Mazoyer et al., 2002). White matter tracts were identified according to the diffusion tensor imaging (DTI)-based atlas by Catani and Thiebaut de Schotten (2012). In addition, probabilistic cytoarchitectonic maps of the white matter fiber tracts from the JuBrain atlas (Bürgel et al., 2006), implemented in the Anatomy Toolbox of the Juelich Research Center, were consulted to safeguard against possible differences in fiber tract labeling due to methodological differences in preparing different atlases.

RESULTS

Figure 1 illustrates the conventional lesion density plot for all $n = 45$ patients with either LHD or RHD.

At the behavioral level, in the LHD group, 2 patients (L09, L13) showed a deficit in multiplication, 6 patients (L01, L04, L08, L13, L15, L16, L19) were impaired in single-digit addition, and 4 patients (L01, L04, L13, L15) in single-digit subtraction. In the RHD patient group, 3 patients (R09, R15, and R23) were impaired in single-digit multiplication. No patient with RHD was impaired in single-digit addition or subtraction. Nevertheless, overall performance was good, as is apparent from the raw data presented in **Table 2**. Testing was aborted due to five consecutive incorrect responses only in patient L01 for addition and subtraction, and patient L13 for multiplication.

Additionally, dissociation in performance between the three arithmetic operations was observed. Details are given in the upper panel of **Table 3**.

Several patients suffered from aphasia. To further explore the relationship of language and arithmetic deficits we tested for

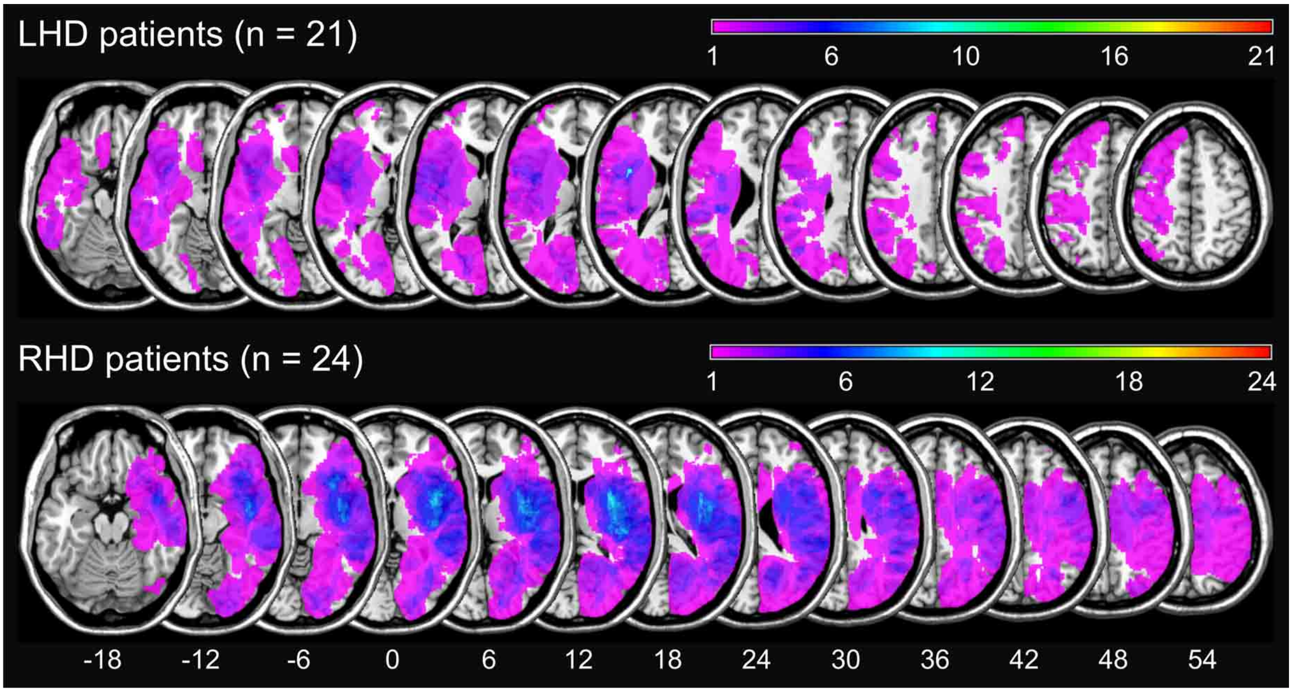

FIGURE 1 | Simple lesion-overlap for the LHD resp. the RHD patient group. The number of overlapping lesions is color-coded with increasing frequencies from violet ($n = 1$) to red ($n =$ maximum observed).

Table 3 | Patients with dissociations between arithmetic and language tasks.

Dissociation	Classical[a]	Strong[b]
Addition > Subtraction	L04	
Multiplication > Subtraction	L01	
Subtraction > Multiplication		L13
Multiplication > Addition	L01	
Addition > Multiplication	R09, R15	L13
Picture naming > Addition		L01
Addition > Picture naming	L16, L18	L10
Picture naming > Subtraction		L01
Subtraction > Picture naming	L10, L18	
Picture naming > Multiplication		L13

[a] *Only one function impaired and significantly poorer than the non-impaired function.*

[b] *Both functions impaired, but significantly different from each other (cf. Crawford and Garthwaite, 2005).*

dissociations between performance in each arithmetic operation and the picture naming task. The results are presented in the lower panel of **Table 3**. In fact, there were patients (L09 for multiplication, L04 for subtraction) who were impaired on an arithmetic task despite no language deficit. However, these differences were not large enough to qualify as dissociation. All of the cases reported in **Table 3** who performed significantly better on a language task than on an arithmetic task showed a strong dissociation.

Results of the VLBM analyses are presented in **Figure 2**. In the LHD patient group, deficits in *single-digit multiplication* were significantly associated with a lesion in the superior longitudinal fascicle II (SLF II) according to the JuBrain atlas (Bürgel et al., 2006). This fiber bundle corresponds to the structure termed longitudinal segment of the arcuate fascicle (AF) according to the Atlas of Human Brain Connections (Catani and Thiebaut de Schotten, 2012) as apparent when overlaying these two pathway maps on the same template. In contrast, deficits in *single-digit addition* were significantly related to lesions of the insula, Rolandic operculum, Heschl's gyri, inferior frontal operculum, external capsule and the AF, but not the SLF II as described by the JuBrain atlas. For *single-digit subtraction* significant lesion-behavior correlations were found for insula, external/extreme capsule (EC/EmC)-system, and putamen. In the RHD patient group no significant correlations were observed[1].

According to Catani and Thiebaut de Schotten (2012) the AF can be further partitioned into three segments: anterior, long and posterior segment. The long segment corresponds to the dorsal pathway directly connecting frontal, parietal and temporal cortices, whereas the anterior segment corresponds to more dorsal (superior) fibers connecting temporal and frontal regions via the parietal Geschwind's area. With regard to this partition of the

[1] For the size of the current sample dichotomous analysis is the method of choice (Rorden et al., 2007). Yet, an analysis of continuous data (i.e., without cut-offs) using the Brunner-Munzel test revealed similar results; however, for addition the result was only marginally significant due to the overall small sample.

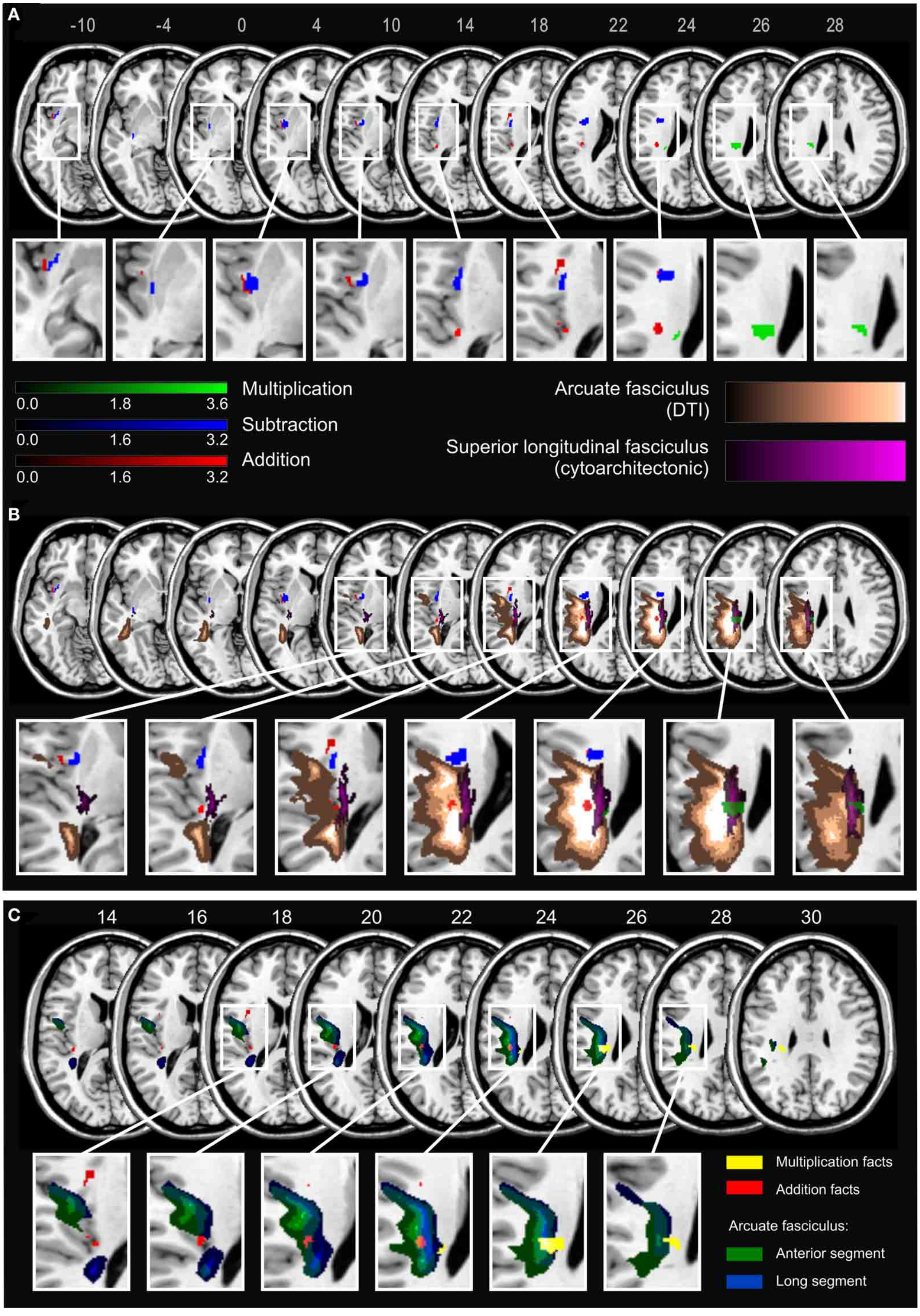

FIGURE 2 | (A) Statistical voxel-wise lesion-behavior mapping (VLBM) analyses using the Liebermeister-test statistic for the dichotomous deficit yes/no-criterion in the three arithmetic tasks in 21 LHD patients. Plotted are voxels that survived a permutation-test based FEW correction at $p < 0.05$. Areas in red are associated with deficits in addition, in blue with subtraction, and in green with multiplication. Color bars indicate z-scores. MNI coordinates of transversal sections *(Continued)*

FIGURE 2 | Continued
are indicated. **(B)** Significant lesion areas from panel **A** overlaid on white matter pathways: in orange the AF according to the DTI-based atlas by Catani and Thiebaut de Schotten (2012), and in violet the SLF according to the probabilistic cytoarchitectonic JuBrain atlas (Bürgel et al., 2006). The graded shadowing represents the probability of a given voxel belonging to the particular fascicle, where brighter color indicates higher probability. **(C)** Results of the VLBM analyses for multiplication facts (yellow) and addition facts (red) overlaid on two segments of the AF according to Catani and Thiebaut de Schotten (2012). The anterior segment of the AF is depicted in green and the long segment in blue. Note that the segments overlap partially. MNI coordinates of transversal sections are also indicated.

AF the current results suggest that the part of the lesion map related to multiplication facts primarily comprised the long segment, while the map for addition mostly covered the anterior segment (**Figure 2C**). However, because the two segments of the AF partially overlap, a clear-cut distinction was not possible.

DISCUSSION

The objective of the current study was to identify brain regions critical for solving single-digit arithmetic tasks employing VLBM in a sizeable group of stroke patients. In particular, we aimed to explore differences in brain areas subserving this process in different arithmetic operations.

DISSOCIATION OF OPERATIONS

The present results show dissociations on the behavioral level, as well as distinct pathways for solving of single-digit tasks in different arithmetic operations. Deficits in multiplication were associated with more superior lesions in the SLF II according to the JuBrain atlas (Bürgel et al., 2006) than deficits in addition, although they both involved the AF according to the atlas by Catani and Thiebaut de Schotten (2012). In contrast, significant lesion maps for subtraction involved the external/extreme capsule-system and insular cortex. These results support the notion of relative distinctness of arithmetic operations, suggesting that even single-digit additions and subtractions are processed differently than multiplication table facts.

This finding is in line with the evidence from functional neuroimaging studies suggesting partly distinct processing patterns for different arithmetic tasks, as revealed by the meta-analysis of Arsalidou and Taylor (2011). However, Rosenberg-Lee et al. (2011) found different patterns of activations for addition, subtraction and multiplication tasks (carefully matched for difficulty and processing speed) only in the right hemisphere, whereas the left hemisphere activations were overlapping. In contrast, in the current study we observed differences between operations in a group of LHD patients, while no significant lesion-symptom correlations were found for the RHD patients group. Similar discrepancies have repeatedly been shown for language functions, where the regions of fMRI-activations in language processing did not exactly correspond to regions critical and necessary for the execution of a particular function (Binder et al., 2009). Another possible explanation for our divergent findings is the type of task employed, which was a verification task in the study by Rosenberg-Lee et al. (2011) in contrast to a production task in the current study.

The question about the source of discrepancies among arithmetic operations remains open. It has previously been argued that the neural correlates underlying the representations of multiplication table facts and subtraction problems diverge, because they rely on different solution strategies (Dehaene and Cohen, 1997; Lee, 2000; Tschentscher and Hauk, 2014). Although single-digit addition and subtraction problems analyzed in the current study are commonly considered all to be solved predominantly via retrieval from long term memory in a normal population (Campbell and Xue, 2001; Grabner et al., 2009), the results of Fayol and Thevenot (2012) and Barrouillet and Thevenot (2013) contradict this view: the reported solution times suggest that—in contrast to multiplication—even single-digit addition and subtraction might not be directly retrieved from memory. In the present study we measured patients' performance only with respect to accuracy. This approach is more common in neuropsychological testing, as RTs are highly variable and less informative in acute stroke patients. Thus, it remains ambiguous whether the observed dissociations result from distinct semantic representations or processing strategies underlying different arithmetic operations. Another source of differences might be the relative problem size of different arithmetic operations. While products of single-digit addition and subtraction task remain relatively small, single-digit multiplication yields much larger results. Several studies aiming at a comparison of different arithmetic operations have struggled with this issue (e.g., Dehaene and Cohen, 1991, 1997; Kazui et al., 2000; Lee, 2000; Van Harskamp and Cipolotti, 2001; Kawashima et al., 2004; Delazer et al., 2006; Ischebeck et al., 2006; Zhou et al., 2006). However, we are confident that for the specific question of the current study—single-digit tasks that could possibly be retrieved as rote facts—the issue of problem size is less dramatic than in case of complex calculation tasks.

Altogether, along with several neuroimaging studies (e.g., Arsalidou and Taylor, 2011; Prado et al., 2011; Fayol and Thevenot, 2012; Tschentscher and Hauk, 2014) our results challenge the traditional cognitive psychology models (Ashcraft, 1992; Campbell and Oliphant, 1992; Siegler and Shipley, 1995) assuming that single-digit addition, subtraction and multiplication are all solved through the use of very similar retrieval strategies.

DISCONNECTION AS A SOURCE OF ARITHMETIC DEFICITS

Interestingly, the main sites revealed by the VLBM analyses were located in the white matter of the brain. This may be explained by the fact that the cortical areas critical for arithmetic were to a large extent spared in our patient sample: most importantly, the angular gyrus (AG).

One of the claims of the TCM is the involvement of the left AG in language-based retrieval of arithmetic facts. This was confirmed in both neuroimaging studies in healthy participants (Delazer et al., 2003; Grabner et al., 2009; Zamarian et al., 2009), and several brain damaged patients (Cohen et al., 2000; Lee, 2000). Results of an fMRI study based on self-reports about calculation strategies further point to involvement of the left AG in fact-retrieval (Grabner et al., 2009). However, Zaunmüller et al. (2009) reported a diverging finding in a patient with severe multiplication fact-retrieval deficits, although his brain lesion did not involve the left AG (see also Van Harskamp et al., 2005 for a similar case). Another patient with preserved single digit multiplication, despite a lesion extending to the supramarginal gyrus and part of the left AG, was also described (Van Harskamp and Cipolotti, 2001). However, this latter patient suffered from dementia and brain atrophy.

Apparently, disruption of white matter tracts can be critical for arithmetic. This lesion locus seemed to be present in some single case studies (e.g., Van Harskamp et al., 2005; Zaunmüller et al., 2009) as well. For instance, a case of pure Gerstmann syndrome has been described after a subcortical lesion beneath, but sparing the AG itself (Mayer et al., 1999). In (Zaunmüller et al., 2009) both a lesion of the ventral external/extreme capsule system and the dorsal SLF II were reported but not discussed. While we agree with the original interpretation that a lesion of the basal ganglia may have added to the severe multiplication impairment of the patient, we want to suggest that even though the left AG was not affected by the lesion, this area was no longer connected to frontal areas such as Broca's area. Therefore, the observed disconnections of both dorsal and ventral fiber pathway systems may also account for the observed multiplication impairment.

Whereas cortical substrates of numerical cognition have been investigated extensively, white matter pathways mediating the complex, multimodal processes of calculation have not yet been attended to systematically. Individual differences in white matter integrity have been shown to predict arithmetic skills in children. In particular, arithmetic approximation skills correlated with fractional anisotropy in the anterior portion of the SLF (Tsang et al., 2009) and performance on a basic equations test correlated with fractional anisotropy in the left inferior lateral fascicle (Van Eimeren et al., 2008). In adolescents, fractional anisotropy and radial diffusivity of the left SLF, left superior corona radiata (as labeled by the JHU-atlas Mori et al., 2009 corresponding to the AF in Catani and Thiebaut de Schotten, 2012), and the left cortico-spinal tract correlated with performance on the math subtest of the Preliminary Scholastic Aptitude Test, which is a nationwide administered scholastic measure, including word problems, geometry, algebraic equations, and complex arithmetic (Matejko et al., 2013). In adults, a combined fMRI-DTI study revealed a correlation between gray matter activation during calculation (all four arithmetic operations taken together) and the microstructure of the adjacent white matter (Van Eimeren et al., 2010). Activation of the left AG correlated significantly with the fractional anisotropy values of left superior corona radiata. For small (product < 25) but not for large problem size items, the correlation was significant for the superior coronae radiatae bilaterally. Thus, some evidence pointing to the importance of white matter connections for arithmetic functioning has already been published.

RELATION TO THE TCM

The current results challenge the traditional psychological models of arithmetic but also the currently most popular neuropsychological model—the TCM (Dehaene et al., 2003).

Figure 1 reveals that the most important anatomical structures implied by the TCM, i.e., the left angular gyrus and the intraparietal sulcus (IPS) bilaterally, were not covered in our sample of LHD patients. Therefore, based on the premises of the modular cognitive neuropsychological TCM, one should not expect any of the patients to present with deficits in calculation, which were nonetheless observed in our sample.

One of the central postulates of the TCM is the general distinction between a mental number magnitude representation on the one side and verbally mediated fact retrieval processes on the other side. According to the TCM, arithmetic problems can be solved via two basic routes. First, rote and overlearned arithmetic facts can be retrieved from long-term memory without relying on quantity information via the so-called *direct* route. Alternatively, the arithmetic problem gets related to quantity information via the *indirect* semantic route in the bilateral intraparietal cortex and only then submitted to left perisylvian regions, in particular the left angular gyrus, and finally linked to a number word to be uttered (e.g., in case of more difficult tasks).

The TCM (Dehaene et al., 2003) does not yet specify the neuroanatomical connections between the proposed modules. The first attempt to systematically investigate white matter pathways involved in numerical cognition was made by Klein et al. (2013). The authors performed probabilistic DTI-based fiber tracking, taking as seed points areas of activation for easy and complex addition tasks (assumed to represent fact-retrieval and number magnitude-based processing, respectively). The resulting network included all major sites predicted by the TCM plus several other areas previously proposed as an amendment to the TCM. The authors identified two separate networks for easy and more difficult calculation, both involving dorsal (SLF) and ventral pathways (external/extreme capsule system) connecting frontal and parietal regions. Regions involved primarily in easy arithmetic tasks were connected predominantly by ventral fibers belonging to the middle longitudinal fascicle, converging in the sub-insular white matter near the claustrum as well as superior and medial part of the external and/or extreme capsule (Klein et al., 2013). These ventral pathways were also crucial for single-digit subtraction in the present study.

In contrast, deficits in addition and multiplication fact retrieval were associated with lesions of the dorsal pathways. In particular, lesion maps for multiplication facts involved major parts of the long segment of the AF (corresponding to the SLF II in other atlases), which constitutes a major, direct connection between temporal and temporo-parietal areas involved in arithmetic fact retrieval. For addition, the significant lesion map did not involve the SLF II bundle, although it also overlapped with the AF—most probably with its anterior part. This pattern is consistent with the TCM, which regards addition as a mixed operation, relying on both fact retrieval and (intraparietal) number magnitude processing. Accordingly, we observed that addition deficits may relate to lesions of both, the long segment and the anterior segment of the AF, which connects frontal with temporal parts indirectly via intraparietal areas (Catani et al., 2005). Multiplication, based primarily on rote fact retrieval, seems to rely rather on pathways directly connecting frontal and temporoparietal areas.

Thereby, our results also demonstrate the importance of white matter pathway connections in the human brain. A recent atlas guiding glioma surgery suggests that most white matter pathways are not resectable (i.e., resection would most probably cause functional loss; Ius et al., 2011). Thus, for the interpretation of impairments in behavior observed in single-case studies or voxel-based lesion mapping studies not only gray matter lesions should be investigated but also disconnections of white matter fiber pathways.

LIMITATIONS AND FUTURE PERSPECTIVES

Although the significant lesion maps for addition and subtraction deficits did not overlap, current statistical methods implemented in MRIcron do not yet allow for a direct (multivariate) comparison, which is a common problem in neuroscience research (Nieuwenhuis et al., 2011). The dissociation of operations can thus only be tested based on the behavioral data, where in fact significant differences were found.

However, at the behavioral level, three out of four patients impaired in subtraction also showed a deficit in addition. Since addition and subtraction are complementary operations, addition may be used as a back-up strategy to solve subtraction tasks and vice versa. In case one operation is impaired, the complementary back-up strategy is missing, thus making the other operation more error-prone. Alternatively, an association between impairments may be due to other reasons.

The cortical parts of the maps with significant lesion-performance association for addition and subtraction both encompass the insula, which has recently been suggested by Arsalidou and Taylor (2011) to be included in the TCM. These authors argue that the insula plays a rather non-specific role, being involved in switching between working memory and default states during problem solving, since in other studies the insula was associated with error processing (Hester et al., 2004) or the execution of responses (Huettel et al., 2001).

The current results suggest that solving single-digit arithmetic operations is subserved predominantly by the left hemisphere. This conforms with the majority of previous clinical evidence, on which the earlier version of the TCM is based (e.g., Dehaene and Cohen, 1997). In the fMRI literature left-lateralization of activation patterns is particularly evident for multiplication, whereas other arithmetic operations have been found to activate both hemispheres (e.g., Chochon et al., 1999; Zhou et al., 2007; Klein et al., 2010). Nonetheless, some of the RHD patients also did show deficits in solving arithmetic tasks: all of them only in multiplication.

Further, in the current study we used a standardized neuropsychological test. Like many other clinical assessments it only appraises accuracy and not solution times, because response times of acute patients are not as reliable and informative as in the healthy population. However, it is also possible that a lesion to a region causes slowing down of responses but no drop in accuracy.

In the current study we investigated a group of patients in acute stroke phase to avoid the influence of compensation and brain reorganization processes. The next step would be to investigate longitudinal aspects of brain damage at different stages of recovery from acute stroke to chronic phase. This would inform about the stability of observed structure-behavior correlations in light of spontaneous neural recovery and compensatory brain plasticity.

CONCLUSIONS

In the present study, we provide first evidence from a voxel-based lesion mapping analysis in a sizeable group of acute stroke patients for distinct neural processing pathways in different arithmetic operations. We identified different white matter pathways that lead to arithmetic fact-retrieval deficits in different arithmetic operations when disrupted. Our findings contribute to reconciling diverging evidence about involvement of the AG in arithmetic fact retrieval, by showing that a disconnection of a cortical structure through a white matter lesion can be associated with deficits comparable to those after damage of the cortical structure itself. Our results also argue for further amendments of the anatomo-functional TCM, which does not yet provide inclusion of white matter interconnections of the multiple (cortical) processing modules it describes.

AUTHOR CONTRIBUTIONS

Elise Klein, Klaus Willmes, and Hans-Otto Karnath designed the study. Urszula Mihulowicz conducted the study. Urszula Mihulowicz and Elise Klein performed the analyses. Urszula Mihulowicz prepared the figures. Urszula Mihulowicz, Elise Klein, Klaus Willmes, and Hans-Otto Karnath wrote the article.

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SUPPLEMENTARY MATERIAL

The Supplementary Material for this article can be found online at: http://www.frontiersin.org/journal/10.3389/fnhum.2014.00286/abstract

Supplementary Table 1 | Demographic and clinical data of all patients.

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Young children's use of derived fact strategies for addition and subtraction

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Forty-four children between 6;0 and 7;11 took part in a study of derived fact strategy use. They were assigned to addition and subtraction levels on the basis of calculation pretests. They were then given Dowker's (1998) test of derived fact strategies in addition, involving strategies based on the Identity, Commutativity, Addend +1, Addend −1, and addition/subtraction Inverse principles; and test of derived fact strategies in subtraction, involving strategies based on the Identity, Minuend +1, Minuend −1, Subtrahend +1, Subtrahend −1, Complement and addition/subtraction Inverse principles. The exact arithmetic problems given varied according to the child's previously assessed calculation level and were selected to be just a little too difficult for the child to solve unaided. Children were given the answer to a problem and then asked to solve another problem that could be solved quickly by using this answer, together with the principle being assessed. The children also took the WISC Arithmetic subtest. Strategies differed greatly in difficulty, with Identity being the easiest, and the Inverse and Complement principles being most difficult. The Subtrahend +1 and Subtrahend −1 problems often elicited incorrect strategies based on an overextension of the principles of addition to subtraction. It was concluded that children may have difficulty with understanding and applying the relationships between addition and subtraction. Derived fact strategy use was significantly related to both calculation level and to WISC Arithmetic scaled score.

Keywords: young children, mathematical development, arithmetical reasoning, derived fact strategies, addition, subtraction

INTRODUCTION

There have been a number of studies of children's use of derived fact strategies in addition and subtraction (Baroody et al., 1983; Russell and Ginsburg, 1984; Beishuizen et al., 1997; Carpenter et al., 1997; Dowker, 1998, 2009; Blote et al., 2000; Star and Rittle-Johnson, 2008; Jordan et al., 2009; Torbeyns et al., 2009; Cowan et al., 2011). Certain derived-fact strategies appear very early (Baroody and Gannon, 1984; Carpenter and Moser, 1984; Siegler and Jenkins, 1989; Cowan and Renton, 1996). One of the earliest is the "counting-on-from-larger" concrete addition strategy, whereby the child adds two numbers (e.g., $2 + 6$), by representing the larger number (e.g., with fingers) first, and then "counting-on" the smaller number: "6, 7, 8–it's 8!" This involves implicit use (with or without an explicit knowledge) of the commutativity principle (Baroody and Gannon, 1984; Cowan and Renton, 1996). By contrast, there are many sophisticated strategies involving the use of decomposition and decomposition for multi-digit arithmetic that appear late and appear to characterize unusually skilled mental calculators (Hope and Sherrill, 1987).

