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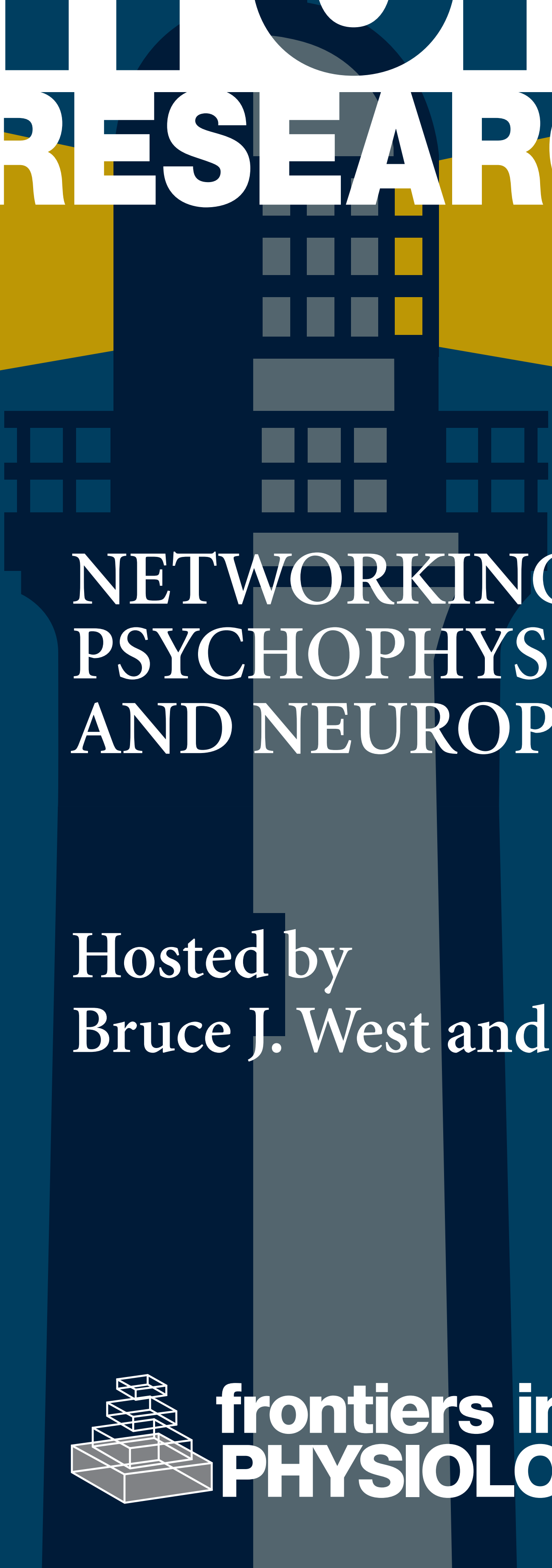

NETWORKING OF PSYCHOPHYSICS, PSYCHOLOGY AND NEUROPHYSIOLOGY

Hosted by
Bruce J. West and Paolo Grigolini

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NETWORKING OF PSYCHOPHYSICS, PSYCHOLOGY AND NEUROPHYSIOLOGY

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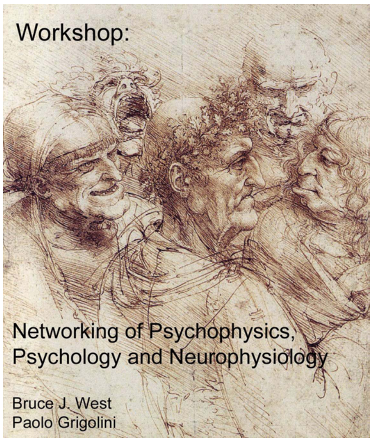

To many scientists the gap between the nineteenth century views of consciousness proposed by the psychologist William James and that developed by the inventor of psychophysics Gustav Fechner has never seemed wider. However the twentieth century concept of collective/cooperative behavior within the brain has partially reconciled these diverging perspectives suggesting the notion of consciousness as a physical phenomenon. A kernel of twenty-first century investigators bases their investigations on physiological fluctuations experiments. These fluctuations, although apparently erratic, when analyzed with advanced methods of fractal statistical analysis reveal the emergence of complex behavior, intermediate between complete order and total randomness, a property usually referred to as temporal complexity. Others, with the help of modern technologies, such MRI, establish a more direct analysis of brain dynamics, and focus on the brain's topological complexity. Consequently the two groups adopt different approaches, the former being based on phenomenological and macroscopic considerations, and the latter resting on the crucial role of neuron interactions. The neurophysiology research work has an increasing overlap with the emerging field of complex networks, whereas the behavior psychology experiments have until recently ignored the complex cooperative dynamics that are proved by increasing experimental evidence to characterize the brain function.

It is crucial to examine both the experimental and theoretical studies that support and those that challenge the view that it is an emergent collective property that allows the healthy brain to function. What needs to be discussed are new ways to understand the transport of information through complex networks sharing the same dynamical properties as the brain. In addition we need to understand information transfer between complex networks, say between the brain and a controlled experimental stimulus. Experiments suggest that brain excitation is described by inverse power-law distributions and recent studies in network dynamics indicate that this distribution is the result of phase transitions due to neuron network dynamics. It is important to stress that the development of dynamic networking establishes a connection between topological and temporal complexity, establishing that a scale-free distribution of links is generated by the dynamic correlation between dynamic elements located at very large Euclidean distances from one another. Dynamic networking and dynamics networks suggest a new way to transfer information: the long-distance communication through local cooperative interaction. It is anticipated that the contributed discussions will clarify how the global intelligence of a complex network emerges from the local cooperation of units and the role played by critical phase transitions in the observed persistence of this cooperation.

Table of Contents

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EDITORIAL
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Networking of psychophysics, psychology, and neurophysiology

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The Army Research Laboratory program on the *Network Science of Human Decision Making* brought together researchers from a variety of disciplines to work on a complex research problem that defies confinement within any single discipline. Consequently, new and rewarding solutions have been obtained for problems of importance to society and the Army in the human dimension of complex networks. This program is reviewed by West (2011) in *Overview 2010 of ARL program on network science for human decision making* wherein he investigates the basic research foundation of a science of networks supporting the linkage between the cognitive and social domains as they relate to human decision making. In the same spirit as the present workshop the research strategy extends recent methods of non-equilibrium statistical physics to non-stationary, renewal stochastic processes characteristic of the interactions among nodes in complex networks. The theoretical analyses of complex networks, although mathematically rigorous, often elude analytic solutions and require simulation and computation to analyze the underlying dynamic process.

Dynamic networking and dynamic networks suggest new ways to transfer information utilizing the long-distance communication through local cooperative interaction. The papers contributed to the workshop clarified how the global intelligence of a complex network emerges from the local cooperation of units, whether these units are neurons or people and emphasizes the role played by critical phase transitions in the observed persistence of this cooperation.

To many scientists the gap between the nineteenth century views of consciousness proposed by the psychologist William James and that developed by the inventor of psychophysics Gustav Fechner has never seemed wider. However the chasm may not be as large as believed as (Hawkins, 2011) explains in *William James, Gustav Fechner, and Early Psychophysics*. The twenty-first century concept of collective/cooperative behavior within the brain has partially reconciled these diverging perspectives suggesting the notion of consciousness as a physical phenomenon as so eloquently explained by the late Gerhard Werner (Werner, 2011) in *Letting the Brian Speak for Itself*. He recognized that the self-referential mode of function and the propensity for self-organization to critical states requires a fundamentally new orientation, based on Complex System Dynamics as non-ergodic, non-stationary processes with inverse-power-law statistical distributions.

According to an increasing number of researchers intelligence emerges from criticality as a consequence of locality breakdown and the onset of long-range correlation, well-known properties of phase transition processes. Turalska et al. (2012) in *Cooperation-induced topological complexity: a promising road to fault tolerance and Hebbian learning* study a model of interacting units, as an idealization of real cooperative systems such as the brain or a flock of birds, for the purpose of discussing the emergence of long-range correlation from the coupling of any unit with its nearest neighbors. They focus on the critical condition that has been recently shown to maximize information transport and study the topological structure of the network of dynamically linked nodes.

Some investigators base the focus of their discussion on understanding the apparent contamination of social dynamics by erratic fluctuations. *Social interactions model and adaptability of human behavior* by Zhao and Bianconi (2011) concerns human social networks and their evolution on the fast timescale of face-to-face interactions and of interactions mediated by technology such as telephone calls and video conferences. The resulting networks have a strong dynamical component that changes significantly the properties of dynamical processes. They study a general model of pair wise human social interaction intended to model both face-to-face interactions and mobile-phone communication.

Fluctuations in social and brain dynamics, although apparently erratic, when analyzed with advanced methods of fractal statistical analysis reveal the emergence of complex behavior, intermediate between complete order and total randomness, a property usually referred to as temporal complexity. *Renormalization group for critical phenomena in complex networks* (Boettcher and Brunson, 2011) provides a detailed, pedagogical introduction to the application of renormalization group theory to the understanding of criticality in complex networks.

The physical singularity of life phenomena is analyzed in *The inert vs. the living state of matter: extended criticality, time geometry, anti-entropy – an overview* by Longo and Montévil (2012) by means of comparison with the driving concepts of theories of the inert. They outline conceptual analogies, transferals of methodologies and theoretical instruments between physics and biology, in addition to indicating significant differences and sometimes logical dualities. In order to make biological phenomenalities intelligible, they introduce theoretical extensions to certain physical theories such as criticality.

Others, with the help of modern technologies, such as functional magnetic resonance imaging (fMRI), establish a more direct analysis of brain dynamics, and focus on the brain's topological complexity. In *Criticality in large-scale brain fMRI dynamics unveiled by a novel point process analysis* (Tagliazucchi et al., 2012) introduces a theoretical framework in terms of an order and control parameter derived from fMRI data, where the dynamical regime can be interpreted as one corresponding to a system close to the critical point of a second order phase transition. The analysis demonstrates that the resting brain spends most of the time near the critical point of such transition and exhibits avalanches of activity ruled by the same dynamical and statistical properties described for neuronal events at smaller scales.

Neurophysiology research work has an increasing overlap with the emerging field of complex networks, and the behavior psychology experiments have until recently ignored the complex cooperative dynamics that are proved by increasing experimental evidence to characterize the brain function. Lovecchio et al. (2012) in *From self-organized to extended criticality* implemented the notion of extended criticality, which is realized through a wide set of critical points rather than emerging as a singularity from a unique value of the control parameter. Their approach explained the experimental observation that neuronal avalanches occur in time with surprisingly regularity, in apparent conflict with the temporal complexity of physical critical points.

Gallos et al. (2012) address the problem of the hierarchical organization in the brain through network analysis. In *The conundrum of functional brain networks: small-world efficiency or fractal modularity* their analysis identified functional brain modules of fractal structure that were inter-connected in a small-world topology. They provide details on the use of network science tools to elaborate on this network behavior and indicate the importance of using percolation theory to highlight the modular character of the functional brain network. These modules present a fractal, self-similar topology, identified through fractal network methods.

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Overview 2010 of ARL program on network science for human decision making

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The Army Research Laboratory program on the *Network Science of Human Decision Making* brings together researchers from a variety of disciplines to work on a complex research problem that defies confinement within any single discipline. Consequently, new and rewarding solutions have been obtained for a problem of importance to society and the Army, that being, the human dimension of complex networks. This program investigates the basic research foundation of a science of networks supporting the linkage between the cognitive and social domains as they relate to human decision making. The research strategy extends recent methods of non-equilibrium statistical physics to non-stationary, renewal stochastic processes characteristic of the interactions among nodes in complex networks. The theoretical analyses of complex networks, although mathematically rigorous, often elude analytic solutions and require simulation and computation to analyze the underlying dynamic process. The information transfer between two complex networks is calculated using the principle of complexity management as well as direct numerical calculation of the decision making model developed within the project.

Keywords: complex networks, principle complexity management, decision making model

INTRODUCTION

The modern world is an interconnected mesh of networks satisfying a myriad of functions: transportation, electrical power, food distribution, finance, and health care to name a few. The interoperability of these networks developed as part of urban evolution over the past century such that these and other webs connect to national and/or global networks (National Research Council of the National Academies, 2005). This is the engineered webbing of humanity, but there are comparable natural structures in the spheres of biology, ecology, sociology, and physiology.

This modernity is manifest in the military through the development of network-centric warfare (NCW) which takes cognizance of human behavior in a networked environment of organized actions directed toward political, social, and military ends and is the basis of a new theory of war (Office of Force Transformation, 2004; Garstka and Alberts, unpublished). Thus, NCW has at its core a shift in focus from military platforms such as ships and tanks to networks having platforms as members. Army scientists need to understand the dynamics, controllability, and predictability of generic non-linear complex networks in order to realize their goal of supporting both society and the soldier through research and the development of new technologies.

It is not only our external world that is cluttered with networks, but our internal world as well. The neuronal network carrying the brain's signal to the body's physiological networks is even more complex than the modern city or a typical ecological network. Thus, the basic research into network science must span and encompass a multitude of disciplines; understanding each sheds light on the others.

The problem addressed within this program is to develop the basic research foundation of a science of networks that supports the linkage between the cognitive and social domains as they relate to decision making. This approach is not directed at the totality of developing a Network Science, but has the more modest goal of understanding the deeply interdependent human networks of crucial importance to society as a whole and to the Army in particular. Even such a restricted problem is a significant challenge due to the multiply interconnecting networks buttressing the common decision making objective.

On the one hand, the military is proactive in that networked forces can operate in an agile manner to promote decision making superiority. On the other hand, the Army is reactive in the need to respond to enemies who are also using the power of networks against United States interests. The research program provides insight to allow the Army to anticipate the enemy's use of network strategy and thereby reduce the reactive mode of operation. In Grigolini and West (2011) we reviewed what is presently known about complex networks, regardless of the disciplinary context and adapted that understanding to the decision making paradigm. Moreover, the barriers to further understanding and to filling the gaps in knowledge of the linkages between social and human decision making networks were addressed.

The research strategy of the ARL program is based on theory, computation/simulation, and experiment/observation. This is a cyclic interactive process in which new theory stimulates unique simulations, yielding insight into parameter values and network configurations, which in turn suggests specific experiments, whose outcome guides the refinement and development of theory. This modern approach to scientific research is applied to the phenomenon of human decision making with a view for eventual application to NCW. The core group of Army scientists is the focal point for external researchers requiring militarily relevant challenges and internal ARL efforts.

One of the mysteries of human social interaction is how agreements are reached and cooperative alliances are made. Individuals become part of social groups or networks in a number of ways: choice, peer pressure, and subliminal seduction; but always through a sequence of decisions, either conscious or not. Network characteristics cannot be deduced from the properties of individuals; they emerge during the formation and growth of the network. Consequently we need mathematics to quantify the strength of the interactions between the network components, as well as to describe how a network develops in time and responds to perturbations (stimulation). This has been done through the construction of the decision making model (DMM) that for very weak coupling is much like the Ising model of cooperative behavior, but for strong coupling can be very different (Turalska et al., 2009); see Grigolini and West (2011) for an overview.

Ubiquitous aspects of complex networks are the appearance of non-stationary, non-ergodic, and renewal statistical processes. These properties are manifest through inverse power–law statistical distributions that not only challenge traditional understanding of complexity in physical networks, but require new strategies for understanding how information is exchanged between networks (West et al., 2008; West and Grigolini, 2011), as in the case of interest here among human networks including cognitive networks. The approach is to adapt the methods of non-equilibrium statistical physics that have been used to characterize the dynamics of complex phenomena and phase transitions. These methods were extended to the study of such social phenomena as linguistics, biofeedback techniques, and the brain's response to music (Bianco et al., 2007) and to further develop them to model decision making with incomplete information in an uncertain environment.

The research into decision making has been addressed using a variety of strategies. The mathematics of complex networks has been examined using the newly developed DMM to understand consensus (Turalska et al., 2009); a psychophysical model of how individuals make decision and then irrationally change their minds shows agreement between theory and experiments (West and Grigolini, 2010a); renewal statistics reveal how we habituate to the familiar (West and Grigolini, 2010b) and forget the uninteresting (West and Grigolini, 2010c); $1/f$ variability captures how the brain processes tasks of increasing complexity during decision making (Grigolini et al., 2009); and finally we have determined how all these various pieces fit into the overall picture of exchanging information between complex networks (West et al., 2008; Grigolini and West, 2011; West and Grigolini, 2011).

The major accomplishment of the present research program has been the identification of the first universal principle in the science of networks, that being, the principle of complexity management (PCM) discussed in Grigolini and West (2011), West and Grigolini (2011). PCM states that the maximum information is exchanged between two complex networks when there is compatibility of the complexity of the two networks. A mathematical proof of this principle has been constructed over the past year (Aquino et al., 2010, 2011).

PRINCIPLE OF COMPLEXITY MANAGEMENT

The mathematician Norbert Wiener speculated that the transfer of influence from a complex network high in information to one low in information even though the latter may be higher in energy represents a new kind of interaction (Wiener, 1985), which we called Wiener's Rule (Grigolini and West, 2011). His insight was vindicated a half century later (Aquino et al., 2010, 2011) and required the generalization of a number of concepts from statistical physics (Allegrini et al., 2007, 2011; Aquino et al., 2007; Budini and Grigolini, 2009) resulting in the PCM (West et al., 2008; West and Grigolini, 2011) as we discussed last year (Grigolini and West, 2011).

One measure of the information content of a network is provided by the probability density most often used in the determination of the negative entropy of Shannon and Wiener. An apparently ubiquitous distribution in the description of empirical complex networks is the hyperbolic, having the survival probability:

$$\Psi(t) = \frac{T^{\mu-1}}{(T+t)^{\mu-1}}, \tag{1}$$

which asymptotically becomes an inverse power–law. The average time between events in complex webs such as power grid blackouts, heartbeats, time between earthquakes (West and Grigolini, 2011) of a given magnitude can be determined using the probability density $\psi(t) = -d\Psi(t)/dt$, to be

$$\langle t \rangle = \int_0^\infty t\Psi(t)dt = \begin{cases} \frac{T}{\mu-2}; \mu > 2 \text{ ergodic} \\ \infty; \mu > 2 \text{ non ergodic} \end{cases} \tag{2}$$

It is interesting that when the power–law index is in the interval $2 < \mu < 3$ the distribution has a finite first moment and the statistics are ergodic, meaning that the time average and ensemble averages yield the same result. However when the power–law index is $\mu < 2$ there are no finite integer moments and the time and ensemble averages are not the same, that is, the process is non-ergodic. We shall have more to say about non-ergodicity subsequently.

One measure of the information transfer between two complex networks is the cross-correlation between a complex network P and a complex network S being perturbed by P with ε the strength of the perturbation. For our purposes it is sufficient to apply the generalized linear response theory (LRT; Aquino et al., 2010, 2011) we previously developed the normalized cross-correlation function:

$$\Phi(t) \equiv \frac{C(t)}{\varepsilon} = \int_0^t R_x\left(t'\right)\Psi_s\left(t-t'\right)\Psi_p\left(t,t'\right)dt'. \tag{3}$$

The perturbing complex network P is characterized by the non-stationary autocorrelation function $\Psi_p\ (t,\ t')$, which depends separately on the time of the last perturbation t' and time of

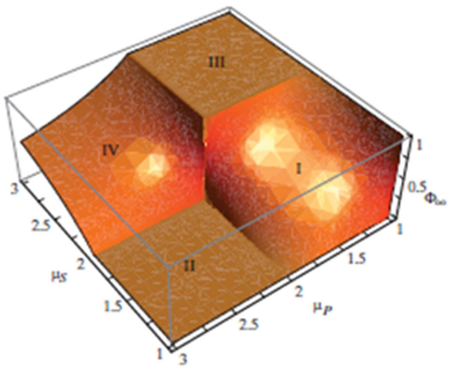

FIGURE 1 | The cross-correlation cube. The asymptotic cross-correlation function defined by Eq. 3 is graphed as a function of the two power–law indices of the perturbed network *S* and the perturbing network *P*. (From West and Grigolini, 2011, with permission.)

the measurement *t*. The function $R_S(t')$ is the rate of generating perturbing events at time t' within the network being perturbed and is based on renewal theory (West and Grigolini, 2011). The perturbed network *S* is characterized by the stationary autocorrelation function $\Psi_S\ (t - t')$, which depends only on the difference in times from the last perturbation to the measurement.

In **Figure 1** the asymptotic cross-correlation function normalized to the strength of the perturbation is graphed as a function of the power–law indices of the two networks to form a cross-correlation cube. The cube displays a number of remarkable properties: (1) when the power–law indices are both equal to two there is an abrupt jump from zero correlation in region II to perfect consensus in region III; (2) the upper plateau region III indicates that when *P* is non-ergodic $1 < \mu_P < 2$ and *S* is ergodic $2 < \mu_S < 3$ there is an information response in which the perturbed network tracks the perturbing network exactly and the information transfer is maximal; (3) when *P* is ergodic $2 < \mu_P < 3$ and *S* is non-ergodic $1 < \mu_S < 2$ there is no response asymptotically and the information transfer is minimal as shown in region II. How a complex network responds to a perturbation by another complex network is determined by the kind of mismatch that exists in the complexity of the fluctuations in the two networks.

Wiener's Rule describes the influence of the perturbing network outside the lower plateau region of the cross-correlation cube. In all regions except this one the weak perturbation significantly modifies the properties of the complex network being perturbed. In the upper plateau region the perturbation by network *P* actually dominates the properties of the perturbed network *S* and reorganizes it, just as Wiener anticipated. The PCM embodied in the cross-correlation cube therefore subsumes Wiener's Rule as we reviewed in West et al. (2008). In addition we showed the application of PCM to the phenomenon of habituation and other activities involving the human brain.

SYNCHRONIZATION AND INFORMATION EXCHANGE

Aquino et al. (2011) observe that the growing interest in the dynamics of complex networks is shifting research attention from the synchronization of two stochastic units (Pecora and Carroll, 1990) to the synchronization of large numbers of units (Wang, 2002), an interesting phenomenon that is closely related to the very popular model of Kuramoto (1984). The single units of the processes of chaos synchronization are chaotic and they surprisingly synchronize while maintaining the erratic dynamics that they have in isolation. Although the single units of the Kuramoto model are regular, it is becoming increasingly evident that the emergence of a global synchronization is a condition independent of whether the single units are regular or stochastic. The single units of the work of Bianco et al. (2008), Turalska et al. (2011) are Poisson processes and if one of them drives the other, they would obey the principle of aperiodic stochastic resonance (Lukovi'c et al., 2008). If the two units are bi-directionally coupled they are expected to undergo a condition of perfect synchronization if the coupling is sufficiently intense. When the number of interacting units is very large a phase transition occurs from the non-cooperative to the cooperative behavior (Bianco et al., 2008; Turalska et al., 2011).

It is important to stress that at criticality no permanent consensus is reached, and the mean value of the global field vanishes. Yet, this condition is strikingly different from the non-cooperative condition. The whole network remains in the "yes" ("no") state for an extended time before making a transition to the "no" ("yes") state.

It is surprising that the phase transition literature seems to have overlooked, with only a few exceptions (Contoyiannis and Diakonos, 2000; Bianco et al., 2008; Turalska et al., 2011), that the transitions from the "yes" ("no") to the "no" ("yes") state occurring at criticality are the "crucial" events defined in Section I of Aquino et al. (2011). In other words, the time interval between two consecutive transitions is derived from a *pdf* that has the asymptotic time structure of Eq. 1 with a power index μ fitting the inequality condition $1 < \mu < 3$. Some authors (Bianco et al., 2008; Turalska et al., 2011) argue that $\mu = 1.5$ and others (Frantsuzov et al., 2009), releasing the condition that all the units share the same Poisson rate, generate a global condition with crucial events characterized by $\mu < 2$, but significantly departing from the value $\mu = 1.5$. Note that the theoretical arguments of Turalska et al. (2009), might yield the misleading impression that the crucial value of μ is a consequence of ordinary statistical physics.

An important result of Aquino et al. (2011) is the discovery of a promising road to settle the problem of information transmission from one to another complex network. In fact, if the inner synchronization corresponds to a criticality condition and criticality generates crucial events with a power–law index in the interval $1 < \mu < 3$, then a complex network at criticality is a generator of $1/f$ noise, with a power spectrum $S(f) \propto 1/f^{3-\mu}$. Thus, the problem of information transmission from one to another complex network becomes equivalent to the phenomenon of $1/f$ resonance illustrated in Aquino et al. (2011).

Aquino et al. (2011) distinguish between a phenomenological and dynamic LRT. The experiments (Onsager, 1944; Silvestri et al., 2009) support the dynamical rather than the phenomenological LRT. It is important to stress that phenomenological LRT is a natural consequence of adopting the asymptotic time perspective replacing the waiting-times *pdf* $\psi(\tau)$ of Eq. 1 with $\psi(\tau) \propto 1/\tau^{\mu}$.