There has been rather less research on children's subtraction strategies than on their addition strategies. The use of derived fact strategies might seem even more important with regard to subtraction than addition, since children are generally less able to retrieve subtraction facts than addition facts (Barouillet et al., 2008), so could benefit more from alternative strategies. Yet it may be more difficult for children to use derived fact strategies for subtraction than addition, both because their relative lack of known facts gives them less of a base from which to use them, and because some derived fact strategies for subtraction, such as the "subtraction by addition" strategy (DeSmedt et al., 2010; Peters et al., 2013) depend on some understanding of the inverse relationship between addition and subtraction, which some studies suggest is difficult for children (see below).

Most studies of derived fact strategies have not adjusted the difficulty of the arithmetic problems to the child's arithmetical level, thereby risking on the one hand that some children may find it easier to calculate or retrieve a solution directly than to derive it on the basis of a principle, and on the other hand that they may find the problems so difficult that they refuse to attempt them at all, or make wild guesses. The present study aimed to adapt the problems given to individual children to their previously assessed calculation ability, and to present them with problems just a little too difficult for them to solve unaided. More generally, most studies have not looked at the relationship between derived fact strategy use and arithmetical ability, but have focused more on chronological age differences. The present study looks at relationships between the use of derived fact strategies and both calculation performance level and performance on a standardized arithmetic test emphasizing reasoning. Previous work by this author (Dowker, 1998, 2005, 2009) has focused on individual differences in general readiness to use derived fact strategies (i.e., the total number of such strategies used in a task), whereas this study focuses more on the use of particular strategies.

Thus, the present study investigated 6- and 7–year-olds' ability to use derived fact strategies, based on a range of principles, for both addition and subtraction. The principles were selected for their applicability across a fairly wide range of difficulty. Some derived fact strategies, such as most counting-based strategies or those based on the use of doubles, are mainly applicable to single-digit arithmetic (Carpenter and Moser, 1984; Baroody, 1987); others, including certain decomposition strategies (Beentjes and Jonker, 1987; Beishuizen, 1993; Beishuizen et al., 1997; Carpenter et al., 1997; Fuson et al., 1997) are mainly applicable to multi-digit arithmetic. Important as these strategies are, the present study restricted the strategies under consideration to those that may be used for both single-and multi-digit arithmetic

The *Identity principle*, which is here investigated for both addition and subtraction, is the most basic of arithmetical principles: that if an arithmetical operation produces a given result, then the repetition of the same arithmetical principle will produce the same result. Its use in predicting the result of an arithmetical operation is properly speaking not a "derived-fact strategy" but a "same-fact" strategy. Thus, its inclusion in the study is intended to investigate whether children tend to use the result of one operation to predict the result of another at all, over and above the particular principles that they are able to use in such predictions. This principle has received relatively little attention, but would appear to be a cornerstone of the ability to use derived fact strategies. It was predicted that while the majority of children would use this strategy, a significant number would not.

The *Commutativity principle* is a crucial addition principle, and one which appears to be used with some frequency by primary school children (Baroody et al., 1983; Russell and Ginsburg, 1984; Canobi et al., 1998; Dowker, 1998, 2009; Canobi, 2005). Strategies based on commutativity only hold for addition and therefore are only investigated for that operation.

Simple associativity-based strategies, involving the addition and subtraction of 1, are also investigated. The *N + 1 principle* for addition is the simplest of the assumptions that result from the broader associativity principle. This is the principle that if one of the addends is increased by 1, then the sum will also be increased by 1. Other related principles, also to be investigated here, include:

The *N − 1 principle* for addition: that if one of the addends is *decreased* by 1, then the sum will also be decreased by 1.
The *Minuend + 1 principle* for subtraction: that if the minuend is increased by 1, then the remainder will also be increased by 1.
The *Minuend −1 principle* for subtraction: that if the minuend is decreased by 1, then the remainder will also be decreased by 1.
The *Subtrahend +1 principle* for subtraction: that if the subtrahend is increased by 1, then the remainder will be *decreased* by 1.
The *Subtrahend −1 principle* for subtraction: that if the subtrahend is decreased by 1, then the remainder will be *increased* by 1.

Finally, this study investigates strategies based on the inverse relationship between addition and subtraction. Most studies (Bisanz and LeFevre, 1990; Demby, 1993) suggest that strategies based on the *addition/subtraction Inverse principle* ($a + b - b = a$; if $a + b = c$, then $c - b = a$) are among the later-developing derived-fact strategies, and are not typically used until the age of about 10. However, Baroody et al. (1983) found that many 7-and 8-year-olds used this strategy, and that it typically preceded the $N + 1$ strategy. Gilmore and Bryant (2006, 2008) and Robinson and Dubé (2009, 2013) found considerable individual differences in elementary school children's use of this strategy, but some 6-to 9-year-olds pupils used it effectively. A strategy logically related to inversion strategies is the *Complement principle*: if $a - b = c$, then $a - c = b$. This has not been much investigated, at least with regard to children in this age range, and will be considered here. It was predicted that neither the addition/subtraction Inverse principle nor the complement principle would be used by a majority of children.

METHODS

PARTICIPANTS

144 children ranging from 6;0 to 7;11 were tested individually. They came from two state primary schools in Oxford. 79 were boys and 65 were girls. Their mean age was 81.95 months ($SD = 6.23$).

PROCEDURE

Use of principles task

The task was Dowker's (1998, 2009) test of use of arithmetical principles in derived fact strategies. It included tests of strategy use in both addition and subtraction.

Addition principles task

In order to evaluate the children's competence in addition calculations, each child was given the mental addition test previously devised to assess children's arithmetical performance prior to an estimation task (Dowker, 1997). It consisted of a list of 20 addition sums graduated in difficulty from 4 + 5, 7 + 1, etc., to 235 + 349. These sums were simultaneously presented orally and visually in a horizontal format. The children's answers were oral.

The sums were as follows:

(1)	6 + 3	(11)	31 + 57
(2)	4 + 5	(12)	68 + 21
(3)	8 + 2	(13)	52 + 39
(4)	7 + 1	(14)	45 + 28
(5)	4 + 9	(15)	33 + 49
(6)	7 + 5	(16)	26 + 67
(7)	8 + 6	(17)	235 + 142
(8)	9 + 8	(18)	613 + 324
(9)	26 + 72	(19)	523 + 168
(10)	23 + 44	(20)	349 + 234

Testing continued with each child until (s) he had failed to give a correct response to six successive items.

The children were then divided into five levels according to their performance on the mental calculation task. The levels were: Beginning Arithmetic (unable to deal reliably with single-digit addition); Facts to 10 (passed items 1–4 but failed at least 2 of the next 4 items); Facts to 25 (passed items 1–8, but failed at least 2 of the next 4 items); 2-Digit Addition- No Carrying (passed items 1–12, but failed at least 2 of the next 4 items); and 2-Digit Addition –Carrying (passed items 1–16, but failed at least 2 of the final 4 items). **Table 1** in the Results section gives the numbers of

Table 1 | Addition strategies used at different levels.

Level	Beginning arithmetic	Facts to 10	Facts to 25	2-Digit (Carrying)	2-Digit (No carrying)	Total		
Problem within range	2 + 2	5 + 3	8 + 6	23 + 44	52 + 39			
Problem just outside range	5 + 3	8 + 6	23 + 44	52 + 39	523 + 168			
n	11	34	63	16	20	144		
Mean age in months	79.88 (6.63)	80.98 (6.5)	82.04 (6.3)	83.54 (3.82)	84.65 (5.98)	81.95 (6.23)		
Mean arithmetic scaled score	3.86 (1.07)	8.89 (2.31)	10.62 (3.09)	10.7 (3.68)	12.19 (4.12)	9.97 (3.59)	χ^2	*p*
Identity	22%	56%	80%	88%	95%	73%	25.66	0.000**
Commutativity	9%	38%	65%	82%	70%	57%	28.00	0.000**
Addend +1	0%	24%	51%	71%	75%	56%	28.04**	0.000**
Addend −1	0%	18%	40%	59%	65%	37%	22.06	0.000**
Inverse	0%	18%	6%	35%	25%	14%	9.59	0.031*

*$p < 0.05$; **$p < 0.01$.

children at each level, and examples of items that would be within and just outside of their range.

They were then given an arithmetical reasoning test involving *use of arithmetical principles in derived fact strategies*. The technique was used of giving children the answer to a problem and then asking them to solve another problem that could be solved quickly by using this answer, together with the principle under consideration. Problems preceded by answers to numerically unrelated problems were given as controls. The exact arithmetic problems given varied according to the child's previously assessed calculation level of the child, and were selected to be just a little too difficult for the child to solve unaided. Such a set of problems is here termed, as in earlier studies (Dowker, 1998, 2009), the child's *base corresponding set*).

Each child was shown the addition problems, while the experimenter simultaneously read them to him/her. Children were asked to respond orally. The children received three addition problems per principle. The questions about the principles were grouped around the addition problems, so that the children received 6 questions (involving 5 principles and a control question) for one addition problem; then 6 questions for the second addition problem; then 6 questions for the third addition problem.

The principles investigated were as follows:

(1) The *Identity principle* (e.g., if one is told that $8 + 6 = 14$, then one can automatically give the answer "14," without calculating, if asked "What is $8 + 6$?").
(2) The *Commutativity principle* (e.g., if $9 + 4 = 13$, $4 + 9$ must also be 13).
(3) The *N ± 1 principle* (e.g., if $23 + 44 = 67$, $23 + 45$ must be 68).
(4) The *N − 1 principle* (e.g., if $9 + 8 = 17$, $9 + 7$ must be $17 - 1$ or 16).
(5) The *addition/subtraction Inverse principle* (e.g., if $46 + 27 = 73$, then $73 - 27$ must be 46).

For one of the addition problems in each set, the order of presentation of principles was:

Commutativity, Identity, $N + 1$, $N - 1$, Control, Inverse.
For a second problem in each set, the order was:
Inverse, $N + 1$, $N - 1$, Commutativity, Identity, Control.
For the third problem in each set, the order was:
Control, Inverse, $N - 1$, Identity, Commutativity, $N + 1$.
The order of presentation of the addition problems was varied systematically.
Children were allowed 30 s to begin answering a question; if they did not give an answer within that time, the researcher moved on to the next question.

A child was deemed to be able to use a principle if (s) he could explain it and/or used it to derive at least 2 out of 3 unknown arithmetical facts, while being unable to calculate *any* sums of similar difficulty when there was no opportunity to use the principle.

Subtraction principles test

The *subtraction principles* part of the Use of Principles Task was also preceded by a calculation pretest, which consisted of a list of 20 subtraction problems, as follows:

(1)	$6 - 2$	(11)	$68 - 42$
(2)	$8 - 4$	(12)	$86 - 44$
(3)	$10 - 3$	(13)	$62 - 14$
(4)	$9 - 5$	(14)	$43 - 17$
(5)	$15 - 7$	(15)	$75 - 38$
(6)	$13 - 6$	(16)	$84 - 59$
(7)	$12 - 4$	(17)	$326 - 125$
(8)	$15 - 7$	(18)	$894 + 513$
(9)	$37 - 23$	(19)	$681 - 214$
(10)	$55 - 32$	(20)	$572 - 348$

The children were then divided into four levels according to their performance on the mental calculation task. The levels were: Beginning Arithmetic (unable to deal reliably with single-digit subtraction); Facts to 10 (passed items 1–4 but failed at least 2 of the next 4 items); Facts to 25 (passed items 1–8, but failed at least 2 of the next 4 items); and 2-Digit Subtraction (passed items 1–12, but failed at least 2 of items 13–16 and/or of items 17–20). Originally, the 2-Digit Subtraction group was divided into two groups, as with addition: 2-Digit-No Borrowing, and 2-Digit-Borrowing. However, as only 8 children would have met criteria for the 2-Digit-Borrowing group, they were grouped together, for the purposes of the present study, with those who could only carry out 2-digit subtraction when borrowing was not involved. **Table 3** in the Results section gives the numbers of children at each level, and examples of items that would be within and just outside of their range.

The questions about the principles were grouped around the subtraction problems, so that the children received 8 questions (involving 7 principles and a control question) for one addition problem; then 8 questions for the second addition problem; then 8 questions for the third addition problem

The principles investigated for *subtraction* were as follows, in order of their difficulty for the children:

(1) The *Identity principle* (e.g., if one is told that $12 - 5 = 7$, then one can automatically give the answer "7," without calculating, if asked "What is $12 - 5$?").
(2) The *Minuend ±1 principle* (e.g., if $67 - 45 = 22$, $68 - 45$ must be 23).
(3) The *Minuend −1 principle* (e.g., if $572 - 348 = 224$, $571 - 348$ must be 223).
(4) The *Subtrahend ±1 principle* (e.g., if $9 - 6 = 3$, $9 - 7$ must be 2).
(5) The *Subtrahend −1 principle* (e.g., if $37 - 23 = 14$, $37 - 22$ must be 15).
(6) The *addition/subtraction Inverse principle* (e.g., if $681 - 214 = 467$, then $214 + 467$ must be 681.
(7) The *Complement* principle (e.g., if $11 - 3 = 8$, $11 - 8$ must be 3).

For one of the subtraction problems in each set, the order of presentation of principles was:

Complement, Minuend + 1, Subtrahend + 1, Inverse, Minuend −1, Subtrahend −1, Identity, Control.

For a second problem in each set, the order was:

Minuend−1, Subtrahend +1, Minuend +1, Inverse, Identity, Minuend −1, Control, Complement.

For the third problem in each set, the order was:

Control, Identity, Subtrahend +1, Minuend +1, Subtrahend −1, Complement, Minuend −1, Inverse.

The order of presentation of the subtraction problems was randomly varied.

Children were allowed 30 s to begin answering a question; if they did not give an answer within that time, the researcher moved on to the next question.

The order of presentation of addition and subtraction was randomly varied.

In addition, the children were given the Arithmetic subtest of the Wechsler Intelligence Scale for Children or WISC (Wechsler, 1991).

RESULTS

As no children calculated the answers to the control questions within the time given, responses to control questions will not be analyzed here.

RESULTS FOR ADDITION

Table 1 gives the percentage of responses at each level using each principle in derived fact strategies for addition.

Chi-square tests were carried out to investigate whether there were significant differences between the different levels as regards the frequency of each strategy. The chi-square value and p value are given in the final two columns of **Table 1**. In all the chi-square comparisons, there were 4° of freedom.

Post-hoc tests were then carried out to investigate which group differences were causing the significant effects. For the Identity principle and the Addend −1 principle, the significant differences were between Beginning Arithmetic and each of the other levels except for the Facts to 10 level; and between the Facts to 10 level and the 2-Digit (Carrying) level. For the Commutativity principle and the Addend +1 principle, the significant differences were between Beginning Arithmetic and each of the other levels except for the Facts to 10 levels; and between the Facts to 10 level and both the 2-Digit (No Carrying) the 2-Digit (Carrying) levels. For the Inverse principle, there was a borderline significant difference between the Beginning Arithmetic and the 2-Digit (No Carrying) level, and no other group differences reached significance.

Entry method nominal logistic regressions were carried out with each principle (Used or Did Not Use) as the dependent variable. The covariates were Age in months and WISC Arithmetic (Scaled Score). The chi-square and p-values for these regressions are given in **Table 2**.

RESULTS FOR SUBTRACTION

Table 3 gives the percentage of responses at each level using each principle in derived fact strategies for subtraction. With regard to the Subtrahend +1 and Subtrahend −1 strategies, two percentages are given. The first percentage given is that for use of a common but incorrect strategy: that of assuming that if $a - b = c$, then $a - (b + 1) = c + 1$ (instead of $c - 1$), or that if $a - b = c$, then $a - (b - 1) = c - 1$ (instead of $c + 1$). The second percentage is for the use of the correct strategy.

Chi-square tests were carried out to investigate whether there were significant differences between the different levels as regards the frequency of each strategy. The chi-square value and p value are given in the final two columns of **Table 3**. In the case of the Minuend +1 and Minuend −1 strategies, two comparisons were made: one taking only the correct strategy into account; and one combining use of the correct strategy and the common incorrect strategy. In all the chi-square comparisons, there were 3° of freedom.

Table 2 | Results of nominal logistic regressions on the use of addition strategies with age and arithmetic scaled score as covariates.

Principle used	Age in months: χ^2	Age in months: *p*	Arithmetic scaled score: χ^2	Arithmetic: scaled score: *p*
Identity	4.505	0.034*	4.92	0.034*
Commutativity	4.66	0.031*	3.885	0.049*
Addend +1	2.73	0.099	8.045	0.005**
Addend −1	3.32	0.069	7.64	0.006**
Inverse	1.43	0.232	0.027	0.87

$^*p < 0.05$; $^{**}p < 0.01$.
In all chi-square comparisons, df = 1.

Table 3 | Subtraction strategies used at different levels.

Level	Beginning arithmetic	Facts to 10	Facts to 25	2-Digit subtraction	Total		
Problem within range	?	6–3	12–5	58–34			
Problem just outside range	6–3	12–5	58–34	82–26			
n	18	56	48	22	144		
Mean age in months	79.88 (6.63)	80.98 (6.5)	82.87 (5.84)	85.63 (4.59)	81.95 (6.23)		
Mean arithmetic scaled score	4.82 (1.94)	9.39 (2.58)	11.4 (3.33)	12.31 (3.61)	9.97 (3.59)	χ^2	*p*
Identity	17%	61%	77%	86%	65%	29.49	0.000**
Minuend +1	0%	23%	54%	71%	38%	35.26	0.000**
Minuend −1	0%	21%	50%	71%	56%	9.42	0.022*
Subtrahend +1	0% + 6%	20% + 4%	60% + 4%	55% + 22%	38% + 6%	1.92[a] 9.66[b]	0.775[a] 0.02*[b]
Subtrahend -1	0% + 6%	20% + 2%	54% + 6%	43% + 29%	33% + 7%	2.45[a] 11.23[b]	0.57[a] 0.009**[b]
Complement	0%	18%	6%	35%	14%	9.43	0.022*
Inverse	0%	7%	17%	27%	12%	8.56	0.026*

$^*p < 0.05$ $^{**}p < 0.01$
[a]Analysis for correct strategy only.
[b]Analysis for combination of correct strategy with common incorrect strategy.

Post-hoc tests were then carried out to investigate which group differences were causing the significant effects. For the Identity principle, the significant differences were between Beginning Arithmetic and each of the other levels. For the Subtrahend +1 principle and the Subtrahend −1 principle, the significant differences were between Beginning Arithmetic and each of the other levels; and between the Facts to 10 level and each of the other levels. Two different *post-hoc* analyses were carried out for the Subtrahend +1 and Subtrahend −1 principles: for the correct strategy alone, and for the correct strategy combined with the common incorrect strategy. For the correct strategy alone, no group differences were significant for these principles. For the combination of the correct and the common incorrect strategy, the significant differences, in the case of both the principles, were between the 2-Digit Subtraction level and every other level. For the Complement principle and the Inverse principle, there were significant differences between the Beginning Arithmetic and the 2-Digit Subtraction level, and no other group differences reached significance.

Entry method nominal logistic regressions were carried out with each principle as the dependent variable. The dependent variable was binary (Used vs. Did Not Use). In the case of the Minuend +1 and Minuend −1 principles, two different analyses were done: (a) for Correct Strategy Use alone and (b) for Correct Strategy Use combined with Common Incorrect Strategy Use. The covariates were Age in months and WISC Arithmetic (Scaled Score). The chi-square and p-values for these regressions are given in **Table 4**.

CHILDREN'S JUSTIFICATION OF THEIR ANSWERS

Most (91%) of children classed as using the principles were able to justify their answers.

Typical justifications included:

(Identity); "It's the same!"
(Commutativity): "Those numbers are just the same, but the other way round."
($N + 1$ principle for addition; Minuend +1 principle for subtraction): "It's just one more."
($N - 1$ principle for subtraction; Minuend −1 principle for subtraction): "It's just one less."
(Subtrahend +1 principle for subtraction): (Usually, incorrectly): "It's just one more." (Correctly): "That's one more, so the answer has to be one less."
(Subtrahend −1 principle for subtraction): (Usually, incorrectly): "It's just one less." (Correctly): "That's one less, so the answer has to be one more."
(× 10) principle: "You just add on a 0."

Table 4 | Results of nominal logistic regression on use of subtraction strategies with age and arithmetic scaled score as covariates.

	Age in months: χ^2	Age in months: *p*	Arithmetic scaled score: χ^2	Arithmetic: scaled score: *p*
Identity	4.86	0.041*	6.84	0.009**
Minuend +1	8.265	0.004**	14.77	0.000**
Minuend −1	3.3	0.068	10.9	0.001**
Subtrahend +1	1.86[a]; 3.3[b]	0.24[a]; 0.07[b]	0.12[a]; 1.37[b]	0.73[a]; 0.24[b]
Subtrahend −1	4.42[a]; 4.43[b]	0.035*[a]; 0.035*[b]	0.57[a]; 0.7[b]	0.45[a]; 0.4[b]
Complement	4.915	0.027*	2.13	0.145
Inverse	0.88	0.348	5.45	0.02*

$^*p < 0.05$; $^{**}p < 0.01$.

[a] *Analysis for correct strategy only.*

[b] *Analysis for combination of correct strategy with common incorrect strategy.*

(Inverse principle): "Because that (a + b) = c, so c take away that (a) must be (b)." (Of course the child used the actual numbers rather than letters.)

(Complement principle): "If that (a) take away that (b) = c, then that (a) take away that (c) must be b."

DISCUSSION

This study shows that many 6- and 7-year-olds children can make explicit use of derived fact strategies in addition and subtraction. There is, however, a great deal of variation in the use of such strategies in this age range, influenced by both by the particular strategies involved, and by children's calculation ability.

USE OF PARTICULAR STRATEGIES

The most basic principle, Identity, was used with by far the greatest frequency; and is the only strategy that was used more than once or twice at the Beginning Arithmetic level. It was still only used by a minority of children at this level, however; and was not used universally even at the higher levels. This was followed by commutativity of addition, supporting other studies that suggest that this principle is used earlier than most other arithmetical principles (Baroody et al., 1983; Cowan and Renton, 1996; Canobi et al., 1998, 2003).

The strategy of using commutativity is followed in frequency by strategies that involve adding, or (to a lesser extent) subtracting, 1 from a problem component and thereby to the result.

Strategies of the latter type could be, and often were, used incorrectly as well as correctly. When used for addition, they tended to be used correctly; but this was not the case for subtraction, where the Subtrahend +1 and Subtrahend −1 problems were more likely to lead to incorrect than correct strategy use. Children are more likely, if told that $a - b = c$, to deduce that $a - (b + 1) = c + 1$, than correctly that $a - (b + 1) = c - 1$. In other words, when using this class of strategies, they often fail to make appropriate use of compensation. This may in part reflect procedural difficulties, perhaps relating to working memory limitations. However, when considered in conjunction with the children's common failure to use the addition-subtraction inverse principle for addition or subtraction, or the complement principle for subtraction, it probably also reflects a difficulty in understanding the relationships between addition and subtraction. The arithmetical relationships most accessible to children appear to be those appropriate to addition, and these are sometimes inappropriately extended to subtraction. It may be that the same is true of relationships between addition and other arithmetical operations; e.g., MacCuish (1986) found that 9- and 10-year-olds children overextended certain addition principles to multiplication.

With regard to strategies involving use of the inverse relationship between addition and subtraction, results of the present study are far more consistent with those of Bisanz and LeFevre (1990) than with those of Baroody et al. (1983), in that strategies of this nature were used very infrequently. The logically related complement strategy for subtraction was used even more rarely.

This is particularly striking, since these children were being taught mathematics according to the National Numeracy Strategy (DfEE, 1999), which explicitly recommended teaching children to understand the inverse relationship between addition and subtraction from the second year of primary school onwards. Nevertheless, the principle was only used by about one in ten children, similar to findings for a sample studied before the explicit introduction of this concept into the English school curriculum (Dowker, 1998). This suggests that children, at least under the age of 8, do not readily make use of this principle in arithmetic.

However, this may not be the case for *all* arithmetical tasks. Gilmore and Bryant (2006, 2008) found that 6-to 9-year-olds children did often make use of derived fact strategies involving inverse relationships between addition and subtraction. They performed better and more accurately on such problems as "$15 + 12 - 12 = \square$" than on control problems such $11 + 11 - 7 = \square$." An explanation for the discrepancy in results might be that children are better at noticing and making use of relationships between addition and subtraction *within* an arithmetic problem than *between* two arithmetic problems. If there is an addition and a subtraction within the *same* problem it is perhaps harder to treat them as unrelated—"one's adding and one's taking away"—than if the task involves perceiving and using a relationship between an addition problem and a subtraction problem. This provides further evidence that the ability to use derived fact strategies is not "all or nothing" and may be highly dependent on context and mode of presentation of a task.

DO SUCCESS AND FAILURE IN THE DERIVED FACT STRATEGY TEST ALWAYS REFLECT USE OF PRINCIPLES?

So far in this paper, "use of principles" and "use of derived fact strategies" have been discussed almost as though they were synonymous; but of course the relationship between the two is likely to be far more complex. With all of the arithmetical principles discussed here, there are two separate issues: whether a child understands an arithmetical concept or principle, and whether they use this principle appropriately in an arithmetical strategy. Some principles may not be used in derived fact strategies because the children have no access to the principles. On the other hand, children may understand an arithmetical principle or relationship, but not apply it appropriately.

The present study involved explicit use of derived fact strategies in a task involving arithmetic problems presented in symbolic format, and not embedded in a practical or social context. Some studies have suggested that children may be more likely to use derived fact strategies when problems are presented in concrete form (Bryant et al., 1999) or if the task requires only implicit rather than explicit use of the principle (Siegler and Stern, 1998). Canobi et al. (1998) studied 6-to 8-year-olds' use of derived fact strategies based on commutativity and associativity, and their evaluations of puppets using these strategies. They were considerably better at judging and justifying the appropriateness of a puppet's use of such strategies than at using the strategies themselves. It is therefore likely that the present study gives a somewhat conservative estimate of the extent of derived fact strategy use in young children.

However, studies also suggest that elicited use of derived fact strategies is not the *most* difficult task. Children and even adults tend to be better at using derived fact strategies appropriately when these are instructed or directly elicited, than at using them spontaneously, though there is a strong correlation between elicited and spontaneous use of such strategies (Gaschler et al., 2013).

A review by Prather and Alibali (2009) indicates that context and mode of assessment may have a significant impact on whether children use such strategies. Moreover, it is possible that children may sometimes have failed to use a strategy because of a coincidental procedural error or momentary distraction, rather than because of a failure to understand the principle. The fact that the criterion for success on a principle was use of the relevant strategy for two out of three arithmetic problems (rather than all three) reduces this risk, but does not eliminate it completely.

The question also arises of whether the reverse may have happened at times: could children have responded correctly to some items because they calculated from scratch and did so accurately, rather than because they used the principle? However, while this possibility can never be totally ruled out, it is unlikely to have occurred in most cases because (1) the sets of problems given to individual children were selected on the basis of the pretest indicating that they would be too difficult for them to calculate mentally; (2) they were not able to calculate the control problems mentally; (3) in the vast majority of cases, they were able to justify their correct answers.

RELATIONSHIPS BETWEEN DERIVED FACT STRATEGIES AND ARITHMETICAL ABILITY

Although discrepancies can and do occur, in both directions, between calculation performance level and extent of derived fact strategy use (Dowker, 1998, 2009), the two are very strongly associated (see **Tables 1**, **3**). This was true despite the fact that the difficulty of the arithmetic problems given was adjusted according to the children's calculation performance levels. Only a minority of children at the Beginning Arithmetic levels for addition and subtraction used any derived fact strategies. The use of such strategies became more frequent at the Facts to 10 levels, and increased sharply as children reached the Facts to 25 level and beyond. This increase with calculation performance level was found for both addition and subtraction; and was significant for all strategies except for the Complement principle for subtraction, perhaps due to floor effects for this principle.