This way of proceeding, although generating the elegant mathematics of fractional derivatives, has as an ultimate effect the misleading discovery of the death of linear response. We do not adopt the asymptotic time perspective but the special form of Eq. 1. This is not a unique way of connecting the longtime to the short-time regime. However, whatever form we adopt we are convinced that there will be a parameter corresponding to the parameter T of Eq. 1. It is reasonable to assume that an external perturbation may perturb either T or μ, or both. However, the perturbation of μ is incompatible with the assumption of a weak stimulus. In fact, μ is a consequence of the cooperation among the units of the network, and a perturbation may affect μ only if its strength is large enough to influence the interaction among the units of the network. Thus, an external weak perturbation can only have an effect on T, thereby making the dynamical LRT become the proper way to study the response of a complex network to a weak external stimulus, in accordance with the experimental results (Onsager, 1944; Allegrini et al., 2009; Silvestri et al., 2009).

For these reasons, we can conclude that **Figure 1** is an original, and important, result of this research program. We hope that the application of the PCM may open the door to solving the problem of information transmission from one complex network to another, a research topic that is still in its infancy.

DECISION MAKING MODEL AND PHASE TRANSITIONS

In order to better understand the transfer of information between complex networks last year we numerically analyzing networks consisting of a large number of non-linearly interacting nodes. The properties of the DMM developed by Turalska et al. (2009) using a master equation formalism (Turalska et al., 2011) were discussed where each element of the network is a two state oscillator and is described by a master equation of the form

$$\begin{aligned}\frac{dp_1(t)}{dt} &= -g_{12}(t)\,p_1(t) + g_{21}(t)\,p_2(t),\\ \frac{dp_2(t)}{dt} &= -g_{21}(t)\,p_2(t) + g_{12}(t)\,p_1(t)\end{aligned} \tag{4}$$

and $p_j(t)$ is the probability of being in the state $j = \pm 1$; $p_1 + p_2 = 1$. The coupling functions are time-independent for non-interacting members of the network.

Subsequently, we considered a network consisting of L discrete variables located at the nodes of a 2D square lattice. Each unit s_i is a stochastic oscillator and can be found in either of the above two states. For the dynamic complex lattice each element s_i interacts with each of its nearest neighbors and is updated in an elementary time step with transition rate g:

$$g_{12} = g\left(s_i^{+1} \to s_i^{-1}\right) = g_0 \exp\left[\frac{K}{M}\left(M_{+1} - M_{-1}\right)\right] \tag{5}$$

Here M is the total number of nearest neighbors; M_{+1} and M_{-1} are the number nearest neighbors that have made the decision "yes" and "no," respectively. The single individual changes opinion, and as a consequence these numbers are variables fluctuating in time, while the total number of elements is constant. When $K > 0$ a unit who is in the state "yes" ("no") makes a transition to the state "no" ("yes") faster or slower according to whether the majority of the elements are in the state "no" ("yes") or "yes" ("no"), respectively.

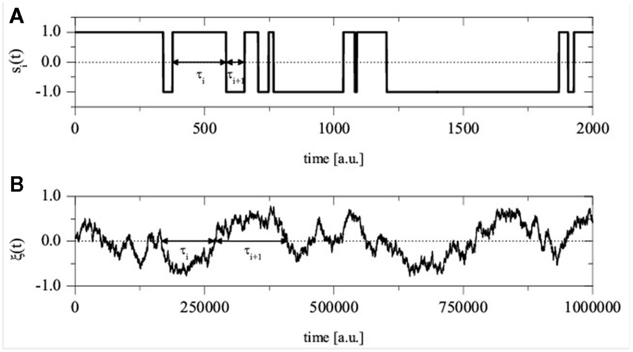

FIGURE 2 | (A) Temporal evolution of a single unit and **(B)** of the global order parameter for the DMM realized on a square lattice with $L = 50$, $g_0 = 0.01$, and $K = 1.70$. Notice the different time scales on the two plots. (From Turalska et al., 2011, with permission.)

Turalska et al. (2011) do all calculations on a $L \times L$ lattice with periodic boundary conditions. They characterize the network in terms of the global order parameter

$$\xi(t) = \frac{1}{L}\sum_{j=1}^{L} s_j, \tag{6}$$

whose variability is not dichotomous. In **Figure 2** an example of the temporal evolution for a single unit is compared with that of the global order parameter.

Note that the amplitude of the global order parameter depends on the value of the coupling constant K. When $K = 0$, single units of the network are independent. When $K > 0$, single units are less and less independent, resulting in a non-zero average. The quantity K_c is the critical value of the control parameter K, at which point a phase transition to a global majority state occurs. In numerical calculations they use the time average $\xi_{eq} \equiv \langle|\xi(t)|\rangle$ as a measure of the global majority. More precisely after an initial million time steps, which is sufficient time to suppress any transients, an average is taken over the same number of consecutive time steps in the DMM.

They find that in the special case when M is the same for all the nodes and $g_0 \ll 1$, that DMM generates the same kind of phase transition as is observed in the 2D Ising model discussed in Onsager's seminal paper (Onsager, 1944). The phase transition for the global variable ξ_{eq} is indicated in **Figure 3** under various conditions. It is evident that the DMM phase transition on a lattice is only equivalent to the Ising model under very restricted conditions. The apparent equivalence between the Ising and DM models is merely formal, since the DMM does not have a Hamiltonian origin and its elements are not in contact with a thermal bath (Turalska et al., 2011). These differences explain why the equivalence requires the transition rate to vanish, so as to freeze the dynamics of the single units, in the absence of cooperation.

When the transition rate assumes a finite value the equivalence between DMM and the Ising model is lost. Turalska et al.

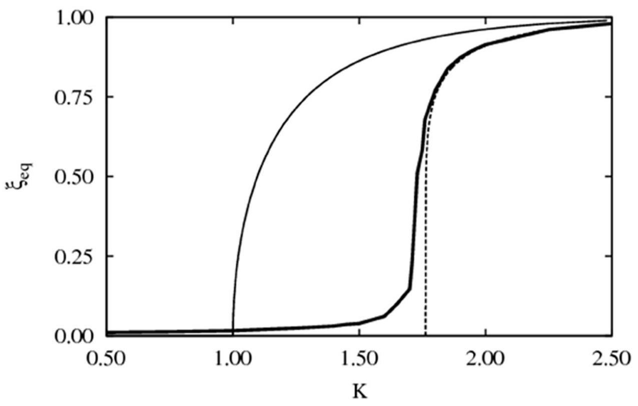

FIGURE 3 | The phase diagram for the global variable ξ_{eq}. The thin solid line and dashed line are theoretical predictions for the fully connected and 2D regular networks, respectively. In both cases $L = \infty$ and the latter case is the Onsager prediction for a 2D regular lattice. The thick solid line corresponds to the global states observed for a 2D regular lattice with $L = 100$ and $g_0 = 0.01$. Periodic boundary conditions were applied. (From Turalska et al., 2011, with permission.)

(2011) investigate the parameter phase space to determine the domain of phase transitions and find that they can occur for values of K below that of the theoretical K_c. There is also a situation for relative high transition rates in which every unit is surrounded by nearest neighbors in the opposite state, yielding an update of its state at every time step and generating the condition in which the order parameter is exactly zero at all times.

Turalska et al. (2011) conjecture that the crossings of the origin by the global order parameter are the significant events to observe. As illustrated in **Figure 2** they interpret the time interval τ between two consecutive crossings as the time duration of a given decision, even if this decision may rest on a slight and fluctuating majority. They evaluate the distribution density of decision–time duration τ, $\psi(\tau)$, and the corresponding survival probability $\Psi(\tau)$, where

$$\Psi(t) = \int_t^\infty d\tau \psi(\tau) . \tag{7}$$

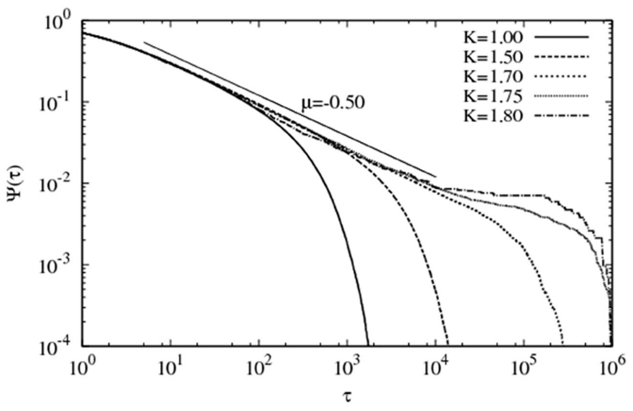

FIGURE 4 | Survival probability function $\Psi(\tau)$ for the global order parameter evaluated on a 2D lattice with $L = 50$, $g_0 = 0.01$ and increasing values of the coupling constant K. The straight line corresponds to a slope of −0.50, namely $\mu = 1.5$ since $\Psi(\tau) \propto 1/\tau^{\mu-1}$. (From Turalska et al., 2011, with permission.)

Although emerging from a simple regular lattice, that is, one with no structural complexity, the survival probability presented in **Figure 4** shows a scale-free property that extends over more than four orders of magnitude in time for $K \approx K_c$ A further increase in the coupling strength does not affect the power–law region.

CONCLUSION

Wiener's Rule maintains that a network with high information can organize one with low information. For example a tightly coupled organization, with rules and policies to cover all contingencies, changes little over time and therefore is low in information. CPM quantifies Wiener's Rule by introducing a measure of complexity allowing us to compare the level of information in interacting complex networks. This measure is determined by the power–law index of the hyperbolic distribution and a generalization of LRT enabled us to construct the cross-correlation cube to determine the degree of asymptotic influence one network has on another. In this way the $1/f$ variability of stimuli is found to resonate with the human brain (Grigolini et al., 2009), as when we are entranced by music or irritated by a dripping faucet (Grigolini and West, 2011; West and Grigolini, 2011).

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William James, Gustav Fechner, and Early Psychophysics

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American psychologist and philosopher William James devoted the entirety of his career to exploring the nature of volition, as expressed by such phenomena as will, attention, and belief. As part of that endeavor, James's unorthodox scientific pursuits, from his experiments with nitrous oxide and hallucinogenic drugs to his investigation of spiritualist mediums, represent his attempt to address the "hard problems" of consciousness for which his training in brain physiology and experimental psychology could not entirely account. As a student, James's reading in chemistry and physics had sparked his interest in the concepts of energy and force, terms that he later deployed in his writing about consciousness and in his arguments against philosophical monism and scientific materialism, as he developed his "radically empiricist" ideas privileging discontinuity and plurality. Despite James's long campaign against scientific materialism, he was, however, convinced of the existence of a naturalistic explanation for the more "wayward and fitful" aspects of mind, including transcendent experiences associated with hysteria, genius, and religious ecstasy. In this paper, I examine aspects of James's thought that are still important for contemporary debates in psychology and neuroscience: his "transmission theory" of consciousness, his ideas on the "knowing of things together," and, finally, the related concept of "the compounding of consciousness," which postulates the theoretical possibility for individual entities within a conscious system of thought to "know" the thoughts of others within the system. Taken together, these ideas suggest that James, in spite of, or perhaps because of, his forays into metaphysics, was working toward a naturalistic understanding of consciousness, what I will term a "distributive model," based on his understanding of consciousness as an "awareness" that interacts dynamically within, and in relation to, its environment.

Keywords: history of psychology, William James, Gustav Fechner, psychophysics, neuroscience, philosophy

Those familiar with the legacy of William James (1842–1910) know him best as the father of American psychology, founded the first experimental psychology laboratory at Harvard, bestowed the first Ph.D. degree in psychology to his student, G. Stanley Hall (1844–1924), and popularized a new philosophical method called Pragmatism. But this list of "firsts" yields an incomplete picture of James the iconoclast who devoted his intellectual life to intractable problems. As one of his earliest biographers aptly suggests, James possessed "the kind of mind which requires an anvil to its hammer, a resistance to overcome – whether unmitigated evil for which to devise a remedy, or stubborn facts on which to think" (Perry, 1967, p. 64). In his lifetime, James persistently defended the scientific study of what he termed the "*wild* facts" of human subjectivity (James, 1983, p. 249). For James, these consisted of mediumistic trance, hallucinations, and religious ecstasy. As he would write, "Anyone will renovate his science who will steadily look after the irregular phenomena" (James, 1979, p. 223). And that is exactly what James did, building an entire philosophy, termed "radical empiricism," around the pluralistic, disjointed, discontinuous range of human novelty he had discovered as a psychologist. Crucially, his philosophy argued for indeterminacy and uncertainty, based on his scientific investigation of the idiosyncrasies of consciousness and the influence of personality on individual free will and volition.

In this regard, James has much in common, both personally and professionally, with his elder nineteenth-century contemporary, German physicist Gustav Fechner (1801–1887) who founded psychophysics, a new field that undertook the empirical measurement and correlation of brain states with sensory experience. Both men were the sons of deeply religious fathers. James's father was a follower of the Swiss mystic Immanuel Swedenborg, while Fechner's father was a minister. Both James and Fechner studied the natural sciences and took formal degrees in medicine but did not practice it. Both ended up professors in fields where neither had taken a doctoral degree. Fechner became a professor of physics at the University of Leipzig, while James became a Harvard professor of philosophy and psychology, based on knowledge independently earned by his obsessive reading in natural science, philosophy, and brain physiology. Both men represented their psychological and philosophical worldviews in ways they believed were compatible with Darwinian evolutionary biology. Finally, since both men's intellectual development took place at a time when science and philosophy were not yet institutionally distinct as disciplines, their contributions were indebted to both scientific exploration and philosophical speculation.

There is much to connect James and Fechner, then, intellectually. Both rejected strictly materialist scientific accounts of the mind–brain relationship and postulated new theoretical scientific frameworks to account for spontaneity, novelty, and evolutionary change within individuals and larger systems. Both were led by scientific questions into metaphysical terrain as a means of helping them to forge new frameworks to account for the novelty they encountered. In keeping with the positivistic spirit underlying the science of their age, both James and Fechner aimed to distil a range of philosophical and scientific ideas concerning the composition of nature and of experienced reality into to a few underlying principles, but, most crucially, without deterministic consequences for individuals. Science, for both men, was based on the entirety of human experience; nothing that could be experienced, therefore, would be excluded from the domain of scientific inquiry. From a Jamesean perspective, this is the very definition of his "radical empiricism." "To be radical," James wrote, "an empiricism must neither admit into its constructions any element that is not directly experienced, nor exclude from them any element that is directly experienced" (James, 1904, p. 315). James thus cultivated a philosophy that would be true to reality as it was experienced: a world of discontinuous flux and novelty. In keeping with that experience, he called radical empiricism "a mosaic philosophy, a philosophy of plural facts" (James, 1904, 315).

This essay follows a similarly mosaic-like structure, providing a brief account of Fechner's psychophysics and its influence on James, in the context of the early history of experimental psychology, the field that served as the final wedge dividing philosophy from psychology. As a cultural historian, I am primarily interested in the sociological consequences of Fechner's and James's contributions to experimental psychology and neuroscience. The controversies each figure inspired highlight pervasive tensions between scientific and speculative epistemologies that continue to have significant consequences for those working in the embattled terrain of the mind sciences. The discussion that follows illuminates James and Fechner's shared scientific and philosophic interests as well as the historical and social contexts for their complex *weltanschauungen*, while suggesting ways in which contemporary neuroscientists continue to draw upon ideas that originated with Fechner and James.

FECHNER AND JAMES: SCIENTIST–PHILOSOPHERS

Though historians of science have described Fechner's epistemology as monistic, devoted to identifying the natural world with single origins, whether of a transcendent or mechanical order (Marshall, 1974), his science and philosophy are in fact closer to James's pluralism. German science historian and neuroscientist Michael Heidelberger, who has written the most recent and most comprehensive monograph on Fechner's intellectual origins and the significance of his scientific and metaphysical works, describes Fechner as "a radical empiricist with a phenomenalist outlook" (Heidelberger, 2004, p. 73). For Fechner, empirical observations came first. Subsequently, these observations became the basis for his metaphysical and natural-philosophic speculations regarding the constitution of the universe and the nature of human perception. For Fechner, the psychical (or mental) and the physical (or material) were different modalities of experience. Like the opposing sides of a single coin, the psychical and the physical were functionally parallel; they operated simultaneously, yet maintained an interdependency that was not linked by causality (Marshall, 1982; Heidelberger, 2004). What has been termed Fechner's "double-aspect" view of the psychical and the physical, first described in his metaphysical work *Zend-Avesta* [*Zend-Avesta oder über die Dinge des Himmels und des Jensits. Vom Standpunkt der Naturebetrachtung*] (1851), postulated a functional relationship between human experience and perception. Fechner declared that this idea came to him in a flash of intuitive insight in the wee hours of October 22, 1850, when he awakened to the realization that "the functional relation between mental and physical might be construed logarithmically." The term Fechner coined, "psychophysics," stood for "physiological bodily processes immediately accompanying psychical events" (Marshall, 1982, p. 71, 80). His subsequent work, *Elements of Psychophysics* (1860) [*Elemente der Psychophysik*] delineated a mathematical means (later refined by Weber) of expressing this relationship that influenced mathematicians, such as James's friend Charles Sanders Peirce, in the USA, and the founders of German experimental psychology, Ernst Mach and Wilhelm Wundt, whose Leipzig laboratory was the training ground for a generation of American experimental psychologists who followed James.

James's knowledge of Fechner's psychophysics, chronicled in his student notebooks, began as early as 1868 while studying psychology under Wundt and Herman von Helmholtz in Germany. As Marshall has shown, James was not entirely at ease with Fechner's formula for correlating physical sensations with mental cognition. In his 1876 essay, "The Teaching of Philosophy in Our Colleges," James dismissed Fechner's psychophysical formula, writing, "It is more than doubtful whether Fechner's 'psychophysic law' (that sensation is proportional to the logarithm of its stimulus) is of any great *psychological* importance" (qtd. in Marshall, 1982). Subsequently, James wrote condescendingly of Fechner, calling him that "dear old man," in his *Principles of Psychology* (1890). No amount of statistical measurement, James argued, could overcome the problem of retrospective analysis of perception and external stimuli, a problem that, for James, stemmed from "the misleading influence of speech" (James, 1981, p. 193). In other words, the brain cannot speak for itself; it requires human agents to interpret its responses to stimuli. While James concluded with the experimental psychologists that "introspection is no sure guide to truths," he nonetheless found fault with the strict statistical empiricism of the experimentalists on the grounds that "the poverty of the psychological vocabulary leads us to drop out certain states from our consideration, and to treat others as if they knew themselves and their objects as the psychologist knows both" (p. 196).

In part because of James's original dismissal of Fechner's psychophysics in *Principles of Psychology* (1890), we have overlooked both James's later indebtedness to Fechner for his radical empiricism and, consequently, James's potential contributions to contemporary neuroscience. By the time James delivered his Hibbert lectures comprising *A Pluralistic Universe* (1904), in which he devoted his fourth lecture to the "compounding of consciousness" and Fechner's panpsychic worldview, much had changed in psychology and in modern physics that would make James more receptive to Fechnerian thought. Almost a decade earlier, in his lecture "On Human Immortality" (1896), James made the German physicist his intellectual ally in exploring the most intractable of problems facing the mind sciences: the mind's relation to the brain and that of consciousness to human embodiment. James was most attracted to Fechner's metaphysical ideas, set forth in such works as *Little Book of Life After Death* [*Das Büchlein vom Leben nach dem Tode*] (1835; to which James wrote the introduction to the 1904 English translation) and *Zend-Avesta*, subtitled, " On the Things of Heaven and the Afterlife: From the Standpoint of Meditating on Nature" (1851). These metaphysical works lay the groundwork for Fechner's important 1861 *Elements of Psychophysics* (Marshall, 1982; Heidelberger, 2004), and influenced James's radical empiricism, a philosophy that promotes a theoretical middle ground between strictly materialist and strictly metaphysical means of addressing the mind–brain problem.

This essay not only provides a cultural and historical framework for situating James's psychology in the context of psychophysics, the field Fechner pioneered, it also traces a genealogy of descent from Fechner to James and to James's modern descendents in the mind sciences. This requires us to look back upon the nineteenth-century divide between philosophy and psychology as a necessary starting point for understanding the on-going tensions and debates between contemporary psychology and neurophysiology. As experimental psychology in the USA was coming into its own as a scientific discipline in the late nineteenth century, it opened up disputed borders between the natural sciences and philosophy, raising significant questions about whether or not philosophy could meaningfully contribute to scientific progress (Bordogna, 2008). In the following section, I will first discuss James's formal and informal education in the natural sciences and his investigation of brain physiology before going on to show how Fechner's psychophysics relates to James's most significant psychological theories for contemporary neuroscience: "transmission theory," "the compounding of consciousness," or co-consciousness, and, finally, the radical empiricist scientific framework James developed before his death in 1910. Finally, I will conclude by presenting the ideas of several contemporary psychologists, neuroscientists, and philosophers whose thought is indebted to James's unique understanding of psychophysics, while further suggesting ways that James's philosophical work remains significant for the mind sciences.

THE FORCE OF HUMAN WILL

The "zig–zag" course with which James's career is said to have unfolded is not quite as inconsistent as accounts by his biographers and intellectual historians – or even James himself – would have us believe (Perry, 1967; Simon, 1998; Feinstein, 1999; Richardson, 2007). In fact, James's earliest diaries and reading notebooks show a remarkable consistency in his study of "energy" and "force," concepts from physics to which James attributed a psychological dimension, in part as a consequence of his daring fusion of philosophical idealism with British and Germanic strains of scientific materialism. Will, habit, and attention form the Jamesean triumvirate of psychological inquiry concerning subjectivity and volition, dating from his earliest days as a student. James's artistic training with the renowned painter William Hunt cultivated his keen eye for empirical observation and accounts for the significance the role of attentive perception plays in his philosophical and psychological works (Feinstein, 1999; Leary, 2002). Between 1864, the year James enrolled at Harvard's medical school, and 1875, the year he launched Harvard's and the nation's first experimental psychology laboratory, he had read Laplace in mathematics; Newton, Maxwell, and Planck in physics; Hughlings-Jackson in neurology; Spinoza, Leibnitz, Descartes, and Schopenhauer in philosophy; and Galton, Spencer, and Wundt in psychology (Taylor, 1996, p. 73). He was well versed in neurology and was deeply influenced by German laboratory science, for he had studied physiology and experimental neurology in Berlin, and experimental psychology at Heidelberg under Wundt and Helmholtz. In 1875, the same year James opened his experimental psychology laboratory at Harvard, he also gave a series of 10 lectures at Johns Hopkins on "The Brain and the Mind."

Any consideration of James's renewed interest in Fechner's psychophysics in his later years, then, must begin with James's life-long study of energy and force as components of human intellectual endeavor, from the ravings of the insane to the inspiring cognitive leaps of genius. James enrolled at Harvard the same year James Clerk Maxwell's (1831–1879) *A Dynamical Theory of the Electromagnetic Field* (1861), inaugurated a unified "field theory" of electromagnetism. At Harvard, he studied chemistry and read Michael Faraday's (1791–1867) *Experimental Researches in Chemistry and Physics* (1859), the landmark work describing his experiments with "electromagnetic induction" (Richardson, 2007, p. 51). In his second year at Harvard's Lawrence Scientific School, James began independently studying philosophical and scientific works related to matter and force. Among these were British physicist William Robert Grove's (1811–1896) *On the Correlation of Physical Forces* (1846), a book that anticipates by 1 year Hermann von Helmholtz's ground-breaking theory on the conservation of energy. James read German orientalist and philologist Max Müller (1823–1900), and Ludwig Büchner's (1824–1899) *Kraft und Stoff: Empirisch-naturphilosophische Studien* (*Force and Matter: Empiricophilosophical Studies*, 1855). From Müller he took the idea that the mind as a kind of force that possesses a little understood power of "synthesis," or "of joining two or more ideas and contemplating them in their mutual relations as one."[1] From Büchner, James copied down the materialist maxims that "force and matter are inseparable" and that "matter is imperishable." (Croce, 1995, p. 108). Years later, James gave the concepts energy and force the following psychological formulation: "matter is motion, motion is force, force is will" (Richardson, 2007, p. 51). In the midst of all this heady reading in physics, James also read Charles Darwin on the origin of species and Jonathan Edwards on original sin. Natural science, physics, and Christian theology supplied the intellectual and philosophical ballast for James's understanding of consciousness as a component of the personal subjectivity known as the "self," and the cultivation of the will, which formed the basis for individual beliefs and subsequent actions. The study of what motivates individual and group choices and actions, then, form the core of James's person-centered investigation of consciousness.

What James recognized, more than anything, was the powerful force of ideation, both socially and epistemologically. Moreover, when it came to the study of human consciousness, he underscored

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[1] "Reading Notes and Observations; Sketches," bMS Am 1092.9 (4497). William James Papers (MS Am 1092.9–1092.12). Houghton Library, Harvard University.

the impossibility of overcoming first-person narration. No matter how carefully one attempts to purge figurative speech and metaphor from scientific discourse, a human agent (with all its attendant messiness and subjectivity) is at the center of it. Furthermore, the translation and interpretation of observed or experienced facts into scientifically meaningful "events" – particularly in the case of the mind sciences – necessarily reduces complex inner states to static principles and formulae that describe physiological functions, while providing little account of how or why complex mental states come into being (James, 1981). The problem for James, as it was for Fechner, in his philosophy and in his psychophysical formula, was how to connect the subjective experience of inner psychological states with the so-called "external" facts of perception and sensory experience. This is where an understanding of James's interest in physics allows us to pick up the lost thread of the more technical and scientific aspects of his philosophical thought.