Scaled score on an arithmetical reasoning task was also a strong predictor of most strategies, showing a significant relationship to use of all strategies except some of the more difficult ones: the Inverse principle in the addition task; and the Subtrahend $+1$, Subtrahend -1 and Complement principles in the Subtraction task. Thus, the use of most derived fact strategies is closely related to arithmetical ability. The relationship to chronological age is less strong, but is present for Identity and Commutativity in addition and for Identity, Minuend $+1$ and Subtrahend -1 in subtraction. These children were all within a relatively limited age range (6;0 to 7;11) and age might be found to have a stronger influence in a group with a wider age range.

The relationships that were found between derived fact strategy use and both calculation performance levels and WISC Arithmetic could indicate that a certain level of arithmetical knowledge is a prerequisite for the use of such strategies. Alternatively, the derived fact strategies may develop first, and contribute to an improvement in calculation performance.

As pointed out by Dowker (2009), these alternative possibilities have some parallels with the "some principles first" and "skills first" theories of the relationship between counting principles and procedures. Findings (Dowker, 2008) with regard to the existence of both a strong correlation and the existence of discrepancies in both directions in individual children suggest some degree of "mutual development" or iterative relationship between the two (Baroody and Ginsburg, 1986; Cowan et al., 1996). Rittle-Johnson et al. (2001) have suggested that this extends to the iterative development of principles and skills in the later development of arithmetic. This would be consistent with the results of this study, showing a strong relationship between derived fact strategy use and arithmetical ability as measured both by addition and subtraction performance levels and by the WISC Arithmetic test, but at the same time, showing discrepant performance (e.g., **Tables 1**, **3** show that some children at the Facts to 10 level used the addition/subtraction inverse strategy, and some at the higher levels failed to use Identity—though the latter was rare, and might possibly be explainable on the basis of momentary distraction or procedural error).

Dowker (2009) found the relationships between derived fact strategy use and performance on standardized arithmetic tests to be less strong in children with mathematical difficulties than in other children, which may indicate that the iterative integrative process occurs less effectively in this group than among typically achieving children.

Further research is needed to investigate the extent to which both age and level of mathematical achievement may influence the relationships between calculation and derived fact strategy use. Certainly, the evidence suggests that there are children, both among low and typical achievers in mathematics, whose derived fact strategy use is considerably better than would be expected from their calculation ability (Dowker, 1998; Gilmore and Bryant, 2006, 2008). Further studies of the characteristics of such children might give us a greater understanding of the levels of functional independence and interdependence between derived fact strategy use and other arithmetical abilities.

OTHER AREAS FOR FURTHER RESEARCH

Much more research, and in particular longitudinal research, is needed if we are to fully understand the nature, foundations and development of derived fact strategies. This must involve research into the order in which such strategies develop, and whether any particular strategies are prerequisites for any other strategies. It must also involve studying the nature and direction of predictive relationships between derived fact strategies, calculation performance, and arithmetical concepts; and, in particular, whether derived fact strategies are more dependent on principled knowledge or the ability to implement strategies in arithmetic. Intervention studies would be crucial here: would training in derived fact strategies lead to improvement in calculation, and/or vice versa?

With regard to this issue, it is also important to investigate the effects of context and task presentation mode on performance. Research should also go beyond arithmetic in examining whether any domain-general abilities have a specific role to play in the development and use of derived fact strategies.

Moreover, it would be desirable to investigate the neural mechanisms involved in understanding and using derived fact strategies. Studies of patients have indicated that double dissociations can occur between retrieval and derived fact strategy use (Warrington, 1982; Delazer, 2003). Now that it is increasingly feasible to carry out brain imaging studies with children, researchers should investigate whether the network of brain areas involved in the use of derived fact strategies differs in any way from that involved in other aspects of arithmetic, and whether this changes with development.

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Decimal fraction representations are not distinct from natural number representations – evidence from a combined eye-tracking and computational modeling approach

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Decimal fractions comply with the base-10 notational system of natural Arabic numbers. Nevertheless, recent research suggested that decimal fractions may be represented differently than natural numbers because two number processing effects (i.e., semantic interference and compatibility effects) differed in their size between decimal fractions and natural numbers. In the present study, we examined whether these differences indeed indicate that decimal fractions are represented differently from natural numbers. Therefore, we provided an alternative explanation for the semantic congruity effect, namely a string length congruity effect. Moreover, we suggest that the smaller compatibility effect for decimal fractions compared to natural numbers was driven by differences in processing strategy (sequential vs. parallel). To evaluate this claim, we manipulated the tenth and hundredth digits in a magnitude comparison task with participants' eye movements recorded, while the unit digits remained identical. In addition, we evaluated whether our empirical findings could be simulated by an extended version of our computational model originally developed to simulate magnitude comparisons of two-digit natural numbers. In the eye-tracking study, we found evidence that participants processed decimal fractions more sequentially than natural numbers because of the identical leading digit. Importantly, our model was able to account for the smaller compatibility effect found for decimal fractions. Moreover, string length congruity was an alternative account for the prolonged reaction times for incongruent decimal pairs. Consequently, we suggest that representations of natural numbers and decimal fractions do not differ.

Keywords: number comparison, decimal fractions, compatibility effect, string length congruity effect, computational modeling, artificial neural network

INTRODUCTION

In recent years, there was increased research interest in the cognitive mechanisms underlying multi-digit number processing (see Nuerk et al., 2011 for a review). Nevertheless, while considerable progress has been accomplished in understanding the processing of natural multi-digit numbers and also fractions, the cognitive mechanisms involved when processing decimal fractions have largely been neglected so far. This is particularly noteworthy because decimal fractions – just like multi-digit natural numbers – comply with the general base-10 place-value structure of the Arabic number system: the numerical value of each individual digit in a multi-digit Arabic number is determined by its position within the respective digit string (i.e., units, 10, 100, etc.). Any number can thus be written as a linear combination of powers of 10, each weighted with one from the set of 10 symbols (i.e., the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9). For instance, 639 can be expressed as $6 \times 10^2 + 3 \times 10^1 + 9 \times 10^0$. Different from natural numbers, decimal fractions are also composed of weighted powers of 10 with integer exponents smaller than zero, separated from the components with integer exponents larger than or equal to zero by the so-called decimal point[1] (e.g., $63.9 = 6 \times 10^1 + 3 \times 10^0 + 9 \times 10^{-1}$). Decimal fractions may thus be considered just an extension of natural numbers.

Despite these structural similarities there are at least two important differences when individuals have to process decimal fractions, for instance in a number magnitude comparison task: (i) different from natural numbers the mere number of digits is not an indicator for the overall magnitude of decimal fractions (e.g., 2.45 is larger than 1.532, although 2.45 is the shorter digit string). (ii) The role of zeros in decimal fractions differs from their role in natural numbers: while zeros to the right of the decimal point with one or more non-zero digits further to the right (e.g., 6.07) do change the value of a decimal fraction, adding one or more zeroes at the right end of a decimal fraction does not change its magnitude

[1] Please note that different cultures use different symbols for the decimal mark. The two most common decimal marks are a dot "." and a comma ",".

(e.g., 6.0 = 6.00, but 60 < 600). Importantly, these differences lead to characteristic errors observed in numerical development. Desmet et al. (2010) investigated misconceptions about decimal fractions in children from grade 3 to 6 using a number magnitude comparison task. Younger children, in particular, tended to overgeneralize their previously acquired knowledge about natural numbers and assumed systematically (but mistakenly) that the more digits a number has, the larger its value. Moreover, children assigned zero the same role as in natural numbers: on the one hand, they implied that adding a zero at the end of a decimal fraction would make it larger. On the other hand, they assumed that adding a zero at the tenths position would not change a decimal fraction's value.

The processing of decimal fractions in adults has only recently been examined in cognitive psychology. Based on the observation that the numerical distance effect (i.e., faster and less error-prone responses when comparing relatively distant numbers, e.g., 1 vs. 9 as compared to close numbers, e.g., 4 vs. 5, Moyer and Landauer, 1967) did not differ between natural numbers and decimal fractions, Dewolf et al. (2013) concluded that decimal fractions are processed similar to natural numbers. This corroborates the natural number conversion hypothesis (Dewolf et al., 2013) assuming that whenever participants have to compare the magnitude of decimal fractions they might convert the decimal fractions into natural number expressions by simply ignoring the decimal points and then comparing the corresponding natural number.

In contrast to this view, Varma and Karl (2013) suggested that participants do not resort to natural numbers. The authors investigated differences between decimal fraction and natural number processing, proposing two effects other than the distance effect: (i) a syntactic interference effect and (ii) a semantic interference effect. The syntactic interference effect is just another label, introduced by Varma and Karl (2013), for the processing property described by unit-decade compatibility of two-digit numbers (Nuerk et al., 2001). The unit-decade compatibility effect states that magnitude comparisons of unit-decade-compatible natural number pairs (e.g., 42 vs. 57, with 4 < 5 and 2 < 7) are executed faster and less error-prone than comparisons of unit-decade-incompatible pairs (e.g., 47 vs. 62, with 4 < 6, but 7 > 2). Thereby, the unit-decade compatibility effect designates an influence of decision-irrelevant units on the comparison process of the whole numbers. This suggests that the numerical magnitude of a number is represented componentially via the magnitudes of units, 10, 100, etc., complying with the base-10 place-value structure of the Arabic number system (cf. Nuerk et al., 2011 for a review). Varma and Karl (2013) compared this compatibility effect for two-digit numbers with a similar compatibility effect for tenths and hundredths of decimal fractions (e.g., compatible: 0.42 vs. 0.57 and incompatible: 0.47 vs. 0.62). The authors found that the compatibility effect was smaller for two-digit decimal fractions than for two-digit natural numbers and interpreted this finding to imply that different representations are used for decimal fraction and natural number comparison.

Additionally, Varma and Karl (2013) also observed a semantic interference effect: response times for comparisons with congruent numerical magnitude relations between pairs of decimal fractions, respectively natural numbers (e.g., 0.2 vs. 0.53; 0.2 < 0.53 and 2 < 53), were faster than comparisons for incongruent pairs (e.g., 0.23 vs. 0.5; 0.23 < 0.5, but 23 > 5). The authors interpreted this semantic interference effect as a consequence of parallel access to the individual digits, implying that decimal fractions also activate natural number representations in addition to decimal fraction representations. Interestingly, Varma and Karl (2013) compared the semantic interference effect in decimal fractions with the semantic interference effect in natural numbers by converting the decimal fractions into natural numbers by deleting the leading zero and attaching it to the right end of the number (e.g., 0.23 was converted to 23.0). The semantic interference effect was larger in decimal fractions, supporting their hypothesis that decimal fractions are represented differently than natural numbers.

Taken together, recent research seems to suggest that decimal fraction representations are accessed exclusively when comparing decimal fractions with an identical number of digits in the digital string, whereas natural number representations seem to interfere in case of decimal fractions with an unequal number of digits to the right of the decimal point.

In the present study, we will point out that the rejection of the natural number conversion hypothesis by Varma and Karl (2013) might be premature by offering an alternative explanation for their findings of a reduced compatibility effect and an increased semantic interference effect. First, Varma and Karl (2013) suggested other explanations for the smaller compatibility effect for decimal fractions than natural numbers. One of these accounts was that in their experiment the position of the decision-relevant digits was confounded with whether natural numbers or decimal fractions had to be compared with respect to overall number magnitude. Thus, the smaller compatibility effect can also be explained by the difference between natural numbers and decimal fractions regarding the position of the digits decisive for the number magnitude comparison process. For natural numbers the first (leftmost) digit is (primarily) decisive (e.g., in 21 vs. 87, 2 and 8 are relevant), whereas for decimal fractions (<1) the second digit is decisive[2] (e.g., in 0.21 vs. 0.87, 0 is irrelevant, but 2 and 8 are relevant). Thus, we argue that the important difference between natural numbers and decimal fractions is notational. We suggest that padded natural numbers (like 021 vs. 087) should be processed similarly to decimal numbers (0.21 vs. 0.87) and when leading zeros of decimal numbers are omitted, these should be processed similarly to positive numbers. However, it has not been shown yet that there are similar compatibility effects, when three digit natural numbers – either with differing first digits or with identical first digits but differing second digits – have to be compared (see Nuerk et al., 2011; Klein et al., 2013 for reviews). Hence, we suggest that the observed difference in compatibility effects by Varma and Karl (2013) might be due to different notations used to examine the compatibility effect in natural numbers and decimal fractions.

Second, we also propose an alternative account for the semantic interference effect. In particular, we suggest that the semantic interference effect might alternatively be explained in terms of a

[2]Please note that all decimal fractions in the study of Varma and Karl (2013) were smaller than 1 and had 0 as the leading digit.

congruity effect between the numerical magnitude of the single digits constituting the respective number and the number of digits constituting the digital string. Henceforth referred to as *string length congruity effect*: the comparison of the decisive digits of two decimal fractions can be either congruent (e.g., 2.7 vs. 2.91 with 7 < 9 and 2 vs. 3 digits) or incongruent with the comparison of the string lengths (e.g., 7.14 vs. 7.6 with 1 < 6, but 3 vs. 2 digits). Importantly, first evidence on a somewhat similar effect has already been reported in previous studies. Naparstek and Henik (2010, 2012) and Pansky and Algom (2002) observed that – at least for circular or matrix presentation – the number of digits interfered with the numerical value of the digit. In the numerical value block of Naparstek and Henik (2010), participants had to compare the magnitude of a digit out of a set composed of a varying number of identical digits (e.g., 333) and letter fillers to the standard five. Although the number of digits was irrelevant, participants processed congruent items (number of digits and numerical value being the same: e.g., 333) faster than incongruent items (number of digits and numerical value different: e.g., 3333). Hence, this finding suggests that the number of digits interfered with the processing of the numerical value of a digit. Moreover, in numerical cognition, a common assumption is that numerical and physical magnitudes are not processed independently (e.g., Walsh, 2003; Bueti and Walsh, 2009). Because incongruent items differ from congruent items also in their physical magnitude (i.e., continuous magnitude dimensions such as total surface area, and total "white" color over black background, see Leibovich and Henik, 2013), not only the number of digits, but also their physical magnitude might interfere with processing of the numerical magnitude.

Thus, prolonged reaction times for length incongruent decimal fractions (e.g., 7.14 vs. 7.6 with 1 < 6, but 3 digits vs. 2 digits; Varma and Karl, 2013) might also be explained in terms of interference between the numerical magnitude of the digits constituting the number and the string length of the number. As a consequence, the semantic interference effect observed by Varma and Karl (2013) might be caused by a purely structural difference between natural numbers and decimal fractions, which cannot be matched across item types: as argued above, the mere number of digits is not a valid indicator for the overall magnitude of decimals, whereas it is always a valid indicator for natural numbers. Varma and Karl (2013) tried to account for different visual aspects by adding a zero at the end of natural numbers, but, nevertheless, this difference cannot be controlled for (e.g., 3 < 15 and 3.0 < 15). Therefore, we suggest that the semantic interference effects observed for natural numbers and decimal fractions might have different origins. In the case of natural numbers, numbers with more digits are always larger than numbers with fewer digits, even when the magnitude of the single digits constituting the number with fewer digits is larger than the magnitude of the digits constituting the number with more digits. For instance, when comparing "9 vs. 27," "27" contains more digits indicating that it is larger than "9." However, "9" is larger than "2" and "7" and therefore, a componential comparison of 9 with "2" or "7" suggests that "9" should be larger resulting in the proposed semantic interference effect for natural numbers (Varma and Karl, 2013). Thus, for natural numbers, the numerical magnitudes of the single digits interfere with each other. In contrast, in the case of decimal fractions, string length may interfere with numerical magnitude. For instance, when comparing "0.9" with "0.27" and "0.9" is larger than "0.27," because "9" is larger than "2." However, the number "0.27" contains more digits than the number "0.9," which in case of natural numbers would indicate the larger number - and thus, interferes with the decision-relevant comparison of "9" and "2" resulting in the proposed string length congruity effect.

The alternative accounts regarding the reduced compatibility effect for decimal fractions as well as the semantic interference effect observed by Varma and Karl (2013) rely heavily on the notion of componential processing of the individual digits of any multi-digit number as, for instance, indicated by the (unit-decade) compatibility effect (e.g., Nuerk et al., 2001). We want to argue that the processing of decimal fractions may well be integrated into the model of componential number processing. In the present study, we will corroborate this claim by means of a combined empirical and computational modeling approach. In a first step, we will appraise participants' eye-fixation behavior, while engaged in a number magnitude comparison task involving decimal fractions (e.g., 2.91 vs. 2.43; see also **Table 1** for an overview of different decimal fraction types employed in the present study). This method is used because the registration of eye movements allows for a more fine-grained online evaluation of the comparison process itself. According to the eye-mind hypothesis, the location of eye-fixations and their duration are valid and reliable indicators of what part of a stimulus (i.e., which digit) is processed at a given moment in time and how long this processing lasts (e.g., Rayner and Pollatsek, 1989; Rayner, 1998). In a second step, we aimed at evaluating whether an adapted version of our computational model for multi-digit natural number comparison (Huber et al., 2013a) can also account for the processing of decimal fractions. Thus, the main focus of the present study lies on the computational model, with which we want to show that the findings of Varma and Karl (2013) can be explained by the natural number conversion hypothesis.

More specifically, the aim of the eye-tracking experiment was threefold: first, we wanted to explore why the compatibility effect in the study of Varma and Karl (2013) was smaller for decimal fractions than for natural numbers. A recent study by Huber et al. (2013a) showed that in two-digit number comparison the size of the unit-decade compatibility effect increased with the percentage of within-decade filler items (e.g., 43 vs. 47) relative to between-decade critical items. Correspondingly, the authors found that fixations on the irrelevant unit relative to the relevant 10 digits increased, the more within-decade fillers were included. Similarly, the number (and duration) of fixations on irrelevant digits might be indicative of the size of the compatibility effect. Transferring these findings to the case of decimal fraction processing, the smaller compatibility effect for decimal fractions should be associated with fewer fixations on the non-decisive hundredth digits. Such a fixation pattern would indicate that participants process decimal fractions with identical first digits more sequentially than natural numbers with differing first digits. A sequential processing strategy can be identified, when most fixations are on relevant digits and only few fixations on irrelevant digits. Conversely, when participants process digits in

Table 1 | Examples for compatible and incompatible and congruent and incongruent decimal fraction pairs for decimal types a.0c, a.b0, a.bc, and a.b, respectively.

Decimal type	Compatible			Incompatible		
	Numbers	Tenth digit	Hundredth digit	Numbers	Tenth digit	Hundredth digit
a.0c	9.07 vs. 9.39	0 < 3	7 < 9	1.09 vs. 1.51	0 < 5	9 > 1
a.b0	8.10 vs. 8.97	1 < 9	0 < 9	6.54 vs. 6.90	5 < 9	4 > 0
a.bc	4.25 vs. 4.69	2 < 6	5 < 9	3.29 vs. 3.67	2 < 6	9 > 7
	Congruent			Incongruent		
	Numbers	Tenth digit	Number of digits	Numbers	Tenth digit	Number of digits
a.b	2.7 vs. 2.91	7 < 9	2 vs. 3 digits	7.14 vs. 7.6	1 < 6	3 vs. 2 digits

parallel, the number of fixations on relevant and irrelevant digits would be more balanced (Moeller et al., 2009). A combination of sequential and parallel processing strategies was already observed for multi-digit number comparison beyond the two-digit number range (Meyerhoff et al., 2012) and therefore, might also be present when comparing decimals. Accordingly, we would expect the tenth digits to be fixated much longer than the hundredth digits as measured by total reading time (TRT; i.e., the time spent fixating a digit).

Second, Varma and Karl (2013) observed that decimal fractions with zeros at the rightmost position (e.g., "0.30") were responded to faster than other decimals. However, the authors did not investigate whether processing of decimal fractions with zeros directly to the right of the decimal point (e.g., "0.03") might also be processed faster, providing further support for a privileged role of the digit zero in the comparison of decimal fractions. Similar to faster response times, we would expect that the total fixation time for decimal fractions containing at least one zero will be shorter than for decimal fractions without a zero.

Third, we used the proportion of fixations on tenth and hundredth digits obtained from the empirical study for attentional weighting of the respective digits in the computational model. In the computational model, relevance of the respective digits has to be prespecified in a task demand layer. In principle, two heuristics are possible to obtain suitable values: (i) a trial-and-error approach using starting random values and (ii) an approach using proportions of digits as starting values and adjusting them such that error rates of the simulated data corresponded to the error rates of the empirical data. As we had available proportions of fixations on tenth and hundredth digits from the empirical study, we employed the second approach. Finally, we used reaction times and error rates from the eye-tracking experiment to validate our computational model.

The particular aim of the computational modeling study was to examine whether an extended version of our model for two-digit number comparison (Moeller et al., 2011; Huber et al., 2013c) can account for the findings of the eye-tracking study, which would corroborate the natural number conversion hypothesis for decimal fractions. Therefore, we adapted the existing model to specifically account for the effects observed by Varma and Karl (2013). Most importantly, we extended the network such that three-digit numbers could be compared (as it was already done in the study of Huber et al., 2013b) in order to consider the string lengths of the numbers to be compared. With these measures, a general model of multi-digit number processing encompassing natural as well as decimal fractions can be developed and tested empirically, as done in the current study.

EMPIRICAL STUDY

METHODS

Participants

Twenty five students of the University of Tuebingen participated in the study (15 female, 10 male) for course credits. Average age was 24.8 years with a standard deviation (SD) of 2.67 years (range 21–33 years). All participants reported normal or corrected-to-normal vision. The study was approved by the local ethics committee of the Medical Faculty of the Eberhard Karls University of Tuebingen. All participants gave their written informed consent.

Apparatus

Eye-fixation behavior was recorded by an EyeLink 1000 tracking device (SR-Research, Kanata, ON, Canada). Following 9-point calibration at the start of the experiment as well as drift corrections before each trial, the spatial resolution of the eye-tracking device was less than 0.5 degrees of visual angle at a sampling rate of 1000 Hz. A 20″ monitor set at a resolution of 1024 × 768 pixels and driven at a refresh rate of 120 Hz was used to present stimuli. Viewing distance was about 60 cm. The experiment was programmed using the Experimental Builder software (SR-Research, Kanata, ON, Canada).

Stimuli and design

We created 320 pairs of decimal fractions. Decimal fractions were either two- or three-digit numbers ranging from 1.04 to 9.96. Unit digits were identical for number pairs and ranged from 1 to 9. Digits at the tenth and hundredth position ranged from 0 to 9. Participants had to compare four different types of decimal fraction pairs (i.e., 80 decimal fraction pairs for each type). Whereas one of the decimal fractions always consisted of non-zero digits (e.g., 2.91), the other one was generated considering the following constraints: the decimal fraction involved (i) a zero at the tenth position (a.0c; e.g., 2.04), (ii) a zero at the hundredth position (a.b0; e.g., 2.40), (iii) no digit at the hundredth position (a.b; e.g., 2.4), and (iv) no zeros at all (a.bc; e.g., 2.43). Furthermore, we manipulated compatibility and string length congruity (see **Table 1**). To increase the relevance of the hundredth digit, we further included 120 filler number pairs with identical digits at the tenth position (e.g., 7.91 vs. 7.98 or 2.83 vs. 2.8) which resulted in a total of 440 items. Moreover, all digits except for units differed for decimal fraction pairs. Importantly, we balanced overall distance, tenth distance and hundredth distance across all stimulus groups and problem size across all stimulus groups except stimuli with zero at the hundredth position, which had to have a lower overall problem size than the other groups because of its definitional properties.

Stimuli were displayed as white digits on a black background in Courier New (size: 48, style: bold). By using this non-proportional font we ensured that all digits had the same width. X/Y coordinates of decimal fraction pairs were 496/384 and 528/649 pixels or 528/384 and 496/649 pixels, such that leading digits were not presented above each other in order to prevent column-wise processing (see Meyerhoff et al., 2012 for a similar layout). The coordinates of the fixation cross were 512/150 pixels. Decimal fractions consisting of three digits extended to a visual angle of 4.8, horizontally, and 1.2, vertically.

Procedure

Participants were assessed individually in a dimly lit room. Instructions requested participants to indicate the larger of two decimal fractions as accurately and fast as possible. When the larger decimal fraction was at the top of the screen, the upper button on a gamepad had to be pressed with the right thumb. Otherwise, when the larger decimal fraction was at the bottom of the screen, the lower button had to press with the left thumb. The position of the larger decimal fraction was counterbalanced for each type of decimal fraction pairs. Each participant completed five practice trials to become familiar with task requirements. Trial order was pseudo-randomized ensuring that the same button was not pressed more than three times in a row. After fixation of the fixation point had been checked by the experimenter, the next item was presented until the participant pressed one of the response buttons, which was immediately followed by the presentation of the fixation point for the next trial.

Analysis

Unfortunately, because of a programming error only 320 of the 440 items were presented. These items were randomly drawn from all 440 items such that each participant worked on a different subset of the 440 items. About 85.5 (SD = 3.76) items of the 320 number pairs were filler items (i.e., about 27% like in the original set). Moreover, since the items were a random subset of all items, matching was not affected substantially (see **Table 2**).

Three participants exhibited error rates higher than 25% (50% guessing rate) in the string length incongruent condition and were excluded. All subsequent analyses were run on the data of the remaining 22 participants. Additionally, we only included trials followed by a correct answer with response latencies longer than 200 ms and within ±3 standard deviations from the individual mean RT. This trimming procedure resulted in a loss of 6.52% of the data. Only items considered in the RT analysis were also included in the analysis of the eye-fixation data. Moreover, error rates were subjected to an inverse sine transformation before analysis to approximate their binomial distribution with a normal distribution (see e.g., Kirk, 2013, p. 103).

For the analysis of participants' eye fixation behavior interest areas around each digit and the decimal point were defined with a height of 120 pixels and a width of 39 pixels. We defined the interest areas to be quite narrow, such that the interest areas of each digit and decimal mark were of equal size and interest areas did not overlap. Therefore, it might be possible that participants processed the tenth digit parafoveally while fixating the decimal mark. We exclusively analyzed TRT on the relevant digits (i.e., tenth and hundredth digit) and on critical decimal fractions. Thus, we included only critical decimal fractions with either a zero at the tenth position (e.g., 2.04), a zero at the hundredth place (e.g., 2.40) or only two-digit decimal fractions (e.g., 2.4), respectively, whereas

Table 2 | Mean (SD in parentheses) overall distance, tenth distance, hundredth distance, problem size, and number of items for compatible/congruent and incompatible/incongruent decimal fraction pairs for decimal types a.0c, a.b0, a.bc, and a.b, respectively.

Decimal type	Compatibility/ congruity	Overall distance	Tenth distance	Hundredth distance	Problem size	Number of items
a.0c	Compatible	0.40 (0.02)	3.66 (0.25)	3.48 (0.15)	10.62 (0.63)	29.56 (2.83)
	Incompatible	0.41 (0.02)	4.42 (0.22)	3.44 (0.26)	10.50 (0.47)	28.76 (2.71)
a.b0	Compatible	0.40 (0.02)	3.69 (0.25)	3.62 (0.15)	11.36 (0.63)	28.60 (2.33)
	Incompatible	0.40 (0.02)	4.37 (0.22)	3.56 (0.26)	10.85 (0.47)	29.76 (1.94)
a.bc	Compatible	0.40 (0.02)	3.67 (0.23)	3.50 (0.23)	11.26 (0.50)	28.56 (2.83)
	Incompatible	0.40 (0.02)	4.39 (0.17)	3.48 (0.14)	10.79 (0.35)	28.88 (2.82)
a.b	Congruent	0.40 (0.02)	3.67 (0.23)	3.77 (0.23)	11.23 (0.50)	30.20 (2.66)
	Incongruent	0.40 (0.02)	4.39 (0.17)	3.69 (0.14)	10.80 (0.35)	30.00 (2.87)

Please note that participants were presented different numbers of items due to a programming error.

the additional decimal fraction in numbers of the a.bc type was not included (e.g., 2.91), because there is no critical single decimal fraction. For these latter decimal fractions, mean TRTs from both tenth and hundredth digits were calculated.

Reaction times, error rates, and TRT were analyzed by running separate repeated measures analyses of variance (ANOVA). In case of violation of the sphericity assumption for repeated measures ANOVA, the Greenhouse-Geisser (GG) correction was applied to adjust the degrees of freedom. For reasons of readability, the original *df* together with the *GG* coefficient are reported.

RESULTS

Reaction times and error rates

Processing of zeros and tenth–hundredth compatibility. First, we examined, whether we could replicate the relatively smaller compatibility effect found in the study of Varma and Karl (2013) and how zeros in different positions of decimal fractions influenced processing of these decimal fractions. Therefore, we analyzed reaction times and error rates by conducting repeated-measures 3 × 2 ANOVAs with factors decimal type (a.0c, a.b0, and a.bc) and tenth–hundredth compatibility (compatible and incompatible).

The ANOVA for RT revealed a significant main effect of decimal type [$F(2,42) = 42.61$, $p < 0.001$, $\eta_p^2 = 0.67$]. Pairwise comparisons indicated that participants compared decimal type a.0c fastest, followed by a.b0 and decimal type a.bc slowest (all $p < 0.001$; a.0c: $M = 805$ ms; a.b0: $M = 832$ ms; a.bc: $M = 851$ ms; see also **Figure 1A**). Thus, we found that zeros facilitated participants' comparisons of decimal fractions. The main effect of compatibility and the interaction between group and compatibility were not significant, indicating sequential processing of decimal fractions (both $p > 0.14$). Moreover, for error rates we did not find significant main or interaction effects (all $p > 0.28$).

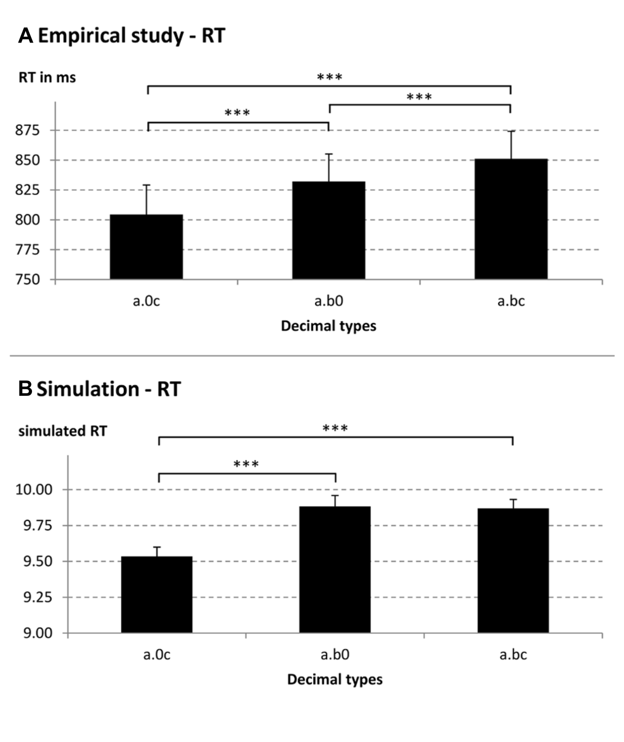

FIGURE 1 | Reaction times (RT; A, empirical, B, simulated) for different decimal fraction pair types (e.g., a.0c = 2.04 vs. 2.91, a.b0 = 2.40 vs. 2.91, and a.bc = 2.43 vs. 2.91). ***$p < 0.001$.

String length congruity. String length congruity effects on RT and error rates were analyzed by running ANOVAs with congruity as independent variable. Both ANOVAs yielded significant results, indicating shorter RT and lower ER for length congruent than incongruent decimal fraction pairs [RT: $M = 819$ vs. 886 ms; $F(1,21) = 43.42$, $p < 0.001$, $\eta_p^2 = 0.67$; ER: $M = 2.29$ vs. 9.24%; $F(1,21) = 34.33$, $p < 0.001$, $\eta_p^2 = 0.62$; see also **Figures 2A,C**].

Eye-tracking data

Processing of zeros and tenth–hundredth compatibility. Furthermore, we explored whether participants processed the digits in the decimal fractions sequentially or in parallel. Therefore, we ran a repeated-measures 3 × 2 × 2 ANOVA with factors decimal type (a.0c, a.b0, and a.bc), tenth–hundredth compatibility (compatible and incompatible) as well as digit position (tenth and hundredth digit) and TRT as dependent variable. Results will be reported starting with the three-way interaction, followed by its constituting two-way interactions before main effects will finally be described. Mean TRT for all factors are provided in **Table 3**.

We observed a significant three-way interaction [$F(2,42) = 5.17$, $p < 0.05$, $\eta_p^2 = 0.20$, $GG = 0.74$; see **Figure 3**]. This interaction was broken down by conducting two 2 × 2 ANOVAs with factors decimal type and tenth–hundredth compatibility separately per digit position (i.e., tenth and hundredth digit).

For the tenth digit (see **Figure 3A**), the interaction between decimal type and tenth–hundredth compatibility was significant, indicating that compatibility effects differed at the tenth digit position [$F(2,42) = 11.55$, $p < 0.001$, $\eta_p^2 = 0.36$, $GG = 0.79$]. The two-way interaction was further analyzed by conducting (i) two univariate ANOVAs with the factor decimal type for compatible and incompatible items separately and (ii) three univariate ANOVAs with the factor tenth–hundredth compatibility for each decimal type.

For compatible number pairs, we found a significant main effect of decimal type [$F(2,42) = 38.42$, $p < 0.001$, $\eta_p^2 = 0.65$]. Participants fixated the tenth digits of decimal type a.bc longer than that of other types (both $p < 0.001$), whereas TRT on the tenth digits of decimal types containing a zero did not differ significantly ($p = 0.38$). For incompatible number pairs, the main effect of decimal type was also significant [$F(2,42) = 32.62$, $p < 0.001$, $\eta_p^2 = 0.61$]. However, different from compatible number pairs, participants fixated the tenth digit of decimal type a.0c shorter than that of other types (both $p < 0.001$), while TRT on the tenth digit of decimal types a.b0 and a.bc did not differ significantly ($p = 1.00$).

Subsequently, we evaluated tenth–hundredth compatibility effects on participants TRT for each of the three decimal types separately. Results indicated that the main effect of tenth–hundredth compatibility was significant for decimal type a.b0 [$F(1,21) = 22.96$, $p < 0.001$, $\eta_p^2 = 0.52$], but not for the other decimal types (both $F < 1$). While the tenth digits were fixated 45 ms

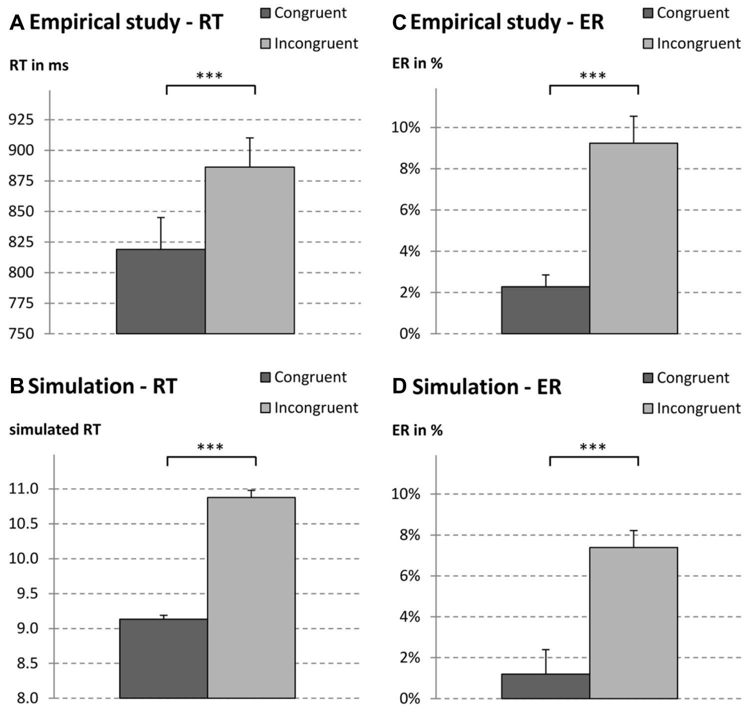

FIGURE 2 | Reaction times (RT; A, empirical; B, simulated) and error rates (ER; C, empirical; D, simulated) for decimal fraction pair type a.b (e.g., 2.4 vs. 2.91). The decimal fraction with three digits was either larger (i.e., congruent: 3.98 vs. 3.6) or smaller than the decimal fraction with two digits (i.e., incongruent: 7.63 vs. 7.9). ***$p < 0.001$.

Table 3 | Mean (SD in parentheses) TRT in ms on tenth and hundredth digits for compatible/congruent and incompatible/incongruent decimal fraction pairs for decimal types a.0c, a.b0, a.bc, and a.b, respectively.

	Tenth digit		Hundredth digit	
Decimal type	**Compatible/ congruent**	**Incompatible/ incongruent**	**Compatible/ congruent**	**Incompatible/ incongruent**
a.0c	181 (42)	178 (40)	32 (28)	34 (27)
a.b0	194 (43)	239 (51)	27 (26)	32 (36)
a.bc	234 (35)	237 (41)	39 (33)	42 (44)
a.b	91 (57)	137 (62)	1 (4)	1 (2)

less for compatible than for incompatible a.b0 number pairs, mean compatibility effects for decimal types a.0c and a.bc were −3 and 3 ms, respectively (see also **Figure 3A**).

For the hundredth digit (see **Figure 3B**), the main effect of decimal type was significant [$F(1,21) = 5.19$, $p < 0.05$, $\eta_p^2 = 0.20$]. Pairwise comparisons revealed that participants fixated the zero of the decimal type a.b0 less than the hundredth digit of decimal types a.bc ($p < 0.05$). All other pairwise comparisons were not significant (all $p > 0.17$). The main effect of tenth–hundredth compatibility and the two-way interaction were not significant (both $F < 1.24$, $p > 0.54$).

Taken together, the three-way interaction revealed a significant compatibility effect on tenth digits for decimal type a.b0. Moreover, we found that zeros were fixated less often than other digits: tenth digits of decimal type a.0c were fixated less than tenth digits of decimal type a.bc in case of compatible number pairs and less than tenth digits of decimal types a.b0 and a.bc in case of incompatible number pairs. Similarly, the zero of decimal type a.b0 (i.e., the hundredth digit) was fixated less than the hundredth digits of other decimal types.

All three two-way interactions were significant [decimal type × compatibility: $F(2,42) = 18.08$, $p < 0.001$, $\eta_p^2 = 0.46$; decimal type × digit position: $F(2,42) = 23.06$, $p < 0.001$, $\eta_p^2 = 0.52$; digit position × compatibility: $F(1,21) = 4.49$, $p < 0.05$, ${\eta_p}^2 = 0.18$]. In the above analyses of the three-way interaction, we already described the interaction between decimal type and tenth–hundredth compatibility for each digit position separately (i.e., for tenth and hundredth digit separately). Moreover, we already

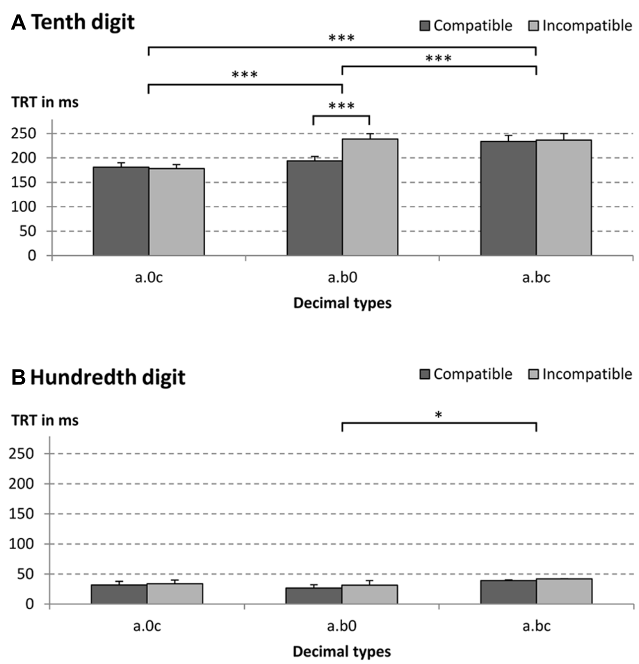

FIGURE 3 | Mean total reading times (TRT) for (A) tenth and (B) hundredth digit for different decimal fraction pair types (e.g., a.0c = 2.04 vs. 2.91, a.b0 = 2.40 vs. 2.91, and a.bc = 2.43 vs. 2.91). Decimal fraction pairs were either compatible (e.g., 4.21 vs. 4.67) or incompatible (e.g., 2.45 vs. 2.91). ***$p < 0.001$, *$p < 0.05$.

analyzed differences between decimal types for each digit positions (as indicated by the two-way interaction between decimal type and digit position). Finally, tenth–hundredth compatibility effects were already discerned for tenth and hundredth digit. Thus, only the analysis of the digit position effects remains to be presented (TRT on tenths − TRT on hundredths) for each decimal type (as indicated by the two-way interaction between decimal type and digit position). First, we analyzed digit position effects for each decimal type separately. Then, we compared digit position effects of decimal types against each other. Generally, the tenth digits were fixated longer than the hundredth digits for all decimal types (all $p < 0.001$) with mean differences between TRT on tenth and hundredth digits for a.0c, a.b0, and a.bc being 147, 187, and 195 ms, respectively. The digit position effect of decimal type a.0c was significantly smaller than the effect of other types ($p < 0.001$), but decimal types a.b0 and a.bc did not differ significantly ($p = 1.00$). Thus, irrespective of decimal type, participants fixated hundredth digits far less than tenth digits indicating an (at least partially) sequential processing strategy.

Finally, all main effects were significant [decimal type: $F(2,42) = 96.29$, $p < 0.001$, $\eta_p^2 = 0.82$; compatibility: $F(1,21) = 55.27$, $p < 0.001$, $\eta_p^2 = 0.73$; digit position: $F(1,21) = 281.66$, $p < 0.001$, $\eta_p^2 = 0.93$]. Comparable to the RT analysis, pairwise comparisons revealed that participants fixated decimal type a.0c shortest, followed by a.b0 and a.bc (all $p < 0.001$; a.0c: $M = 106$ ms; a.b0: $M = 123$ ms; a.bc: $M = 138$ ms). Moreover, the significant compatibility effect indicated shorter TRT for compatible than incompatible number pairs ($M = 118$ vs. 127 ms). Finally, we found longer TRT on relevant tenth digits than on irrelevant hundredth digits ($M = 210$ vs. 34 ms).

String length congruity. Since the critical digit for decimal type a.b did not contain a digit at the hundredth's position, we analyzed TRT of tenth digits separately by conduction a paired t-test. Thereby, we evaluated whether string length congruity also affected participants' fixation pattern in addition to response times. The paired t-test for the string length congruity effect of decimal type a.b revealed a significant congruity effect. Participants fixated the tenth digits of incongruent number pairs longer than those of congruent number pairs [congruent: $M = 91$ ms vs. incongruent $M = 137$ ms; $F(1,21) = 40.06$, $p < 0.001$, $\eta_p^2 = 0.66$].

SIMULATION

MODEL FOR TWO-DIGIT NUMBER COMPARISON

To simulate the processing of decimal fractions, we adapted the computational model of Huber et al. (2013c), which simulates the comparison of two-digit numbers using an artificial neural network (see **Figure 4**). This model consists of two single-digit comparison networks for tens and units and a cognitive control network, which was inspired by the cognitive control network of Verguts and Notebaert (2008).

The single-digit comparison networks are composed of an input layer and a comparison layer. In the input layer the representation of digits is modeled using a place coding system with a fixed Gaussian distribution for each digit (see Verguts et al., 2005, for a similar approach). Thus, for each digit there is a unit which is activated most and units coding digits of a similar magnitude are activated to a lesser degree depending on their numerical distance to the respective digit [i.e., $f(i,j) = \exp(-10 * |i - (j+1)|)$ for node i and digit j]. The units of the input layer are connected via forward connections to two comparison nodes, one coding "left digit larger" and the other one coding "right digit larger." The activation of comparison nodes is calculated by the weighted sum of input nodes reduced by inhibitory connections between comparison nodes with weights $w^{inh} = -2$. The activation function of the comparison nodes is a sigmoid function.

Before single-digit comparison networks were integrated into the cognitive control network, they were trained using the delta rule (Widrow and Hoff, 1960). To do so, weights between input and comparison layer were initialized by generating pseudo-random values in the interval [−1; 1]. The training comprised 100,000 trials, after which the network compared all combinations of single-digit number pairs correctly. Only one network was trained. Connection weights were reused in the second network.

In the cognitive control network, activity of input nodes is propagated to the comparison nodes following formula A1 of Verguts and Notebaert (2008). However, instead of an indicator function we use the activation propagated from the input layer to the comparison layer because in our model the input of the cognitive control network is the output of the single-digit comparison networks. Hence, instead of using color information like in a Stroop task (see Verguts and Notebaert, 2008),

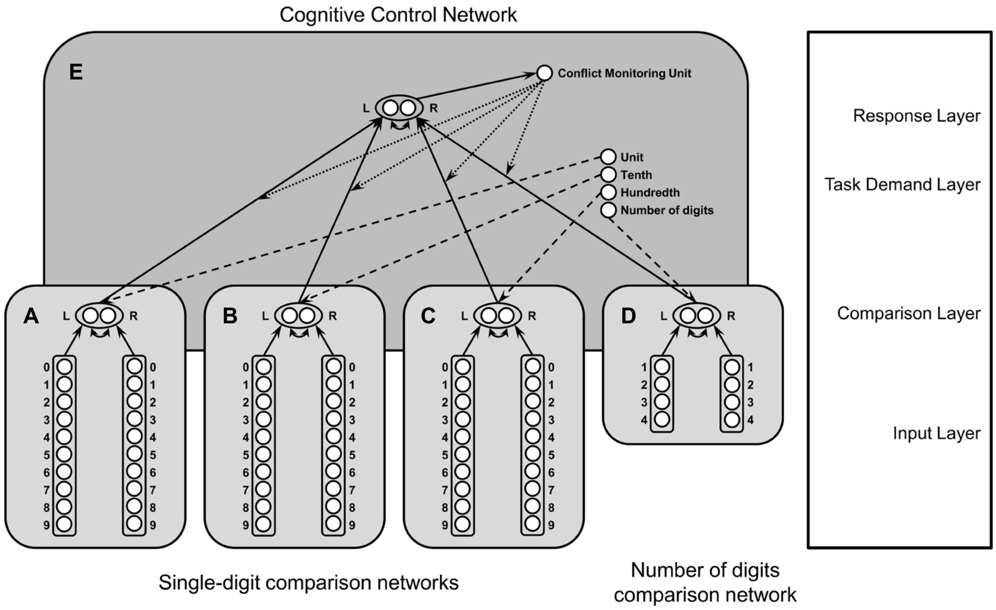

FIGURE 4 | Architecture of the neural network model for decimal fraction processing: networks (A) to (C) depict digit comparison networks for (A) units, (B) tenth digits and (C) hundredth digits, and (D) the number of digits comparison network. These networks are integrated into the neural network of cognitive control **(E)** as suggested by Verguts and Notebaert (2008). L, left digit larger node; R, right digit larger node.

we use the comparison information of one of the digits being larger. This information is not prespecified in the input layer (as it would be when using the original network architecture of Verguts and Notebaert, 2008), but is generated by the single-digit comparison networks. Comparison nodes are connected to two response nodes via forward connections. Again, one node codes "left digit larger" and the other one codes "right digit larger" and response nodes are connected to each other via inhibitory connections with $w^{inh} = -0.5$. Moreover, activation in the comparison layer is modulated by task nodes for the two tasks of comparing either tens or units, as described by equation A2 in Verguts and Notebaert (2008).

Effects of cognitive control are simulated via a conflict monitoring unit. The degree of conflict detected by the conflict monitoring unit is used to adapt the connection weights between task demand nodes and corresponding comparison nodes according to the Hebbian learning rule in equation A3 of Verguts and Notebaert (2008).

Response times are simulated by counting the number of steps needed until one of the comparison nodes reaches a fixed threshold θ of 0.8. Since it is possible that this threshold is never reached, the maximum value for simulated response times was set to 200.

MODIFICATIONS OF THE EXTENDED MODEL FOR DECIMAL FRACTIONS

The model for two-digit numbers could be extended to simulate decimal fraction comparison without introducing further qualitative changes to the original model structure (see **Figure 4**). Since decimal fractions consisted of up to three digits, we simply added another single digit comparison network. Moreover, to simulate the comparison of numbers comprising different numbers of digits (i.e., two vs. three digits), we added another network for the comparison of the number of digits, which was very similar to the single-digit comparison networks (see **Figure 4C**). The comparison of the number of digits can be simulated by the very same network architecture as the comparison of digits (e.g., Verguts and Fias, 2004; Santens and Verguts, 2011). However, tuning curves have to be broader for non-symbolic comparisons (i.e., number of digits) than symbolic comparisons (i.e., comparison of numerical magnitude; Santens and Verguts, 2011) using the following activation function: $f(i, j) = \exp(-|i - (j + 1)|)$ for node i and number of digits j. Moreover, the number of nodes was reduced to four nodes for the comparison of numbers with up to four digits (see **Figure 4D**). Four digits were chosen to extend the network easily for comparisons of numbers with up to four digits, but also three or five or more digits would have been feasible. We also had to extend the task demand layer. Differing from the two-digit comparison network, there are nodes for the comparison of units, tenths, hundredths, and the number of digits.

Furthermore, activation of task nodes and connection weights between comparison layer and response layer were modified to simulate different attentional weighting of the respective digits of decimal fractions. Attentional weighting values were inspired by the relative frequency of fixations on tenth and hundredth digits. Activity of the task node for units was set to 0 as a negligible number of fixations fell on unit digits, namely 2.5% of all fixations. The task node for the tenth digits was set to the largest value of 1.5, because they were fixated most (i.e., 84% of all fixations). To implement the very weak interference of hundredth digits, with only 13% of all fixations on hundredth digits, activation was set to the very low value of 0.01. A similar pattern of values was chosen for the connection weights between comparison layer and response layer: 0.1, 1.0, 0.1, for unit, tenth and hundredth digits, respectively. A different weighting of relevant and irrelevant tasks was also suggested by Santens and Verguts (2011). In their computational model the representational layer of the irrelevant dimension was multiplied by a parameter Θ with the value of 0.15, which determined the size of the size congruity effect (see also Schwarz and Ischebeck, 2003).

Moreover, to capture the increased impact of zero in decimal fraction processing (Varma and Karl, 2013), single-digit comparison networks were not trained using a distribution obtained from a Google-survey (see also Verguts and Fias, 2006; Moeller et al., 2011; Huber et al., 2013b). Instead, the frequency of occurrence of 0 in the training set of single-digit numbers was 15% larger and the frequency of occurrence of 1 was 5% larger than the frequency of occurrences of the other digits. By increasing the frequency of occurrence of 0, comparison with 0 will be trained more often, thereby; increasing corresponding weights between input nodes and comparison nodes, thus leading to faster comparisons with zero (see also Verguts et al., in press, for frequency effects). We also tried only to increase the frequency of 0, but this was not as effective as additionally increasing the frequency of 1. However, other values might also have resulted in a similar effect. Moreover, we slightly increased the training phase to 120,000 trials (in contrast to 100,000 trials in Huber et al., 2013c).

Attentional weights for the comparison of the number of digits were set to slightly lower values than the attentional weights for the comparison of tenth digits (i.e., activation of task node: 1.0 and connection weight between comparison and response layer: 0.7). However, each comparison of the number of digits was trained more often than the comparison of single digits in the single-digit comparison networks (i.e., 100,000 trials for 4 nodes vs. 120,000 trials for 10 nodes of the other single-digit comparison networks). Thereby, the comparison of number of digits is faster than the magnitude comparison of digits, resulting in a more pronounced string length congruity effect.

Other parameters of the cognitive control network are mostly identical to the ones used by Huber et al. (2013c) and Verguts and Notebaert (2008): $\tau = 0.8$, $\beta_{in} = 0.2$, $w^{inh} = -0.5$, $C = 0.7$, $\beta_{\text{con}} = 1$, $\lambda_{\text{con}} = 0.8$, $\lambda_w = 0.7$, $\alpha_w = 1$, and $\beta_w = 0.5$.

PROCEDURE AND ANALYSIS

The same stimuli as in the empirical study were used. However, artificial networks were presented with the entire set of 440 items[3]. We simulated 22 participants by creating 22 randomizations of trial orders. Since we added random Gaussian noise at each time step ($M = 0$, $SD = 0.11$), simulated RT and error rates were different for each simulated participant. Similar to the empirical study, we excluded simulated RT of trials which were not solved correctly from further analyses resulting in a loss of 2.6% of the data. Moreover, error rates were subjected to the inverse sine transformation prior to analyses to approximate a normal distribution.

[3]Please note that results were comparable when analyzing only a random subset of 320 items of the 440 items corresponding to 320 items which were presented to the participants in the empirical study.

RESULTS

Processing of zeros and tenth–hundredth compatibility

To examine the tenth–hundredth compatibility effect and how zeros influence processing of decimal fractions, we analyzed simulated RT and error rates by conducting two repeated-measures 3 × 2 ANOVAs with factors decimal type (a.0c vs. a.b0 vs. a.bc) and tenth–hundredth compatibility (compatible vs. incompatible). For RT data, the main effect of decimal type was significant [decimal type: $F(2,42) = 18.03$, $p < 0.001$, $\eta^2_p = 0.46$]. Mean RT for a.0c, a.b0, and a.bc were: 9.53, 9.88, and 9.87, respectively. Similar to the findings for the behavioral RT, we found that decimal type a.0c was compared faster than the other decimal types (all $p < 0.001$). However, different from the empirical RT findings, decimal type a.b0 did not differ significantly from decimal type a.bc ($p = 1.00$, corrected for multiple comparisons). The main effect of tenth–hundredth compatibility and the interaction between decimal type and tenth–hundredth compatibility were not significant (both $p > 0.24$). Moreover, in line with the empirical findings, we did not find any significant main or interaction effects for error rates (all $p > 0.05$). Thus, as indicated in **Figure 1**, simulated RT replicated the finding of faster RT for decimal type a.0c compared to the other decimal types studied. However, the model could not account for faster RT for decimal type a.b0 compared to decimal type a.bc.

String length congruity

String length congruity effects for simulated RT and error rates were analysed by running two paired *t*-tests. Both *t*-tests were significant indicating shorter simulated RT and lower simulated ER for length congruent than incongruent decimal fraction pairs [RT: $M = 9.14$ vs. 10.87; $F(1,21) = 265.43$, $p < 0.001$, $\eta^2_p = 0.93$; ER: $M = 1.02$ vs. 7.05%; $F(1,21) = 65.98$, $p < 0.001$, $\eta^2_p = 0.76$]. Thus, as depicted in **Figure 2** (for RT see **Figures 2A,B** and for ER see **Figures 2C,D**), simulated RT as well as ER were in accordance with the empirical findings.

DISCUSSION

In the present study, we aimed at providing an alternative explanation for the findings of Varma and Karl (2013), who had reported a smaller compatibility effect and a larger semantic interference effect for decimal fractions as compared to natural numbers. These two findings would be consistent with the hypothesis that decimal fractions are represented differently and, thus, decimal fractions would be processed differently when compared to natural numbers. Therefore, we examined whether rejecting the natural number conversion hypothesis, stating that decimal fractions are processed similar to natural numbers, may have been premature. Our results indicated that both findings can also be explained by relying on componential processing of multi-digit natural numbers in line with the natural number conversion hypothesis (see also Dewolf et al., 2013). It provides a more parsimonious explanation for findings in decimal fraction comparison, because additional decimal fraction representations do not have to be assumed in order to explain how participants compare decimal fractions. Thus, the present study supports the notion that natural numbers and decimal fractions are processed similarly.