THE DEMISE OF THE PHILOSOPHER–SCIENTIST AND THE RISE OF THE NEW PHYSICS

In the early twentieth century, the philosopher's displacement by the natural scientist as an authoritative public spokesperson for secular values played an essential role in the modernization of American intellectual life. James was a transitional figure in this movement, as he worked between shifting disciplinary borders, namely physiology and medicine on the one hand and the philosophy and psychology of religious and transcendent experience on the other. Throughout his lifetime, the respect James would earn in one domain often came at the expense of the other. For example, many psychologists maintain that after James published his landmark two-volume work, *Principles of Psychology* in 1890, he made no further contributions to the field (see Taylor, 1992, 2002, 2003, 2010). James had always to negotiate between the "professional" standards of his scientist colleagues and the unorthodox research he believed was necessary for psychology's advancement, but that ultimately undermined his professional scientific authority (Bjork, 1983). The "wild facts" of human experience that interested James, were, from the standpoint of scientific positivism, not facts at all, only epiphenomena, rogue brain activity that was not only unclassifiable, but also unworthy of sustained scientific investigation.

The larger problem for James, however, was how to overcome the epistemological and methodological problems related to identifying how interior states of consciousness correspond to physiological processes, an effort that necessarily would rely on accurate self-reporting and careful observation by investigators. From James's perspective, investigators would have to be self-observers as well, attuned to the ways in which their own biases might influence and predetermine results. In today's terms, an investigator serves as a witness to the self-reporting of phenomena, in tandem with seemingly "objective" visualizing technologies, such as EEG, or other means of visually representing internal cognitive processes. But these are merely descriptive of processes and fail to address the more complex actions that lead to individual and collective decisions, including the decision-making of researchers themselves. To address this problem, investigators have called for a more "phenomenologically oriented psychology," one that focuses on "the phenomenology of the science-making process itself, and the experimenter as the new confounding variable in the conduct of experiments" (Taylor, 2010, p. 411). The study of "neurophenomenology," writes Taylor, would address an epistemological divide between the neuroscientific and philosophy of mind approaches. The problem is that the brain is physical, while the "mind" is impossible to locate; to be more accurate, "the mind is a metaphor for experience" (Taylor, 2010, pp. 421–422). And this experience, for James, was riddled with inconsistencies. There is no transcendently true experience that holds for all individuals at all times.

James's championing of the discontinuity, indeterminacy, and flux that characterized his psychical research was anathema to psychologists invested in systematizing psychology by promoting experimental methods with reproducible, certain results, identifying psychophysical laws, and charting the brain's neurophysiologic coordinates for mental response. Indeed by the 1890s, James was declared the "nemesis" of all self-respecting psychologists invested in having psychology taken seriously as a scientific discipline. Those, like Hall, who had initially been his allies in founding the American Society for Psychical Research (ASPR) in 1884, fled its ranks to launch the American Psychological Association in 1890, what would become the bastion of scientific respectability for the new field (Coon, 2002, p. 129).

Experimental psychology promoted by Hall at Clark University, James Rowland Angell at the University of Chicago, Hugo Münsterberg at Harvard University, and Edward Bradford Titchener at Oxford and Cornell Universities – and later canonized by Edwin G. Boring's monumental *History of Experimental Psychology* (1929) – displaced an older American tradition of introspective self-scrutiny going back to the Puritans. The reigning narrative set forth by Boring of the American experimental psychologist's descent from a German laboratory tradition, as psychology historians suggest, has occluded the significant role that the religiously schooled early American "mental philosophers" played in the development of American psychology; moreover, transcendentalist and Swedenborgian elements pervade James's revaluation of ecstatic religious experience as a valid source of intuitive knowledge (Fuchs, 2002, pp. 79–84; Taylor, 1996, p. 182; Taylor, 2002). Something else, however, accounts for James's invigorated metaphysical speculations after 1896. As religious historian Catherine Albanese has shown, the unknown and mysterious new forces unleashed by the discoveries of modern physics breathed new life into these older, mystical traditions belonging to the early Americas (Albanese, 2007).

If we track James's major publications with the discoveries that distinguish modern physics from the mechanistic, Newtonian worldview, the influence of the new physics on James's thinking becomes clear. James developed his "transmission theory" of consciousness in 1897, in which he described the brain's transmissive function in terms of invisible "rays," a mere 2 years after Wilhelm Roentgen's discovery of the X-ray. In 1902, the year Marie and Pierre Curie discovered the invisible element, radium; James lectured on mystical experience and the "reality of the unseen," later published as *The Varieties of Religious Experience.* James's heavily annotated copy of mathematician and philosopher of science Karl Pearson's second edition of *The Grammar of Science* (1900; a text that deeply influenced Einstein) highlights the ways in which James's thought had taken a relativistic turn. Indeed, the lectures comprising *A Pluralistic Universe*, forming the basis for his philosophy called "radical empiricism," were published in 1904, just 1 year before Einstein arrived at his special theory of relativity. What I want to suggest is that these new discoveries in physics gave James a lexicon for describing consciousness, and reality itself, as palpably physical yet luminously immaterial. Moreover, what the historian Henry Adams at the turn-of-the nineteenth-century described as the "supersensual" domains disclosed by modern physics gave James a conceptual underpinning for his *Pluralistic Universe*, a universe honoring novelty, discontinuity, and ceaseless change within subjective experience. As he would write of consciousness, "motion there obeys no Newton's laws" (1922, p. 34). Building on the electromagnetic "field theory," which represented physical reality as interpenetrating, "continuous fields" (Einstein 269), James's 1890 *Principles of Psychology* described consciousness in terms of a spherical structure, composed of a "halo," a "penumbra," or a "fringe" radiating outward from a central awareness, what we might think of as a unified "self."

While James turned to physics for insights regarding the "force" of the human mind, he turned to philosophy for explanations. The kinds of questions James pursued in his physiological study of the brain led him to philosophy and metaphysics for answers. James's multidisciplinary approach to the study of mind combined his knowledge of natural history, psychology of religious experience and abnormal mental states to affirm a non-reductive materialism, a "softer" positivism, similar to that of Fechner. Radical empiricism, furthermore, marked James's attempt to refute the positivism of his skeptical peers with a philosophical framework that would justify the scientific investigation of dissociative trance, abnormal and ordinary subjective mental states, associated with volition. James's radical empiricism was ahead of its time in suggesting that what we think of as "mind" is a consequence of many interpenetrating systems, a result of the brain's interactions with the environment, but not reduced to brain physiology or external stimuli alone. Though James had the philosophical framework in place, he lacked the technical scientific background to make it useful to the scientific study of consciousness. Therefore he turned to Fechner for the means of substantiating his theory of transmission and co-consciousness.

FECHNER'S PSYCHOPHYSICAL THRESHOLD AND JAMES'S COMPOUNDING OF CONSCIOUSNESS

First off, let us review a brief chronology of James's concern with the problem of the compounding of consciousness. Two versions of compounding appear in James's thought: (1) the compenetration of individual thoughts within the "stream" of consciousness; in other words, "compounding" describes the process by which thoughts and perceptions are filtered and influenced by a past history of perception and succeed each other in time. (2) The compenetration of individual fields of consciousness within a larger panpsychic world system, a hypothesis that presumes that all organic systems are perceiving and sensate. James initially launched a discussion of the first version of the phenomenon he called "compounding" in his "mind stuff" chapter of the *Principles of Psychology* (1890), in which he endeavored to purge psychology of all metaphysical speculations, focusing only on empirical data relating to brain physiology and sensations. As James understood it then, "compounding" had to do with the question of whether simpler mental states or perceptions could give rise to more complex mental states. James's answer to this question was a decisive "no." Rather than perceive individual thought or sensations as discretely separate parts split off from each other, James represented "consciousness" as a meandering "stream" in which thoughts and sensations are not disjointed but "flow" successively one into the next. James revisited the theme of compounding in his 1894 presidential address to the American Psychological Association on "The Knowing of Things Together." Try as he might, however, James could not altogether divorce psychology from metaphysical questions. "On Human Immortality" (1898) invokes the second version of compounding, which involves considering the collective compenetration of individual minds or consciousnesses; this form of compounding is possible, James suggests, if we take the brain to be a "transmissive," rather than productive organ; Lecture V in a *Pluralistic Universe* (1904) continued this theme by postulating the existence of compenetration fields of individual awareness, building on his theory articulated in "A World of Pure Experience" (1904) that thoughts compenetrate other thoughts in the form as "co-conscious" transitions "by which one experience passes into another when both belong to the same self" (James, 1922). Collectively these essays represent what I would like to call James's populist "metaphysics for the masses," ideas that would challenge, yet ultimately not disturb individual religious needs. James's Pragmatism, after all, was not concerned with proving the ultimate truth of individual beliefs but affirmed their ethical value for helping individuals lead more fulfilling and meaningful lives. Yet these popular lectures and essays, which have received the most sustained critical attention, belie James's stringent efforts to fulfill the strict empiricist criteria of Fechner's psychophysics.

While James's use of terms culled from physics was more poetical than technical, his ideas anticipate more recent understandings on the part of contemporary neuroscientists that mind is an emergent property of the nervous system's engagement with its environment. James would have agreed with the recent consensus that identifying the neuronal correlates to consciousness alone will not address the "hard problems" concerning the *how* and *why* of subjective experience. The mind theorists whose ideas most resonate with those of James – from the Australian philosopher David Chalmers, to science historian and Buddhist practitioner B. Alan Wallace, phenomenologist Evan Thompson, and biomedical engineer Paul Nuñez – each postulate an interdependency of consciousness on the structure of reality itself. They approach the hard problem of consciousness by focusing on the "explanatory gap" between consciousness and the natural world. To understand the manifold attributes of consciousness, in relation to, but not reducible to neuronal networks, they argue, requires taking a closer look at the structure of reality. In recognition that consciousness and reality are co-constitutive, researchers are turning to dynamic systems, or complexity, theory to synthesize the efforts of neurobiology, phenomenology, and psychology in order to arrive at a better understanding of consciousness as a constituent component of reality itself.

JAMES'S TRANSMISSION THEORY OF CONSCIOUSNESS

James anticipates these scientists by positing a structure of reality more in keeping with the discoveries of modern physics: reality was more than what the eye itself could see. Material substance, far from being physically inert, was composed of invisibly moving, highly charged particles, and permeated by invisible rays. In keeping with these new understandings of reality as composed of invisible substances, James's transmission theory of consciousness builds upon his earlier writing in which James described consciousness's outer barrier as a "haze," "penumbra," or "halo." Consciousness (that awareness that we think of as the "self") is encompassed and surrounded by a permeable "fringe," suggesting a model for consciousness that is both broad and diffuse, in touch with environmental phenomena of which individual persons are not always consciously aware. Potentially, for James, as we shall later see, the outer "fringes" or "fields" of each individual consciousness touch upon other fields in ways that multiply or compound fields within fields of other subjective experience (Barnard, 1997).

James presented his "transmission theory" in the most unlikely of contexts. He had been asked by Harvard to give the annual Ingersoll Lecture, named after a deeply religious alumnus, one Caroline Haskell Ingersoll, who bequeathed money to Harvard to advance the study of the afterlife. Bemused by his selection, and hardly feeling himself an appropriate choice, James admitted dryly that he was selected not "because he is known as an enthusiastic messenger of the future life," but "apparently because he is a university official" (James, 1992b, p. 1100). James's rhetorical stance in "On Human Immortality: Two Supposed Objections to the Doctrine" (1898/1900) was a strategic one; as a scientist tasked with making psychology into a respected science, he could not risk throwing it back into a mire of metaphysical speculation. Therefore, James asked his audience to take as gospel "the great psycho-physiological formula: *Thought is a function of the brain*" (James, 1992b, p. 1104). If we take this formulation as a given, James asked, "Does this doctrine logically compel us to disbelieve in immortality?" James then bases the rest of his lecture on a philosophical thought experiment in which he uses the concept of immortality, or the survival of human consciousness beyond bodily death, to hypothesize a possible structure of consciousness in relation to the human brain quite apart from functional dependency. In place of the production theory, James argued for the brain's "permissive" or "transmissive" potential, in which the brain acts as a filter to information coming from outside.

As James himself argued, "My thesis now is this: that, when we think of the law that thought is a function of the brain, we are not required to think of productive function only: *we are entitled also to consider permissive or transmissive function.* And this the ordinary psycho-physiologist leaves out of his account" (James, 1992b, p. 1110, emphasis in original). In describing the brain as analogous to a "prism, or a refracting lens," which transmits light, or to a pipe organ, through which air produces sounds, but is not itself "engendered in the organ" (James, 1992b, pp. 1109–1110), James argued that "mind is not generated by the brain but instead focused, limited, and constrained by it" (Kelly et al., 2007, p. xxx). Postulating that "*our brains* are such thin and half-transparent places in the veil" of nature, James went on to suggest that "the genuine matter of reality. . . will break through our several brains into this world in all sorts of restricted forms, and with all the imperfections and queernesses that characterize our finite individualities here below" (James, 1992b, p. 1111). For James, the phenomenon we think of as "mind," cognition, or mental awareness, is a consequence of the brain's behaving as a kind of receiving station to "the genuine matter of reality" transmitted by the environment. There is, of course, a peculiarly Jamesean legerdemain in not naming the substance of this reality, except through suggestive metaphors: invisible light, the trajectory of an arrow shooting through air, air passing through the apparatus of a pipe organ or, more poetically as a "white radiance." Consciousness was a "sphere of being" that is "continuous" with "that more real world" (James, 1992b, p. 1111). Of what invisible substance "genuine" reality was composed, James would leave to others to discover.

James was not the first to argue that the brain functioned as a "filter" to consciousness, or to argue for experiential flux as a pervasive aspect of reality. As James himself acknowledged, philosophers Immanuel Kant and F. C. S. Schiller made similar arguments. Kant, for example, maintained that the body restricts the intellectual function of the brain, which only comes into full flower after death. Schiller similarly argued that matter restricts "the consciousness which it encases" (James, 1992b, p. 1119, n9). James's "transmission theory" was indebted to the ideas of at least two other key individuals: psychophysicist Fechner and Frederic H. Myers, founder of the British Society for Psychical Research. James's transmission theory was modeled in part on Fechner's "conception of a fluctuating psychophysical threshold" (Kelly et al., 2007, p. 29), while his notion of the self as an entity that contains a plurality of possible mental states and secondary "personalities," drew upon Myers's concept of a "subliminal" or "supraliminal" Self – an entity that encompasses a field of broader awareness coexistent with a subject's more narrow sense of a coherent self, but that is not necessarily restricted by or even known to that primary self.

In developing his "transmission theory," James had refined Myers's theory of the Subliminal Self by being the first to explicitly link "notions of transmission and filtering with the brain" (through the metaphor of the "prism" through which light passes), only to come out on "the other side filtered, reduced, focused, redirected, or otherwise altered in some systematic fashion" (Kelly et al., 2007, p. 606). On the face of it, James's "transmission theory" with its metaphors of a prismatic dome and pipe organ may sound like outlandish metaphysical claptrap, but, in fact, these metaphors suggest models that resemble more recent conceptions of mind–brain dynamics. James's model of the brain as a "filter," or, in contemporary terms, a "nested hierarchy" (Nuñez, 2010, p. 11), for processing information from the environment posits the mind and environment as co-dynamic, mutually constitutive entities. In a different context, James would describe this "permissive" or "transmissive" function of the brain as a kind of "Marconi station" (James, 1986, p. 359). Making no reference to James's transmission theory, biomedical engineer Paul Nuñez then goes on to posit "a highly speculative" account of consciousness that is nonetheless dramatically similar to that of James when he describes how "whole brains or special parts of brains might behave like antenna systems sensitive to an unknown physical field or other entity that, for want of a better name, may be called Mind" (Nuñez, 2010, p. 274). In this way, James's account of the brain's "transmissive" properties resembles more contemporary accounts assigning the mind–brain specific temporal–spatial dimensions and a hierarchical structure.

THE "KNOWING OF THINGS TOGETHER" AND THE COMPOUNDING OF CONSCIOUSNESS

When James discussed the "knowing of things together" he was initially thinking of the phenomenon of how individuals experience the sensation of their thought as one continuous succession, as ideas co-penetrate, as attention wanders, or as one shifts one's awareness to a new introspective thought or aspect of the environment. In "A World of Pure Experience," James described the "conjunctive relation," or "co-conscious transition... by which one experience passes into another when both belong to the same self," as that which "has given the most trouble to philosophy." He would go on to say that my and your experiences may be "with" each other externally, "but mine pass into mine, and yours pass into yours in a way in which yours and mine never pass into one another. Within each of our personal histories, subject, object, interest and purpose are continuous or may be continuous. Personal histories are processes of change in time, and the change itself is one of the things immediately experienced" (James, 1922). Philosopher David Chalmers's "double-aspect theory of information" makes a similar point. Adopting "information" as a basic principle of consciousness, Chalmers maintains that "information" represents the "basic structure of [a] *difference* [of] relations between its elements, characterizing the ways in which different elements in a space are similar or different, possibly in complex ways" (Chalmers, 2010, p. 25). For Chalmers, as for James, what consciousness perceives or apprehends, then, is actually a *difference* among relations.

When James later discussed the "compounding of consciousness" in the fifth chapter of his *Pluralistic Universe*, however, he extended this idea to include that of other consciousnesses, writing,

> My present field of consciousness is a centre surrounded by a fringe that shades insensibly into a subconscious more... The centre works in one way while the margins work in another, and presently overpower the centre and are central themselves. What we conceptually identify ourselves with and say we are thinking of at any time is the centre; but our *full* self is the whole field, with all those indefinitely radiating subconscious possibilities of increase that we can only feel without conceiving, and can hardly begin to analyze (1977, p. 130).

In writing this, James was thinking expressly of Fechner's psychophysical threshold, now known as the Weber–Fechner law, postulating that "consciousness" is the threshold at which subjective perception and subjective sensation coincide. James was less interested in the mathematical formulation for this law than he was in the assigning of temporal–spatial movement to consciousness. These "movements," as James would write in his introduction to the English translation of Fechner's *Little Book of Life and Death*, "can be superimposed and compounded, the smaller on the greater, as wavelets upon waves. This is as true in the mental as in the physical sphere. Speaking psychologically, we may say that a general wave of consciousness rises out of a subconscious background, and that certain portions of it catch the emphasis, as wavelets catch the light... On the physical side we say that the brain-processes that corresponded to it altered permanently the future mode of action of the brain" (1904, p. xv). What James was arguing – drawing upon Fechner's model of the threshold of consciousness as a sinusoidal wave – is richly suggestive of dynamical systems. James's point of view similarly accords with that of phenomenologist Evan Thompson, who collaborated with the late Francisco Varela to write *Mind in Life* (2007). In this phenomenological account of neurophysiological processes, Thompson understands "dynamical systems" as "a collection of related entities or processes that stands out from a background as a single whole, as some observer sees and conceptualizes things" (Thompson, 2007, p. 39). The solar system is one such example, but James's transmission theory offers the example of the social environment, in which one consciousness coexists among many others. In a very real sense, the compounding of consciousness suggests the co-penetration of individual consciousnesses within ever larger and interpenetrating systems.

This idea that consciousnesses themselves co-penetrate is made explicit in an even earlier passage, from the first lecture in *A Pluralistic Universe*. In distinguishing monism from his philosophical pluralism, James writes: "My thoughts animate and actuate this very body which you see and hear, and thereby influence your thoughts. The dynamic current somehow does get from me to you, however numerous the intermediary conductors may have to be. Distinctions may be insulators in logic as much as they like, but in life distinct things can and do commune together every moment" (James, 1977, pp. 115–116). The world of a *Pluralistic Universe*, is just such a dynamical system comprised of a world of interconnecting relations, of "complexity-in-unity" enveloped by a surrounding "earth-consciousness" (James, 1977, p. 73; James, 1909, 1910). And here we finally arrive at the panpsychic view James adopted later in life and attributed to Fechner. What exactly panpsychism means, particularly for James has been the source of much misunderstanding in James scholarship.

Just what is this "panpsychic view" and how does it correspond to contemporary neuroscientific debates about consciousness? James scholar David Lamberth distinguishes James's "moderate" panpsychism from the "strong" or "idealistic" versions held by his contemporaries. The basic tenet of panpsychism is that nature is animate. More rigid versions are dualistic, positing an essential correspondence between the psyche and nature. The "pluralistic panpsychism" that James embraced allowed him to develop "a pluralistic metaphysics of pure experience and a correspondingly pluralistic notion of causality" (Lamberth, 1997, p. 250). This philosophical position of James's strongly accords with the contemporary neuroscientific theory of "dynamic co-emergence," held by Thompson and Varela, in which living and mental processes are understood as "unities or structured wholes rather than simply as multiplicities of events external to each other, bound together by efficient causal relations" (Lamberth, 1997, p. 67). In phenomenological terms, this means revising our understanding nature as "not pure exteriority," but rather as possessing "its own interiority." Thompson is careful to distinguish this perspective from "metaphysical idealism," the argument for a "preexistent consciousness." Instead, it implies a "transcendental orientation" by which we understand that "the world is never given to us as a brute fact detachable from our conceptual frameworks. Rather, it shows up in all the describable ways it does thanks to the structure of our subjectivity and our intentional activities" (Lamberth, 1997, p. 82). James would understand this in terms of an inherent intimacy of relations between the self and the world with which the self engages. Consciousness itself is "transcendent," in Thompson's terms, in part because, as he says, it "is always already presupposed as an invariant condition of possibility for the disclosure of any object[;] there is no way to step outside, as it were, of experiencing subjectivity, so as to effect a one-to-one mapping of it onto an external reality purged of any and all subjectivity" (Lamberth, 1997, p. 87). Consciousness seems defined then by some variable movement or change in time that is perceived differently in relation to one's location in time and space, and that also depends upon one's particular role and orientation toward the experiment, that is, whether one is experiencing mental phenomena as a subject in an experiment or as the witnessing and recording observer. In light of Thompson's phenomenological orientation toward the mind–brain conundrum, it is this intersubjective dimension that becomes most salient to the future of contemporary mind–brain research.

James's metaphors of "stream," "halo," and "penumbra" to describe what has been termed a "fringe" consciousness describe a structure for consciousness that is, in my words, a "distributed" one. To explain what I envision by the term "distributed," I will use a familiar metaphor from the natural world. Imagine a tree in winter: a single trunk gives rise to smaller branches, forming the essential architecture of the tree; from these branches, smaller ones grow, giving rise to even smaller, finer branches as the tree extends upward and outward. Imagine, if you will, a whole forest of such trees, whose branches co-penetrate to a greater or lesser extent, depending on their proximity to one another, or upon other natural forces in the environment: a gust of wind, birds alighting, rain or snow falling on the branches. It would not be hard to imagine this "system" of co-penetrating branches in still other naturalistic forms: a flock of birds, a school of fish, a moving crowd, or bundles of neurons within a human brain, as each individual within the larger system imperceptibly shifts in relation to the subtle movements communicated at a subconscious level. These images are not hierarchical and they are not necessarily linear, for, at any point within the system a single movement, or a random complex of movements among disparate individual parts could produce something like the perception, to an observer, of cooperative "decision" within the system as a whole. But the observer is also part of the system, and we now arrive at a problem that links physics indelibly to consciousness as part of the "measurement problem" in quantum physics.

The observer's volitional role of visually arresting an object in space in the act of perception is deeply problematized by the phenomenon known as the "collapse of [the] wave function" in quantum physics. As B. Alan Wallace explains, "quantum measurement entails the 'collapse of a wave function,'" in which measurement itself involves selecting one alternative from "a range of probabilities." This selection thus forces a "reduction" in which "all the alternatives vanish." This "reduction postulate" attempts to "describe what is actually observed in the measurements of quantum systems using classical methods" (Wallace, 2007, p. 81). Building on Michael Mensky's "many-worlds interpretation," Wallace argues for an abandonment of classical methods and a recognition that "Consciousness does not mechanically cause the wave function to collapse or influence physical particles. Rather, the observer's brain and the observed system are synchronously entangled" (Wallace, 2007, p. 82). The measurement problem has brought increased attention to the role that the observer's "cognitive frame of reference" plays in studies of consciousness, particularly in acquiring the first-person accounts necessary for an empirical study of subjectivity. As a Buddhist adept, Wallace maintains that scientific observers should integrate "contemplative methods of inquiry" into the study of mind; only by acquiring heightened powers of mental concentration, will scientists develop more reliable first-person accounts of subjectivity (Wallace, 2007, p. 105). Thompson, whose phenomenological approach to the mind–brain problem we have just seen, similarly argues for the need for observers to "suspend or refrain from judgment," and "to develop more explicitly the pragmatics" of such practice "as a first-person method for investigating consciousness" (Thompson, 2007, p. 20). James's concluding remarks in his *Pluralistic Universe*, anticipates the words of both Wallace and Thompson, when he urges his listeners to "discriminate 'theoretic' or scientific knowledge from the deeper 'speculative' knowledge aspired to by most philosophers, and concede that theoretic knowledge, which is knowledge about things, as distinguished from living contemplation or sympathetic acquaintance with them, touches only the outer surface of reality" (James, 1977, p. 111). This more philosophic attitude of receptivity, delineated by Thompson, is one that James pioneered in his radical empiricist philosophy and in his life-long willingness to attend to the less clear-cut aspects of individual psychological experience.