COMPATIBILITY EFFECT

We did not find a significant compatibility effect for reaction times or error rates data in our behavioral experiment. This result suggests that hundredth digits interfere less, when comparing decimal fractions than when comparing natural numbers with respect to their magnitude. At first glance, this pattern of results is in line with the interpretation of Varma and Karl (2013), suggesting distinct representations for decimal fractions. However, there is another explanation for the absence of the compatibility effect in our study: participants may have compared the decimal fractions – at least in part – sequentially (e.g., Moeller et al., 2009; Meyerhoff et al., 2012). This assumption could be tested systematically in the present study by evaluating participants' eye fixation behavior. In particular, Moeller et al. (2009) hypothesized that a sequential comparison of two two-digit numbers would primarily lead to fixations on tens and to only a very small number of fixations on units. Moreover, fixations should not differ between compatible and incompatible number pairs. Analogously, sequential processing of decimal fractions in our study would result in long TRT on the tenth digits and only very short TRT on the hundredth digits. In fact, we actually found that pattern for number pairs including no zeros (i.e., a.bc) or zero at the tenth position (i.e., a.0c). There was no compatibility effect for hundredth digits, and TRT on tenth digits were about six times longer (ranging from 1.6 to 223 times for individual participants) than TRT on hundredth digits. In order for (automatic) processing of the hundredth digits to interfere sufficiently with the processing of tenth digits (i.e., to elicit a significant compatibility effect), participants have to fixate the hundredth digits to a certain extent (cf. Huber et al., 2013a). Thus, our findings indicate that hundredth digits could be ignored more easily, when comparing decimal fractions with identical unit digits. The most probable reason for this is that participants processed decimal fractions sequentially and not in parallel, which is typical for two-digit numbers.

Nevertheless, we found a significant compatibility effect for TRT on the tenth digits of decimal fractions having zero at the hundredth position (i.e., a.b0). Thus, whereas compatibility did not affect participants' reaction times and error rates, it modulated participants' fixations on the tenth digits. Therefore, zero at the hundredth position might mislead participants to assume that this decimal fraction is smaller than the other one. As a consequence, participants had to fixate the tenth digit longer in incompatible than in compatible decimal fraction pairs to overcome this bias.

Finally, also in our computational simulations we did not find a significant compatibility effect, although we simulated the comparison of decimal fractions, using a fully componential model. As outlined above, this finding is obviously a consequence of the very low attentional weighting of the hundredth digits, eliminating the compatibility effect. Increasing the activity of the task demand nodes and the weights of the connection between task demand nodes and comparison layer for the hundredth digits would result in reliable compatibility effects for all decimal fraction types. Similarly, we would predict that increasing the relevance of the hundredth digit in an empirical study would result in a reliable compatibility effect. One way to achieve this would be to increase the number of filler items. Macizo and Herrera (2011; see also Huber et al., 2013a) found that the size of the unit-decade compatibility effect in two-digit numbers depends on the number of filler items: the more within-decade filler items, the larger the compatibility effect. Therefore, it is possible that increasing the number of filler items would have led to a significant compatibility effect. To conclude, also our computational modeling suggested that a smaller compatibility effect or even the absence of a compatibility effect for decimal fractions does not necessarily imply a distinct representation of decimal fractions.

STRING LENGTH CONGRUITY EFFECT

Similar to the results of Varma and Karl (2013), string length congruity had a very strong impact on the comparison of decimal fractions. In particular, we found reliably longer reaction times and higher error rates for length incongruent than length congruent decimal fraction pairs (i.e. congruent: 2.7 vs. 2.91 with $7 < 9$ and 1 vs. 2 digits; incongruent: 7.14 vs. 7.6 with $1 < 6$, but 2 vs. 1 digits). We even had to exclude three participants from the analysis because of their very high error rates (almost at chance level) when comparing string length incongruent items. A possible explanation may be that these three participants confused, for instance, 2.06 with 2.60. However, if so, they made this error not systematically. Otherwise, we would have observed a more systematic error pattern and error rates close to 100% for incongruent pairs, which was not the case. Instead, error rates were close to chance level (i.e., 50%). Moreover, our participants were university students who not only should have learnt the decimal notation in school, but are also confronted with it in their statistics courses. Therefore, we are confident that they should at least have had a basic understanding of decimal number notation. Nevertheless, a further study would be required to investigate whether a poor understanding of decimal number notation might explain the poor performance of some students when comparing decimals.

Varma and Karl (2013) suggested that the string length congruity effect is caused by semantic interference of natural and decimal fraction mental representations. However, our simulation provides an alternative explanation. We did not include specific representations for decimal fractions in our network architecture. Instead, we added representations for the number of digits of decimal fractions. Thereby, we were able to simulate the observation of longer reaction times for incongruent than for congruent decimal fraction pairs (i.e., the string length congruity effect). The string length congruity effect may thus be just another example for the assumption that numerical magnitude and physical magnitude (as reflected by the number of digits) are not processed independently (Pansky and Algom, 2002; Naparstek and Henik, 2010, 2012). Numbers with more digits also have a larger physical (i.e., horizontal) extension. Thus, continuous magnitude dimensions (e.g., total surface area, and total "white" color over black background) might interfere with the processing of numerical magnitudes (see also Leibovich and Henik, 2013, for a similar suggestion regarding numerosities). This notion is further supported by the fact that we used a very similar network architecture as in the study of Santens and Verguts (2011), who simulated the size congruity effect (Henik and Tzelgov, 1982) using a dual route model with separate representations for numerical magnitude and physical magnitude. In their model the shared decision account was implemented, according to which the interaction between comparison of numerical and physical magnitude takes place at the decision level (e.g., Schwarz and Heinze, 1998; Santens and Verguts, 2011). Contrarily, the shared representation account suggests that numerical and physical magnitude share the same representation (e.g., Walsh, 2003; Bueti and Walsh, 2009). We adapted the approach of Santens and Verguts (2011) by creating separate representations for numerical magnitude and number of digits, the latter implemented as a discrete measure of horizontal extent. Thereby, the network architecture of the present study was motivated by the shared decision account for the interaction of numerical magnitude and continuous physical magnitude as proposed by Santens and Verguts (2011). Hence, we suggest that not only numerical and continuous physical magnitude may interfere at the decision level (as for the size congruity effect), but also numerical magnitude and the discrete number of digits.

Taken together, our simulation study indicates that the number of digits interferes with the comparison of numerical magnitudes (as measured by the string length congruity effect). However, this influence of the number of digits might indicate an influence of physical magnitude (i.e., vertical extension) on the processing of number magnitude. Further studies are needed to disentangle these two possible origins of the string length congruity effect.

ROLE OF ZERO

Interestingly, we observed that decimal fractions with a zero at the tenth position were processed fastest. Importantly, however, we were only able to simulate this finding by increasing the frequency of occurrence of zeros in single-digit number comparison. Yet, this modification did not explain why decimal fractions with ending zeros were compared faster than decimal fractions without zeros. Varma and Karl (2013) suggested that zeros may be privileged in cognitive processing. Our simulation, implementing such a privileged processing of zeros, suggested that this privileged processing affected decimal fractions with zeros at the tenth digit position, but not those with zeros at the hundredth position, which was more or less neglected by the participants in our study and therefore, should not have affected response times. However, an alternative account might be that participants did not process decimal fractions with zeros at the end componentially. Instead, they might have processed the fractional part of the decimal fraction holistically. For instance, when comparing 2.91 and 2.40, participants ignored the unit digits and compared 91 and 40 instead. Specific processing advantages (as indicated be faster response times) for multiples of ten have already been reported before (Brysbaert, 1995; Nuerk et al., 2002). Thus, faster responses for decimal fractions with zeros at the end might not necessarily indicate a privileged role of zeros but a privileged role of multiples of ten, which a model of strictly componential processing cannot account for.

PERSPECTIVES

By assuming that processing of decimal fractions is not different from processing of natural numbers, our computational model also allows for predictions about the processing of natural numbers and decimal fractions within one model framework. In accordance with our findings for decimal fractions, we would expect that the unit-decade compatibility effect is smaller for three-digit numbers with identical hundred digits as compared to the hundred-decade compatibility effect for three-digit numbers.

Necessarily, the natural number conversion hypothesis and the model architecture employed in the present study suggest that the number of digits should influence the comparison of numbers not only when comparing decimal fractions, but also when comparing two natural numbers. However, differing from decimal fractions in natural numbers, the number of digits is always congruent with numerical magnitude. Therefore, different numbers of digits can only facilitate the comparison of two natural numbers, but never interfere with the comparison of two natural numbers. Nevertheless, the computational model architecture predicts that natural numbers containing different numbers of digits should be compared faster than natural numbers with the same number of digits, even if distance and problem size are matched.

Moreover, the computational model, which served as a basis for the extended network presented in the current study, was developed to simulate effects of cognitive control observed in two-digit number processing. It was able to account for the proportion congruity effect found by Macizo and Herrera (2013) and predicted a Gratton effect (Gratton et al., 1992) in two-digit number comparison. Hence, the computational model predicts that also the comparison of decimal fractions should be under cognitive control modulating the relevance of tenth and hundredth digits. Regarding the tenth–hundredth compatibility effect, we would expect it to be more pronounced in a stimulus set with a smaller proportion of incompatible relative to compatible number pairs, as found by Macizo and Herrera (2013) for the case of two-digit numbers. Moreover, in the original computational model (Huber et al., 2013c) cognitive control was implemented to act locally on a trial-by-trial basis and, thereby, it was able to simulate the Gratton effect in two-digit number comparison. Transferred to the case of decimals, the model thus predicts that participants should adapt to different proportions of incompatible trials when comparing decimal fractions on a trial-by-trial basis as well. However, the computational model suggests that not only the relevance of digits should be influenced by processes of cognitive control, but also the inferential influence of the number of digits. This means that the inferential influence of the number of digits should depend on the proportion of string length incongruent to congruent number pairs (in the sense of a proportion congruity effect). In the current computational model, the network for comparing different numbers of digits was added in the same way as the network for the magnitude comparison of digits. Therefore, the same processes of cognitive control, which modulate the relevance of tenth and hundredth digits, should also modulate the influence of the number of digits. In a similar vein as for the relevance of tenth and hundredth digits, the computational model predicts that the influence of the number of digits should be smaller for higher proportions of incongruent trials. More specifically, the string length congruity effect should be larger in a condition with only 25% incongruent trials than in a condition with 75% incongruent trials (see Macizo and Herrera, 2013, for a similar proportion congruity manipulation on the unit-decade compatibility effect). Moreover, as cognitive control was implemented to act locally on a trial-by-trial basis, the computational model also predicts a Gratton effect for the string length congruity. These predictions are a direct result of the network architecture employed to simulate the string length congruity effect and have to be demonstrated in future empirical studies.

Furthermore, we simulated the processing of decimal fractions using a fully componential model. With this architecture, however, the model was not able to account for the observed faster responses for decimal fractions with zero at the rightmost position. We suggested that the fractional part of these decimal fractions might be processed holistically because of its similarity to multiples of ten (e.g., the fractional part of 2.40 is 40). In the study of Moeller et al. (2011), the fully componential model was favored, because it was more parsimonious than the hybrid model assuming that there exit both a componential as well as a holistic representation of two-digit numbers (e.g., Nuerk and Willmes, 2005). However, the present study suggests that at least some two-digit numbers (i.e., whole 10) might be processed holistically favoring a hybrid model of two-digit number processing.

CONCLUSION

The present study aimed at investigating the processing of decimal fractions. Currently, there is a debate on whether decimal fractions are processed like natural numbers (natural number conversion hypothesis) or whether there exist mental representations of decimal fractions, which are distinct from those of natural numbers. The latter suggestion of distinct representations for decimal fractions was supported (i) by the finding of a smaller compatibility effect in decimal fraction than in natural number comparison and (ii) by a semantic interference effect indicating that natural number representations interfere with the comparison of decimal fraction representations. In the present study, we investigated whether these differences indeed indicate that decimal fractions are processed differently from natural numbers. To do so, we provided another account for the semantic interference effect. We proposed that a string length congruity effect evoked by an incongruity between comparison of the magnitude of digits and the physical length could also account for the semantic interference effect. To evaluate this suggestion, we conducted an eye-tracking study and simulated the empirical findings using a computational model. Importantly, in the computational model we did not implement specific decimal fraction representations. Instead, our model was an extension of our fully componential model for two-digit number comparison. To account for the proposed string length congruity effect, we added a network for the comparison of the number of digits. The computational model could account for the smaller compatibility effect in decimal fraction comparison and for the string length congruity effect providing further support for the natural number conversion hypothesis.

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Optimized gamma synchronization enhances functional binding of fronto-parietal cortices in mathematically gifted adolescents during deductive reasoning

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As enhanced fronto-parietal network has been suggested to support reasoning ability of math-gifted adolescents, the main goal of this EEG source analysis is to investigate the temporal binding of the gamma-band (30–60 Hz) synchronization between frontal and parietal cortices in adolescents with exceptional mathematical ability, including the functional connectivity of gamma neurocognitive network, the temporal dynamics of fronto-parietal network (phase-locking durations and network lability in time domain), and the self-organized criticality of synchronizing oscillation. Compared with the average-ability subjects, the math-gifted adolescents show a highly integrated fronto-parietal network due to distant gamma phase-locking oscillations, which is indicated by lower modularity of the global network topology, more "connector bridges" between the frontal and parietal cortices and less "connector hubs" in the sensorimotor cortex. The time domain analysis finds that, while maintaining more stable phase dynamics of the fronto-parietal coupling, the math-gifted adolescents are characterized by more extensive fronto-parietal connection reconfiguration. The results from sample fitting in the power-law model further find that the phase-locking durations in the math-gifted brain abides by a wider interval of the power-law distribution. This phase-lock distribution mechanism could represent a relatively optimized pattern for the functional binding of frontal–parietal network, which underlies stable fronto-parietal connectivity and increases flexibility of timely network reconfiguration.

Keywords: mathematically gifted adolescents, fronto-parietal functional binding, EEG cortical network, gamma phase-locking duration, power-law model

INTRODUCTION

In the fields of education and psychology, exceptional logical reasoning and visual-spatial imagery abilities are regarded as the main characteristics of mathematically gifted adolescents. Numerous neuroscience studies have reached an agreement that the gifted mathematical thinking abilities are supported by a cooperative fronto-parietal network (O'Boyle et al., 2005; Lee et al., 2006; Wartenburger et al., 2009; Prescott et al., 2010; Desco et al., 2011; Hoppe et al., 2012), including the widespread activation of fronto-parietal cortices, the heightened intrahemispheric frontal–parietal connectivity, and the enhanced interhemispheric frontal connectivity between the dorsolateral prefrontal and premotor cortices (Prescott et al., 2010). Some empirical studies have further suggested that the functional facilitation of the fronto-parietal network is driven by the extensively activated posterior parietal cortices (Lee et al., 2006; Desco et al., 2011). Besides, math-gifted adolescents were found having a larger number of fronto-parietal connections within the right hemisphere as compared with the left hemisphere (Prescott et al., 2010). Based on the highly developed right hemisphere and well-developed interhemispheric interaction, math-gifted adolescents can activate a "bilateral" fronto-parietal network during the cognitive processing related to mathematical thinking (Alexander et al., 1996; Sternberg, 2003; O'Boyle et al., 2005; O'Boyle, 2008; Desco et al., 2011). Therefore, the heightened "interplay" of anterior/posterior accompanied with the enhanced interhemispheric frontal connectivity, the extensive parietal activation and the bilateral fronto-parietal network have been suggested as the important neural mechanisms of the math-gifted brain (Singh and O'Boyle, 2004; O'Boyle et al., 2005; Lee et al., 2006; Prescott et al., 2010; Desco et al., 2011).

The parieto-frontal integration theory (P-FIT) on individual differences in reasoning competence emphasizes the crucial process of information communication between association cortices within the parietal and frontal brain regions (Jung and Haier, 2007). Neural oscillations and synchronization represent important mechanisms for interneuronal communication and binding of information among distributed brain regions. Specifically, gamma oscillations (30–60 Hz) are considered as the important building blocks of the electrical activity of the brain and possibly represent a universal code of information communication in the central nervous system (Basar et al., 1999, 2001). Gamma-band modulation in spectral power shows spatial correspondence with the fMRI blood oxygenation level dependent (BOLD) variation in the activated regions of the brain (Niessing et al., 2005; Lachaux et al., 2007). Gamma oscillation is also highly involved in sensation, perception, and cognition, and is correlated with high-order cognition, working memory load, and decision-making, etc. (Karakas et al., 2001; Howard et al., 2003; Fitzgibbon et al., 2004; Herrmann et al., 2010). As low-frequency oscillations coordinate long-range functional connectivity, gamma synchronization oscillation is more spatially restricted and reflects high-density local information processing (Brovelli et al., 2005; Bassett et al., 2006), which has been proposed as a crucial mechanism for the short-lasting functional binding between discrete brain regions (Koenig et al., 2005). Furthermore, the gamma binding-by-synchrony activity among neuronal populations constitutes a transient, large-scale, and task-specific functional neurocognitive network (Basar-Eroglu et al., 1996; Doesburg et al., 2008; Uhlhaas et al., 2011).

On the other hand, the network with dynamic binding not only depends on the transient coupling between neural assembles, but also requires the timely reconfiguration of connections to adapt to external stimuli and inner perturbation. As a representation of functional coupling strength between adjacent or distant brain areas, the synchronization between neuronal assembles is actually operated in a metastable dynamic system (Werner, 2007). For example, EEG phase synchronization (PS) is a mixture of episodic phase-locking durations interrupted by phase-shifts (desynchronization) in spontaneous EEG (Freeman and Rogers, 2002; Chialvo, 2004; Thatcher et al., 2009a). As continuous phase-locks enhance the functional coupling between neuronal populations and lead to the emergence of connections in neuronal networks, phase-shifts mark the beginning of a different set of connections and the occurrence of network reconfiguration (Thatcher, 2012). Moreover, these phase-locking durations have been discovered to conform the rule of power-law distribution, which has been widely accepted as a typical empirical signature of non-equilibrium systems in self-organized critical states (Kitzbichler et al., 2009). The gamma network in particular has been found having the highest global synchronizability in the fractal networks of the brain, suggesting that the gamma synchronizing network is dynamically located at a critical edge in transit to desynchronization. The highly critical state of the gamma network increases its adaptiveness to cater for changing environmental requirements through rapid reconfiguration of connections (Bassett et al., 2006).

Through EEG source analysis of the gamma cortical network, the present study aims to find the giftedness-related capacity of functional binding in the crucial fronto-parietal network of reasoning, by assessing the task-related functional connectivity and adaptive network reconfiguration. The study first compared the basic cortical network topologies constituted by gamma phase-locking oscillations in math-gifted and average-ability adolescents while they were performing a deductive reasoning task. Furthermore, at a neural-mechanistic level of analysis, the study investigated the temporal dynamics of the fronto-parietal network, including the phase-locking intervals/durations (PLI) and the lability of fronto-parietal network reorganization. Then, the parameter fitting of the PLIs in the power-law model was conducted to assess the criticality of phase-locking durations, which could construct an association between the functional connectivity and adaptive reconfiguration of fronto-parietal network. After that, the relationship among the enhanced fronto-parietal connectivity, the extensive reorganization of fronto-parietal connections, and the high criticality of PLIs in the math-gifted brain was analyzed and discussed.

MATERIALS AND METHODS

SUBJECTS

Two groups of subjects were enrolled in this study. The math-gifted group included 11 adolescents (eight males and three females) aged 15–18 years (mean ± SD: 16.3 ± 0.6), who were from the Science and Engineering Experimental Class at Southeast University (Nanjing, China). The class was composed of adolescents who had been recruited through a special college entrance examination aiming at gifted students under 15 years old with exceptional abilities in mathematics and natural sciences. Three criteria were employed to select math-gifted subjects from the class according to the definition of "school giftedness" (Renzulli, 1978; Heller, 1989): (1) nomination: they were recommended by their teacher according to their behavioral performance; (2) academic performance: they should have been awarded prizes in nationwide or provincial mathematical competitions; (3) intelligence score: their scores of Raven Advanced Progressive Matrices (RAPM) test were higher than 32 (mean ± SD: 33.6 ± 0.8). For the control group, 13 subjects were recruited from the Fourth High School of Nanjing, using the following criteria: (1) they were matched with the math-gifted group for age (mean ± SD: 15.9 ± 0.7) and gender (eight males and five females); (2) they had average-level performance in mathematical class tests; (3) their scores of RAPM test were <32 (mean ± SD: 23.5 ± 4.5).

The exclusion criteria adopted included left handedness, medical, neurological or psychiatric illness, and history of brain injury or surgery. To avoid the long-term training effect on the human brain activity, students who had received special training course of Mathematical Olympiad were excluded from this experiment. All the subjects were given informed consent and the study was approved by the Academic Committee of the Research Center for Learning Science, Southeast University, China. The subjects received financial compensation for their participation.

EXPERIMENTAL PARADIGM

As the essential mathematical skill and the standard type of deductive reasoning, a categorical syllogism task of analytic type (verbal–logical way) was adopted in this study. Categorical syllogism is constituted by a major premise, a minor premise, and a conclusion. The actual reasoning process has been considered to emerge during the presentation of the minor premise and remain active until the validation of the conclusion (Fangmeier et al., 2006; Rodriguez-Moreno and Hirsch, 2009). Neuroimaging studies have identified that frontal, parietal, temporal, and occipital complexes are involved in deductive reasoning tasks (Goel et al., 2000; Goel and Dolan, 2001; Knauff et al., 2002; Goel, 2007). Particularly, the activations in the left inferior frontal gyrus, bilateral precentral gyrus of the left fronto-parietal system, and the left basal ganglia have been consistently reported to be specific to categorical syllogism (Prado et al., 2011).

The syllogistic sentences without specific content include three basic items: "S," "M," and "P." "M" is the medium item and is presented in both the major premise and the minor premise. "S" and "M" constitute the major premise, and "M" and "P" the minor premise. From the two premises, the inferred relationship between "S" and "P" forms the conclusion (**Figures 1A,B**).

The experiment adopted a three-block paradigm, included a valid block (32 trials), an invalid block (32 trials), and a baseline block (40 trials). The combinations of syllogistic sentences following the true logical rules constituted a valid block, which employed the logic expressions proposed by Evans et al. (1993). An invalid block was constituted by the invalid combinations of syllogistic sentences, in which there was inconclusive relationship between two premises or incorrect conclusion under clear premises. A baseline block consisted of the trials including the same letter items in each sentence, in which there was no need for subjects to think of the relationship between the items. The letters used in the syllogistic sentences were randomly selected from the 26 letters of the English alphabet. Some samples are shown in **Figure 1B**.

The trials of the three blocks were presented in a random order, which was performed by the E-Prime 2.0 experimental procedure. The stimuli presentations of all the trials took about 30 min. The major premise, minor premise, and conclusion were presented sequentially along the timeline (**Figure 1C**). When the minor premise was shown, subjects were asked to draw a logical conclusion to judge whether the subsequent conclusion was valid or invalid (the ratio of the numbers of valid and invalid trials was 1:1). Subjects put their left index finger on "D" key and right index finger on "K" key at the beginning of a trial. They were asked to respond as accurately as possible by pressing "D" for "invalid" and "K" for "valid" within 3000 ms after the presentation of the conclusion. The time length of a reasoning process is 9000 ms.

Before the formal experiment, a practice session including five trials was conducted by each subject. After that, they decided whether to practice again or enter the following formal procedure. The sentences included Chinese characters and English letters, which were white on black background to avoid visual fatigue.

EEG RECORDING AND PREPROCESSING

The EEG data were recorded using the Neuroscan system at sampling rate 1000 Hz, with 60 scalp electrodes placed according to the international 10–20 system (**Figure 2**). Additionally, bilateral mastoids were used to place the reference electrodes. To monitor ocular movements and eye blinks, electro-oculographic (EOG) signals were simultaneously recorded by four surface electrodes, with one pair placed over the higher and lower left eyelids and the other pair placed 1 cm lateral to the outer corners of the left and right orbits.

By using the Scan 4.3 data preprocessing software, the continuous EEG signals with correct responses were band-pass filtered between 1 and 100 Hz. The epoch of each trial was extracted using a time window of 9500 ms (500 ms pre-stimulus and 9000 ms post-stimulus), and was baseline-corrected according to the pre-stimulus time interval. Ocular artifacts were removed according

A

Rule		
Major premise	Minor premise	Conclusion
S-M	M-P	S-P
S-M	P-M	S-P
M-S	M-P	S-P
M-S	P-M	S-P

Form			
A	E	I	O
All S are P	No S is P	Some S are P	Some S are not P

B

Valid sample	All S are M All M are P Therefore, all S are P
Invalid sample	Some S are M Some M are P Therefore, some S are P
Baseline sample	Some S are S All M are M Therefore, some P are P

C

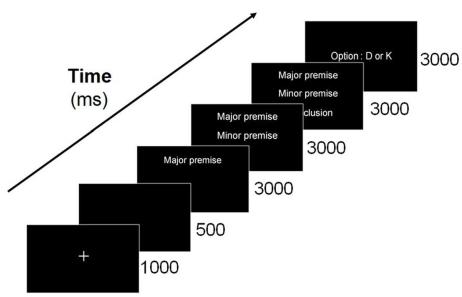

FIGURE 1 | Experiment protocol: **(A)** rules and forms for cross-combination in logical syllogism. The valid forms utilized in this experiment are AAA, AII, EAE, EIO, AEE, EAE, EIO, AOO, AAI EAO, IAI, OAO, which are adapted from Evans et al. (1993). For example, a valid combination of EAE and the first rule is "No S is M; All M are P; Therefore, No S is P"; **(B)** some samples of valid, invalid, and baseline trials; **(C)** timeline of the stimuli.

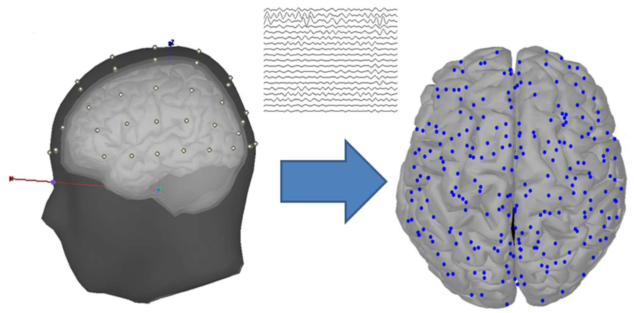

FIGURE 2 | Head model and cortical vertices. The diagram located at the left of the arrow shows the head model with four layers (scalp, outer skull, inner skull, and cortex), and the scalp is placed with 60 EEG data channels. The diagram located at the right of the arrow shows the cortical vertices that are transformed from the EEG channel locations through a source estimation procedure.

to the simultaneously recorded EOG signals. After the artifact rejection with the thresholds ranging from 50 to 75 μV, the blink and electrocardiogram noises were excluded. Finally, 18–22 trials were retained for each math-gifted subject and 15–25 trials were retained for each control subject. In addition, the independent component analysis (ICA) in the EEGLAB Toolbox was used to further clear the visible artifacts, such as the components of possible ocular and muscle movements. Since the emergence of the minor premise in the syllogistic sentence was viewed as the beginning of the actual reasoning process, the time interval 3000–9000 ms (presentation time of the minor premise and conclusion) of the artifact-free EEG signal was selected as the event-related time window. Because of the individual differences in response speed and completion time of each trial, the interval 4000–8000 ms was further extracted as the time window for data analysis.

GAMMA-BAND RESPONSE AND CORTICAL SOURCE ESTIMATION

Gamma-band response

Task-induced response at the gamma frequency of the human brain activity was first assessed in each EEG channel by calculating event-related synchronization/desynchronization (ERS/ERD), which was expressed as the percentage of power increase/decrease relative to the baseline resting state:

$$\text{ERS/ERD}\left(f\right) = \left[A\left(f\right) - R\left(f\right)/R(f)\right] \times 100\% \qquad (1)$$

where f indicates the gamma frequency band, $A(f)$ is the power spectrum density (PSD) of an EEG signal in the task period and $R(f)$ is the PSD in the pre-stimulus interval of the signal. Positive value is ERS and negative value represents ERD.

Cortical current estimation

There is a limitation that the EEG-based brain connectivity analysis was influenced by the volume conduction, which was caused by the variation of the electrical conductivity among the different head layers (Langer et al., 2012; Klados et al., 2013). To avoid this problem, the scalp-recorded EEG signals were transformed into the source space, which was performed by using the source estimation procedure of the Brainstorm Toolbox that is documented and freely available at http://neuroimage.usc.edu/brainstorm (Tadel et al., 2011). In the source estimation, the EEG signals were assumed to be mainly determined by a block of electric dipoles located at the surface of the cortex. Based on an averaged realistic head model that was constituted by four layers, i.e., scalp, outer skull, inner skull, and cortex, the symmetric Boundary Element Method (BEM) in the open-source software (http://www-sop.inria.fr/athena/software/OpenMEEG/) (Gramfort et al., 2010) was applied to the EEG electrode locations to obtain the volume conductor modeling of the subjects, i.e., the forward model matrix. Through an inverse kernel matrix produced by the standardized Low Resolution Brain Electromagnetic Tomography (sLORETA) and the forward model, the raw EEG signals were transformed into the current sources located at the cortical surface. By applying a downsampling procedure to the original sources, 248 cortical vertices were selected to serve as the nodes in the following graph theory analysis (**Figure 2**).