CONCLUSION: RADICAL PHENOMENALISM

Contemporary neuroscientists attempt to resolve the "explanatory gap" between mind and nature. James attempted this linguistically by adopting metaphors for the structure of consciousness that served to reconcile Darwinian evolutionary theory with discoveries in the physics of his day. Both models helped him explore intractable, yet fundamental, epistemological, and ontological questions: Was the universe self-unifying and ordered according to absolute metaphysical or mechanical causes, or was it inherently discontinuous with human perception? Correspondingly, he asked, What is the nature of human consciousness itself, and how do we account for our awareness of our thoughts or of the sensation of possessing a unified "Self?" To put the matter as succinctly as possible, as a philosopher and as a psychologist James was interested in understanding the relationship between the one and the many, the "each form," as he termed it, and the "universal," or "all form" (1977, p. 20). His writings emphasize Darwinian "variety," and "struggle," while invoking invisible particles and forces resonant with Faraday, Maxwell, and Hertz's representation of physical reality as interpenetrating, continuous fields. A passage from his 1890 *Principles of Psychology*, illustrates this fusion of scientific world views that pervades James's thought:

> The mind… works on the data it receives very much as a sculptor works on his block of stone. In a sense the statue

> stood there from eternity. But there were a thousand different ones beside it, and the sculptor alone is to thank for having extricated this one from the rest. Just so the world of each of us, howsoever different from our several views of it may be, all lay embedded in the primordial chaos of sensations, which gave the mere matter to the thought of all of us indifferently. We may, if we like, by our reasonings unwind things back to that black and jointless continuity of space and moving clouds of swarming atoms which science calls the only real world. But all the while the world we feel and live in will be that which our ancestors and we, by slowly cumulative strokes of choice, have extricated out of this, like sculptors, by simply rejecting certain portions of the given stuff. Other sculptors, other statues from the same stone! Other minds, other worlds from the same monotonous and inexpressive chaos! My world is but one in a million alike embedded, alike real to those who may abstract them. How different must be the worlds in the consciousness of ant, cuttlefish, or crab!
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> (James, 1981, pp. 277–278).

In this extended metaphor for consciousness, the mind, playing the role of "sculptor," participates in natural selection. It emerges randomly from the "primordial chaos of sensations," and evolves "by slowly cumulative strokes of choice." Yet each organism, from cuttlefish to crab, represents a variety of sentient forms evolving from the same primordial chaos. Born in a Darwinist age to a father who was a Swedenborgian mystic and close friend of the transcendentalist Ralph Waldo Emerson, James himself embodied the deeply rooted conflict so many of his generation felt between the indisputable facts of evolutionary biology and, as James later wrote in an address delivered to the Harvard Young Men's Christian Association in 1895, "the craving of the heart to believe that behind nature there is a spirit whose expression nature is" (James, 2000, p. 225). If scientifically proving the existence of a transcendent aspect of organic life was the longed-for "invisible reality" of James's era, identifying a phenomenal link between consciousness, experience, and the natural world seems to be that of ours.

For his own part, Fechner's "day view" speculations attempted to bridge these realms by hypothesizing that organic life is interconnected by a "divine consciousness," which represents the "inner side" of the natural world. Fechner's panpsychism was attractive to James because it allowed him to develop a coherent philosophy for the science of psychology, a science that in his view would blend the personal and humanistic attributes of individual experience with biological principles common to the human species. James belonged to an era that needed a more optimistic philosophy than was found in the period's social Darwinism and scientific positivism; his radical empiricism provided an ethics grounded in understanding reality as a complex of interconnected systems, founded on individual responsibility to larger communities, whether in the natural world or a global community. Like Fechner, he sought a naturalistic understanding of consciousness that could account for the spontaneity and novelty of individual minds – their flashes of insight and bursts of genius – the very expressions of individual creativity that appear to distinguish human forms of cognition from that of other species. According to Heidelberger, although Fechner's metaphysics is often dismissed as "antimodern" and "backward" from the perspective of mechanistic materialists, he "sketches a new sort of epistemology, explaining the reality of the mental and the organic, bridging the cleft that separates nature and consciousness, reality and perceptual appearance, and combining science with direct human experience" (Heidelberger, 2004, p. 65). Both Marshall and Heidelberger point out that Fechner and James were philosopher–scientists who felt morally and ethically bound to "to understand science in a way that reunites science with the real world of people, with all the ethical and esthetic implications involved, instead of excluding them from it, as mechanistic materialism does" (Heidelberger, 2004, p. 65). Fechner's "day view," like James's radical empiricism, sought a social role for his psychophysics; both carried out a phenomenological investigation of knowledge perception and construction, believing it was essential to ethical scientific inquiry, for the advancement of all the sciences, and for the mind sciences in particular.

Like James in his own cultural moment, recent contemporary discussions of the mind–brain problem similarly try to bridge divergent biological, psychological, and philosophical approaches. I like to call these "combinatory approaches," that aim for some "middle ground" between the mind–brain as productive and the mind–brain as transmissive. Each new theory requires a correspondingly new definition of reality, one that makes consciousness, or experience, or information, awareness, or criticality, an emergent quality of the universe, and which all living things to a greater or lesser extent seem to possess. What today's mind theorists invested in tackling the "hard" problem of consciousness share with James is his pluralistic conception of mind as an entity composed of, but not limited by, physical reality. A "disseminated, distributed, or incompletely unified appearance," writes James, "is the only form that reality may yet have achieved" (1977, p. 25). Underlying their concept is a conviction that reality – invisible or otherwise – may be discovered to have a subtler structure consonant with that of consciousness itself.

Science historians have aptly described James as a "serial transgressor" of orthodox and unorthodox intellectual and disciplinary boundaries (Cotkin, 1990; Bordogna, 2008). I would suggest, however, that James did not so much make a deliberate program of transgressing boundaries, as he sought knowledge from a constellation of disciplines that he felt would best address his intellectual concerns. In so doing, he also recognized the possibility for fruitful interdisciplinary collaboration on resolving challenging problems in the mind sciences. Though the methodologies of philosophy and experimental psychology in the era of James and Fechner were sometimes antagonistic, more often than not, they facilitated rather than hindered each other's pursuits. Although Fechner endorsed the liberation of natural science from philosophy, he nonetheless believed his own philosophical interests to be compatible with his scientific ones. Intellectually, James collaborated with an international cohort of scientist–philosophers – psychologists, physicists, and physiologists – who not only rejected the growing disciplinary divide between philosophy and the natural sciences, but who also disputed the opposition between science and metaphysics. Not all practitioners of experimental psychology in the late nineteenth and early twentieth centuries agreed that science could so easily be divorced from speculative philosophy.

Neither an empiricist nor a philosophical approach need be absolutist to James's way of thinking. The same could be said for all the scientist–philosophers with whom James enjoyed a rich correspondence: in England, figures like Frederick Meyers; in France, Pierre Janet and Theodore Flournoy; in Germany, psychologist Carl Stumpf, and physicist Ernst Mach – to name only a few. James's affinity for both Fechner and the French philosopher Henri Bergson derived from his sense that Bergson was a philosopher who respected science and that Fechner was a scientist who valued the ways in which speculative philosophy could supply a theoretical framework for the hard facts and formulas later discovered by science. In this regard, James and Fechner alike were figures who not only thought deeply about how volitional and subjective aspects of consciousness influence scientific hypotheses, but who also believed that science should not lose sight of the larger human issues: the reverence for mystery and meaning in individual lives.

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Letting the brain speak for itself

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Metaphors of Computation and Information tended to detract attention from the intrinsic modes of neural system functions, uncontaminated by the observer's role in collection, and interpretation of experimental data. Recognizing the self-referential mode of function, and the propensity for self-organization to critical states requires a fundamentally new orientation, based on Complex System Dynamics as non-ergodic, non-stationary processes with inverse-power-law statistical distributions. Accordingly, local cooperative processes, intrinsic to neural structures, and of fractal nature, call for applying Fractional Calculus and models of Random Walks with long-term memory in Theoretical Neuroscience studies.

Keywords: cybernetics, complex networks, criticality, fractional calculus

INTRODUCTION

Conceptual frames in which we ordinarily think and interact are, in general, fundamentally metaphorical in nature (Lakoff and Johnson, 1980): familiar patterns are sources of organizing the understanding of novel situations, offer convenient locutions for application to less defined or ill understood circumstances, and assist with selecting decisions and actions (Ortony, 1993). In Science, metaphors have traditionally influenced the formation of scientific concepts and theories, and supplied evocative terms for their formulation (Harre, 1995). They are also often credited with stimulating novel ideas and generalizations, and suggesting useful experimental approaches (Paton, 1996). Yet, caution is in order: beware the bearers of false gifts! The correspondence between a given situation or problem area, as the target, with the source of a metaphor thought to offer a suitable metaphoric relation, is in general based on surface appearance. At a deeper level, the source may derive its validity from underlying assumptions and embedded conditions that diverge from, and may in fact conflict with, the conceptual structure of your target domain. You may find yourself now unwittingly applying the conceptually deep structure of your source to the target: you imported a deep structure for which you did not bargain. In Werner (2004), I illustrated errors and constraints that can thereby arise for the interpretation of observations in certain neurophysiological experiments. In the least, commitment to the metaphor may lead one to overlooking or disregarding more pertinent alternatives. In this spirit, Eliasmith and Anderson (2003) called for "moving beyond metaphors," citing symbolicism, connectionism, and dynamicism as three pervasive metaphoric domains of traditional System Neuroscience. In the following, I will at first briefly review my own list of metaphoric obstacles (partly divergent from Eliasmith), with emphasis on their historical origin, and the nature of constraints they have imposed on conceptualizing System Neuroscience. I will then review what I believe we have learned in the last 20 years by recognizing the brain as a self-organizing complex dynamical system in a state of criticality.

THE METAPHORICAL BRAIN OF CYBERNETIC ORIGIN

This section's heading is also title of Arbib (1972) seminal book which presented authoritatively the range and scope of Cybernetics' impact on thinking in the Neuroscience. Those old enough to have witnessed the rise and consolidation of Cybernetics in the decades of the 1940s and 1950s will recall the excitement, fervor of novelty, and the promise of new horizons for conceptualizing the nervous system. In a short span of time, Wiener's Cybernetics with the notions of feedback and control, Shannon's information theory, the concept of the Turing machine and the Church–Turing thesis, and von Neumann's invention of programmability of electronic computers appeared as the goldmines that would allow making sense of the activity of spiking neurons for the brain's information processing strategies, and ultimately suggest generalizations of societal import. For exploring the scientific and social implications of these innovations, the Macy Foundation sponsored the annual conferences of the "Cybernetics Group," beginning in 1943 and extending over 10 consecutive years. Composition of the group varied from year to year, ensuring the inclusion of a wide array of disciplines. A principal mover of these meetings was Warren McCulloch, to whom I will return later. The fascinating story of this group's wide ranging and often contentious deliberations is told in books by Heims (1991), Dupuy (1994), and in sections of Hayles (1999) book. Transcripts of the proceedings were finally published by Pias (2003).

We owe the Metaphorical Brain two specific Metaphors: one concerning computation in and by the brain, the other considering the brain as an information processor. The next two sections will address them, in this order. While this essay is, sadly, in part an account of "sic transit gloria mundi," it also aims to show the role of the Cybernetics movement in the dialectic to the next steps of conceptualizations.

THE COMPUTATION METAPHOR

This topic was reviewed in some detail at an earlier occasions (Werner, 2007a,b) and I will therefore limit myself here to a few essential aspects that bear on the specific objective of this essay. There are two aspects to the computation metaphor: programmable Symbolic computation on Turing machines on the one hand, and Connectionism, on the other. The former lost its glamor for the practicing Neuroscientist fairly quickly: Turing machine computation (like any other form of programmable computation) transforms abstract objects by syntactic rules. Both, rules and semantics, must be supplied by the user. Hence, this form of computation is not part of a natural (i.e., user–observer independent) Ontology (Hayes et al., 1992). Moreover, almost all of physics is framed in Real Numbers which, due to computing errors by rounding and truncation, are not materially executable with the advertised accuracy (Landauer, 1999; see also Cartwright, 1983) "Why the laws of Physics lie." Most importantly, this form of Computationalism raised the uncomfortable question of representation: in Artificial Intelligence applications, and in many areas of Cognitive Science, the semantic (propositional) content is supplied by the user (Scheutz, 2002). The Neuroscientist, on the other hand, needs to account for a natural origin of the representations on which neural systems would perform computational transformations. An "Internalist's Semantics" would be required for conferring semantic import to neural states, appealing exclusively to mechanisms internal to the brain. Failing, there may be a way around this: drawing on constructs from Control Theory and Signal Processing, Grush (2004, 2009) suggests that the organism's sensorimotor engagement could deliver the functionality required for Computationalism to work. This seems of importance for generating Internal representation in Artificial Intelligence and Robotics for which Neurophysiology and Cognitive Theory are expected to provide useful heuristics (see for instance: Shanahan, 2005). But for the Neurophysiologist interested in the principles of brain function, representations pose a major hurdle: ultimately they must originate from the sensory signals the brain receives, whether by sensorimotor activity or simply by receiving sensory signals. Under the heading of "Neural coding," this subject will be addressed in the context of the Information Metaphor.

Neural Networks seemed to offer a new perspective on computation in the brain: Connectionism's appeal for the Neurophysiologist are due to its sharing -at least on a superficial level- some basic features of neural systems: a densely interconnected network of processing units ("neurons") that interact with one another by sending and receiving signals modulated by the weights associated with the connections between them. Two signal contributions have set tone and problematic of computation in these stylized Neural Networks (Hertz et al., 1991): considering simple model neurons as binary threshold units for computing weighted sums of their inputs, McCulloch and Pitts (1943) proved that an asynchronous assembly of such elements performs, in principle, Universal Computation (given certain choices of weights). Forty years later, Hopfield (1982) initiated the burgeoning era of Neural Network computation which, ultimately, rests on the definition of energy as a state function over a network of threshold elements; together, they display emergent collective computational abilities. Not only can networks of more realistic (e.g., spiking) neurons of various formal properties realize any Turing computable function (Siegelmann and Sonntag, 1995); under certain conditions, they can even outperform them (Siegelmann, 2003).

Broadly speaking, the thrust of the virtually incessant stream of publications on Neural Network Computation falls into two categories: one, to conceptualize a however tenuous connection to the Symbolic Computation paradigm. Smolensky (1987) "on the proper treatment of Connectionism" is a valuable repository of the attempts to align symbolic and neural computation, introducing a subsymbolic paradigm as a kind of half-way measure: rather than hard syntactic rules, cooperation among "soft constraints" would collectively deliver inferences by a kind of parallel relaxation, conceivably emulating some features of cognitive processes. Cognitive Science has made extensive use of this principle's elaboration and extension (see, for instance: Feldman and Ballard, 1982). The other, neurobiologically more important category takes its directive from Hopfield's original conceptual alignment with Statistical Mechanics: it stresses the cooperative behavior and emergent computational properties of connected networks of simple processors (e.g., Amit, 1980; Sompolinsky, 1981), including their propensity for forming stable attractor states (Rolls, 2010), and attractor networks (Albantakis and Deco, 2011). The capacity of such networks for self-organization (Linsker, 1988; Hoshino et al., 1996), phase transitions (Opper and Kinzel, 1996; Kinzel, 1997), and their natural interpretation as vector-to-vector transformers places the resources of dynamical systems at their disposal (Pellionisz and Llinas, 1982). Endowed with plastic synapses for changing synaptic weights, and capitalizing on the representational capacity of State Spaces, such vector spaces provide the required flexibility for multiple processing layers and recurrency. In this perspective, and at a higher level of abstraction, the entire pattern of the network's neuron activity is represented as a point in state space; activity patterns generated by an input vector that the network has learned to group together cluster to a circumscriptive cloud in the state space; and learning traces a trajectory in state space along the error dimension. Representations (concepts) can be portrayed as State Space partitions (Churchland, 1987, 1989, 1995).

The bulk of recent and current studies with neural networks emphasize a dual allegiance, true to their name: to neural (or neuron-like) elements on the one hand, and to network dynamics, on the other. Pursuing this latter avenue has uncovered surprising results. Consider, for instance that neural networks can self-organize to critical states (Bornholdt and Rohl, 2003; Levina et al., 2007a) display avalanche dynamics (Levina et al., 2007b); and how synaptic plasticity can drive self-organizing neural networks toward criticality (Meisel and Gross, 2009). In another series of studies, de Arcangelis and Herrmann (2010) showed that avalanches formed in self-organizing neural networks can learn complex rules at phase transition, as result of a collective process. Vogels et al. (2005) direct attention to various forms of network dynamics and complex patterns of signal propagation with interrelations between stimulus driven and internally sustained network activity. These and similar phenomena violate the intent of the "framers" of the Computation Metaphor for whom computation was discrete (and generally synchronous) in the programmable case, and continuous in Neural Networks. Aspects of non-linear dynamics which are the reason for these violations will be discussed in later sections. They invite forcefully the timely question "Are biological systems poised at criticality" (Moran and Bialek, 2010)? In any case, it appears amply justified to extricate Connectionism from its affiliation with the Computation Metaphor, and resurrect it in the perspective of "Computation on Networks," or "message-passing on graphs" (Mezard and Mora, 2009). However, as "old soldiers never die," it behooves one to be alert to the fact that the Computation Metaphor continues to influence, often subliminally, thought, and speech patterns among practicing Neuroscientists. Viewing Connectionisms in the framework of the Computation Metaphor lets you forget its most distinctive features: propensity for discontinuous state transitions and self-organization. For the engineer, they are dreaded like the plague, and must be avoided. For the brain, they may the essential mode of operation. Accordingly, Connectionism in Neurophysiology should more appropriately viewed as a stage in the "Dynamic Turn" of a later section in this essay.

THE INFORMATION METAPHOR

Information was among the most embattled and controversial topics in the discussions of the Cybernetics Group. As is well-known, Shannon (1948) developed the mathematical theory of communication (MTC) which delivers a quantitative measure of the accuracy (and correspondingly, of uncertainty) with which a message from a sender can be received by a receiver. It is based on the selection by the receiver of one of the elements in a predefined ensemble. Shannon also established the formal equivalence of the uncertainty with physical Entropy in closed thermodynamic systems. Studying control system, Wiener (1948) arrived independently at the same mathematical result. In sometimes acrimonious deliberations of the Cybernetics Group, MacKay (1969), proposed to complement Shannon's "selective information" with a "structural information" which would capture the stepwise accretion of elements to a composite (like viewing a picture in a sequence of scans), and "semantic information" as the selective operation which a signal performs on the recipient's set of possible states of behavioral readiness (MacKay, 1954). For a more detailed discussion Werner (1989, 2007a). However, Shannon carried the day in the Group's deliberations: the neat quantification of selective information was just too seductive to compete with "muddy" meaning and semantics (Adams, 2003).

The one aspect of these debates that continues to be particularly relevant for Neuroscience pertains to "Neural Coding": for the Computationalist of the programmable version, this was to be the source of internal representations; for the connectionist, it provided the input to the neural nets. As indicated before, activity in individual nerve fibers or neurons can be considered binary. It was then virtually irresistible to view the relation between input and output of a neuron in the framework of Shannon's information transmission from a sender to a receiver. This became what one may call the "hegemony of the digital doctrine" in Neuroscience. Interestingly and with a twist of irony, R. Gerard, the only Neurophysiologist of the Group, most strenuously objected to this notion, despite the fact that he and Li were in fact the first to record single neuron spikes from cortex. He argued that undue emphasis on patterns of single neuron activity would defeat appreciating the genuine nature of brain events.

Nevertheless, "Neural Coding" prevailed and has triggered a flood of theoretical and experimental studies. This field was most competently reviewed by Rieke et al. (1997) but the stream of new investigations still persists incessantly. In the foundational context of Information Theory, Neural Coding, Representation, and Information Processing came to constitute a closely interrelated nexus of investigative targets (Borst and Theunissen, 1999). Two questions are directive: one, which feature of a neural spike train (rate, interval statistics, correlations, etc) carries (encodes) the message (in Shannon's sense)?; and second, how do downstream neurons "evaluate" (decode) a putative message that may be encoded in a spike pattern (see, e.g., de Charms and Zador, 2000; Jacobs et al., 2009) or in time-dependent signals (Bialek et al., 1991)?. Concerning the first issue, Perkel and Bullock (1968) listed 15 aspects of neural spike trains that could conceivably function as codes.

Failing to obtain in the short run any conclusive answer to both question, some investigators turned to an alternative approach: comparing neural activity elicited by natural stimuli with known behavioral or psychophysical measures, the idea being that whatever measure of neural spike trains compares best with corresponding perceptual–cognitive activity is then presumably a "neural code." For illustration of the basic pattern of this approach: an early study of this type, involving cutaneous touch receptors and using firing rate as response measure, determined that the spike count (rate code) in peripheral efferent fibers suffices for reliably distinguishing eight different stimulus intensities (Werner and Mountcastle, 1965). This amounts to a capacity for transmitting three bits of information, which is also equal to the limit of cognitive processing in human subjects (Miller, 1956). For a review of numerous comparable studies examining correspondences between scales of neural activity and Psychophysics for different sense modalities, see Werner (1968). The same principle of seeking correlations between neural and perceptual–cognitive activity became also the target of innumerable investigations with the 60-Hz cycles in sensory neurons, extending over several decades. For a recent summary and overview, see: von der Malsburg et al. (2010).

What matters for the principal thesis of this essay is, however, that the views regarding the Coding Problem diverged in time into two radically different directions: one, applying mathematical approaches of increasing sophistication to analysis, and generation, of spike trains in individual, or ensembles of, neurons; the other, calling the very notion of "neural code" in question. One recent example of the former category is the elegant work of Haslinger et al. (2009): their approach determines a spike train's causal state model (i.e., a minimal hidden Markov model) that generates time series which are statistically identical with the original spike trains. This enables a novel view to the coding issue, for it is then possible to relate the covariates to the causal states as generators of the spike train, rather than to the spike train itself. There was then also the question whether groups of discharging neurons might carry a message: for instance, Yu et al. (2010) were interested in the information delivery rate from a population of neurons, with attention to redundancy of information within and between functional neuron classes. It also turned out that ongoing network states are effective determinants of an individual neuron's spiking activity (e.g., Harris, 2005; Shlens et al., 2006; Tang et al., 2008), and Truccolo et al. (2009) reported that the ensemble history is a better predictor for a neuron's spiking than is the ensemble's instantaneous state. These few recent and representative examples are indicative of the efforts to characterize statistical properties of spike trains, but they do not by themselves contribute to the question of message transfer on afferent pathways. However, the frequently applied measure of "Mutual Information" is an effective way for determining the degree of independence of two data sets; it is independent of the underlying distributions and requires minimal assumptions about dynamics and coupling of systems (Schreiber, 2000; Tkacic, 2010).

The second branch of the Coding Problem's history takes an entirely different view. Its origin can be traced to debates and controversies among the member of the original Cybernetics Group: the pivotal point was the contentious issue of the observer. As a paradigmatic situation, consider the usual experimental condition for studying "coding": typically, one applies stimuli whose metric one chooses as plausible, and then evaluates the neural (or behavioral) responses they evoked by a metric one chooses, such that one obtains a statistically valid (or otherwise to the experimenter meaningful) stimulus–response relationship. One then claims that the metric of the neural response "encodes" the stimulus, being tempted to conclude that the organism is "processing information" in the MTC paradigm. But recall that this paradigm deals with *selective* information; that is: the receiver needs to have available a known ensemble of stimuli from which to select the message. Moreover, this procedure does not permit one to know whether any of the metrics applied is of *intrinsic* significance to the organism under study: the observer made the choices on pragmatic grounds (Werner, 1988). In reality, once triggered by an external input, all that is accessible to the nervous system are the states of activity of its own neurons; hence it must be viewed as self-referring system.

In a broader context, self-reference and self-organization became the rallying point of a successor to Cybernetics, generally known as "Second-order Cybernetics," with Heinz von Foerster as leading proponent. Closely related is Maturana (1970) idea of Autopoiesis: in some ways a premonition of things that needed another 50 years to mature, as the Section on Non-analytic Dynamics will show. In this view, and in contrast to the route of coding, stimuli from the periphery are thought to perturb the central nervous system's structure and internal organization according to its own internal dynamics (see also: Maturana and Varela, 1980). Coding and representing have in this framework lost legitimacy.

The application of MTC in Neurophysiology is also vulnerable on other grounds. MTC and its generalizations to Information Theory and "information processing" are predicated on the assumption of normalcy of data distributions, and ergodicity of the data generating process. But the abundant evidence for fractal patterns and structures, and self-similarity at all levels of neural organization violates this assumption (Werner, 2010). More about this in the section on Non-analytic Dynamics. For different reasons, the Neuronal Group Selection theory takes the view that the complexity, variability, and unpredictability of the world precludes the notion of preexisting information, applicable to all situations, which the selective information paradigm of MTC requires (Edelman and Finkel, 1984).

At the time of its conception, MTC blended beautifully with the theory of programmable computation to forge an alliance that has molded an immensely influential Metaphor; the colloquial "Information" of prevailing linguistic use undoubtedly fostering its ready acceptance. This entailed forgetting that Information (technically speaking) and Computation (of the Turing type) are observer constructs. Lacking an intrinsic ontology, Information refers to a *description* of reality, and Computation to an user-relative semantics. Nevertheless, bearing this in mind, both metaphors can serve useful purposes (Cox, 1946; Knuth, 2010). Information and computation metaphor are valuable constructs in the service of neurocomputational engineering developments (Eliasmith and Anderson, 2003): in these applications, *You* are the master in your house: *You* design and build an object whose Ontology is therefore transparent to You, and to which *You* can choose the epistemic access to suit your purpose. But as Neurophysiologist, you face an unknown Ontology; the best you can do is to try finding an epistemic access that, by some criterion of your choice, is optimal: a kind of inverse problem. As I tried to show before: metaphors are likely to be treacherous guides in this endeavor.

ANALYTIC BRAIN DYNAMICS

The starting point was W. Freeman's insight that electrical activity recorded from aggregates of neurons (neuron masses, in his originally terminology) can be interpreted as meaningful spatio-temporal patterns, related to sensory perception and learned behavior: from electroencephalographic records of the olfactory bulb in rabbits, he identified odor specific, stable activity patterns which reflected odor discriminations acquired by prior training. For each discriminated odorant, a learned limit cycle attractor is formed, differentiated from others by its basin and its spatial amplitude pattern. This ground-breaking conceptualization (and its multiple implications and extensions) was summarized in Freeman and Skarda (1985), Skarda and Freeman (1987) as the result of some 12 years of work, and is also splendidly reviewed by McKenna et al. (1994) in the broader context of viewing the brain as a physical system. This initial work introduced non-linear dynamic analysis of neuronal system. Based on this work, Freeman confronted the Information Metaphor head on: here are some excerpts from Freeman and Skarda (1985): "context and meaning of representations are invariably in the brain of the observer and not of the observed..... In the language of representations, the olfactory bulb extracts features, encodes information, recognizes stimulus patterns, . . .These are seductive phrases, ... but in animal physiology they are empty rhetoric."

In the following years, Freeman's work continued in this vein with ever more sophisticated applications of non-linear dynamics to characterizing records of human Electroencephalogram (EEG), which led to identifying the role of chaotic phase transitions (Freeman, 2000) and, eventually, to proposing that the non-linear brain dynamics is a macroscopic manifestation of a many-body field dynamics (Freeman and Vitiello, 2006). But this gets ahead of some developments in Physics with far reaching implications for virtually every field of knowledge, including, of course, the Neurosciences.

In the wake of the opening moves by Nicolis and Prigogine (1970) and Haken (1975, 1983), the study of non-equilibrium dynamic systems in Physics yielded a massive body of knowledge and experiences that eventually came to constitute the modern theory of critical phenomena. Stanley (1987, 1999) offers succinct accounts of the key concepts, as are the comments on theoretical foundations of particular relevance to neurobiology by Le Van Quyen (2003). Recall that a phase space is a mathematical construct: it is understood as abstract space with independent coordinates representing the dynamic variables needed to specify the instantaneous state of the system, which is represented as a point in that space. A system's state may be described at a microscopic level in terms of its constituent elements. It can also be characterized at a macroscopic level: that is how it appears to an observer who "coarse grains" the microscopic state by lumping elements to larger aggregates. Non-linear dynamic evolution equations determine the trajectory's motion in phase space. At certain points along the trajectory, singularities (discontinuities, bifurcations) of the evolution equations may be encountered which cause the system to undergo abruptly a phase transition to a qualitatively new and different state. This may occur spontaneously, or in response to external perturbation. For applying this framework to Neuroscience, it must be understood that the differential equations for describing the state space dynamics stand in actual reality for the local physical and chemical processes that are in effect between neuronal elements; they are thought of as cooperative and collective interactions. In this spirit, the underlying neuronal dynamics is entirely internal to the modeled system, involving merely local collective processes among the constituting elements. If computational simulations adequately correspond to actual observations with real neural system, it is then thought that the dynamics of the real system is also sustained by system-intrinsic processes, safe of course for perturbations of external origin.

The plausibility of the dynamic view of brain function received forceful support from the discovery of spatially irregularly occurring patterns of propagated neuron discharges sequences in neural tissue: on the basis of detailed quantitative–statistical analysis, Beggs and Plenz (2003, 2004) identified these patterns with the "avalanches," characterized by Bak et al. (1987, 1988) as evidence for a physical system having attained by self-organization a persistent critical state. Such avalanches are bursts of activity with size and duration obeying power-laws, created by spontaneous (intrinsic) fluctuations of local activity (Chialvo, 2004, 2008). The principles underlying this claim are supported by numerous theoretical and experimental studies, and detailed aspects of the dynamics have been clarified by, for instance, Dickman et al. (2000) and summarized in Sornette (2000). Criticality signifies here that minor perturbations, possibly spontaneous random noise, will trigger avalanches that correspond to (continuous) phase transitions in the Theory of Critical Phase Transitions, referred to earlier. Plenz and Thiagarajan (2007) propose to view such avalanches as dynamic cell assemblies in neural tissue. Numerous investigations of human fMRI under various conditions, summarized recently by Chialvo (2010) and Tagliazucchi and Chialvo (2011) also solidly affirm the evidence for the type of complex emergent phenomena in brain that are typical of systems poised near to or at a critical state of (second-order) phase transitions. Physics of condensed and excitable matter provides the theory of this phenomenon (e.g., Kadanoff et al., 1989; Sethna, 2006), with the well-known Ising model serving as one of the physical prototypes: there, the critical state is evident as long-range correlation links in scale-free networks, with the same characteristic features that are observed with functional magnetic imaging in brains (Fraiman et al., 2009; Kitzbichler et al., 2009; Expert et al., 2010). Thus, the notion of brain criticality rests on relatively solid grounds, not as a Metaphor, but as the result of intrinsic physical mechanism. However, it is then also important to be aware that the mathematical tools for describing criticality in statistical systems is in general quite different from the language used when working with dynamical systems (Moran and Bialek, 2010). This will be pursued in the Section on non-analytic Dynamics.

The implications of the dynamic conceptualization and criticality in brain physiology are discussed and illustrated in Werner (2007b, 2009a,b,c). Haken's approach, cited above, spawned the extensive investigations that eventually consolidated to the field of Coordination Dynamics (Haken et al., 1985; Kelso et al., 1992; Kelso, 1995). It established in different types of experiments and with careful analytic methods the concurrence of phase transitions in motor and perceptual–cognitive performance with brain electrical activity. In the course of these investigations, Kelso arrived at the view that the critical state in brain dynamics should be understood as "a space for exploring competing, perhaps conflicting, dynamic regimes" (Kelso and Engstroem, 2006). This may also bear on the proposal by Bailly and Longo (2011) that a kind of "extended criticality" should be considered, possibly reflecting an entanglement of coexisting levels of order at the macroscopic. For the sake of completeness, recall that a "dynamic turn" occurred also in Cognitive Science, where it was essentially set in motion by Port and van Gelder (1995) book "Mind in Motion."

What is the significance of brain criticality? In the first place, it is associated with the establishment of long-range correlation for integration of activity across distant regions of neural tissue, displaying conspicuous fractal properties. The nature of this profound neuronal reorganization was ascertained under various experimental conditions and in numerous human EEG and fMRI studies (for detailed citations, see: Turcotte, 1999; Werner, 2010): in this process, neurons assemble to new clusters whose size distribution scales with a power function of a negative exponent smaller than two, and exhibits self-similarity. Accordingly, the elements of the system assume a fractal order which is associated with entirely new properties: obeying, at the system's macroscopic level, new laws, and requiring new descriptors, which cannot be simply deduced from the prior state (West, 2004). We say then that a new ontology originates with phase transitions, which requires new epistemic criteria for its description and interpretation (Werner, submitted).

Here, then is the drastic difference to the cybernetic Information Metaphor: neural systems do not process information; rather, being perturbed by external events impinging on them, neural systems rearrange themselves by discontinuous phase transitions to new ontic states, formed by self-organization according to their internal dynamics. Internal to the system, these new ontic states are the "raw material" for the ongoing system dynamics, by feedback from, or forward projection to other levels of organization. Looking at the dynamics from the outside, these dynamic states are objects to which the observer can do no more than apply his/her observer-dependent interpretation of accessible system observables. Clearly, if seen in this way, the metaphors of cybernetic origin and the brain dynamics sketched here are incommensurable: albeit interacting, inside and outside are two worlds, apart. How to deal with the (fractal) dialect the brain speaks in its own internal world is the subject of the next section.

BRAIN COMPLEXITY WITH NON-ANALYTIC DYNAMICS

Following this path will require delving into the burgeoning field of Complexity management (West, 1999). But first, a new look at recent findings of neuronal brain dynamics, based on EEG scalp records of normal subjects. Applying the time evolution of a minimum spanning tree method (Kruskal, 1956) to analyzing human EEG, Bianco et al. (2007) noted the intermittently occurring abrupt changes of topology, designated as "events," which obeyed a renewal statistics. In the further course of the analysis, these authors concluded that EEG reflects a non-ergodic, non-Poisson, renewal process with a power-law index <2 (Bianco et al., 2008). Comparing this observation with results of an earlier study of the fluorescence intermittency in blinking quantum dots, Bianco et al. (2005) suggested that the recorded events can be attributed in both cases to a cooperative dynamics with emerging self-organized coupling of many interacting units (Grigolini, 2005; Bianco et al., 2008). In an independent series of experiments, Allegrini et al. (2009) recorded coincidences of events, occurring simultaneously among two or more scalp electrodes. The waiting time distribution between consecutive events presented an inverse-power-law index of approximately 2, corresponding to a perfect 1/f noise. These authors also proved that the coincidences are driven by a renewal process. Allegrini et al. (2010) carried this line of research one step further by studying the rapid transition processes (RTP) in EEG which Fingelkurts and Fingelkurts (2004) had examined in great detail. RTP's are not only evidence for intermittent global metastable transitions, but they also display multichannel avalanches (see Beggs and Plenz, 2003) which appear as simultaneously occurring RTP's at several EEG recording sites. Statistical measures of multichannel avalanches exhibit inverse-power-law statistics and, thus, attest to the state of self-organized criticality of the entire cortex. Evidently, the units of activity underlying the recorded events are in these cases clusters of neurons (neuronal assemblies) whose dynamical interactions constitute networks.

Considering the case of two clusters of element (sub-networks) embedded in a network, Bianco et al. (2008) found that cluster-to-cluster interaction is facilitated if the clusters are self-organized. Interactions occur then abruptly (designates as "crucial vents"), with the statistics of inter-event times indicating a non-ergodic, non-Poisson, renewal process (Turalska et al., 2009). West et al. (2008) examined the conditions for maximizing information exchange between two complex networks, taking into account their characteristics as non-stationary processes with inverse-power-law statistics. Viewing the power-law exponent as measure of complexity, it turned out that information exchange is optimal when the complexities of perturbed and perturbing network are equal. Subsequently, it became apparent that perturbing and perturbed complex network must also share their respective temporal complexities, i.e., their non-stationary, non-ergodic fluctuations at the onset of phase transitions (Turalska et al., 2011). Obviously, conditions for "information sharing" among complex systems are quite stringent, and much more work will be required to identify all relevant parameters. Surprisingly, complex systems do not respond to external stimuli at all unless they are not also complex (Turalska et al., 2009; Aquino et al., 2011).

In the Complex Network literature, it is customary to speak of "Information flow" or "Information exchange" in and between complex networks with the clear understanding that what is under discussion is NOT Shannon-type information, but rather Mutual Information, at best. My own preference is to view the relationship between complex networks as "perturbation," with a perturbing network affecting topology and dynamics of a perturbed network, according to the latter's internal dynamics. Note the drastic difference between this outlook, and the notion of Information transmission between neurons (and neuronal systems) by encoded messages in the Cybernetic Information Metaphor.

An equally fundamental distinction is due to the complex webs being fractal: this renders ordinary or partial differential equations of motion no longer adequate for characterizing system dynamics (West, 2006). Instead, long-term memory in the dynamics of complex phenomena must be taken into account in the form of deterministic or stochastic fractional Dynamics. Fractional Gaussian noise was identified by Maxim et al. (2005) in fMRI records of human brains, and Achard et al. (2008) traced the fractal connectivity of long-memory networks. For other applications of fractional calculus in Neurophysiology, see for instance: Scafetta et al. (2009) work on postural control, and Lundstrom et al. (2008) on fractional differentiation by neocortical pyramidal neurons. Suitable texts for this area of Mathematics are, for instance: West et al. (2003) and Miller and Ross (1993). Computational modeling on this basis may correspond in the Physics of Brain to Random Walks with long-term memory (Montroll and Shlesinger, 1984; West and Grigolini, 2011; Ch. 4). Random Walks were used for tracing pair-wise local interactions along fractal neuronal connections by Sporns (2006), Fontoura Costa and Sporns (2006), and Fontoura Costa et al. (2011). Bieberich (2002) advocated the candidacy of the self-similar branching structure of recurrent fractal neural networks for the brain's local and global "Information processing" (*Sic*!).

The observations and conclusions reported in this Section are in accord with the brain being in a critical state (Chialvo, 2004, 2008, 2010; Chialvo et al., 2008). In addition, they underscore the fractal nature of the critical state that results from the brain's self-organization. The brain's critical state can thus be viewed as a complex network of neuronal clusters on multiple scales: the clusters (neuronal assemblies) being the network nodes whose links enable coordinating their activity states. In virtue of the long-range connections between clusters, all parts of the system act in the critical state as if they can potentially communicate with each other, yet actual interactions are strictly local and constrained by seemingly stringent conditions of complexity matching (West et al., 2008).

SUMMARY AND CONCLUSION

Based on tracing notable landmarks in the history of Theoretical Neuroscience of the past six decades, evidence is presented that the metaphors of Computation and Information have stood in the way of gaining access to the intrinsic modes of neural system operation. Connectionism, originally subsumed under the Computation Metaphor, can more appropriately be viewed in the framework of Complex Networks. More importantly, Neural Dynamics must take into account the fractal nature of phase transitions and criticality in complex systems, requiring the mathematical tools of Fractional Calculus and its physical models of Random Walks with long-term memory: the overriding issue is the fractality in the context of Complex System Dynamics. On account of this, self-similarity in neural organizations and dynamics poses one of the most intriguing and puzzling phenomenon, with potentially immense significance for efficient management of neural events on multiple spatial and temporal scales. Answering the challenge posed in the title: "*fractal spoken here.*"

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Cooperation-induced topological complexity: a promising road to fault tolerance and Hebbian learning

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According to an increasing number of researchers intelligence emerges from criticality as a consequence of locality breakdown and long-range correlation, well known properties of phase transition processes. We study a model of interacting units, as an idealization of real cooperative systems such as the brain or a flock of birds, for the purpose of discussing the emergence of long-range correlation from the coupling of any unit with its nearest neighbors. We focus on the critical condition that has been recently shown to maximize information transport and we study the topological structure of the network of dynamically linked nodes. Although the topology of this network depends on the arbitrary choice of correlation threshold, namely the correlation intensity selected to establish a link between two nodes; the numerical calculations of this paper afford some important indications on the dynamically induced topology. The first important property is the emergence of a perception length as large as the flock size, thanks to some nodes with a large number of links, thus playing the leadership role. All the units are equivalent and leadership moves in time from one to another set of nodes, thereby insuring fault tolerance. Then we focus on the correlation threshold generating a scale-free topology with power index $\nu \approx 1$ and we find that if this topological structure is selected to establish consensus through the linked nodes, the control parameter necessary to generate criticality is close to the critical value corresponding to the all-to-all coupling condition. We find that criticality in this case generates also a third state, corresponding to a total lack of consensus. However, we make a numerical analysis of the dynamically induced network, and we find that it consists of two almost independent structures, each of which is equivalent to a network in the all-to-all coupling condition. This observation confirms that cooperation makes the system evolve toward favoring consensus topological structures. We argue that these results are compatible with both Hebbian learning and fault tolerance.

Keywords: criticality, cooperation, complex topology, inverse power law

INTRODUCTION

The issue of defining a robust performance measure (Boccaletti et al., 2006; Qiang and Nagurney, 2008) to assess network efficiency is one of the main problems of the emerging field of complex networks. As noted in Qiang and Nagurney (2008), problems akin to network vulnerability, which are linked to events such as 9/11, hurricane Katrina and the biggest blackout in North America (8/14/03), cannot be satisfactorily addressed without a measure of global network efficiency. Network efficiency was originally studied in connection with topological structure (Boccaletti et al., 2006), and more recently with measures capturing the flow of information to determine the criticality of nodes and links (Qiang and Nagurney, 2008).

In the last few years increasing attention has been devoted to discussing the connection between synchronization and topology (Arenas et al., 2006, 2008; Gómez-Gardeñes et al., 2007; Díaz-Guilera, 2008; Liang et al., 2009). Using sociological perspective (Castellano et al., 2009) synchronization can be thought of as a form of consensus. Consequently the efficiency of a network can be expressed as a quantity inversely proportional the critical value of the control parameter K_C, at which a phase transition of the network to an organized state occurs. Herein that organized state is identified as consensus, where most of the nodes of the network are found to share the same opinion. Simultaneously, expressing network efficiency through consensus has the effect of establishing a close connection between network topology and the ubiquitous natural phenomenon of synchronization. In this way a number of investigators have concluded that topology plays an important role in biology, ecology, climatology, and sociology (Pikovsky et al., 2001; Arenas et al., 2008; West et al., 2008; Castellano et al., 2009).

Some investigators (Arenas et al., 2006) employ a local order parameter to determine that synchronization can be used to detect communities. More recent work (Liang et al., 2009) follows a similar path, using the response of the surrounding nodes to a weak signal generated by a given node of the network. The paper of Gómez-Gardeñes et al. (2007) uses synchronization as a way to compare the efficiency of a scale-free network to the random network of Erdos and Renyi. All this is extensively discussed in an excellent review paper by Arenas et al. (2008) showing very interesting comparisons of Erdos–Renyi networks and scale-free networks as a function of the control parameter K. The influence of complex network topology on synchronization has been studied (Restrepo et al., 2006) and these authors found that although the heterogeneous network is less efficient than the all-to-all coupled network, the phase transition remains sharp, thereby qualitatively confirming the results of earlier work (Moreno and Pacheco, 2004). The latter authors studied the Kuramoto model (Kuramoto, 1984) embedded in a scale-free network and found that, quite surprisingly, a phase transition emerges at a critical value of the control parameter K with no need to use all-to-all coupling. Another interesting result (Moreno and Pacheco, 2004) is that single nodes, with k links, regress to synchronization with the mean time $\langle \tau \rangle \propto 1/k$, thereby making the hubs very stable. This observation led those authors to conjecture that the scale-free condition may be a consequence of an optimization mechanism. As a main result of this paper, we substantiate this conjecture with an example where the hubs have a dynamical origin.

A limitation of the earlier work on the connection between global efficiency and topological complexity, with one exception (Moreno and Pacheco, 2004), is that the dynamical origin of topological complexity is not addressed. We examine this connection between topology and dynamics herein as follows: (a) We distinguish between a *resting* and a *dynamic* network; the former being the network where the constitutive units are at rest, and the latter being the network generated by the self-organization of the units located on the structure of the resting network. (b) We explore the possible benefits of the dynamic network by using it as a resting network on which to activate our cooperation model. (c) We argue that these results establish a connection between criticality and the famous Donald Hebb's neurophysiological postulate (Hebb, 1949).

MATERIALS AND METHODS

We adopt a decision making model (DMM; Bianco et al., 2008; Turalska et al., 2009), which uses a social paradigm of decision makers who choose between "yes" (+) and "no" (−) at each point in time. Note that the DMM is expected to have a range of application that is not limited to social processes. In fact, the DMM is usefully applied to account for the phenomenon of intermittency of colloidal quantum dots (Frantsuzov et al., 2009). In DMM the dynamics of each node s_i is determined by the following master equation

$$\begin{aligned} \frac{d}{dt}p_1 &= -g_{12}p_1 + g_{21}p_2 \\ \frac{d}{dt}p_2 &= -g_{21}p_2 + g_{12}p_1 \end{aligned} \tag{1}$$

The cooperation among the units in the network is realized by setting the coupling coefficients to the time-dependent form

$$g_{12}(t) \equiv g e^{K\frac{M_2(t)-M_1(t)}{M}} \tag{2}$$

and

$$g_{21}(t) \equiv g e^{-K\frac{M_2(t)-M_1(t)}{M}} \tag{3}$$

Here M denotes the total number of nearest neighbors, and $M_1(t)$ and $M_2(t)$ are the count of nearest neighbors who are making the decision "yes" and "no," respectively. The single individuals change opinion, and as a consequence $M_1(t)$ and $M_2(t)$ are fluctuating in time variables, while, of course, the total number of nearest neighbors is conserved: $M_1(t) + M_2(t) = M$. The single unit in isolation, $K = 0$, would fluctuate between "yes" and "no," with the rate g. Once the value of the coupling increases, $K > 0$, single in the state "yes" ("no") is less and less independent and makes transition to the state "no" ("yes") faster or slower according to whether $M_2 > M_1$ ($M_1 > M_2$) or $M_2 < M_1$ ($M_1 < M_2$), respectively. We define the global fluctuation $\xi(t) \equiv (N_1(t) - N_2(t))/N$, where N is the total number of nodes, and $N_1(t)$ and $N_2(t)$ are the units in the state "yes" and "no" at time t, respectively. Additionally, we use the time average $\xi_{eq} = \langle |\xi(t)| \rangle$ as a measure of the global majority.

It is important to note that the master equation of each site is a fluctuating master equation. This property emerges from the fact that the transition rates $g_{12}(t)$ ($g_{21}(t)$) depend on the stochastic evolution of each unit's environment. Only in the thermodynamical condition when the number of nodes creating the network is infinite, $N = \infty$, and all nodes are directly connected with each other, $M = \infty$, the ratios $M_1(t)/M$ and $M_2(t)/M$ are equivalent to the probabilities $p_1(t)$ and $p_2(t)$ (Turalska et al., 2009). However, this correspondence is violated once the number of nodes is finite or the structure of the network departures from all-to-all coupling. Both those conditions introduce randomness into the dynamic evolution of a single unit. In the most general case this stochastic fluctuations can be realized as a difference between the estimate of a random walker position after finite number of steps (what corresponds to given number of neighbors each node has) and the position obtained after an infinite time.

Here we consider a topology of a simple two-dimensional regular lattice, where each node is coupled to four nearest neighbors, thereby setting $M = 4$. We call this structure a *resting network*. The numerical calculations were performed on a lattice of $N = 100 \times 100$ nodes with periodic boundary conditions. At the initial time, $t = 0$, the state of each node was assigned randomly to be either $+1$ or -1, symbolizing decision in "yes" and "no," respectively. Next, in a single time step a run over the whole lattice is performed and for every unit s_i the transition rate of Eqs 2 or 3 is calculated according to which a node is given the possibility to change its state. After initial 10^6 time steps the time average is taken over the same number of consecutive steps of the model to evaluate the global majority measure ξ_{eq}.

RESULTS

As expected, the global order parameter ξ_{eq} shows a transition to the organized state with the respect to the coupling constant K. The critical value of the coupling constant K_C can be found analytically in the case of $N = \infty$ (McCoy and Wu, 1973) and yields $K_C = 2\ln(1+\sqrt{2}) \approx 1.76$. On **Figure 1** we present the phase transition curve obtained for a finite square lattice of 10,000 nodes. We observe that the transition into the organized phase occurs at the

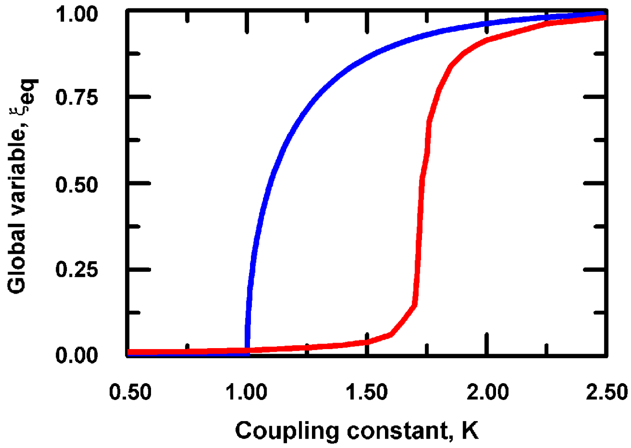

FIGURE 1 | The phase diagram for global variable ξ_{eq}. Solid blue line corresponds to the equilibrium states obtained in the case of fully connected, infinite network $N = \infty$. Red line corresponds to the global states observed for two-dimensional regular lattice ($N = 100 \times 100$) and setting $g = 0.01$.

critical value of $K_C \approx 1.70$, which is significantly larger than the value of the all-to-all coupling case with an infinite number of units (Bianco et al., 2008; Turalska et al., 2009), $K_C = 1$.

To study the dynamically induced network topology, we consider the DMM with the critical value of the coupling, $K_C = 1.70$. After initial 10^6 time steps, we record lattice configurations over 2000 time step windows, registering the dynamics of each node $\{s_i(t)\}$ over that time interval. In the next step we evaluate the linear correlation coefficient between the i-th and the j-th node (Fraiman et al., 2009):

$$r(i,j) = \frac{\langle s_i(t)\, s_j(t)\rangle - \langle s_i(t)\rangle \langle s_j(t)\rangle}{\sqrt{\langle s_i^2(t)\rangle - \langle s_i(t)\rangle^2}\sqrt{\langle s_j^2(t)\rangle - \langle s_j(t)\rangle^2}} \qquad (4)$$

where $\langle \ldots \rangle$ denotes time average. If the correlation intensity between nodes i and j of the square lattice is larger than the arbitrarily chosen threshold value $\Theta = 0.61$, we consider them connected by a link in the dynamically induced topology. The **Figure 2** serves as an illustration of this process.