PHASE SYNCHRONIZATION AND UNDIRECTED GRAPH CONSTRUCTION

To quantify the strength of connectivity, the cortical currents were followed by a phase-locking value (PLV) calculation between each pair of the nodes. PLV is a representative method of PS through obtaining a statistical quantification of the frequency-specific synchronization strength between two neuroelectric signals (Lachaux et al., 1999). The phase-locked neuronal groups can communicate effectively, because the communication windows between these neuronal populations for input and output are open at the same time (Fries, 2005). For two signals $x(t)$ and $y(t)$ with instantaneous phases $\phi_x(t)$ and $\phi_y(t)$, PS is the locking of the phases associated to each signal, i.e., $|\phi_x(t) - \phi_y(t)| = \text{const}$. Phase can be obtained through the Hilbert transform (HT), which is used to constitute an analytical signal as $H\left(t\right) = x\left(t\right) + i\tilde{x}\left(t\right)$. Here, $\tilde{x}\left(t\right)$ is the HT of $x(t)$, defined as $\tilde{x}\left(t\right) = \frac{1}{\pi}\text{PV}\int_{-\infty}^{\infty}\frac{x(t')}{t-t'}dt'$, where PV denotes the Cauchy principal value. The phase of the signal $x(t)$ is defined by $\phi_x\left(t\right) = \arctan\ \tilde{x}\left(t\right)/x\left(t\right)$. The PLV bivariate metric for $\phi_x(t)$ and $\phi_y(t)$ is defined as

$$\text{PLV} = \left|\frac{1}{M}\sum_{j=0}^{M-1}\exp\left(i\left(\phi_x\left(j\Delta t\right) - \phi_y\left(j\Delta t\right)\right)\right)\right| \qquad (2)$$

where Δt is the sampling interval and M is the number of sample points of each signal. The range of PLV is within [0,1], where 1 denotes perfect PS and 0 represents absence of synchronization (Sakkalis, 2011).

After calculating the PLV matrix of size 248 × 248 for all the cortical vertices, a fixed connection density was employed to acquire the adjacency matrix. The connection density of the network was set to $p = 21\text{n}\ n/n$ according to the Erdos–Renyi model (Erdos and Renyi, 1961), where n is the number of the nodes. After that, the graph theory was used to quantify the topological properties of the adjacency matrix (Bullmore and Sporns, 2009; Rubinov and Sporns, 2010).

In the following definitions of the graph-theoretical measures based on an adjacency matrix $[a_{i,j}]$, N is the set of all the nodes in a functional brain network (i, j) represents the link between nodes

i and j ($i, j \in N$). If there is a link (i, j) between nodes i and j, then $a_{i,j} = 1$, which denotes a connection status; otherwise, $a_{i,j} = 0$.

Degree of node i is the number of links connected to it:

$$k_i = \sum_{j \in N} a_{ij} \quad (3)$$

Modularity of a network is defined by

$$Q = \sum_{u \in M} \left[e_{uu} - \left(\sum_{v \in M} e_{uv} \right)^2 \right] \quad (4)$$

where M is a set of non-overlapping modules that the network can be fully divided. e_{uv} is mainly determined by the ratio of the number of the links connecting the nodes in module u with the nodes in module v to the total number of the links in the network.

Characteristic path length is defined by

$$L = \frac{1}{n} \sum_{i \in N} L_i = \frac{1}{n} \sum_{i \in N} \frac{\sum_{j \in N, j \neq i} d_{ij}}{n-1} \quad (5)$$

where L_i is the average distance between node i and other nodes, and d_{ij} is the shortest path length between nodes i and j, which is given by $d_{ij} = \sum_{a_{mn} \in g_{i \to j}} a_{mn}$ ($g_{i \to j}$ is the shortest geodesic path between i and j. For all disconnected pairs i, j, $d_{ij} = \infty$).

Node clustering coefficient is quantified by a proportion of the number of existing connections between the nearest neighbors of a node i to the number of maximally possible connections:

$$C_i = \frac{2t_i}{k_i (k_i - 1)}, \ (C_i = 0 \text{ if } k_i < 2), \quad (6)$$

where t_i is the number of triangles around node i, i.e., $t_i = \frac{1}{2} \sum_{j,h \in N} a_{ij} a_{ih} a_{jh}$, k_i is the degree of the node.

Node betweenness centrality is measured according to the proportion of the number of the shortest paths between all the node pairs passing through a specific node to the total number of shortest paths between all the node pairs, which can assess the communication role of the node within the functional network and is defined as follows:

$$b_i = \frac{1}{(n-1)(n-2)} \sum_{\substack{h,j \in N \\ h \neq j, h \neq i, j \neq i}} \frac{\rho_{hj}(i)}{\rho_{hj}} \quad (7)$$

where ρ_{hj} is the number of the shortest paths between nodes h and j, and $\rho_{hj}(i)$ is the number of the shortest paths between nodes h and j that pass through node i. A node with high betweenness centrality is thus crucial to play the role of "connector hub" in the network.

Edge betweenness centrality is calculated based on how many of the shortest paths between all the node pairs in the network pass through a specific edge:

$$B_{ij} = \frac{1}{(n-1)(n-2)} \sum_{\substack{h,k \in N \\ i \neq j, h \neq k \\ h \neq i, h \neq j \\ k \neq i, k \neq j}} \frac{\rho_{hk}(ij)}{\rho_{hk}}, \ (a_{ij} = 1) \quad (8)$$

where ρ_{hk} is the number of the shortest paths between nodes h and k, and $\rho_{hk}(ij)$ is the number of the shortest paths between nodes h and k passing through edge (i, j). An edge with high betweenness centrality represents a "connector bridge" between two parts of a network, the removal of which might affect the communication between many pairs of nodes through the shortest paths between them.

PHASE-LOCKING DURATION AND NETWORK LABILITY DURING DYNAMIC BINDING PROCESS

Since PLV is the temporal statistic of the intermittent phase-locking durations in a specified time interval, the PLIs between frontal and parietal cortical signals were extracted to further quantify the distribution characteristic of the continuous synchronizations. PLI is the period of time when two oscillators maintain the synchronization activity in their phase difference within a limited range, i.e., $\Delta\phi_{xy}(t) = |\phi_x(t) - \phi_y(t)| <$ const. In this paper, PLI is defined as the length of time during which two signals $x(t)$ and $y(t)$ are synchronized by satisfying the condition of $-\frac{\pi}{4} < \Delta\phi_{xy}(t) < \frac{\pi}{4}$ (Kitzbichler et al., 2009). If this condition does not hold true, the phase-locking oscillation is interrupted and enters into the phase-shifting interval.

On the other hand, to measure the coordinated change of functional coupling states of the synchronizing network during reasoning task, the fronto-parietal lability was calculated in the selected nodes ranging from frontal, sensorimotor to parietal cortices. The lability is quantified by the total number of phase-locking pairs of signals in a dynamic network that can change over time. The number of signal pairs that were phase-locked at any time points can be acquired according to the following preset condition of phase difference:

$$N(t) = \sum_{x<y} b\left(\left| \Delta\phi_{xy}(t) < \frac{\pi}{4} \right| \right) \quad (9)$$

where $b\left(\left| \Delta\phi_{xy}(t) \right| < \frac{\pi}{4} \right) = \begin{cases} 1, & \text{if } \left| \Delta\phi_{xy}(t) \right| < \frac{\pi}{4} \\ 0, & \text{otherwise} \end{cases}$

The lability of a synchronizing network is defined as

$$\Delta^2(t, \Delta t) = |N(t + \Delta t) - N(t)|^2 \quad (10)$$

where the time interval Δt was set to 10, 15, 20, and 25 ms respectively, as 10–30 ms had been proposed as the optimal temporal window for information transmission and storage in cortical circuits (Harris et al., 2003). It is clear that larger $\Delta^2(t, \Delta t)$ represents more extensive change in the fronto-parietal network and more flexible adjustment of the functional connections.

For all the trials, the scattergrams were constituted by the samples with the feature distribution of mean fronto-parietal PLI and mean lability of fronto-parietal network in 10, 15, 20, and 25 ms time intervals. Linear discriminant analysis (LDA) (Webb, 2003) with 10-fold cross-validation was employed to further discover the giftedness-related dynamic functional binding pattern.

CRITICALITY ASSESSMENT OF PHASE-LOCKING DURATIONS

To construct an association between PLI and functional reorganization of network, critical dynamics of the fronto-parietal synchronization is assessed by fitting the PLIs in the "power-law" model. The PLI distributed in a critical interval indicates that a "metastable" synchronization is in effect, which implies the synchronizing state would access "neuronal avalanche" and adaptive reorganization by synaptic interaction in the face of endogenous perturbation and external event (Werner, 2007; Beggs, 2008; Kitzbichler et al., 2009; Thatcher et al., 2009a).

Playing the role of functional integration between posterior parietal and frontal cortices in reasoning (Jung and Haier, 2007), the inter-module connections between frontal and parietal cortical areas are crucial for straightforward coupling. Therefore, the phase-locking durations between 30×30 frontal–parietal node pairs were concatenated to constitute the inter-node PLI sample set.

In this study, the parameter fitting method proposed by Clauset et al. was applied to the PLIs set. The method has been proven valid on various datasets from the natural phenomenon with power-law distribution characteristic (Clauset et al., 2009). Let x represents a discrete set of PLI values, a discrete power-law distribution can be described by the following probability density

$$p(x) = P_{\mathrm{r}}(X = x) = Cx^{-\alpha} \tag{11}$$

where X represents the observed PLI value, C is a normalization constant, and α indicates the power-law exponent. It is clear that smaller α indicates a higher probability of long phase-locking duration. In practice, not all the PLI values obey the power-law, and only the values greater than a minimum value $x_{\min}$ can fit in the power-law distribution with less bias. While the data are drawn from a distribution that follows a power-law exactly for $x \geq x_{\min}$, the scaling parameter α can be estimated correctly. In the special case of $x_{\min} = 1$, the maximum likelihood estimator (MLE) used for appropriate estimation of α is given by the solution to the transcendental equation $\frac{\zeta'(\hat{\alpha})}{\zeta(\hat{\alpha})} = -\frac{1}{n}\sum_{i=1}^{n} \ln x_i$, where ζ is the Riemann zeta function. When $x_{\min} > 1$, the zeta function is replaced by the generalized zeta $\frac{\zeta'(\hat{\alpha}, x_{\min})}{\zeta(\hat{\alpha}, x_{\min})} = -\frac{1}{n}\sum_{i=1}^{n} \ln x_i$. For each possible choice of $x_{\min}$, α was estimated by the MLE. The Kolmogorov–Smirnov (KS) goodness-of-fit statistic was calculated according to $D = \max_{x \geq x_{\min}} |S(x) - P(x)|$, where $S(x)$ is the cumulative distribution function of the data for the observation with the value larger than $x_{\min}$, and $P(x)$ is the cumulative distribution function of the best fitting of data to the power-law model in the region $x \geq x_{\min}$. The optimal estimation of $x_{\min}$ is the one that gives the minimum value of D. Root-mean-square error (RMSE) expressed by $R_{\mathrm{e}} = \sqrt{[\sum d_i{}^2/n]}$ is used to assess the goodness-of-fit of the power-law scaling, where d_i is the deviation between the observed value and the estimated one.

ANOVA STATISTICAL TEST

The single-trial analysis results obtained from 215 samples of the math-gifted group and 252 samples of the control group were examined statistically using the one-way analysis of variance (ANOVA) in the Matlab Statistics Toolbox, with group (gifted/control subjects) serving as the between-subjects factor. At the nodal level of the graph-theoretical analysis, clustering coefficient and node betweenness centrality of each cortical vertex were statically tested by the one-way ANOVA. Moreover, edge betweenness centrality was tested as well for 30×30 links connecting frontal–parietal nodes. The Bonferroni Corrections were used in the multiple statistical tests, with significance level set to 0.05. At the global level of the functional network, the ANOVA was conducted on modularity and characteristic path length, respectively. Additionally, the relevant fitting parameters of PLIs in the power-law model from the single-trial analytical results were statistically compared between the two groups. For the behavioral data, the AVOVA tests were used to identify the group difference in task performances in terms of accuracy and response time.

RESULTS

BEHAVIORAL MEASURE OF TASK PERFORMANCE

In the deductive reasoning task, the math-gifted group has outperformed the control group in average response accuracy (mean $\pm$ SD: $75.14 \pm 12.58\%$ in the math-gifted group and $68.20 \pm 15.29\%$ in the control group). Regarding the reaction time of correct response, significant group difference ($p = 0.0036$) has been observed in the task, in which the math-gifted adolescents showed shorter reaction time than the controls (mean $\pm$ SD: 831 ± 536 ms in the math-gifted group and 994 ± 655 ms in the control group).

ENHANCED FUNCTIONAL INTEGRATION IN THE GAMMA CORTICAL NETWORK

The ERS/ERD based brain topological maps show that the gamma-band response induced by the deductive reasoning task is mainly distributed in the prefrontal, frontal, sensorimotor, parietal, and occipital regions. The math-gifted group in particular has higher gamma-band ERS in the central sensorimotor regions as compared with the average-ability subjects (**Figure 3A**). Corresponding to this result, relatively extensive brain regions with small phase difference are discovered in the math-gifted group, as shown in the phase topologies from the averaged values of the subjects in the time window of data analysis (**Figure 3B**).

From the graph-theoretical analysis results of the gamma cortical network, the basic neurocognitive network topologies of the two groups are primarily composed of the prefrontal, frontotemporal, parietal, occipital, and fronto-parietal modules. With the same connection density employed in the two groups, the gamma synchronization network in the math-gifted group shows an expanded fronto-parietal module that integrates more cortical vertices in frontal, parietal, and sensorimotor regions and the relatively shrinking frontotemporal modules, by using the Louvain method for functional community detection (Blondel et al., 2008). In the comparison between the PLV matrices from the two groups, the intensively increased synchronized node pairs in the gamma cortical network of the math-gifted subjects focus on the fronto-parietal cortical regions, accompanied with the node pairs with decreased synchronization in prefrontal, temporal, and occipital regions (**Figure 4**). Moreover, the ANOVA results for testing the between-groups difference in the individual nodes show that the math-gifted adolescents have significantly high clustering coefficients on the nodes in the fronto-parietal module (adjusted $p < 0.05/248$), especially in the sensorimotor area (**Figure 5A**),

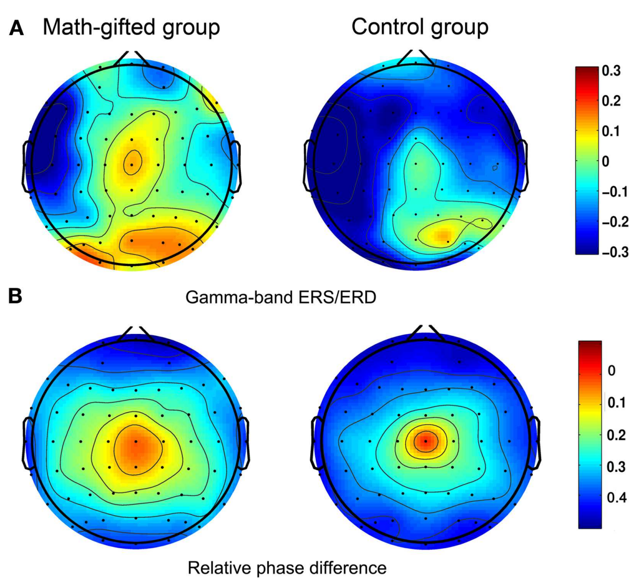

FIGURE 3 | Scalp activities in spectral power and relative phase difference: **(A)** task-induced ERS/ERD at gamma frequency band; **(B)** relative phase topologies, in which the electrode at central location is used as the reference. The left column is from the averaged value of the math-gifted subjects, and the right column is from the control subjects.

which means enhanced local interconnectivity or cliques among the neighbors of the nodes in fronto-parietal cortical area and correlates with higher local efficiency of information transfer and robustness of fronto-parietal network (Bullmore and Sporns, 2009; Power et al., 2010; Kitzbichler et al., 2011).

In the math-gifted brain, the expanded fronto-parietal functional module and enhanced connectivity of the frontal–parietal network are associated with the emergence of more connections between structurally separated frontal and parietal cortical vertices. The ANOVA results indicate that some frontal–parietal links show significantly higher edge betweenness centrality in the cortical network of the math-gifted subjects (adjusted $p < 0.05/900$), suggesting the enhanced role of "connector bridges" of the frontal–parietal connections (**Figure 5B**). The increased direct connections in the fronto-parietal network can make the distant nodes be linked through relatively few intermediate steps, which supports the straightforward information communication and promotes the capacity of parallel information transfer of the fronto-parietal network. Specifically, more fronto-parietal "connector bridges" would decrease the dependence of inter-area information communication on the "connector hubs" and increase the robustness of the gamma network even in the case of the hub lesion. As shown in **Figure 5C**, the cortical vertices with significantly lower node betweenness centrality (adjusted $p < 0.05/248$), i.e., decreased role of "connector hubs," in the math-gifted brain are found being located at the central sensorimotor area, involving some of the cortical vertices in premotor and primary motor regions (**Figure 5C**).

Besides, the ANOVA analysis of the global network further demonstrates that the math-gifted adolescents have significantly lower modularity in the global network topology as compared to the average-ability subjects (**Figure 5D**), which reflects the highly integrated configuration pattern at the level of global topology. However, the longer characteristic path length in the math-gifted group indicates the less economical network configuration, which might be caused by the fixed connection density used in the network analysis that would lead to the disconnected nodes in prefrontal, temporal, and occipital regions (**Figure 5E**) (**Table 1**).

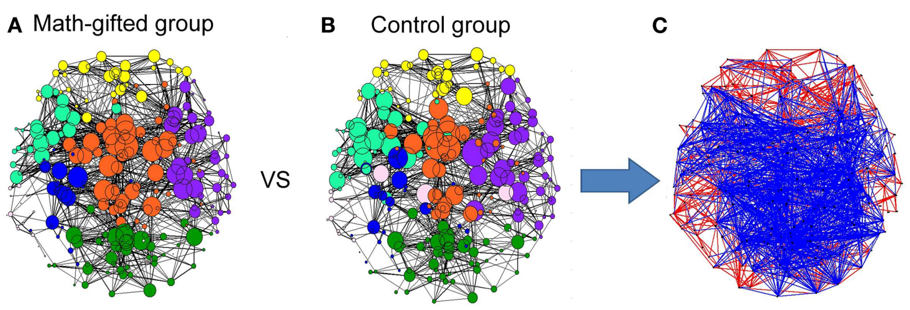

FIGURE 4 | Gamma neurocognitive network topologies and between-groups difference in synchronized node pairs: **(A)** network topology derived from the averaged PLV matrix of the math-gifted subjects; **(B)** network topology derived from the averaged PLV matrix of the control subjects. Each node represents a cortical vertex, and the size of node is proportional to the degree of node. The color of node indicates the membership of topological module, which is segmented by the Louvain method for functional community detection; **(C)** the difference of phase synchronization between the PLV matrices of the math-gifted and control groups. As compared to those of the control group, the blue edges represent the increased synchronizations of the math-gifted group, and the red edges are the decreased synchronizations.

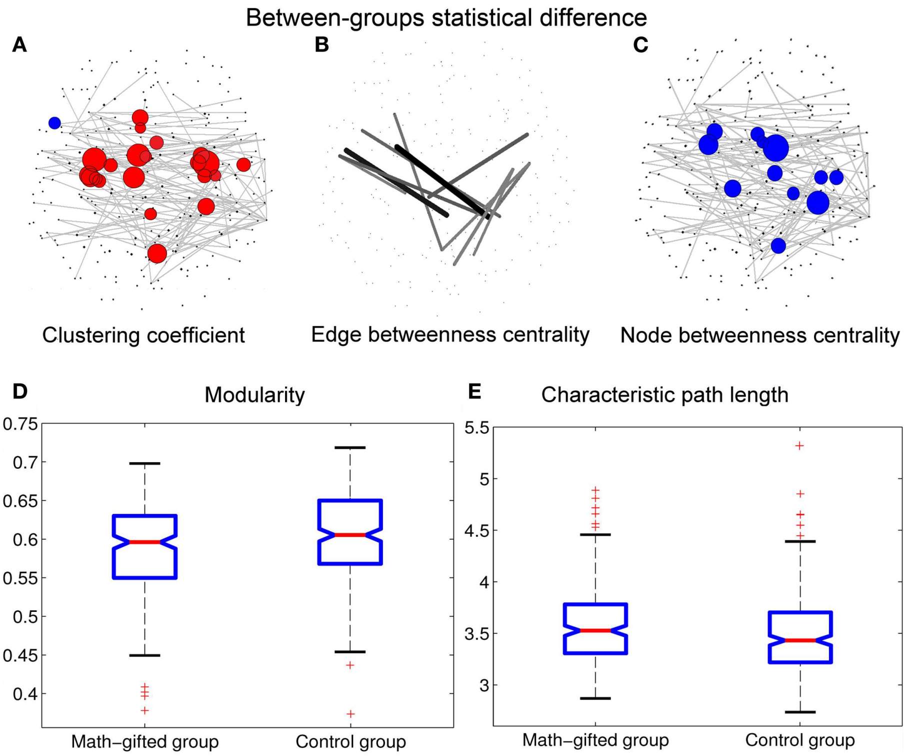

FIGURE 5 | Between-groups statistical differences of topological parameters: when the topological parameters of the math-gifted subjects are compared with those of the control subjects, the statistical differences are shown in (A) the red nodes with significantly higher clustering coefficient and the blue nodes with significantly lower clustering coefficient (adjusted $p < 0.05/248$), (B) the links with significant higher edge betweenness centrality (adjusted $p < 0.05/900$), (C) the cortical vertices with significantly lower node betweenness centrality (adjusted $p < 0.05/248$), (D) the significantly lower modularity of global network ($p < 0.01$), and (E) the significantly longer characteristic path length ($p < 0.05$). The size of node/line corresponds to log *p* value for an ANOVA test with the null hypothesis that between-groups difference is zero.

Table 1 | Between-groups *F*-tests for differences in graph measures of global network topology with fixed connection density: SS, sum of squares; df, degrees of freedom; MS, mean square.

	Source	SS	df	MS	*F*	*P*
Modularity	Group	0.0131	1	0.0131	11.09	$p < 0.01$
	Error	0.5486	465	0.0012		
	Total	0.56169	466			
Characteristic path length	Group	0.0852	1	0.0852	3.91	$p < 0.05$
	Error	10.1389	465	0.0218		
	Total	10.2241	466			

PROLONGED PHASE-LOCKING DURATION AND INCREASED LABILITY OF NETWORK REORGANIZATION

From the result of PLI analysis, the increased inter-module connections of fronto-parietal network can be attributed to stable phase dynamics of synchronization oscillation between distant brain regions (Thatcher et al., 2009a). **Figure 6A** illustrates the episode phase-locks between a pair of frontal–parietal cortical signals and the time-varying process of phase-lock/shift (synchronization/desynchronization) between them. Compared with the average-ability subjects, the longer mean phase-locking duration in the math-gifted adolescents represents a wider range of stable patterns of PS in the time domain, which supports straightforward communication and functional coupling of the frontal–parietal cortical areas (**Figure 8A**).

Although too long phase-locking duration has been surmised to lead to the lack of flexibility of neural activity (Thatcher et al., 2008), **Figure 7** shows a tendency that the prolonged fronto-parietal PLI accompanies with the increase of fronto-parietal network lability. The results of the LDA between the two groups with classification accuracies of 0.8026, 0.7997, 0.7831, and 0.7811, corresponding to different time intervals, indicate that the math-gifted brain could be characterized by longer PLI and higher lability in the fronto-parietal network reorganization, especially for the relatively rapid change in the 10 and 15 ms intervals (**Figures 7A,B**). From the samples of the math-gifted subjects, the long mean PLI helps information processing of network and the extensive adjustment of fronto-parietal connections indicates the widespread connection reorganization to adapt to temporal binding for cognitive event. The phase-lock mechanism in the math-gifted brain represents an optimized synchronization pattern in functional binding of fronto-parietal network, because it simultaneously supports the phase "stability" of functional coupling and the "flexibility" of network connection reorganization.

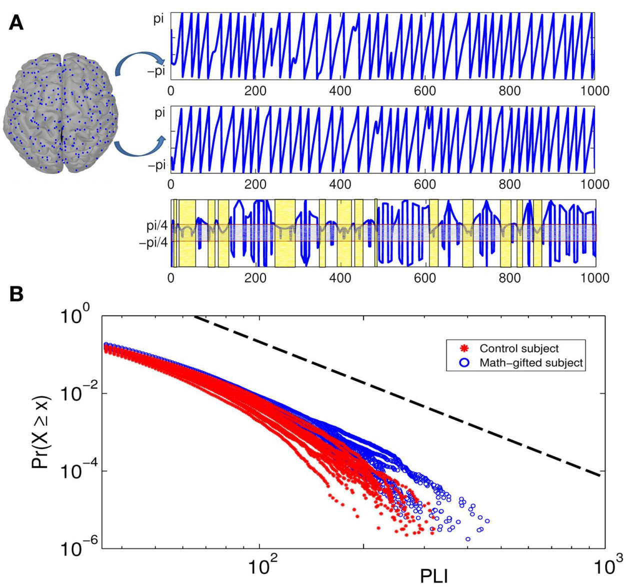

FIGURE 6 | Illustration of phase-locking duration between pairs of cortical signals and power-law distribution diagrams of PLIs at gamma frequency: **(A)** the top of the diagram is the phase signals of the two cortical signals from a frontal–parietal node pair. The bottom of the diagram is the time curve of the phase difference between them. The horizontal axis is time course and the vertical axis represents phase difference between the cortical signals. The light gray box contains the region in which the phase signals are synchronized, and the light yellow boxes represent the PLIs within which the synchronization is unintermittent. **(B)** Cumulative distribution function of PLI (>35 ms) plotted on logarithmic axes. The blue fitting curves are derived from all the math-gifted subjects and the red fitting curves are derived from all the control subjects. The horizontal axis is PLI and the vertical axis is cumulative probability density. The black dotted line represents a power-law rule with exponent $\alpha = 3$.

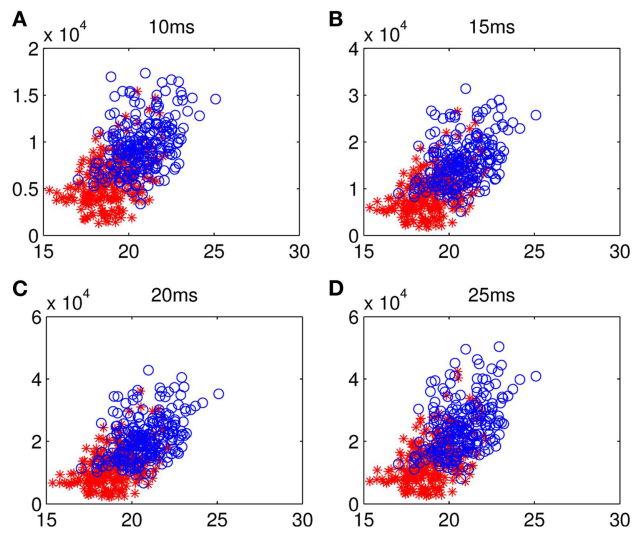

FIGURE 7 | Scattergrams of frontal–parietal PLI and network lability: the blue circles represent the samples from all the math-gifted subjects and the red asterisks are the samples derived from all the control subjects. The horizontal axis is mean value of the frontal–sensorimotor, sensorimotor–parietal, and frontal–parietal PLIs and the vertical axis represents fronto-parietal network lability in **(A)** 10-ms, **(B)** 15-ms, **(C)** 20-ms, and **(D)** 25-ms.