This newly created topology clearly depends both on selected value of the coupling and the threshold value applied to the set of obtained correlation coefficients. If one would consider the dynamics of the system in the subcritical phase, $K < K_C$, where the randomness dominates over the cooperation between units, as a result observed values of $r(i, j)$ would be small, as expected for two independent variables. In the opposite case, when $K > K_C$, strong coupling reduces the variability of a state of a single unit in time, and due to the definition of $r(i, j)$ also leads to small correlation values. In both cases, the distribution created from a set of obtained $r(i, j)$ values is centered at zero, and has very fast decreasing tails. However, once the dynamics approaches criticality, $K \approx K_C$, the coupling between units is just enough to balance the stochasticity. This condition leads to dynamical coupling between units that are not directly connected, and results in much wider distribution of values of $r(i, j)$ than in two previous cases.

In all discussed cases adoption of a low threshold would include most of the correlation pairs $r(i, j)$ and would result in highly connected networks, where almost all nodes are connected to each other. As the threshold Θ increases less and less pairs of nodes would be included in the newly created topology. Since for $K < K_C$ and $K > K_C$ the distribution of correlations decreases much faster than for $K \approx K_C$, the increase of Θ leads to the destruction of the giant component, thus no new structure can be identified. Close to the criticality, the transition between the fully connected structure obtained for low threshold and its destruction into separate modules is more subtle. In this condition, the Dirac-like degree distribution, obtained for low Θ when almost all nodes are directly connected, starts to present a tail once Θ is increased, signifying that some links are eliminated and nodes with degree smaller than the average appear. Even further increase in the value of threshold eliminates enough links to distribute nodes degree as an inverse power law. The inclusion of correlations even higher than this special value leads to the disruption of the giant component.

As mentioned in the previous paragraph, above prescription generates a scale-free network, with the inverse power index $\nu \approx 1$, as shown in **Figure 3**, thereby supporting the observation of Moreno and Pacheco (2004), that the scale-free structure is the result of an optimization process, which is, in the case of this Letter, the realization of a consensus condition. We also evaluate the distribution density $p(l)$ of the Euclidian distance l between two linked nodes and find that the average distance is of the order of 50, namely, of the size of the two-dimensional network 100×100. This average distance implies the emergence of long-range links, essential to realize the fast transfer of information over a complex network (Kleinberg, 2000; Boguñá and Krioukov, 2009; Boguñá et al., 2009; Li et al., 2010).

Now let us turn the dynamically induced network into a *resting network*, called a *dynamically generated resting* (DGR) network and let us study its efficiency running the DMM on it. We find it convenient to compare its cooperative behavior with another seemingly equivalent scale-free networks with the same $\nu \approx 1$. We realize this latter scale-free network using the probabilistic algorithm (Kalisky et al., 2004) and we refer to it as an *ad hoc network*, and run the DMM on it as well as on the DGR network. The phase transition diagrams of the DGR and *ad hoc* networks are illustrated in **Figure 4**, and **Figure 5B**, respectively. We see that the phase transition occurs on both networks at $K = 1$, namely, at the same critical value corresponding to the all-to-all coupling condition. However, in **Figure 4** a new phenomenon is observed, that being the emergence of both a consensus and a non-consensus state. The new state emerges because the self-organization process generates two weakly coupled identical clusters (**Figure 4B**), each cluster being equivalent to an *ad hoc* network with $\nu \approx 1$. These two networks are virtually independent from one another, thereby yielding the states $++$; $--$; $+-$; $-+$ with equal probability. The states $+-$ and $-+$ are the non-consensus state. To support this interpretation we generate two identical *ad hoc* networks with $\nu = 1$ and couple them with a single link. The resulting phase diagram, shown in **Figure 5A**, is very similar to that depicted on the **Figure 4**, thereby establishing that DGR networks may lead to

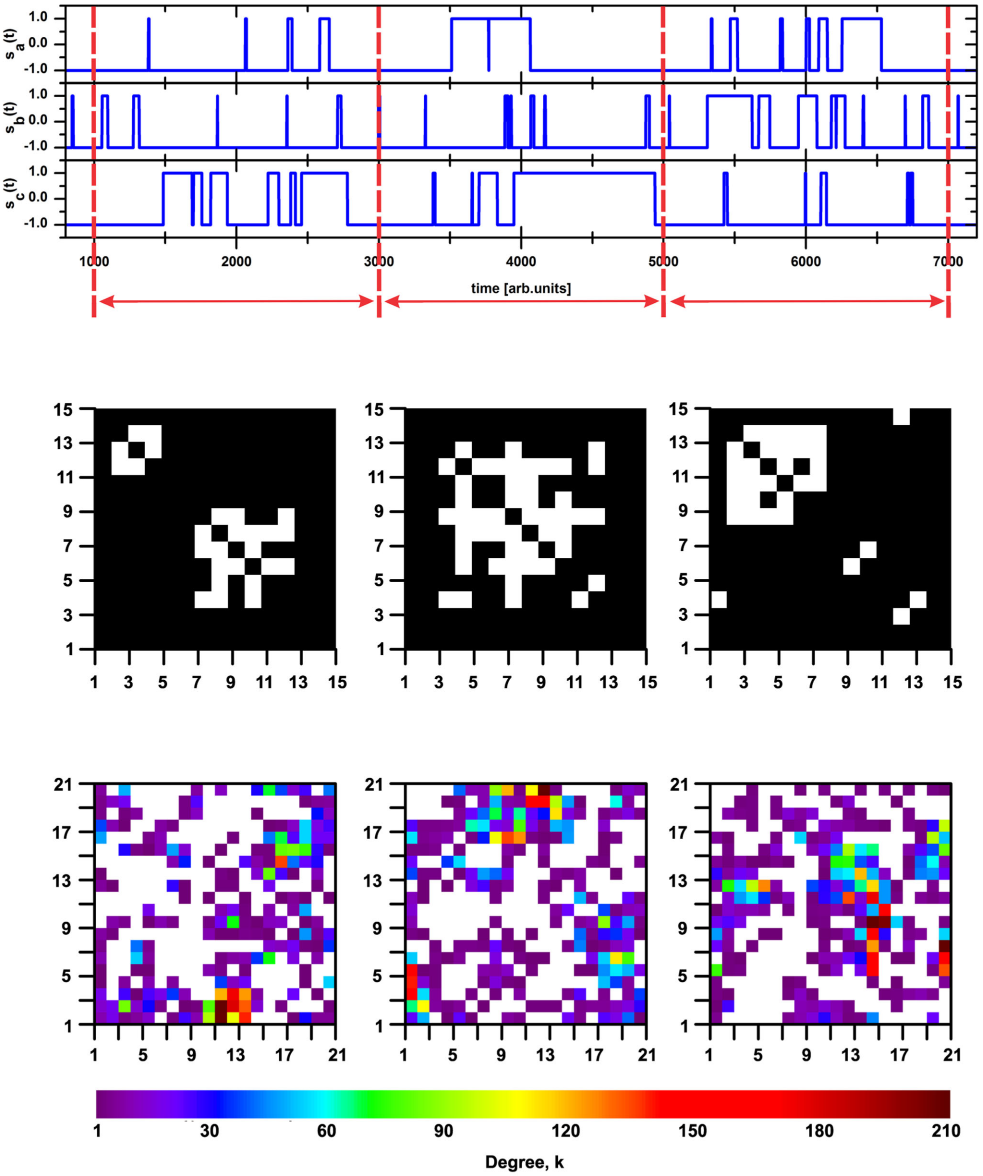

FIGURE 2 | Consecutive steps that lead to the dynamical topology. Top panel shows time evolution of three randomly selected nodes of the square lattice. Arrows denote time intervals over which the correlation index of Eq. 4 is calculated. Middle panel illustrates the fragment of an adjacency matrix of the dynamically created network. It originates by the application of a threshold Θ to the values of correlation all nodes of square lattice. Full size of this adjacency matrix is $N^2 \times N^2$. White squares denote the case when the correlation between unit *i* and *j* is larger than threshold and they correspond to the link between node *i* and *j* in the dynamical network. Lower panel presents fragment of the dynamical topology that arises from above adjacency matrix. Here a single square is a node of the dynamical network. Full dimension of this matrix is $N \times N$. The color scheme denotes the degree of a node. As in the case of the adjacency matrix, this representation confirms that the dynamical network is very sparse when overlaid on the original square lattice (white squares represent nodes of the square lattice that are not a part of the dynamical network). The clarity of figures presented on middle and bottom panels are the only reason for showing fragments of matrices, rather them whole.

coexistence of communities with conflicting opinions, reminiscent of recent results obtained by others (Shao et al., 2009).

DISCUSSION

Note that the scale-free network generated by the mechanism of cooperation is observed in the brain dynamics of neuronal networks (Fraiman et al., 2009). Although these authors use the two-state nodes of the Ising model to explain the source of the scale-free neuronal network, the Ising and the DMM model are essentially equivalent as far as dynamically generated scale-free topology is concerned. This paper, inspired in part by Fraiman et al. (2009), yields the additional discovery that the emergence of consensus produces long-rage links as well scale-free topology, thereby establishing a possible connection between the brain dynamics and the navigation in complex networks (Kleinberg, 2000; Boguñá and Krioukov, 2009; Boguñá et al., 2009; Li et al., 2010).

FAULT TOLERANCE

Let us address the problem of network efficiency and vulnerability. The scale-free topologies are universally judged to be robust against an external random attack, since the key nodes (hubs) are rare and their annihilation is expected to be highly improbable. However, if a unit with a large number of links is annihilated, the efficiency of the network is drastically reduced. Thus, a scale-free network is robust against a random attack but not against an intelligent attack whose targets are the hubs of the network (Albert et al., 2000; Cohen et al., 2000; Gallos et al., 2005).

Now, as illustrated on **Figure 4**, if we consider the emergence of consensus on a DGR scale-free topology two scenarios arise: one in which the evaluation of DMM leads to a state of non-consensus and one in which the consensus depends on the structure of the DGR network. An annihilation of a hub in the latter case would cause disruption of network dynamics and could affect the consensus state. This paper shows that it may be convenient to run the DMM on the original regular network, with $M = 4$, even if the self-organization process would require a higher critical parameter, $K_C = 1.70$ rather than $K_C = 1.00$. The regular network, although dynamically less efficient, is less vulnerable than the scale-free network, either the one dynamically generated or the *ad hoc* network. To substantiate this prediction, let us imagine an intelligent attack annihilating a highly connected hub in the dynamical topology, which action affects the organization state. Since targeted node simultaneously is one of the units of regular lattice, we observe the consequence of the attack also in the regular network, from which dynamical topology emerge. In this case the damage provoked by the intelligent attack is the generation of a defect, and consequently of a network of $N - 1$ nodes, with all of them having four links but four of them that have three links. We expect that for $N \to \infty$ the self-organization process is not severely damaged. To paraphrase, if the flag bearer, guiding a large number of fighters, falls in the battle another fighter will pick up the fallen flag and carry it forward. According to Singer (2005) this property is shared by the brain and is depicted by him as an orchestra with no conductor.

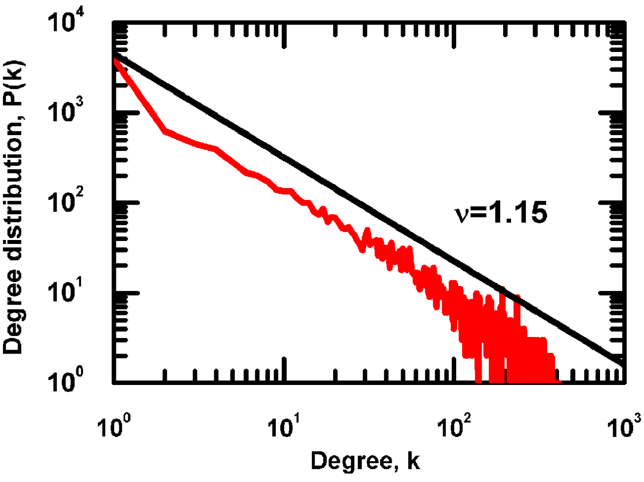

FIGURE 3 | The degree distribution for correlation network created by examining the dynamics of nodes placed on two-dimensional, regular lattice. Examined network had $N = 6740$ nodes and the parameters of DMM were $g = 0.01$ and $K = 1.70$.

HEBBIAN LEARNING

Hebb (1949) proposed the following neurophysiological postulate: "*When an axon of cell A is near enough to excite a cell B and repeatedly or persistently takes part in firing it, some growth process or metabolic change takes place in one or both cells such that A's efficiency, as one of the cells firing b, is increased*," according to which persistence or repetition of neural activity induces lasting cellular changes. We interpret the results of this paper in the light of this fundamental postulate.

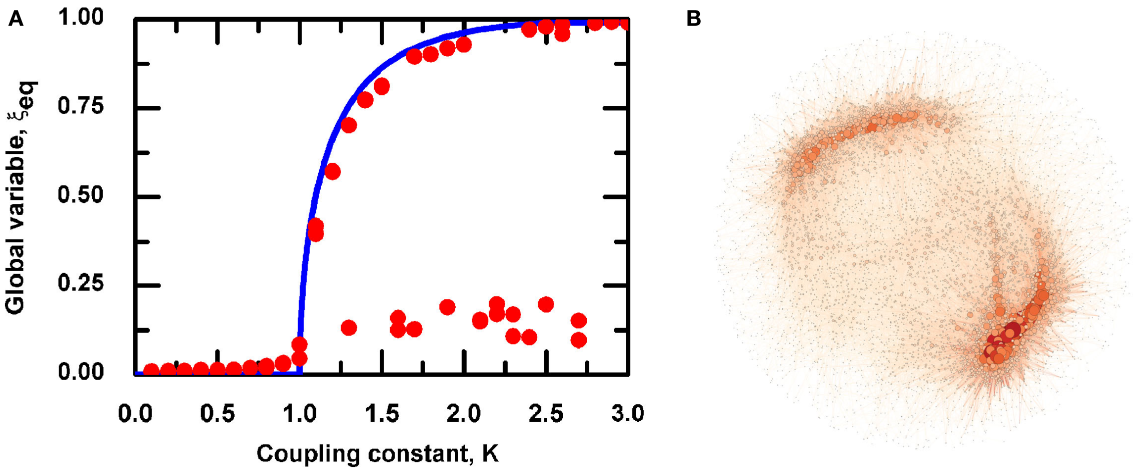

FIGURE 4 | (A) Shows the phase diagram for global variable ξ_{eq}. Solid blue line corresponds to the equilibrium states obtained in the case of fully connected, infinite network ($N = \infty$). Dots correspond to the equilibrium states obtained by evaluating the dynamics of the decision making model on the DGR correlation network with $g = 0.01$. **(B)** Presents graphic depiction of the dynamical network prepared with the help of open-source graph visualization software, Gephi (Bastian et al., 2009).

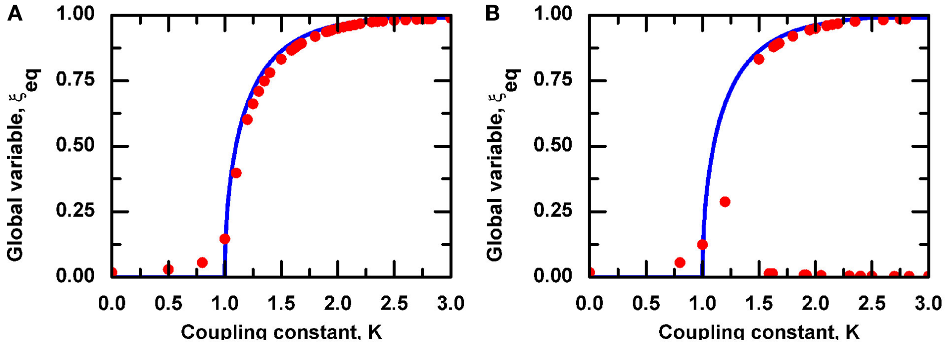

FIGURE 5 | The phase diagram for global variable ξ_{eq}. Solid blue line corresponds to the equilibrium states obtained in the case of fully connected, infinite network ($N = \infty$). **(A)** Dots show the phase diagram for a scale-free *ad hoc* network with $N = 2000$ nodes and scaling exponent of degree distribution $\nu = 1$. **(B)** Dots correspond to the equilibrium states obtained by evaluating the dynamics of the decision making model on a system of two scale-free *ad hoc* networks, each having $N = 1000$ nodes and scaling exponent $\nu = 1$. Those networks are coupled with only one randomly created link. In both cases scale-free *ad hoc* networks were generated according to the probabilistic algorithm of Kalisky et al., 2004.

First, we notice that at criticality the system of cooperating units reaches consensus but the consensus time duration is finite (Turalska et al., 2011). Vanni et al. (2011) have proved that the function of a system of cooperating units benefits from the occurrence of organizational collapses, allowing single units to recover the free-will condition that they have in the absence of cooperation. The consensus time duration T_C is the distance between two consecutive organizational collapses. Using the same procedure as that adopted in the recent work of Turalska et al. (2011), Vanni et al. (2011) find that T_C is proportional to the size of the system. Thus, since dynamically induced links are established on the basis of a temporal measure (Eq. 4), we assume that they correspond to cellular changes lasting for an extended, but not infinite time. Consecutively, we make the conjecture that according to Hebb's postulate, after a number of organizational collapses, reoccurring dynamical links are turned into resting links. This leads to an iterative procedure; where the initial resting network evolves into a dynamical structure (DGR1) which after certain time is used as the resting topology. Evaluation of the DMM on this new topology leads to the generation of a new dynamical network (DGR2), which then can be considered as a starting point.

It is worth noticing that this model for evolutionary network is rather crude, since the DGR2 network is established dynamically only on the basis of positive correlations between the units of the DGR1 network. A more realistic model would imply a random growth of connections, with a selection driven not purely by reinforcing Hebb's rule. However, from the results of this paper, we eventually expect the emergence of a complex network out of any evolution scheme. If we make the plausible hypothesis that increasing the complexity of the network also the dynamics becomes more complex, this evolution has been experimentally established by a recent paper (Smit et al., 2011) by looking at how scale-free indexes in electroencephalograms evolve with brain maturation in children and adolescent.

Herein we proved that, while dynamically induced links evolve into resting links, the thermodynamics of the system change, and the critical coupling significantly decreases. When a resting scale-free network evolves, with the topology herein studied, the coupling coefficient becomes indistinguishable from the minimal possible value, corresponding to all-to-all coupling. This means that the system (the brain) has efficiently decreased the resources necessary to keep working at criticality, both in terms of wiring (number of synapses) and coupling (amount of neurotransmitters). We may conjecture that the hypothesis that the brain works at criticality (Allegrini et al., 2010; Chialvo, 2010; Werner, 2010), or, better, that remains critical while exploring a continuity of different critical states (Longo and Montévil, 2011) is more fundamental than the Hebb's rule itself. In other words, Hebb's rule may have been evolutionary selected due to the cost-saving mechanisms herein suggested.

What about fault tolerance? Apart from the fault tolerance properties exposed in the previous subsection, that characterize the scale-free structure evolving from the Hebb's rule, Hebb's rule also provides network plasticity, so that even hub-oriented attacks can be effectively dealt with. Within our crude evolutionary model, due to the probabilistic nature of the process, it is expected that the links generated dynamically by DGRn do not coincide with its resting nodes, although many DGRn leaders may remain leaders at the DGR($n + 1$) level. Thus we conclude that the discovery of this paper that topological complexity may be dynamically generated is not incompatible with fault tolerance. This is again in accordance with Hebb and his followers, who prevent all synapses from increasing indefinitely by assuming that synaptic effectiveness is reduced when the simultaneous firing of post-abs pre-synaptic impulse does not occur.

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Social interactions model and adaptability of human behavior

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Human social networks evolve on the fast timescale of face-to-face interactions and of interactions mediated by technology such as a telephone calls or video conferences. The resulting networks have a strong dynamical component that changes significantly the properties of dynamical processes. In this paper we study a general model of pairwise human social interaction intended to model both face-to-face interactions and mobile-phone communication. We study the distribution of durations of social interactions in within the model. This distribution in one limit is a power-law, for other values of the parameters of the model this distribution is given by a Weibull function. Therefore the model can be used to model both face-to-face interactions data, where the distribution of duration has been shown to be fat-tailed, and mobile-phone communication data where the distribution of duration is given by a Weibull distribution. The highly adaptable social interaction model propose in this paper has a very simple algorithmic implementation and can be used to simulate dynamical processes occurring in dynamical social interaction networks.

Keywords: social networks, dynamical networks, reinforcement dynamics

INTRODUCTION

In the last 10 years the network theory (Dorogovtsev and Mendes, 2003; Newman, 2003; Boccaletti et al., 2006; Caldarelli, 2007; Barrat et al., 2008) has become the fundamental theory for studying complex systems. Indeed complex networks describe a large variety of technological, biological, and social complex systems. While great attention has been addressed to static networks and the dynamical processes occurring on them, less attention has been paid to dynamical networks and in particular to dynamical social networks.

Social networks (Granovetter, 1973; Wasserman and Faust, 1994) have a very relevant community structure (Palla et al., 2007; Bianconi et al., 2009; Ahn et al., 2010) and are adaptive and extremely dynamical. In the last years several papers have proposed adaptive models of social behavior showing phase transitions in different universality classes (Davidsen et al., 2002; Marsili et al., 2004; Holme and Newman, 2006; Vazquez et al., 2008). In these models social ties are dynamically rewired in a framework of stochastic models. Social ties have in addition to that, a microscopic structure constituted by fast social interactions of the duration of a phone call or of a face-to-face interaction. Dynamical social networks characterize the social interaction at this fast timescale. Recently new network measures are started to be defined for these dynamical networks (Tang et al., 2010) and recent works focus on the implication that the network dynamics has on percolation, epidemic spreading, and opinion dynamics (Holme, 2005; Vázquez et al., 2007; Parshani et al., 2010; Isella et al., 2011; Karsai et al., 2011).

Recently, thanks to the availability of new extensive data on a wide variety of human dynamics (Barabási, 2005; Eagle and Pentland, 2006; Malmgren et al., 2008, 2009; Rybski et al., 2009), human mobility (Brockmann et al., 2006; González et al., 2008; Song et al., 2010), and dynamical social networks (Onnela et al., 2007), it has been recognized that many human activities (Vázquez et al., 2007) are not Poissonian (West et al., 2008). New data on social dynamical networks start to be collected with new technologies such as of Radio Frequency Identification Devices (Cattuto et al., 2010; Isella et al., 2011) and Bluetooth (Eagle and Pentland, 2006). These technologies are able to record the duration of social interactions and report evidence for a bursty nature of social interaction characterized by a fat tail distribution of the duration of face-to-face interactions. This bursty behavior of social networks (Hui et al., 2005; Scherrer et al., 2008; Cattuto et al., 2010; Stehlé et al., 2010; Isella et al., 2011; Zhao et al., 2011) is coexisting with modulations coming from periodic daily (circadian rhythms) or weakly patterns (Jo et al., 2011). Recently new evidence coming from the analysis of mobile-phone communication (K. Zhao, M. Karsai, and Bianconi, in preparation) shows that the duration of mobile-phone interactions is also not Poissonian but has a finite second moment and is described by a Weibull distribution.

Therefore there is a necessity to generate an efficient dynamical model that has the flexibility to account both for scale-free distribution of duration of interactions, as found in face-to-face interactions, and stretched exponential (Weibull distributed) duration of interaction as found in the data of mobile-phone communication.

The fact that the bursty behavior is observed also in social interaction of simple animals (leeches; G. Bisson, G., Bianconi, and V. Torre, in preparation), in the motion of rodents (Anteneodo and Chialvo, 2009), or in the use of words (Altmann et al., 2009), suggests that the underlying origin of this behavior is dictated by the biological and neurological processes underlying the dynamics of the social interaction.

In this paper we use a model with dynamical reinforcement to model the nature of social interaction. Disregarding for the moment the effects of circadian rhythms and weakly patterns, we assume that the decision of the agents to form or leave a group are driven by memory effects dictated by reinforcement dynamics, that can be summarized in the following statements: (i) the longer an agent is interacting in a group the smaller is the probability that he/she will leave the group; (ii) the longer an agent is isolated the smaller is the probability that he/she will form a new group. The biological origin of this principle (which has clear similarity with the Hebbian reinforcement mechanism) will be investigated in future works where it will be studied in the framework of the neuroscience of social interactions in simple animals.

In this paper we show that depending on the nature of the memory kernel used to implement this dynamics the distribution of duration of interaction changes from Weibull distributions to scale-free distributions. This indicates that human social interactions might be determined by an effective use of different memory kernel depending on the different nature of social interactions (e.g., Face-to-face interactions or mobile-phone communication).

THE PROPOSED MODEL OF HUMAN SOCIAL INTERACTIONS

We consider a system consisting of N agents representing interacting agents. The agents are interacting in a social network G representing social ties such as friendships, collaborations, or acquaintances. The network G is weighted with the weights indicating the strength of the social ties between agents. To model the mechanism of human social interaction the agents can call their neighbors in the social network G forming groups of interacting agents. Since at any given time an interaction can be initiated or terminated the network is highly dynamical. For the general model defined above, we use $n_i(t) \in N$ to denote the number of agents in the group of agent i at time t. In this paper, we focus on a simple case with $n_i \leq 2$, such that only pairwise interactions are allowed in the system, or in other words, the agents can only interact with one partner at a time. Indeed, pairwise interaction is the most typical social interaction existing in reality, e.g., phone-call communication or point-to-point on-line connection. Specifically $n_i = 1$ refers to the status of a non-interacting agent i while $n_i = 2$ refers to the status of an interacting agent. Here we give an explicit algorithm for the model of pairwise interaction:

(1) An agent i is selected randomly at time t (initially $t = 0$).
(2) Agent i has two possible subsequent actions:
  - If $n_i(t) = 1$, he/she will start an interaction with probability $f_1(t_i, t)$ where t_i refers to the last time at which agent i has changed his/her state. Once the interaction is initiated, a partner j will be chosen from one of his/her non-interacting neighbors of G with probability proportional to $\Pi(t_j, t)$, and consequently $n_i \rightarrow 2$ and $n_j \rightarrow 2$.
  - If $n_i(t) = 2$, he/she will terminate his/her current interaction with probability $f_2(t_i, t \mid w_{ij})$ where w_{ij} is the weight of the link between i and the neighbor j that is interacting with i, and consequently $n_i \rightarrow 1$ and $n_j \rightarrow 1$.
(3) Time t is updated as $t \rightarrow t + 1/N$ and the process is iterated until $t = T_{max}$.

One should note that, the corporation of t_i in the probability functions f_1, Π and f_2, indeed, reflects the memory effect of agents that their current behavior is depending on the history of their previous behavior. For simplicity, we also assume $f_1 = \Pi$ in this paper, which means the probability of starting an interaction is identical to the probability of replying to an interaction.

MEAN-FIELD SOLUTION TO THE MODEL

The model of pairwise interaction can be solved analytically if we assume G is an uncorrelated and annealed network. The assumption means, at each time step, the links of G are rewired randomly while the degree distribution $p(k)$ and the weight distribution $p(w)$ are remained constant. We denote by $N_1^k(t_0, t)dt_0$ the number of non-interacting agents with degree k at time t that have not interacted with another agent since time $t' \in (t_0, t_0 + 1/N)$. Similarly we denote by $N_2^{k,k',w}(t_0, t)dt_0$ the number of interacting agent pairs (with degree respectively k and k' and weight of the link w) at time t being in the interaction since $t' \in (t_0, t_0 + 1/N)$. In the continuous time limit $N \rightarrow \infty$, the rate equation can be written as a differential form

$$\begin{aligned}
\frac{\partial N_1^k(t_0, t)}{\partial t} &= -N_1^k(t_0, t) f_1(t_0, t) - ckN_1^k(t_0, t) f_1(t_0, t) \\
&\quad + N\pi_{21}^k(t)\delta_{tt_0} \\
\frac{\partial N_2^{k,k',w}(t_0, t)}{\partial t} &= -2N_2^{k,k',w}(t_0, t) f_2(t_0, t|w) + N\pi_{12}^{k,k',w}(t)\delta_{tt_0}.
\end{aligned} \tag{1}$$

The term with normalization constant c in Eq. (1) indicates the rate of non-interacting agents with degree k being contacted by others (If G is uncorrelated, this term should be proportional to k), and c is given by

$$c = \frac{\sum_{k'} \int_0^t dt_0 N_1^{k'}(t_0, t) f_1(t_0, t)}{\sum_{k'} k' \int_0^t dt_0 N_1^{k'}(t_0, t) f_1(t_0, t)}. \tag{2}$$

The rates $\pi_{pq}(t)$ in Eq. (1) indicate the average number of agents changing from state $p = 1$, 2 to state $q = 1$, 2 at time t. These rates can be also expressed in a self-consistent way as

$$\begin{aligned}
\pi_{21}^k(t) &= \frac{2}{N} \sum_{k',w} \int_0^t dt_0 f_2(t_0, t|w) N_2^{k,k',w}(t_0, t) \\
\pi_{12}^{k,k',w}(t) &= \frac{P(w)}{CN} \int_0^t dt_0 \int_0^t dt_0' N_1^k(t_0, t) N_1^{k'}\left(t_0', t\right) f_1(t_0, t) \\
&\quad \times f_1\left(t_0', t\right)\left(k + k'\right).
\end{aligned} \tag{3}$$

where the constant C is given by

$$C = \sum_{k'} \int_0^t dt_0 k' N_1^{k'}(t_0, t) f_1(t_0, t). \tag{4}$$

The solution to Eq. (1) is given by

$$N_1^k\left(t_0,t\right) = N\pi_{21}^k\left(t_0\right)e^{-(1+ck)\int_{t_0}^t f_1(t_0,t)dt}$$
$$N_2^{k,k',w}\left(t_0,t\right) = N\pi_{12}^{k,k',w}\left(t_0\right)e^{-2\int_{t_0}^t f_2(t_0,t|w)dt}. \tag{5}$$

To get a complete solution to the model, we still need to utilize the conservation of the number of agents with degree k, that is

$$\int dt_0\left[N_1^k\left(t_0,t\right)+\sum_{k',w}N_2^{k,k',w}\left(t_0,t\right)\right] = Np(k). \tag{6}$$

Therefore, combining Eqs (5–7), the solution to the model is completely determined.

Especially we are interested in the stationary solutions, which are expected to satisfy

$$N_1^k\left(t_0,t\right) = N_1^k(\tau)$$
$$N_2^{k,k',w}(t_0,t) = N_2^{k,k',w}(\tau)$$
$$\pi_{pq}(t) = \pi_{pq}, \tag{7}$$

where $\tau = t - t_0$ is the duration time elapsed since the last time the agent has changed his/her state. A stationary solution means that, the system reaches a dynamical equilibrium, such that the macro distribution of agents with different states are time-dependent and the transition rate is constant. In the next section, we will see a stationary solution can be achieved by specifically choice of f_1 and f_2 in an asymptotic time limit $t \to \infty$.

Finally, by convenience, we denote by $P_1^k(t_0,t)$ the probability distribution that an agent with degree k is non-interacting for a period from t_0 to t and by $P_2^w(t_0,t)$ the probability that a connection of weight w at time t is active since time t_0. These distributions are obviously proportional to the number of individual in a state $n = 1, 2$ multiplied by the probability of having a change of state, i.e.,

$$P_1^k\left(t_0,t\right) \propto (1+ck)f_1\left(t_0,t\right)N_1^k\left(t_0,t\right)$$
$$P_2^w\left(t_0,t\right) \propto 2f_2\left(t_0,t|w\right)\sum_{k,k'}N_2^{k,k',w}\left(t_0,t\right). \tag{8}$$

STATIONARY SOLUTION OF THE MODEL

In the previous section, one should notice that the form of Eq. (5), the solution to the model of pairwise interaction, is subject to the choice of probability functions f_1 and f_2. A lot of evidence from recent measurements reveals that social interaction and human activities are not merely Poisson process but with bursty effect. The distribution of duration time and inter-event time from the empirical data, turn out to be non-trivial and shows scaling property. To capture these phenomena observed in reality, we define f_1 and f_2 as

$$f_1\left(t_0,t\right) = f_1(\tau) = \frac{b_1}{(1+\tau)^\beta}$$
$$f_2\left(t_0,t|w\right) = f_2(\tau|w) = \frac{b_2 g(w)}{(1+\tau)^\beta}. \tag{9}$$

$b_1 > 0$, $b_2 > 0$, $0 \le \beta \le 1$ are some characteristic parameters and $g(w)$ is an arbitrary positive function. The definition by Eq. (9), indeed reflects a reinforcement dynamics we apply to the model, which could be summarized as "the longer an agent stays in its current state, the more possible it tends to maintain it." The function $g(w)$ is generally chosen as a decreasing function of w, indicating that interacting agent pairs with a stronger weight of tie interact typically for a longer time. In the asymptotic time limit, the necessary condition for the stationary solution to exist is that the summation of self-consistent constraints given by Eqs (2) and (4) together with the conservation law Eq. (6) converge under the stationary assumptions Eq. (7). The convergence depends on the value of the parameters b_0, b_1, β and the choice of function $g(w)$. In particular, when $0 \le \beta < 1$, the convergence is always satisfied. In the following subsections, we will characterize further the stationary state solution of this model in different limiting cases.

CASE $0 < \beta < 1$

The expression for the number of agent in a given state $N_1^k(\tau)$ and $N_2^{k,k',w}(\tau)$ can be obtained by substituting Eq. (9) into the general solution Eq. (5), using the stationary conditions Eq. (7). In this way we get the stationary solution given by

$$N_1^k(\tau) = N\pi_{21}^k e^{\frac{b_1(1+ck)}{1-\beta}\left[1-(1+\tau)^{1-\beta}\right]}$$
$$= N\pi_{21}^k m_1^k(\tau)$$
$$N_2^{k,k',w}(\tau) = N\pi_{12}^{k,k',w}e^{\frac{2b_2 g(w)}{1-\beta}\left[1-(1+\tau)^{1-\beta}\right]}$$
$$= N\pi_{12}^{k,k',w}m_2^w(\tau). \tag{10}$$

To complete the solution is necessary to determine the constants π_{21}^k and $\pi_{12}^{k,k'w}$ in a self-consistent type of solution. To find the expression of $\pi_{12}^{k,k',w}$ as a function of π_{21}^k we substitute Eq. (10) in Eq. (3) and we get

$$\pi_{12}^{k,k',w}(t) = \frac{1}{C}\pi_{21}^k P(w)$$
$$\times\left[k\int_0^t dt_0 m_1^k\left(t_0,t\right)f_1\left(t_0,t\right)\int_0^t dt_0' N_1^{k'}\left(t_0',t\right)f_1\left(t_0',t\right)\right.$$
$$\left.+k'\int_0^t dt_0 m_1^k\left(t_0,t\right)f_1\left(t_0,t\right)\int_0^t dt_0' N_1^{k'}\left(t_0',t\right)f_1\left(t_0',t\right)\right]. \tag{11}$$

Finally we get a closed equation for π_{21}^k by substituting Eq. (11) in Eq. (6) and using the definition of c and C, given respectively by Eq. (2) and Eq. (4). Therefore we get

$$\pi_{21}^k\left[\int_0^\infty m_1^k(\tau)d\tau + \int_{w_{min}}^{w_{max}}P(w)\int_0^\infty m_2^w(\tau)d\tau dw\right.$$
$$\left.\times\left(ck\int_0^\infty m_1^k(\tau)f_1(\tau)d\tau + \int_0^\infty m_1^k(\tau)f_1(\tau)d\tau\right)\right] = p(k). \tag{12}$$

Performing explicitly the last two integrals using the dynamical solution given by Eq. (10), this equation can be simplified as

$$\pi_{21}^{k} = \left[\int_{0}^{\infty} m_1^k(\tau) d\tau + \int_{w_{min}}^{w_{max}} P(w) \int_{0}^{\infty} m_2^w(\tau) d\tau dw \right]^{-1} p(k). \quad (13)$$

Finally the self-consistent solution of the dynamics is solved by expressing Eq. (2) by

$$c = \frac{\sum_k \pi_{21}^k (1+ck)^{-1}}{\sum_k \pi_{21}^k k(1+ck)^{-1}}. \quad (14)$$

Therefore we can use Eqs (13) and (14) to compute the numerical value of π_{21}^k and *c*. Inserting in these equations the expressions for $f_1(\tau), f_2(\tau \mid w)$ given by Eq. (9) and the solutions $N_1^k(\tau), N_2^{k,k',w}(\tau)$ given by Eq. (10) we get

$$\begin{aligned} P_1^k(\tau) &\propto \frac{b_1(1+ck)}{(1+\tau)^{\beta}} e^{-\frac{b_1(1+ck)}{1-\beta}(1+\tau)^{1-\beta}} \\ P_2^w(\tau) &\propto \frac{2b_2 g(w)}{(1+\tau)^{\beta}} e^{-\frac{2b_2 g(w)}{1-\beta}(1+\tau)^{1-\beta}}. \end{aligned} \quad (15)$$

The probability distributions $P_1^k(\tau)$ and $P_2^w(\tau)$, can be manipulating performing a data collapse of the distributions, i.e.,

$$\begin{aligned} \tau_1^{\star}(k) P_1^k \left(x_1 = \frac{\tau}{\tau_1^{\star}(k)} \right) &= A_1 \, x_1^{-\beta} \, e^{-\frac{x_1^{1-\beta}}{1-\beta}} \\ \tau_2^{\star}(w) P_2^w \left(x_2 = \frac{\tau}{\tau_2^{\star}(w)} \right) &= A_2 \, x_2^{-\beta} \, e^{-\frac{x_2^{1-\beta}}{1-\beta}} \end{aligned} \quad (16)$$

with $\tau_1^{\star}(k)$ and $\tau_2^{\star}(w)$ defined as

$$\begin{aligned} \tau_1^{\star}(k) &= [b_1(1+ck)]^{-\frac{1}{1-\beta}} \\ \tau_2^{\star}(w) &= [2b_2 g(w)]^{-\frac{1}{1-\beta}} \end{aligned} \quad (17)$$

where A_1 and A_2 are the normalization factors. The data collapse defined by Eq. (16) of the curves $P_1^k(\tau), P_2^w(\tau)$ and are both described by Weibull distributions.

CASE $\beta = 0$

For $\beta = 0$ the functions $f_1(\tau)$ and $f_2(\tau \mid w)$ given by Eq. (9) reduce to constants, therefore the process of creation of an interaction is a Poisson process and no reinforcement dynamics is taking place in the network. Assigning $\beta = 0$ to Eq. (5), we get the solution

$$\begin{aligned} N_1^k(\tau) &= N\pi_{21}^k e^{-b_1(1+ck)\tau} \\ N_2^{k,k',w}(\tau) &= N\pi_{12}^{k,k',w} e^{-2b_2 g(w)\tau}. \end{aligned} \quad (18)$$

and consequently the distributions of duration of given states Eq. (8) are given by

$$\begin{aligned} P_1^k(\tau) &\propto e^{-b_1(1+ck)\tau} \\ P_2^w(\tau) &\propto e^{-2b_2 g(w)\tau}. \end{aligned} \quad (19)$$

Therefore the probability distributions $P_1^k(\tau)$ and $P_2^w(\tau)$ are exponentials as expected in a Poisson process.

CASE $\beta = 1$

In this section, we discuss the case for $\beta = 1$ such that $f_1^k(\tau) \propto (1+\tau)^{-}1$ and $f_2^w(\tau|w) \propto (1+\tau)^{-1}$. Using Eq. (1) we get the solution

$$\begin{aligned} N_1^k(\tau) &= N\pi_{21}^k (1+\tau)^{-b_1(1+ck)} \\ N_2^{k,k',w}(\tau) &= N\pi_{12}^{k,k',w} (1+\tau)^{-2b_2 g(w)}. \end{aligned} \quad (20)$$

and consequently the distributions of duration of given states Eq. (8) are given by

$$\begin{aligned} P_1^k(\tau) &\propto \pi_{21}^k (1+\tau)^{-b_1(1+ck)-1} \\ P_2^w(\tau) &\propto \pi_{12}^{k,k',w} (1+\tau)^{-2b_2 g(w)-1}. \end{aligned} \quad (21)$$

The probability distributions are power-laws. This result remains valid for every value of the parameters b_1, b_2, $g(w)$ (See Zhao et al., 2011 for a full account of the detailed solution of this model) nevertheless the stationary condition is only valid for

$$\begin{aligned} b_1(1+ck) &> 1 \\ 2b_2 g(w) &> 1. \end{aligned} \quad (22)$$

Indeed this condition ensures that the self-consistent constraints Eqs (2) and (4) and the conservation law Eq. (6) have a stationary solution.

COMPARISON WITH NUMERICAL SIMULATIONS

To check the validity of our annealed approximation versus quenched simulations, we performed a computer simulation according to the dynamical process on a quenched network for the cases $\beta \in (0, 1)$, $\beta = 0$ and $\beta = 1$. In **Figure 1** we compare the results of the simulation for $\beta \in (0, 1)$ with the predictions of the analytical solution. In particular in the reported simulation we have chosen $\beta = 0.5$, $b_1 = 0.02$, $b_2 = 0.05$, and $g(w) = w^{-1}$, the simulation is based on a number of agent $N = 2000$ and for a period of $T_{max} = 10^5$, finally the data are averaged over 10 realizations and the network is Poisson with average $\langle k \rangle = 6$ and weight distribution $p(w) \propto w^{-2}$. In **Figure 1**, we show evidence that the Weibull distribution and the data collapse of $P_2^w(\tau)$ well capture the empirical Weibull behavior observed in the mobile-phone data. The simulated data are also in good agreement with the analytical predictions derived in the annealed approximation for the parameter chosen in the figure. As the network becomes more busy and many agents are in a telephone call, the quenched simulation and the annealed prediction of $P_1^k(\tau)$ differs more significantly. In **Figure 2** we plot the distribution $P_2^w(\tau)$ in the Poisson case $\beta = 0$ and we compare the simulation with the theoretical prediction. In particular in the reported simulation we have chosen $\beta = 0.5$, $b_1 = 0.02$, $b_2 = 0.05$ and $g(w) = w^{-1}$, the simulation is based on a number of agent $N = 2000$ and for a period of $T_{max} = 10^5$, finally the data are averaged over 10 realizations and the network is Poisson with average $\langle k \rangle = 6$ and weight distribution $p(w) \propto w^{-2}$. Finally in **Figure 3** we plot the distribution $P_2^w(\tau)$ in the power-law

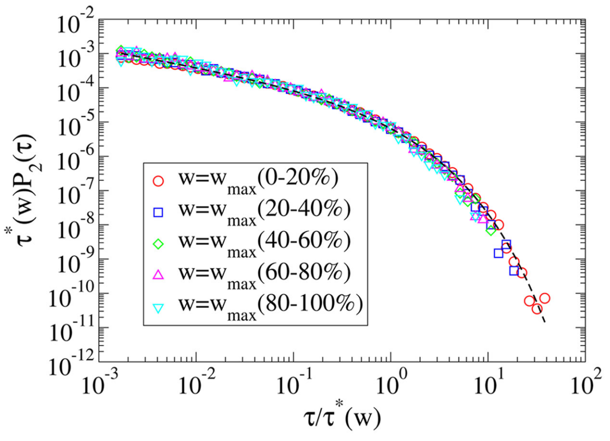

FIGURE 1 | Data collapse of the simulation of the proposed model for cell phone communication with $\beta = 0.5$. In the figure we plot the probability $P_2^w(\tau)$ that in the model a pair of agents with strength w are interacting for a period τ. The collapses data of $P_2^w(\tau)$ is described by Weibull distribution as expected by the theoretical prediction indicated with a solid line.

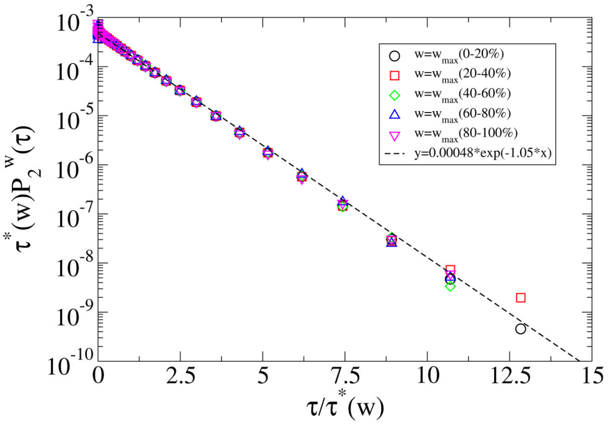

FIGURE 2 | Data collapse of the simulation of the proposed model for cell phone communication with $\beta = 0$. In the figure we plot the probability $P_2^w(\tau)$ that in the model a pair of agents with strength w are interacting for a period τ. The collapses data of $P_2^w(\tau)$ is described by the exponential distribution predicted theoretically and indicated by a solid line.

case $\beta = 1$. In this case we have consider model in the stationary phase and therefore we have taken $b_1 = b_2 = 0.8$. All others parameter of the model are the same as in the precedent simulations. In **Figure 3** we show that in this case the distribution $P_2^2(\tau)$ is a power-law with exponents depending on the weight of the link. The fitted exponents are in very good agreement with the exponents predicted by the mean-field solution of the model.

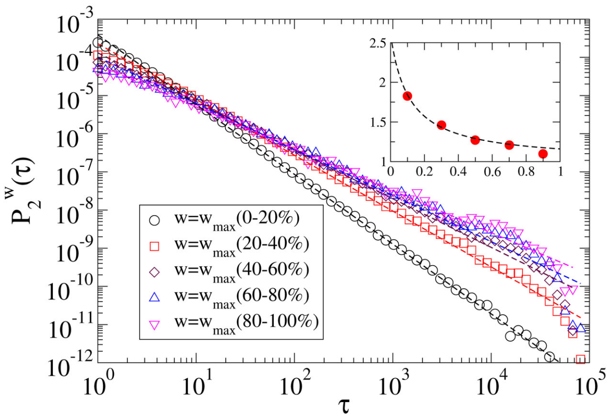

FIGURE 3 | Data of the simulation of the proposed model for cell phone communication for $\beta = 1$. In the figure we plot the probability $P_2^w(\tau)$ that in the model a pair of agents with strength w are interacting for a period τ. The collapses data of $P_2^w(\tau)$ is described by power-law distribution in agreement with the theoretical predictions. In the inset we report the power-law exponent versus the theoretically predicted power-law exponent showing good agreement.

CONCLUSION

In the last 10 years there has been an increasing interest in the structure of complex networks. New universality class has been found characterizing small-word and scale-free networks. Recently new data on social interaction networks are changing the way we look at social networks. In fact the new data is able to record social interactions on the fast timescale of a face-to-face interaction or on the duration of a mobile-phone communication. These new data constitute evidence for the "microscopic structure" or any social tie in a social network. Moreover this data show evidence that human social interactions are dictated by a non-Poissonian dynamics. Here in this paper we discuss a model of social interactions on the fast timescale. This model is able to reproduce both scale-free distribution of duration of interactions and Weibull distribution of duration of interaction as observed respectively in the data of face-to-face and mobile-phone communication. The hypothesis of the paper is that the dynamics of social interaction is mediated by a Hebbian like type of dynamics with memory effects that is able to reproduce the data. In future works we plan to biologically validate this hypothesis in the study of social interactions in simple animals. The model well reproduce the experimental data and is very easy to implement to include the effect of circadian rhythms or the role of an information spreading on the network. This flexibility of the model makes it a perfect platform to use this model in future works for simulating dynamical processes on dynamical social networks.

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Renormalization group for critical phenomena in complex networks

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We discuss the behavior of statistical models on a novel class of complex "Hanoi" networks. Such modeling is often the cornerstone for the understanding of many dynamical processes in complex networks. Hanoi networks are special because they integrate small-world hierarchies common to many social and economical structures with the inevitable geometry of the real world these structures exist in. In addition, their design allows *exact* results to be obtained with the venerable renormalization group (RG). Our treatment will provide a detailed, pedagogical introduction to RG. In particular, we will study the Ising model with RG, for which the fixed points are determined and the RG flow is analyzed. We show that the small-world bonds result in non-universal behavior. It is shown that a diversity of different behaviors can be observed with seemingly small changes in the structure of hierarchical networks generally, and we provide a general theory to describe our findings.

Keywords: renormalization group, critical phenomena, complex networks, Ising model, Hanoi networks

1. INTRODUCTION

The renormalization group (RG; Wilson, 1971; Wilson and Fisher, 1972) is by now a method found in any "classical" statistical physics text book (Goldenfeld, 1992; Plischke and Bergersen, 1994). It has allowed to categorize broad classes of equilibrium systems into an enumerable set of universality classes, each characterized by discrete features, such as their dimension and the symmetries adhered to by their Hamiltonians (Goldenfeld, 1992; Plischke and Bergersen, 1994). Such universality is made possible through the property of "scaling" that is an inherent feature near phase transitions (Kadanoff, 1966), which these systems undergo in certain regions of the space spanned by their physical parameters (couplings). Scaling invariance entails that system-specific details on the microscopic level become irrelevant, as the behavior over many orders in the range of the interactions become self-similar. In this framework, analogous behavior in a surprisingly wide set of phenomena, such as the condensation of fluids, spontaneous magnetization of materials, or the generation of particle mass in the early universe, can be described in a single effective theory; certainly a major intellectual accomplishment of modern physics across all fields (Goldenfeld, 1992).

In the past 15 years, statistical physicist have increasingly applied the ideas of critical phenomena and scaling to problems outside of the immediate material realm, in newly emerging fields such as "Econophysics," "Sociophysics," etc. (Mantegna and Stanley, 1999; Barabasi, 2003; Kleinert, 2004). The considered systems typically feature a large number of interacting agents sharing a finite set of intrinsic properties on account of which they interact. But unlike in a Euclidean defined arrangement of "actors" in a physical system, such as atoms in a material, these systems possess a more complex network of mutual interactions (which may even be directed; Watts and Strogatz, 1998; Boccaletti et al., 2006; Dorogovtsev et al., 2008). Thus, in many respects, the study of these phenomena is inseparable from the understanding of the geometry of networks (Barthelemy, 2011). One major accomplishment of these investigations is the realization that many of the networks that are engineered by some natural or human activity themselves exhibit emergent complex properties, for instance, as found in the scale-free degree distribution of the internet.