POWER-LAW DISTRIBUTION OF LARGE PHASE-LOCKING DURATIONS

The coordination relationship in functional binding of fronto-parietal network can be explained by the power-law distribution of PLIs. Based on a plenty of PLI samples from the trial concatenation for each subject (the sample size $n > 10^6$) (**Table 2**), **Figure 6B** depicts the cumulative distribution functions $P(x)$ of the PLIs of all the subjects when x is >35 ms. It can be seen that each PLI distribution follows the power-law rule (the standard deviation of the estimated values $R_e < 0.5\%$), which is manifested as an exponential fall-off. It is notable that the obvious difference between the two groups is presented in the distribution tail that represents large but rare synchronization and critical behavior as well (Clauset et al., 2009; Kitzbichler et al., 2009).

The basic parameters of the power-law fitting from the single-trial data provide statistic evidence for the difference between the two groups. Corresponding to the higher maximum PLI values, the math-gifted subjects show wider power-law interval of PLIs distribution, i.e., the critical interval, and lower power-law exponent (**Figures 8A–D**) (**Table 3**). In the expanded critical interval, large synchronization durations (>35 ± 3.2 ms) play an important role in maintaining the inter-module connectivity temporally, although they form a small proportion in the total PLI samples. At the same time, the synchronizations in the critical interval are surmised to be tuned to the critical point of state transition, which could make the fronto-parietal synchronizing state "metastable." Additionally, the lower power-law exponent of the math-gifted brain could be viewed as an indicator of higher intrahemispheric frontal–parietal connectivity, as it is found to be correlated to stronger structural connectivity (Kitzbichler et al., 2009).

Critical synchronization can be compatible with the rapid network reorganization in response to temporary perturbation and stimulus, which promotes the adaptive ability of a functional network in spatial reconfiguration of connections (Bassett et al., 2006; Kitzbichler et al., 2009). The adaptive change imposed on a network is realized through local rewiring rules motivated by the activity-dependent synaptic development (Bornholdt and Röhl, 2003). The rich distant connections in fronto-parietal network of the math-gifted brain provide more available links and selection advantage to operate the local rewiring rule, since the adjustment of these connections has been found to be the most salient gamma network change during the adaptive network reconfiguration (Bassett et al., 2006; Kitzbichler et al., 2011). In the math-gifted brain, the phase-locking durations abiding by wider power-law distribution might account for the optimized synchronization pattern of functional binding through achieving a better balance between prolonged PLI and increased network lability.

DISCUSSION

The paradigms used in the previous studies on math-gifted adolescents or children mostly involved visuospatial imagery tasks that were related to mathematical thinking ability, such as RAPM test and mental rotation. As an essential mathematical skill, a cognitive

Table 2 | Basic parameters of the power-law fitting of individual PLI samples between 30 × 30 node pairs from trial concatenation: *n*, sample size; $\langle x \rangle$, mean value of samples; x_{max}, maximum PLI; $\hat{x}_{min}$, estimated minimum PLI of power-law distribution interval; $\hat{\alpha}$, estimated power-law exponent; $n_{tail} = [\hat{x}_{min}, x_{max}]$; R_e, standard deviation of estimated values.

	n	$\langle x \rangle$	x_{max}	$\hat{x}_{min}$	$\hat{\alpha}$	n_{tail}	R_e $(\times 10^{-2})$
MATH-GIFTED SUBJECT							
01	311242	20	316	36	2.89	280	0.31
02	316268	21	438	37	2.83	401	0.26
03	325099	22	332	37	2.86	295	0.27
04	205294	23	376	33	2.87	343	0.33
05	335295	21	355	37	2.80	318	0.23
06	201462	20	347	35	2.82	312	0.25
07	268759	19	334	33	2.92	301	0.29
08	260084	20	265	36	2.9	229	0.29
09	334978	21	432	35	2.92	397	0.30
10	390596	21	456	37	2.86	419	0.21
11	568511	21	401	36	2.85	365	0.28
Mean value	319781	21	368	37	2.86	333	0.27
CONTROL SUBJECTS							
01	318080	18	258	31	2.90	227	0.32
02	322137	19	296	35	2.94	261	0.30
03	252469	18	290	31	2.95	259	0.32
04	313898	17	261	31	2.90	230	0.36
05	307197	20	292	36	2.85	256	0.28
06	321789	19	355	34	2.92	321	0.27
07	312976	21	319	36	2.81	283	0.25
08	383190	19	289	34	2.92	255	0.32
09	297221	17	221	29	2.93	192	0.41
10	401429	20	316	34	2.88	282	0.33
11	396750	19	281	33	2.93	248	0.32
12	362526	19	249	34	2.94	215	0.26
13	275928	19	331	33	2.91	298	0.31
Mean value	328122	19	289	33	2.9	255	0.31

task of the analytic type (verbal–logical way) was designed in this study for determining whether the previous research results were specific to the mathematical thinking or just the general attributions of problem solving. The logical syllogism used in this experiment is viewed as a basic form of mathematically logical thinking and fills the void of the experimental paradigm in neuroscience studies of mathematical giftedness.

To the best of our knowledge, this is the first time that the individual difference between math-gifted and average-level abilities is investigated through EEG dynamic network analysis. With the highest criticality in the fractal networks of the human brain, the cortical network at the classic gamma frequency is assessed by transforming the scalp-recorded EEG signals into the cortical dipoles. According to the results obtained from the graph-theoretical analysis, the math-gifted adolescents demonstrate a highly integrated fronto-parietal network that is supported by the prolonged gamma binding-by-synchrony activity among discrete neuronal assembles, which is in line with the results of the previous fMRI studies and the P-FIT model of reasoning. Furthermore, as the prolonged periods of phase-locking are more likely to occur between the processes within the same functional module (Kitzbichler et al., 2009), the fronto-parietal PLIs in the math-gifted brain might be the consequence of strong structural connectivity of fronto-parietal network. On the other hand, the math-gifted subjects recruited in our experiment might have more practice with this kind of reasoning task by virtue of their exposure to more education. The mental training-related effect might lead to the changes of neuroelectric activities in phase-locking. That is, perhaps the performances of the math-gifted adolescents in gamma synchronization are not solely due to greater innate ability.

Functional connectivity of the phase coherent network is positively related to the phase-locking duration and stability of phase dynamics. In the context of temporally stable fronto-parietal connectivity in the math-gifted brain, the theory of critical dynamics is applied to the realistic data from the high-order cognitive task through the analysis of single-trial samples, which constructs an association between the enhanced functional connectivity and the highly adaptive reconfiguration of the fronto-parietal network in the math-gifted brain. From the perspective of criticality, the existence of power-law distribution of PLIs in the brain puts the large synchronization on a "metastable island"; that is, the longer the PLI is, the higher the desynchronization possibility will be (Werner,

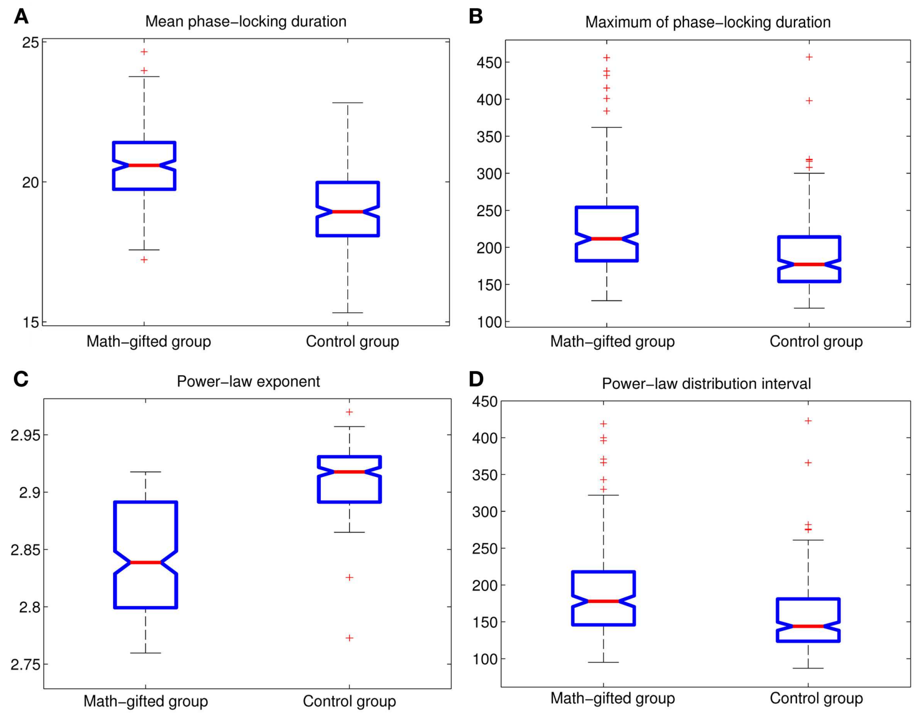

FIGURE 8 | Between-groups AVOVA tests for basic power-law fitting parameters of PLIs from signal-trial data ($p < 0.01$): (A) mean PLI; (B) maximum of PLI; (C) power-law exponent; (D) power-law distribution interval of PLI.

Table 3 | Between-groups *F*-tests for differences in power-law fitting parameters of PLIs: SS, sum of squares; df, degrees of freedom; MS, mean square.

	Source	SS	df	MS	*F*	*P*
Mean phase-locking duration	Group	261.8558	1	261.8558	142.1430	$p < 0.01$
	Error	856.6231	465	1.8422		
	Total	1.1185e + 003	466			
Maximum of phase-locking duration	Group	1.3654e + 005	1	1.3654e + 005	41.4677	$p < 0.01$
	Error	1.5311e + 006	465	3.2927e + 003		
	Total	1.6677e + 006	466			
Power-law exponent	Group	0.1901	1	0.1901	131.0518	$p < 0.01$
	Error	0.6746	465	0.0015		
	Total	0.8647	466			
Power-law distribution interval	Group	1.1235e + 005	1	1.1235e + 005	35.5266	$p < 0.01$
	Error	1.4705e + 006	465	3.1624e + 003		
	Total	1.5829e + 006	466			

2007). The large-sample EEG study conducted in 378 children and adolescents (Thatcher et al., 2008) has suggested that, the "optimal" balance between phase-locking duration and phase-shifting duration benefits the effective allocation of neuronal resources, and is related to high intelligence level that has been consistently considered as a basic factor of mathematical giftedness. The cortical network study in this paper supports the opinion that the math-gifted adolescents can use the well-allocated phase-lock resources to facilitate the functional binding in the fronto-parietal cortices, since the temporal binding between neuronal assembles depends on the transient coupling and adapts to the timely connection redistribution of network. Empirical studies have demonstrated that the significant gamma network reorganization is affected by the motor task, working memory task, cognitive effort, etc. (Bassett et al., 2006; Kitzbichler et al., 2011). In the math-gifted brain, the optimized phase-lock pattern in functional binding would make the synchronizing network flexibly compatible to varying cognitive requirement of the reasoning process. Except the neural correlates of mathematical giftedness, there is evidence that phase-locking and phase-shift durations in EEG low-frequency intervals are significantly different in people with Autism Spectrum Disorder (ASD), with longer periods of phase-lock and fewer phase-shifts (Thatcher et al., 2009b). In addition, the individuals with ASD also have been found showing the abnormal functional connectivity between some regions in default model network (Assaf et al., 2010). As there are frequent reports of the relevance between people with ASD and high mathematical ability, the phase-locking mechanisms in the both populations might follow the similar distribution rule. Perhaps in another aspect of phase-locking duration and network reconfiguration, too long period would also lead to the decreased flexibility of adaptive network reconfiguration, because of the reduced resources available to be operated by the phase-shift mechanism (Thatcher et al., 2008). Due to the difference in network wiring, the locally over-connected functional network in the brain might be related to the deficits seen in ASD.

The optimized synchronization pattern of the fronto-parietal network also plays a key role in information processing. The prolonged fronto-parietal phase-locking durations distributed in a wider critical interval indicate that some optimizations of information processing would occur simultaneously. Firstly, the generally prolonged phase-locking durations enhance the global synchronization of the gamma network through a widespread stability of phase dynamics, which could increase the capacity of information storage of the network. Secondly, the phase-locking duration at a critical state supports effective information communication between neuronal assembles because the long synchronization leads to efficient information transmission. Finally, when the synchronizing activity is maintained at a critical state, it would decrease the stability of the connection but increase the adaptiveness of the network for timely reorganization of connections. In conclusion, the optimizations of the fronto-parietal synchronization enhance the information processing of the math-gifted brain during the deductive reasoning task, and further support the exceptional logical thinking ability of math-gifted adolescents.

AUTHOR CONTRIBUTIONS

John Q. Gan and Haixian Wang designed the research and provided analytic tools; Li Zhang conducted experiment, analyzed data, and wrote the paper; John Q. Gan and Haixian Wang improved the paper.

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Development of abstract mathematical reasoning: the case of algebra

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Algebra typically represents the students' first encounter with abstract mathematical reasoning and it therefore causes significant difficulties for students who still reason concretely. The aim of the present study was to investigate the developmental trajectory of the students' ability to solve simple algebraic equations. 311 participants between the ages of 13 and 17 were given a computerized test of equation rearrangement. Equations consisted of an unknown and two other elements (numbers or letters), and the operations of multiplication/division. The obtained results showed that younger participants are less accurate and slower in solving equations with letters (symbols) than those with numbers. This difference disappeared for older participants (16–17 years), suggesting that they had reached an abstract reasoning level, at least for this simple task. A corresponding conclusion arises from the analysis of their strategies which suggests that younger participants mostly used concrete strategies such as inserting numbers, while older participants typically used more abstract, rule-based strategies. These results indicate that the development of algebraic thinking is a process which unfolds over a long period of time. In agreement with previous research, we can conclude that, on average, children at the age of 15–16 transition from using concrete to abstract strategies while solving the algebra problems addressed within the present study. A better understanding of the timing and speed of students' transition from concrete arithmetic reasoning to abstract algebraic reasoning might help in designing better curricula and teaching materials that would ease that transition.

Keywords: mathematics, education, algebra, problem solving, cognitive development, abstract reasoning, concrete reasoning, strategy

INTRODUCTION

United States National Council of Teachers of Mathematics defines algebra as "a way of thinking and a set of concepts and skills that enable students to generalize, model, and analyze mathematical situations" (National Council of Teachers of Mathematics [NCTM], 2008). This field includes a wide array of topics ranging from elementary linear equation solving to more abstract topics such as modeling given contextual information by formulating complex algebraic expressions. Algebra is usually the first domain in school mathematics that encourages students' abstract reasoning. By making a transition from concrete arithmetic to the symbolic language of algebra, students develop abstract mathematical cognition essential for their further advancement in mathematics and science. Given that understanding fundamental algebra concepts and acquiring the necessary skills for solving algebra problems requires a certain degree of prior knowledge and abstract thinking, algebra is typically introduced in schools after the development of arithmetic reasoning, as its generalization, usually around the age of 12. This is also roughly the age at which, according to Piaget's theory of cognitive development, that had a far-reaching influence on both theory and practice in education, a qualitative change in children's cognitive development occurs (Piaget, 1976). Specifically, this is the age at which most children transition from the concrete operational stage to the formal operational stage (Inhelder and Piaget, 1958; Piaget, 1972). At this time children advance from logical reasoning with concrete to abstract examples, and become able to consider only logical relationships between different elements while ignoring their concrete content. Therefore, this transition from concrete to formal operational stage represents the basis for their further educational advancement. However, many studies have shown that formal reasoning is not developed in most adolescents of that age (Lawson, 1985). Consequently, numerous abstract concepts in mathematics and science curricula are too demanding for the majority of students that remain concrete operational thinkers (Lawson and Renner, 1975). Therefore, it was suggested that teaching abstract concepts should be delayed until the brain maturation permits a transition to the stage of formal operation. Specifically, in the last two decades, brain imaging studies provided new evidence that adolescence represents a period of continued neural development (Blakemore, 2012) that may last longer than would be suggested by Piaget's theory. In particular, maturational changes in some brain regions that are involved in abstract mathematical reasoning, such as the prefrontal cortex, may last until late adolescence (Giedd and Rapoport, 2010). Educational studies confirm that some tests of prefrontal lobe activity highly correlate with scientific reasoning ability and the capacity to reject scientific misconceptions and adopt correct ideas (Kwon and Lawson, 2000). It seems that children can hardly acquire some abstract reasoning skills until certain age.

In line with arguments suggesting that understanding algebra concepts may be difficult for children in primary schools, research has shown that students indeed often face difficulties in moving from the arithmetic to the algebraic form of reasoning (Kieran, 2004). Despite these findings, many researchers argue for an earlier introduction of algebra in mathematics curriculum (e.g., Carraher et al., 2006; Warren et al., 2006). According to these suggestions, developing algebraic skills and exposing students to more demanding abstract tasks would help in enhancing their abstract reasoning, thus facilitating the transition between cognitive phases. This could be done in a gradual fashion, which is in line with modern mathematics curricula that gradually introduce elements of algebraic thinking in the early grades before formally introducing algebra in the later grades (National Council of Teachers of Mathematics [NCTM], 2000). As an example, since the implementation of a National Curriculum in England, algebra is taught earlier compared to the teaching practice 30 years ago. However, this change of practice has not been overly beneficial, as a recent large-scale survey has shown that the present performance in algebra is broadly comparable to that of students 30 years ago (Hodgen et al., 2010). It seems that the early start of algebra teaching gives an initial advantage to students, which appears not sustained at a later age. Overall, despite many efforts to address students' difficulties with formal mathematical reasoning, it seems that little advancement has been made (Hodgen et al., 2010).

A more overarching evaluation of students' success and difficulties in acquiring fundamental algebra concepts is introduced by large international surveys, such as PISA (Program for International Student Assessment) and TIMSS (Trends in International Mathematics and Science Study) that give insights into the quality and efficiency of school systems across many countries. The findings of PISA testing conducted in 2012 with a particular focus on mathematics indicate that students in the highest-performing countries are "more frequently exposed to formal mathematics than students in most of the other PISA-participating countries and economies" (Organisation for Economic Co-operation and Development [OECD], 2013, p. 148). Furthermore, data suggest that the "exposure to more advanced mathematics content, such as algebra and geometry, appears to be related to high performance on the PISA mathematics assessment, even if the causal nature of this relationship cannot be established" (Organisation for Economic Co-operation and Development [OECD], 2013, p. 148). These results indicate a crucial role of algebra in the development of abstract mathematical reasoning.

However, when discussing the acquisition of basic algebra concepts, it is important to highlight that these represent a broad part of school mathematics. As was mentioned earlier, at its fundamental level, algebra includes solving simple algebraic equations that were the focus of the present study. These equations were chosen because equation rearrangement represents a very important skill required for problem solving in many school subjects. Within different teaching frameworks, it is often assumed that, once students learn to solve a simple equations such as, e.g., they can solve such equations for any unknown. This would mean that they are able to solve equivalent simple equations containing both numbers, letters or other symbols. However, physics and chemistry teachers know that students struggle with equations rearrangements, especially for "all-symbol" equations. Küchemann (1981) reported that the majority of the students up to the age of 15 fail to interpret algebraic letters (symbols) as unknowns or generalized numbers, which would be expected from formal operational thinkers. Instead, they still use concrete operational strategies in solving such equations, e.g., ignoring the letters or replacing them with numerical values. This inequivalent treatment of otherwise comparable equations represents only one example of students' inability to apply the learned principle of equation solving on different instantiations of the same equation format. Given such unequivalences, different mathematics education researchers classify equations in different manners. For example, Usiskin (1988) classifies "equations with letters" used in school algebra as a formula ($A = LW$), an equation to solve, ($5x = 40$), an identity ($\sin x = \cos x \tan x$), a property [$1 = n\,(l/n)$], or a function ($y = kx$). Within this, as well as other classifications, it is important to highlight that different types of equations have a different feel not only for students, but also for mathematicians depending on different uses of the idea of a variable (Chazan and Yerushalmy, 2003).

Motivated by these differences, as well as the practical relevance of this topic, the present study was aimed at investigating the developmental trajectory of students' ability to solve simple algebraic equations. Based on the Usiskin's (1988) classification, only formulas and equations to solve were chosen, i.e., we used the equivalent 3-terms equations with numbers or with letters. The participants in the study included primary and secondary school students who were all taught equation rearrangement in mathematics at least one year prior to the testing. In addition, they used formulas in other school subjects such as physics and chemistry. However, we hypothesized that, despite repeated exposure and practice with simple algebraic equations, some students of all grades would still struggle with their rearrangement, especially if equations contained only symbols (letters). Furthermore, we were interested in students' strategies in "all-symbol" equation solving. From our experience and previous studies (Susac et al., 2014, under revision), we assumed that many students use very concrete strategies, such as inserting numbers because it takes time for them to adopt the formal algebraic way of thinking. Consequently, in the present study we explored the age at which the transition from concrete-number-based reasoning to more abstract algebraic reasoning really occurs.

MATERIALS AND METHODS

PARTICIPANTS

The participants in the present study included 331 students from five primary and four secondary state schools in Zagreb. With respect to primary school students, all state primary schools in Croatia have the same curriculum, so their students have comparable experiences with algebra education. With respect to secondary schools, we tested students from two gymnasiums (general education and foreign language type schools) and two technical secondary schools. These schools were chosen to represent the average secondary school population in Zagreb mostly preparing for university studies. Specifically, graduates from the two gymnasiums included in the present study typically continue their education at university, typically studying non-mathematics or science related majors. In comparison, graduates from the tested technical schools often continue their education majoring in technical fields. Students from gymnasiums that specialize in natural sciences and mathematics were not included in this study.

The participants in the present study included students from the seventh grade of primary school (age 13–14 years) to the second grade of secondary school (age 16–17 years). Hence, our sample included the students of four age groups, i.e., different school grades: the 7th and 8th grade of primary, and the 1st and 2nd grade of secondary school. Given that in Croatian schools, equation rearrangement is taught at the end of the sixth grade of primary school roughly corresponding to the students' age of 12–13, all our participants were taught how to solve the task used in the study at least one year prior to this measurement. The number of tested female and male students in each grade is shown in **Table 1**.

The study was approved by the Ethics Committee of the Ministry of Science, Education and Sports, as well as by the schools' headmasters. Each student's parents gave an informed written consent before the child took part in the experiment.

MATERIALS

Raven's Progressive Matrices were used to assess general cognitive ability (Raven, 1941, 1999). The d2 Test of Attention (Brickenkamp, 1962, 1999) was also administered, but the data were not analyzed in the present study.

A computerized test of equation rearrangement was prepared using E-Prime (Psychology Software Tools Inc., Pittsburgh, PA, USA). In each trial, simple equations consisting of three elements (numbers or letters) were presented in the centre of the visual field. The presented numbers and letters were black, displayed in 24 pt size Ariel font on the white background. Participants' task was to make x the subject of the equation. Simultaneously with the equation, a potentially correct or incorrect answer was presented below the equation. The participants were asked to decide if the offered answer was correct or incorrect.

Three types of equations were used in the study:

A equations: $x \cdot a = b$,

B equations: $\frac{x}{a} = b$,

C equations: $\frac{a}{x} = b$.

The offered answers were of the following types: $x \cdot a = b$, $\frac{x}{a} = b$, and $\frac{a}{x} = b$. Within all presented equations, a and b stand for different letters and numbers which all appeared with the same probability during the experiment.

Table 1 | Number of students according to their age and gender.

	7th grade	8th grade	1st grade	2nd grade
Male	36	41	62	54
Female	36	39	33	30
Total	72	80	95	84

PROCEDURE

The participants were tested during two school periods (45 min long). During one school period, Raven's Progressive Matrices and d2 Test of Attention were administered to students in their classrooms. On the same or on another day, students solved the computerized test of equation rearrangement and completed a post-measurement questionnaire in the computer lab.

Before administering the equation rearrangement test, participants were familiarized with the task. They were instructed to respond as quickly as possible by pressing one of the two mouse buttons with their index and middle fingers, corresponding to correct and incorrect answers, respectively. Prior to experimentation, the participants performed a training block consisting of 6 equations equivalent to those used in subsequent experimental trials.

During both practice and experimental trials, each equation was presented until the participant responded, up to a maximum of 30 s. If the participant did not respond within 30 s, the equation disappeared from the screen and another 30 s were available to give an answer. However, these late responses ($<0.1\%$ of all trials) were not included in the analysis. After each response, the next equation was presented after a delay of 1 s. Reaction times (RTs) were measured automatically by the computer from the stimulus onset to the participant's response. No feedback was given to the participants.

During the experiment, the participants were presented with the three previously described types of equations, which were randomized across four blocks. Each block consisted of 15 equations of each equation type, amounting to an overall of 45 presented equations per block. Two blocks contained equations with numbers, while the other two blocks contained equations with letters (symbols). Equations in the first and third blocks contained numbers while those in the second and fourth blocks consisted of letters. The participants could take a break between blocks if needed.

After having finished the computerized test, the participants completed a questionnaire designed for assessing their strategies during equation solving. While responding to these questionnaires, the participants described how they solved each equation type and ranked them by difficulty. In addition, they indicated whether their response depended on the type of the offered answers, and whether they changed their problem solving strategies during the time course of the experiment.

DATA ANALYSIS

For each participant and each condition, reaction time and accuracy were evaluated. Only correct responses were included in the analysis of RTs. Inverse efficiency was also calculated as the ratio of reaction time and accuracy (Townsend and Ashby, 1978). Lower values on this measure indicate higher efficiency on a particular task. Inverse efficiency is used to account for the speed–accuracy tradeoffs, and we used it as a measure of task difficulty.

To determine the effects of age, gender, level of abstraction, repetitions and equation type, a two-way repeated measures analysis of variance (ANOVA) on accuracy and RTs was conducted. Repeated-measures *post hoc* tests using Bonferroni adjustment were used to further assess the differences between different conditions. In addition, a partial correlation coefficient was calculated in order to determine the relation between participants' cognitive abilities and their efficacy in equation rearrangement. A threshold of $p < 0.05$ was used for determining the level of effect significance.

To evaluate participants' strategies in equation solving, we analyzed their answers in the administered *post hoc* questionnaire using the general inductive approach (Thomas, 2003) and descriptive statistical procedures. Each participant's description of how he/she solved each type of equation was categorized. Hence, different categories reflect different student equation solving strategies, some of which were correct, and some incorrect. Some participants used more than one strategy, and were accordingly assigned to two or more categories. To simplify the comparison of used strategies across participants' age, all strategies were divided into concrete and rule-based (more abstract) groups. Each participant was assigned to concrete, rule-based or mixed (concrete and rule-based) group. We also evaluated students' views on equation type difficulty from their ranks provided in the questionnaire.

RESULTS

EFFICACY OF EQUATION SOLVING

Age and gender effects

Two-way ANOVAs with factors Age (7th vs. 8th vs. 1st vs. 2nd grade) and Gender (Male vs. Female) were conducted to compare the mean accuracy and RTs. The obtained results for accuracy indicated a statistically significant main effect of Age [$F(3,323) = 9.43$, $p < 0.001$, $\eta_p^2 = 0.081$] and Gender [$F(1,323) = 6.40$, $p < 0.05$, $\eta_p^2 = 0.019$], while the interaction effect was not significant [$F(3,323) = 1.45$, $p > 0.05$, $\eta_p^2 = 0.013$]. **Figure 1A** shows that accuracy increased with the age of participants. On average, girls were more accurate than boys, and the participants in the 7th grade of primary school were less accurate than those in the 1st and 2nd grade of secondary school, while those in the 8th grade were less accurate than the students in the 2nd grade of secondary school.

A corresponding comparison for RTs revealed a statistically significant main effect of Age [$F(3,322) = 12.91$, $p < 0.001$, $\eta_p^2 = 0.107$] and the interaction effect [$F(3,322) = 4.14$, $p < 0.01$, $\eta_p^2 = 0.037$]. The main effect of Gender was not statistically significant for RTs [$F(1,322) = 0.09$, $p > 0.05$, $\eta_p^2 < 0.0001$]. Average RTs decreased with the age of participants (**Figure 1B**). Boys were faster in equation solving in the first grade of secondary school, whereas girls were faster in the second grade.

Age and abstraction level effects

To test the differences between participants' accuracy and RTs in solving equations with numbers and letters across different age, we used the two-way mixed-design ANOVAs with between-subjects factor Age (7th vs. 8th vs. 1st vs. 2nd grade) and within-subjects factor Abstraction level (numbers vs. letters). With respect to accuracy, the statistically significant main effects of Age [$F(3,327) = 8.37$, $p < 0.001$, $\eta_p^2 = 0.071$] and Abstraction level [$F(1,327) = 47.17$, $p < 0.001$, $\eta_p^2 = 0.126$], as well as the interaction effect [$F(3,327) = 4.89$, $p < 0.01$, $\eta_p^2 = 0.043$], were found. Participants were more accurate on equations with numbers, but only in primary school and in the 1st grade of secondary school (**Figure 2A**). In the 2nd grade of secondary school there was no statistically significant difference in the accuracy of solving equations with numbers and letters.

For the RTs, results revealed a statistically significant main effect of both factors, Age [$F(3,326) = 11.68$, $p < 0.001$, $\eta_p^2 = 0.097$] and Abstraction level [$F(1,326) = 4.45$, $p < 0.05$, $\eta_p^2 = 0.013$], while the interaction was not significant [$F(3,326) = 0.61$, $p > 0.05$, $\eta_p^2 = 0.006$]. RTs deceased with age, the participants in the 2nd grade of secondary school were the fastest, and the students in the 1st grade of secondary school were faster than those in the 8th grade. Similar pattern is present in the RTs data as it is in the accuracy data; differences between equations with numbers and letters decreased with the participants' age (**Figure 2B**).

Age and equation type effects

We have used two-way mixed-design ANOVAs with between-subjects factor Age (7th vs. 8th vs. 1st vs. 2nd grade) and within-subjects factor Equation type (A vs. B vs. C equation) to test the differences between participants' accuracy and RTs for different types of equations across different age. For the accuracy, a significant main effects of both Age [$F(3,327) = 8.37$, $p < 0.001$, $\eta_p^2 = 0.071$] and Equation type [$F(2,654) = 66.59$, $p < 0.001$, $\eta_p^2 = 0.169$] were found, as well as their interaction [$F(6,654) = 2.53$, $p < 0.05$, $\eta_p^2 = 0.023$]. All participants were less accurate on the C equations compared to both the A and B equations, while participants in the 1st grade of secondary school were less accurate on B when compared to A equations (**Figure 3A**).

Corresponding results for the RTs again revealed significant main effects of both Age [$F(3,326) = 11.68$, $p < 0.001$, $\eta_p^2 = 0.097$] and Equation type [$F(2,652) = 41.59$, $p < 0.001$, $\eta_p^2 = 0.113$], as well as their interaction [$F(6,652) = 3.56$, $p < 0.01$, $\eta_p^2 = 0.032$]. Primary school participants solved the A equations faster than the B and C equations, while the secondary school participants were the slowest in solving the C equations. (**Figure 3B**).

Age and repetition effects

Two-way mixed-design ANOVAs with between-subjects factor Age (7th vs. 8th vs. 1st vs. 2nd grade) and within-subjects factor Block (first vs. second block) were used for testing the differences between participants' accuracy and RTs across time course of the experiment. The results showed a statistically significant main effect of both factors, Age [$F(3,327) = 8.37$, $p < 0.001$, $\eta_p^2 = 0.071$] and Block [$F(1,327) = 5.11$, $p < 0.05$, $\eta_p^2 = 0.015$], while the interaction was not significant [$F(3,327) = 1.20$, $p > 0.05$, $\eta_p^2 = 0.011$]. **Figure 4A** illustrates a trend of accuracy increase from the 7th grade of primary school until the 1st grade of secondary school, while pairwise comparisons revealed a statistically significant difference between the accuracy levels of participants in the 7th grade when compared to those in the secondary school, and participants in the 8th grade when compared to participants in the 2nd grade of secondary school.

For the RTs, results indicated corresponding significant main effects of both factors, Age [$F(3,326) = 11.88$, $p < 0.001$,

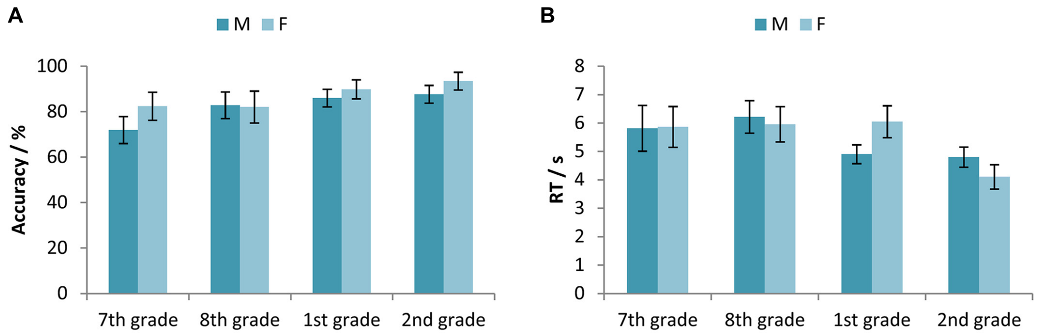

FIGURE 1 | (A) Accuracy (percentage of correct responses) and **(B)** RTs for the participants in the 7th and 8th grade of primary school, and the 1st and 2nd grade of secondary school, separated for male and female participants. Error bars represent 95% confidence intervals.

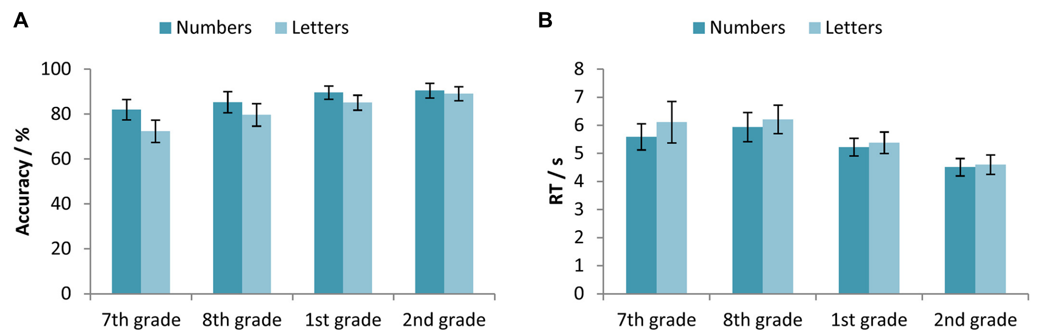

FIGURE 2 | (A) Accuracy (percentage of correct responses) and **(B)** RTs for the participants in the 7th and 8th grade of primary school, and the 1st and 2nd grade of secondary school, separated for the equations with numbers and equations with letters. Error bars represent 95% confidence intervals.

$\eta_p^2 = 0.099$] and Block [$F(1,326) = 312.27$, $p < 0.001$, $\eta_p^2 = 0.489$], while the interaction was not significant [$F(3,326) = 2.58$, $p > 0.05$, $\eta_p^2 = 0.023$]. Participants of all ages became faster in equation solving in the second block (**Figure 4B**).

Equation solving and cognitive abilities

The relation between the students' equation solving efficacy and their cognitive abilities was addressed by calculating the partial correlation coefficient between participants' inverse efficacy and their scores on Raven's Progressive Matrices score, while controlling for the age effects. The obtained results indicate a statistically significant correlation between equation solving efficacy and cognitive abilities [$r(307) = -0.22$ [95% CI: $-0.32, -0.11$], $p < 0.001$], indicating that the participants with higher cognitive abilities were generally more efficient in equation solving.

STRATEGIES USED FOR EQUATION SOLVING

Evaluation of participants' answers in the questionnaires confirmed that they used different strategies for solving equations with letters. We categorized their answers and divided them into two groups – concrete strategies and rule-based strategies. The most frequently used concrete strategy (37% of all participants) was inserting numbers instead of letters. 11% of participants used a "triangle" memory technique and 4% used a "biggest on the top" strategy that is based on a belief that products and numerators are "big." For the equation $a/x = b$, one participant wrote an explanation: "We got b by dividing a by x. Thus, b is smallest and a is biggest. Then we get x by dividing a by b."

The most common rule-based strategy (38% of all participants) was a standard application of multiplication/division operations on the equation. 11% of participants reported correctly moving letters to the other side of the equation and often indicated the operation with arrows. The most frequently used incorrect strategy (6%) was to "move letters other than x on the other side of equation and change the sign" which meant to change multiplication to division and vice versa. This strategy gave correct responses for the A and B, but not for the C equations. 6% of participants used some kind of a learned rule. For example, one participant wrote for $a/x = b$: "If x is a denominator then the solution is the fraction of the remaining factors, given that the nominator of the initial fraction (the one with x) remains the same." For $a/x = b$ equation (C type), some participants (8%) only swapped x and b without performing two steps of multiplication and division.

Figure 5 shows how the proportion of participants who used concrete and rule-based strategies changed with their age. The majority of younger participants (from primary school) used concrete strategies, whereas participants from secondary school mostly used more abstract, rule-based strategies. Some

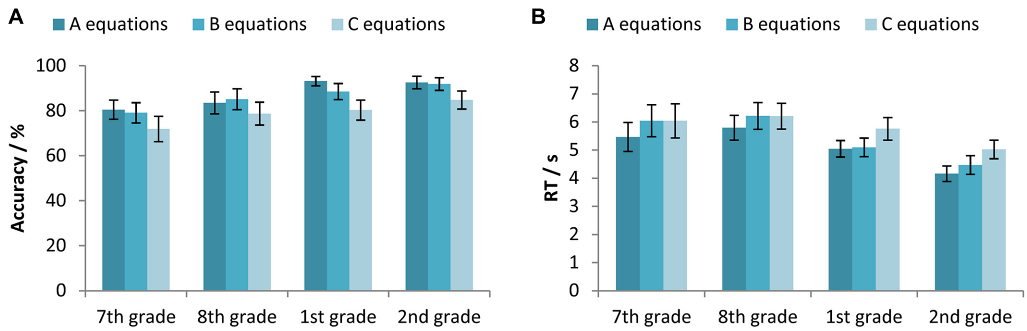

FIGURE 3 | (A) Accuracy (percentage of correct responses) and **(B)** RTs for the participants in the 7th and 8th grade of primary school, and the 1st and 2nd grade of secondary school, separated for the different equation types (the A, B and C equations). Error bars represent 95% confidence intervals.

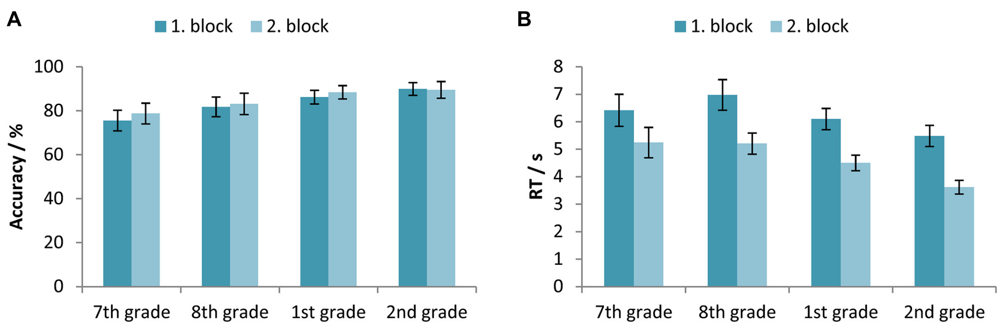

FIGURE 4 | (A) Accuracy (percentage of correct responses) and **(B)** RTs for the participants in the 7th and 8th grade of primary school, and the 1st and 2nd grade of secondary school, separated for the first and second block. Error bars represent 95% confidence intervals.

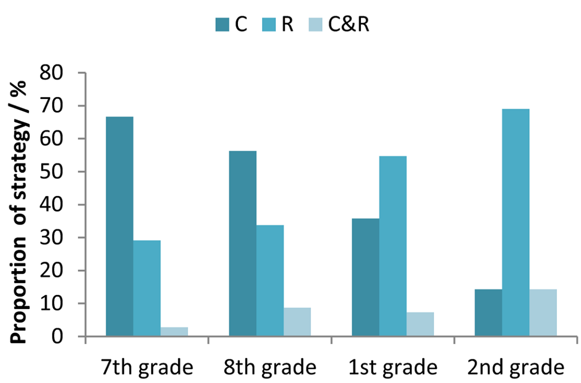

FIGURE 5 | Proportions of used strategy types (C, concrete; R, rule-based; C&R, concrete and rule-based) for the participants in the 7th and 8th grade of primary school, and the 1st and 2nd grade of secondary school.

participants used both concrete and rule-based strategies. For example, one participant used standard multiplication/division procedure for the A and B equations, but she inserted "real numbers" to solve the C equations. Participants who used both concrete and rule-based strategies typically used a concrete strategy to solve the C equations.

EQUATION DIFFICULTY RANKS

Figure 6A shows the inverse efficiency measures for different equation types (all with letters) across different participants' age. To test the differences between participants' inverse efficiency in solving different types of equations a two-way mixed-design ANOVA with between-subjects factor Age (7th vs. 8th vs. 1st vs. 2nd grade) and within-subjects factor Equation type (A vs. B vs. C equation) was used. The obtained results indicate a statistically significant main effect of both factors, Age [$F(3{,}325) = 11.84$, $p < 0.001$, $\eta_p^2 = 0.099$] and Equation type [$F(2{,}650) = 43.72$, $p < 0.05$, $\eta_p^2 = 0.119$], while the interaction was not significant [$F(6, 650) = 0.33$, $p > 0.05$, $\eta_p^2 = 0.003$]. If we adopt inverse efficiency as a measure of task difficulty (Townsend and Ashby, 1978), the results suggest that the C equations were the most difficult. There was no statistically significant difference between the A and the B equations.

Participants ranked different equation types by difficulty in the questionnaires. 28 participants reported that all equation types are equally difficult. Three participants thought that equations with multiplications (A type) are easier than equations with division (B and C). Eight participants did not provide an answer to this question. **Figure 6B** shows the data of the remaining participants across their age groups. Most participants reported that the A equations were the easiest. However, a considerable number of the secondary school students (32%) thought that the B equations

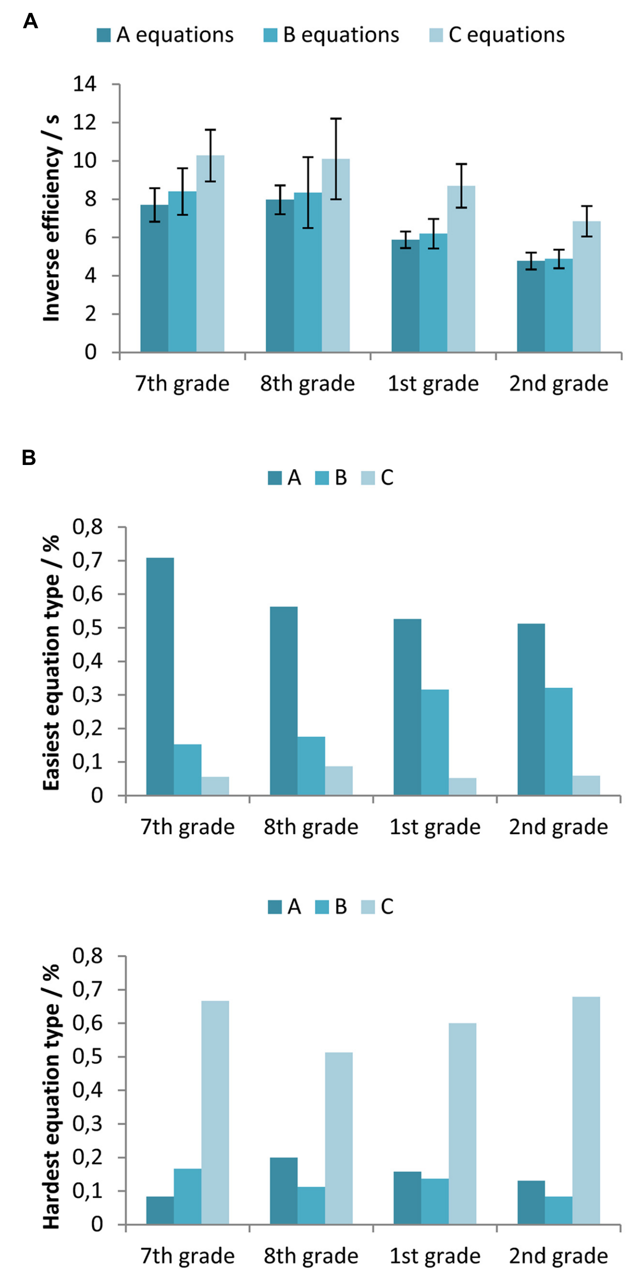

FIGURE 6 | (A) Inverse efficiency on equations with letters for the participants in the 7th and 8th grade of primary school, and the 1st and 2nd grade of secondary school, separated for the different equation types (the A, B, and C equations). Error bars represent 95% confidence intervals. **(B)** Proportion of equation types ranked as the least difficult and the most difficult by the participants in each grade.

were the easiest. Most participants agreed that the C equations were the most difficult.

DISCUSSION

ACCURACY AND SPEED OF EQUATION SOLVING

The results obtained in the present study suggest that the tested students were overall rather successful in equation rearrangement, with accuracy levels amounting to an average of 85%. Although this may seem quite high, if the true-false nature of test items is taken into account this becomes a less satisfactory result, especially for all-symbol equations which were correctly solved by 82% of the participants. However, our data indicate that students become more efficient, i.e., more accurate and faster, in higher school grades.

With respect to gender differences, the girls in our sample were on average more accurate in equation rearrangement than boys, while no significant differences in their speed were revealed. This finding is in disagreement with a common belief that boys are better in mathematics than girls which is based on reports that boys outperform girls on standardized tests like SAT (e.g., Byrnes and Takahira, 1993). However, most studies report no differences between boys and girls on algebra assessments (e.g., Bridgeman and Wendler, 1991). In fact, girls sometimes do even better than boys (e.g., Else-Quest et al., 2010), while male superiority among adolescents is usually related to boys' spatial reasoning and more diverse strategies in problem solving (Geary, 1996). In the present study we observed slightly higher accuracy for girls than for boys, but overall comparable speed of equation solving, which resonates with previous findings suggesting small, if any, gender differences in solving simple algebraic equations.

It is important to emphasize that students' success in solving simple algebra equations differed across different types of equations. Specifically, within the present study we compared equivalent equation formats that contained either symbols or numbers. As expected, the obtained results indicate that the younger participants were more accurate and faster in solving equations with numbers than with letters although these were equivalent. This indicates that younger students still struggle with more abstract equations. In contrast, students in the 2nd grade (age 16–17 years) had a comparable level of accuracy and RTs for equations with numbers and letters. This indicates that they reached an adequate level of formal reasoning (Inhelder and Piaget, 1958), at least for this particular task.

Next, we compared participants' efficacy in solving three different types of equations. The lowest accuracy and the longest RTs obtained for the C equations ($a/x = b$) suggest that this was the most difficult type of equation. Younger students were struggling with this equation type; the accuracy of the 7th grade participants (age 13–14 years) was only 72%. Accuracy on the C equations increased with the participants' age, with 2nd graders (age 16–17 years) reaching 85%. These results reflect the fact that two operations are needed to solve C equations, and only one operation for other two types of equation, thus indicating that the procedural complexity has also a significant effect on efficiency in equation solving. Our data suggest that even our oldest participants, 16–17 years old at the time of the testing, had difficulties with the slightly more difficult, but still very simple equations. This is in agreement with the previous reports on students' difficulties with "all-symbol" equations (Ekenstam and Nilsson, 1979; Küchemann, 1981).

In addition to exploring age, gender and equation type effects, within the present study we also explored practice effects across all equation types. Our participants became faster and more accurate in equation rearrangement during the time course of the measurement. This finding is in agreement with a previous report indicating how children become faster during a 5-day practice in algebra equation solving (Qin et al., 2004). It seems that some of our participants learned how to solve equations as they were repeatedly exposed to them for a short period of time, even without feedback. Even the participants from the 2nd grade of secondary school (age 16–17 years), who had stable high accuracy levels from the beginning until the end of the measurement, became faster in equation rearrangement. This might be an interesting finding for mathematics teachers. However, additional studies are needed to explore a long-term effect of such short and intense practice in equation solving.

Furthermore, our results showed that the participants with higher cognitive abilities were more efficient in equation solving. This is in line with the previous longitudinal testing which indicated that students with higher IQ scores tended to demonstrate higher cognitive levels and made faster progress through algebra levels than students with lower IQ scores (Küchemann, 1981). It has been suggested that on familiar algebra tasks, participants rely on automated routines and acquired facts that are more systematically learned by individuals of higher cognitive abilities (Bornemann et al., 2010). Consequently, they outperform individuals with lower general cognitive abilities, while allocating the same amount, or even less, of cognitive resources to the task. Accordingly, we could conclude that our participants with higher abilities profited from more efficient processes compared to individuals of lower cognitive abilities. However, the general cognitive ability is not the only factor influencing individual's understanding of algebraic equations. Other factors are also important, such as the intuitive assumptions and pragmatic reasoning about a new notation, analogies with familiar symbol systems, interference from new learning in mathematics, and the effects of misleading teaching materials (MacGregor and Stacey, 1997).

CONCRETE AND RULE-BASED STRATEGIES FOR EQUATION SOLVING

Half of the participants used concrete strategies for equation rearrangement and the most frequently used concrete strategy was inserting numbers into equations. When using this strategy, the students think of an equivalent equation with numbers, solve it and then apply the solving algorithm on the equation with symbols. For example, for the A equation ($x \cdot a = b$), they insert numbers so the equation becomes $2 \cdot 3 = 6$ and then conclude that "if $2 = 6/3$, then $x = b/a$." It seems that these participants have not yet reached the formal operational stage and are more comfortable with concrete numbers in equations. This is in agreement with the previous studies on algebraic processing in adolescents (Ekenstam and Nilsson, 1979; Küchemann, 1981; Susac et al., under revision).

A considerable number of participants (11%) used the "triangle" method often taught by physics teachers to "simplify" equation rearrangement for their students. Within this strategy, a triangle is divided into three parts. Two quantities that are multiplied together are written side-by-side at the bottom of the triangle. The remaining quantity (their product) is written at the top. For $x \cdot a = b$ (A equation), x and a are written at the bottom, and b at the top. If we want to make x subject of the equation, x should be covered and what is left, namely "b over a," represents the result. Although this strategy helps students in equation rearrangement, this technique does not develop their formal reasoning.

The "biggest on the top" strategy also has origin in concrete way of thinking. As few participants reported, they always considered product in multiplication equations and numerator in division equations as the biggest object that helped them in the rearrangement. For example, in the A equations ($x \cdot a = b$) they regard b as the biggest object that helped them to form a solution (the biggest goes on the top, therefore $x = b/a$). Although they did not explicitly insert numbers into equations, participants' experience with natural numbers may probably account for their line of reasoning (the biggest number is always the product of two natural numbers, and the numerator is bigger than the denominator and the result of a division). In our previous study we have found that the UK students also use this strategy (Susac et al., under revision).

More than half of the participants (56%) were reasoning more abstractly while solving at least one equation type, i.e., they were applying rules. During the testing, few participants made a transition from concrete substitution of letters by numbers to the recognition of patterns and rules. The most frequently used rule-based strategy was multiplication and division of equation with the "letter next to x." This procedure was performed correctly by the majority of participants who decided to use it. However, the most common incorrect strategy involved the procedure of moving "letter next to x" on the other side of equation and changing the operation, multiplication to division and vice versa. This probably reflects an inappropriate application of the procedure learned for equations with addition/subtraction, indicating that the application of mathematical rules and procedures can be very confusing for students.

As in our previous study (Susac et al., under revision), some participants reported moving letters to the other side of equation. This corroborates findings showing that spatial reasoning is closely related to the number sense (as in the case of mental number line; e.g., Dehaene, 1997) and mathematical operations in general. A number of neuroimaging and neuropsychology studies have demonstrated that the relationship between number and space processing is deeply rooted in the organization of parietal circuits for these capacities (Hubbard et al., 2005). Mathematical experts in our previous study often used spatial terms when explaining their strategies in equation solving (Susac et al., under revision). It seems that the development of spatial reasoning in students might be beneficial even in "non-spatial" areas of mathematics such as algebra. In addition, visualization can be also helpful in developing problem-solving skills in mathematics (Scheiter et al., 2010)

Some participants reported strategies based on some types of rules that they developed by themselves. By repeated exposure to equation rearrangement, they recognized some patterns from which they derived some general rules. Although participants' rules were not always correct, they possibly represent a step in developing more consistent and correct solving strategies. A number of participants recognized that they do not have to perform two steps of multiplication and division for $a/x = b$ equation (C type), and just swapped x and b. In doing so, they developed a new, more efficient strategy during the experiment, through pattern recognition which is of great value in performing algebraic tasks (Orton and Orton, 1999).

Overall, the obtained results suggest that the proportion of concrete strategy usage decreases at the same time as the proportion of rule-based strategies increases with the age of participants. This progression is gradual and it probably continues after the 2nd grade of secondary school (age 16–17 years). Our data confirm that the development of algebraic thinking is a process which unfolds over a long time. Consequently, we can conclude that children at the age 14–15 are in transition from concrete to abstract strategies in algebra that is in agreement with previous research (Küchemann, 1981).

EQUATION DIFFICULTY RANKS

To determine the difficulty of different types of equations with letters, we evaluated inverse efficiency across the age groups of our participants. In all age groups, participants were the least efficient in solving the C equations, which suggests that these are the most difficult equation types. This finding was expected, because the C equations are usually solved in two steps while only one step is needed for the A and B equations. Not all participants performed two operations in solving $a/x = b$ (e.g., some of them swapped x and b). However, for C equations, both equation and solution involve division, which is generally more difficult than multiplication (Hecht et al., 2003).

Inverse efficiency measures indicated that the A equations were of similar difficulty as the B equations. The B equations, $x/a = b$, are probably the easiest because their solution is based on multiplication and the order of the variables in the product is not important. In the A equations, $x \cdot a = b$, the solution includes a division so an additional step to decide the right order of numerator and denominator is needed (as a/b is not the same as b/a). However, it seems that our participants were not fully aware of this pattern, as can be observed in their inverse efficiency results.

It is interesting to note that a large majority of participants reported that the B equations are more difficult than the A equations although this is not supported by the obtained results. Probably their self reports were again influenced by the fact that division is perceived as more difficult than multiplication. However, in judging equation difficulty, participants failed to take into account the fact that correctly solving these equations also includes these operations. Still, the increased number of participants who ranked B equations as easiest among older students suggests that some older participants (from secondary school) became aware of the patterns in the task. In addition, it seems that metacognitive skills improve with age as secondary school students, on average, ranked equation difficulty more accurately than younger participants. This finding concurs the previous reports on the importance of metacognitive activities for success in problem solving in mathematics (Kramarski and Mevarech, 2003; Cohors-Fresenborg et al., 2010).

CONCLUSION

The goal of the present study was to investigate the development of students' abstract reasoning skills on a simple equation rearrangement task. Although all our participants learned equation rearrangement in mathematics at least one year prior to our testing, and were required to solve simple equations in mathematics and science problems, they still had difficulties with some equation types. However, accuracy and speed of equation rearrangement increased with the participants' age. Younger participants were more accurate and faster in solving equations with numbers than with letters, suggesting that they are still concrete thinkers. The difference in the efficacy of solving equations with numbers and letters disappeared for participants from the 2nd grade of secondary school (age 16–17 years), indicating their ability to think more abstractly, at least on our task. The transition from concrete to formal reasoning was also reflected in strategies that the participants used for solving equation with letters. Younger participants from the primary school (age 13–15 years) mostly employed concrete strategies such as inserting numbers, while secondary school participants (age 15–17 years) mainly used rule-based strategies.

Our results indicate that the transition from concrete to abstract reasoning represents quite a long process, even for simple algebraic task used in this study. Teachers and educational policy makers should be aware that it is not enough to learn about equation rearrangement in mathematics once. It should not be presumed that students master this skill quickly and that they can easily apply it in other context such as problem solving in physics. On the contrary, teachers should use every opportunity to encourage students to use formal reasoning – both pattern recognition and effective application of mathematical rules and known procedures.

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