But while these networks, or dynamical systems on them, may behave critically, many of these phenomena were soon found to be non-universal. For instance, in the preferential attachment model for the world-wide net (Barabasi and Albert, 1999), the value of the scaling exponent is tied to microscopic details of the attachment rule. In this sense, it would seem unlikely that any sweeping classification scheme could be devised to categorize this amorphous pile of particulars. Here, we will attempt a foray into such a scheme, albeit limited to those classical equilibrium phenomena, but on a large set of different networks. We suspect that our discussion might help to explain, for example, a number of similar observations of traditionally obscure critical behaviors, such as infinite-order transitions, in very different network models, in and out of equilibrium (Dorogovtsev et al., 2008).

Our classification scheme is best introduced with a variety of hierarchical networks on which RG is exact, and the critical phenomena can be studied in great detail. Metric version of such networks, such as that introduced in the Migdal-Kadanoff RG, provide the classical text-book examples of RG and universality (Berker and Ostlund, 1979; Goldenfeld, 1992; Plischke and Bergersen, 1994; Hinczewski and Berker, 2006). But in the advent of complex networks, many hierarchical designs with non-metric (i.e., small-world or scale-free) properties have been devised and studied (Andrade et al., 2005; Hinczewski and Berker, 2006; Hinczewski, 2007; Boettcher and Goncalves, 2008; Boettcher et al., 2008). Our study shows that criticality in many of these models (percolation, Ising, etc.) is definitely non-universal but falls into a few (here, three) generic regimes, each characterized by its degree of singularity in, say, the correlation length at the critical point (Boettcher and Brunson, 2011). One of these regimes is indeed an infinite-order transition reminiscent of that described by Berezinskii and Kosterlitz and Thouless (BKT; Goldenfeld, 1992; Plischke and Bergersen, 1994). It is flanked on one side by a transition with a "weaker," algebraic divergence, similar to the classical ones of second order (but still non-universal), and on the other by a regime with a fully essential singularity. These regimes are defined through the relative strength of (non-Euclidean) long-range or small-world bonds in one and the same network, with clear demarcations between these regimes as a function of that coupling strength.

In the following Section 2, we describe the Hanoi networks. The analysis of the phase diagrams and the RG flow for the Ising ferromagnet on these networks is discussed in Sections 3 and 4. In Section 5, we introduce families of interpolating networks to reveal a more comprehensive set of regimes, each with its own characteristic type of phase transition, and we conclude with a discussion of our results in their implications in Section 7.

2. GEOMETRY OF THE HANOI NETWORKS

Each of the Hanoi network possesses a simple geometric backbone, a one-dimensional line of sites n, $0 \leq n \leq N = 2^k$ ($k \to \infty$). Each site is connected to its nearest neighbor, ensuring the existence of the $1d$-backbone. To generate the small-world hierarchy in these networks, consider parameterizing any integer n (except for zero) *uniquely* in terms of two other integers (i, j), $i \geq 0$, via

$$n = 2^i \left(2j + 1\right). \tag{1}$$

Here, i denotes the level in the hierarchy whereas $j \geq 0$ labels consecutive sites within each hierarchy. For instance, $i = 0$ refers to all odd integers, $i = 1$ to all integers once divisible by 2 (i.e., 2, 6, 10,...), and so on. Depending on its level of the hierarchy, any site has also small-world (i.e., long-range) bonds to more-distant sites along the backbone, according to some deterministic rule. For example, we obtain a 3-regular network HN3 by connecting also 1 to 3, 5 to 7, 9 to 11, etc., for $i = 0$, next 2 to 6, 10 to 14, etc., for $i = 1$, and 4 to 12, 20 to 28, etc., for $i = 2$, and so on, as depicted in **Figure 1**.

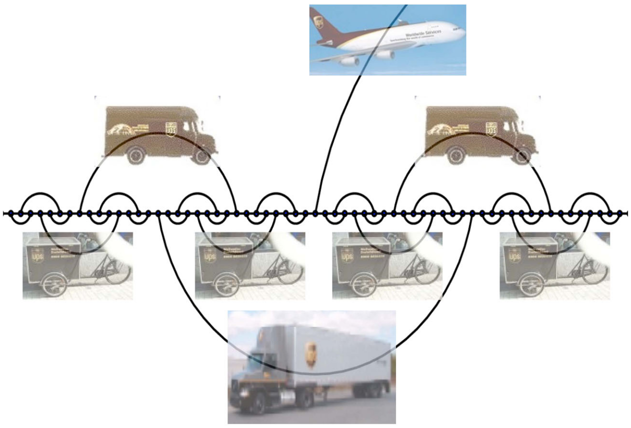

FIGURE 1 | Depiction of the 3-regular network HN3 on a semi-infinite line, here drawn suggestively as a model for hierarchically organized transport. Many goods and services are distributed in a small-world hierarchical manner before they get delivered in a real-world geometry.

While HN3 (and HN4 Boettcher et al., 2008) are of a fixed, finite degree, we introduced here convenient generalizations of HN3 that lead to new, revealing insights into small-world phenomena. First, we can extend HN3 in the following manner to obtain a new planar network of average degree 5, hence called HN5: In addition to the bonds in HN3, in HN5 we also connect all even sites to both nearest sites *within* the same level of the hierarchy $i(\geq 1)$. The resulting network remains planar but now sites have a hierarchy-dependent degree, as shown in **Figure 2**. It is easy to show that the *average* shortest path between any two sites increases $\sim \sqrt{N}$ in HN3, and logarithmically in HN5, with system size N.

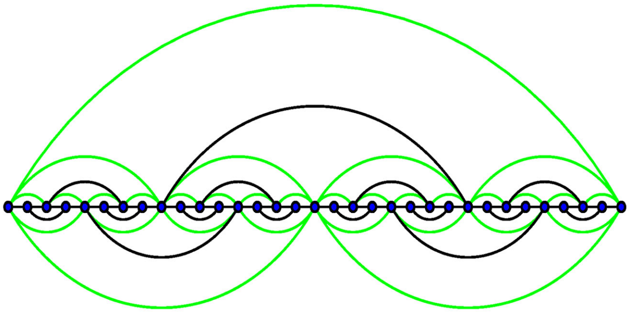

FIGURE 2 | Depiction of the planar network HN5, comprised of HN3 (black lines) with the addition of further long-range bonds (green-shaded lines). Note that sites on the lowest level of the hierarchy have degree 3, then degree 5, 7, etc., comprising a fraction of 1/2, 1/4, 1/8, etc., of all sites, which makes for an average degree 5 in this network.

3. ISING FERROMAGNET ON HN3

The RG consists of recursively tracing out spins level-by-level in the hierarchy (Boettcher et al., 2008). In terms of Eq. (1), we start by tracing out all sites with n odd, i.e., $i = 0$, then those n which are divisible by 2 only once, i.e., $i = 1$, and so on. We can always relabel all sites n after any RG step by $n \to n/2$, so that we trace out the respective odd-relabeled sites at any level. It is apparent, for instance from **Figure 1**, that odd-labeled sites are connected to their even-labeled nearest neighbors on the backbone, say, by a coupling $K_0 (= \beta J_0)$. At any level, each odd-labeled site $x_{n\pm1}$ is also connected to one other such site $x_{n\mp1}$ across an even-labeled site x_n with $n = 2(2j + 1)$ that is exactly *once* divisible by 2. Let us call that coupling $K_1 (= \beta J_1)$. The basic RG step is depicted in **Figure 3** and consists of tracing out the two sites $x_{n\pm1}$ neighboring the site x_n for all j with $n = 2(2j + 1)$.

We can section the Ising Hamiltonian

$$-\beta\mathcal{H} = \sum_{n=1}^{2^{k-2}} \left(-\beta\mathcal{H}_n\right) + \mathcal{R}\left(K_2, K_3, \ldots\right), \tag{2}$$

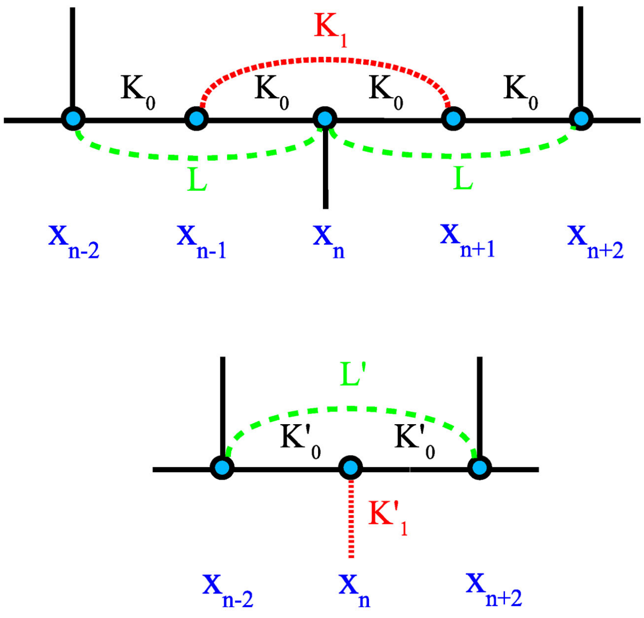

FIGURE 3 | Depiction of the (exact) RG step for the Ising model on HN3. The step consists of tracing out odd-labeled variables $x_{n\pm1}$ in the top plot and expressing the renormalized couplings (L', K_0') on the bottom in terms of the old couplings (L, K_0, K_1). Note that the original network in **Figure 1** does not contain couplings of type (L, L'), but that they certainly become relevant during the process.

where $\mathcal{R}$ contains all coupling terms of higher level in the hierarchy, and each sectional Hamiltonian is given by

$$-\beta\mathcal{H}_n = K_0\left(x_{n-2}x_{n-1} + x_{n-1}x_n + x_nx_{n+1} + x_{n+1}x_{n+2}\right) + K_1x_{n-1}x_{n+1} + L_0\left(x_{n-2}x_n + x_nx_{n+2}\right) + 4I, \tag{3}$$

where (K_0, K_1, L) are the unrenormalized couplings defined in **Figure 3** and I is a constant that fixes the overall energy scale per spin. (There are effectively 4 spins involved in each graph-let, as those at each boundary are equally shared with neighboring graph-lets.) While couplings of the type L_0 between next-nearest even-labeled neighbors emerges that are not part of the network initially in HN3, they do emerge during the RG step (otherwise the system of recursion equations would not close), see **Figure 3**.

To simplify the analysis, we introduce new variables similar to inverse "activities" (Plischke and Bergersen, 1994),

$$C = e^{-4I}, \quad \kappa = e^{-4K_0}, \quad \lambda = e^{-4L_0}, \quad \mu = e^{-2K_1}, \tag{4}$$

which ensure that the RG flow only contains algebraic functions and, for the ferromagnetic model, remains confined within the physical domain $0 \le \kappa, \lambda, \mu \le 1$. Thus, we rewrite Eq. (3) as

$$e^{-\beta\mathcal{H}_n} = C^{-1}\kappa^{-\frac{1}{4}\left(x_{n-2}x_{n-1}+x_{n-1}x_n+x_nx_{n+1}+x_{n+1}x_{n+2}\right)} \lambda^{-\frac{1}{4}\left(x_{n-2}x_n+x_nx_{n+2}\right)}\mu^{-\frac{1}{2}x_{n-1}x_{n+1}}. \tag{5}$$

Tracing out the odd-labeled spins, we have to evaluate

$$\begin{aligned}
&\sum_{\{x_{n-1}=\pm1\}}\ \sum_{\{x_{n+1}=\pm1\}} e^{-\beta\mathcal{H}_n}\\
&= C^{-1}\mu^{-\frac{1}{2}}\lambda^{-\frac{1}{4}\left(x_{n-2}x_n+x_nx_{n+2}\right)}\\
&\quad\left[\kappa^{-\frac{1}{4}\left(x_{n-2}+2x_n+x_{n+2}\right)} + \mu\kappa^{-\frac{1}{4}\left(x_{n-2}-x_{n+2}\right)}\right.\\
&\quad\left.+\mu\kappa^{\frac{1}{4}\left(x_{n-2}-x_{n+2}\right)} + \kappa^{\frac{1}{4}\left(x_{n-2}+2x_n+x_{n+2}\right)}\right]\\
&= \left(C'\right)^{-\frac{1}{2}}\left(\lambda'\right)^{-\frac{1}{4}x_{n-2}x_{n+2}}\left(\kappa'\right)^{-\frac{1}{4}\left(x_{n-2}x_n+x_nx_{n+2}\right)}
\end{aligned} \tag{6}$$

for the remaining spins in terms of the *renormalized* quantities C', κ', λ'. Of the eight possible relations resulting from the combinations x_{n-2}, x_n, $x_{n+2} = \pm1$, only three are independent. After some algebra, we extract from those the RG recursions:

$$\begin{aligned}
\kappa' &= \kappa\lambda\frac{2(1+\mu)}{1+2\mu\kappa+\kappa^2},\\
\lambda' &= \frac{(1+\kappa)^2(1+\mu)}{2\left(1+2\mu\kappa+\kappa^2\right)},\\
C' &= C^2\frac{\kappa\mu}{\sqrt{2}(1+\kappa)(1+\mu)^{\frac{3}{2}}\sqrt{1+2\mu\kappa+\kappa^2}}.
\end{aligned} \tag{7}$$

Note that for couplings in higher levels of the hierarchy it is $K_i' = K_{i+1}$ for $i \ge 1$; correspondingly, these couplings, and hence, μ, will *not* renormalize. Instead, they retain their "bare" value μ^2 determined by the temperature, $kT/J = -2/\ln\mu$. In this sense, we will use μ as a measure of temperature throughout.

Only half of the contribution to the renormalized energy scale is originating with the sectional Hamiltonian in Eq. (5), since at the next level *two* such sections are combined into one, making $C' \propto C^2$. While we do not consider the recursions for C in this paper, they are essential to reconstruct the free energy for each system, and will be analyzed elsewhere (Boettcher and Brunson, unpublished).

Equation (7) provide recursions order-by-order in the RG for the evolution of the effective couplings characterizing increasingly larger scales of the network. To facilitate this RG flow, we need to specify initial conditions for a particular physical situation realized in the unrenormalized, bare network. Here, we restrict ourselves to networks with uniform bonds (although many interesting choices are conceivable, such as distance-dependence; Kotliar et al., 1983; Katzgraber, 2003; Hinczewski and Berker, 2006). For HN3 this implies that we chose $J = 1$ as our energy scale, such that $K_i = \beta J = \beta$ and $I = L_0 = 0$ initially, or in terms of Eq. (4);

$$C^{(0)} = \lambda^{(0)} = 1, \qquad \kappa^{(0)} = \mu^2 = e^{-4\beta}. \tag{8}$$

Searching for fixed points $K_0' = K_0 = K_0^*$ and $L' = L = L^*$, i.e., $\kappa' = \kappa = \kappa^*$ and $\lambda' = \lambda = \lambda^*$ in Eq. (7), immediately provides the trivial, high-temperature solution $\kappa^* = \lambda^* = 1$, i.e., $K_0^* = L^* = 0$. Further analysis yields only a line of (unstable) strong-coupling fixed points,

$$\kappa^* = 0, \quad \lambda^* = \frac{1+\mu}{2}, \tag{9}$$

extending from $\lambda^* = \frac{1}{2}$ for low temperatures, $\mu = 0$, to $\lambda^* = 1$ for $T \to \infty$, where $\mu = 1$. Even at $T = 0$, only the renormalized backbone bonds K_0 provide strong coupling, the emerging long-range bonds L_0 only exert limited coupling strength.

Local analysis near the fixed points with the Ansatz

$$\kappa_n \sim \kappa^* + \epsilon_n, \quad \lambda_n \sim \lambda^* + \delta_n, \quad \epsilon_n, \delta_n \ll 1 \tag{10}$$

reveals that the high-temperature fixed point is always stable and corrections decay exponentially, where the exponential contains a factor of $2^{\frac{n}{2}} = \sqrt{N}$. At the low-temperature line of fixed points in Eq. (9) we find

$$\epsilon_n \sim \epsilon_0 (1+\mu)^{2n}, \quad \delta_n \sim \frac{1-\mu}{1+\mu}\epsilon_0 (1+\mu)^{2n}, \tag{11}$$

which is divergent for all $T > 0$, i.e., $0 < \mu \leq 1$, making the fixed point at $T = 0$ unstable. For any fixed point, there is no linear expansion possible that would yield critical exponents. For the initial conditions in Eq. (8), corresponding to uniform couplings throughout the unrenormalized network, the RG flow always evolves to the high-temperature fixed point. Thus, the ferromagnet on this network behaves similar to a $1d$ Ising model.

4. ISING FERROMAGNET ON HN5

As shown in Section 2, HN5 is basically an extension of HN3, created by adding a new layer of links to each level of the hierarchy. As is apparent from the foregoing discussion in 3, these additions correspond precisely to new renormalizable operators (here, the bonds L) that *inevitably emerge* during the RG of HN3, see **Figure 3**. In HN5, these new operators are simply deemed an original feature of the network, hence, maintaining the RG as an exact procedure. Consequently, the RG itself hardly changes, see **Figure 4**; it merely differs by one extra link in the graph-let, L_1, compared to that for HN3 in **Figure 3**. In Eq. (3), it only adds the term $L_{1 x_{n-2} x_{n+2}}$ to the sectional Hamiltonian and, like L_0 itself, L_1 does not get traced over in the calculation in Eq. (6). We can introduce these new bonds as yet another free, non-renormalizing coupling in the RG and choose, to wit,

$$L_1 = y K_1, \quad \text{i.e.} \quad e^{-4L_1} = \mu^{2y}. \tag{12}$$

This merely contributes a factor of $\mu^{-\frac{y}{4}x_{n-2}x_{n+2}}$ to the unprimed side of Eq. (6), which correspondingly alters only the recursion for λ' in Eq. (7) by a factor of μ^{2y}. Otherwise using the same definitions as in Section 3, we obtain the RG recursions for the Ising ferromagnet on HN5:

$$\begin{aligned} \kappa' &= \kappa\lambda \frac{2(1+\mu)}{1+2\mu\kappa+\kappa^2}, \\ \lambda' &= \mu^{2y} \frac{(1+\kappa)^2(1+\mu)}{2\left(1+2\mu\kappa+\kappa^2\right)}, \\ C' &= C^2 \frac{\kappa\mu}{\sqrt{2}(1+\kappa)(1+\mu)^{\frac{3}{2}}\sqrt{1+2\mu\kappa+\kappa^2}}. \end{aligned} \tag{13}$$

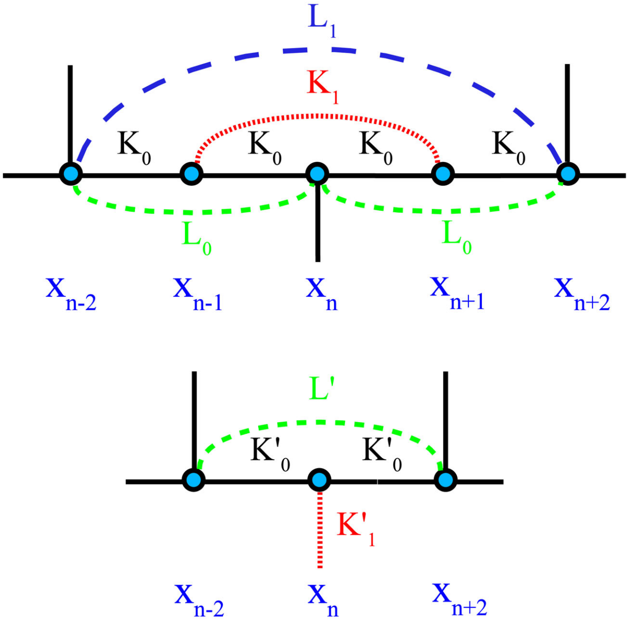

FIGURE 4 | Depiction of the (exact) RG step for the Ising model on HN5. This step is *identical* to that for HN3 in **Figure 3** aside from the extra link L_1 spanning between x_{n-2} and x_{n+2} (top), which contributes to the renormalization of L'_0 (bottom).

Accordingly, due to the bare existence of the L_1 bond, we will have to change the initial conditions from Eq. (8) to

$$C^{(0)} = 1, \quad \kappa^{(0)} = \mu^2 = e^{-4\beta}, \quad \lambda^{(0)} = \mu^{2y}. \tag{14}$$

Analyzing these recursions for fixed points, $\kappa' = \kappa = \kappa^*$ and $\lambda' = \lambda = \lambda^*$, we find that the addition of the extra long-range bond has *eliminated* the high-temperature fixed point found in HN3. At low temperatures, we find similar to Eq. (9) in HN3 a line of fixed points

$$\kappa^* = 0, \quad \lambda^* = \frac{\mu^{2y}}{2}(1+\mu), \tag{15}$$

which here extends over the entire domain for the long-range bonds, $0 \leq \lambda^* \leq 1$ for $0 \leq \mu \leq 1$. Note that although y represents a continuous interpolation between HN3 and HN5, there is a singular limit at $y \to 0$ toward an isolated point corresponding to HN3, see Eq. (9). In the following, we only treat the case of couplings that are homogeneous throughout the unrenormalized network, $y = 1$. Consideration of the rich set of transitions occurring for the family of networks parameterized by interpolating $0 < y \leq 1$ is deferred to Section 5.

Dividing out the $\kappa^* = 0$-solution, further analysis of Eq. (13) for $y = 1$ reveals yet another line of fixed points given by

$$\begin{aligned} \kappa^* &= \frac{1}{2}\left[-(1-\mu)\mu + \sqrt{(1+\mu)\left(\mu^3 - 3\mu^2 + 8\mu - 4\right)}\right], \\ \lambda^* &= \frac{\mu}{4}\left[2 - \mu + \mu^2 + \sqrt{(1+\mu)\left(\mu^3 - 3\mu^2 + 8\mu - 4\right)}\right], \end{aligned} \tag{16}$$

which can be expressed most simply in closed form as

$$\lambda^* = \frac{1}{2}\left[\kappa^* - 1 + \sqrt{5 + 2\kappa^* + 5(\kappa^*)^2 + 4(\kappa^*)^3}\right] \tag{17}$$

by eliminating μ. As we will see, these relations lead to physical fixed points only within a limited range of the temperature μ. The phase diagram for the backbone coupling κ in HN5 at $y=1$ can be found in **Figure 5**. Blue arrows indicate the RG flow for the initial conditions in Eq. (14), which starts on the diagonal, representing all-equal bonds for the homogeneous network. For these initial conditions, the flow always evolves toward smaller values of κ, i.e., stronger coupling. But there is a notable transition where the attained fixed-point jumps from the low-temperature branch in Eq. (15) characterized by $\kappa^*=0$, i.e., a solidly frozen backbone, to the branch given Eq. (17) on which κ^* (as well as λ^*) becomes finite. We obtain this transition point by evaluating Eqs. (16, 17) for $\kappa^*=0$, which yields $\lambda^*=1/2\phi=0.309017\ldots$, giving a critical temperature of

$$\mu_c = \frac{1}{\phi}, \quad \text{or} \quad \frac{kT_c}{J} = -\frac{2}{\ln \mu_c} = 4.15617384247\ldots, \tag{18}$$

where $\phi = \left(\sqrt{5}+1\right)/2 = 1.6180339887\ldots$ is the "golden ratio" (Boettcher and Goncalves, 2008). For bare couplings at this temperature, marked by a blue dot in the $(y=1)$-plot of **Figure 5**, the RG flow marginally reaches the strong-coupling limit. In this network, for these initial conditions the RG flow never reaches an unstable fixed point such as the unstable portion of Eq. (15), marked by a red-shaded line in **Figure 5**. As we will see below, this situation will change when we weaken the impact of long-range couplings.

5. INTERPOLATION BETWEEN HANOI NETWORKS

We have already observed in the construction of HN5 in Section 4 that it is easy to promote the L-couplings that inevitably emerge during the RG to be associated with an actual bond in the network. Here, we will fully exploit this fact to obtain a one-parameter family of problems with various regimes of phase behaviors. In particular, we discover transitions between such regimes as a function of the parameter that will allow us to clarify the connections between the diverse set of behaviors that we have discovered in the previous section.

In Section 4, we argued for the introduction of small-world bonds with couplings L_i and developed the RG recursions in (13) assuming a relative strength of these couplings to those germane to HN3 of the form in Eq. (12). Here, we will now consider the behavior that results from varying the strength parameter y between the two extremes already explored, $y=0$ for HN3 in Section 3 and $y=1$ for HN5 in Section 4.

Analyzing these recursions in Eq. (13) for fixed points, we already found the low-temperature fixed-point line in Eq. (15). Dividing out this $\kappa^*=0$-solution, further analysis of Eq. (13) reveals a line of fixed points,

$$\begin{aligned}\kappa^* &= \frac{1}{2}\left[\mu^y(1+\mu) - 2\mu \pm \sqrt{\mathcal{D}_y(\mu)}\right],\\ \lambda^* &= \frac{\mu^y}{4}\left[2(1-\mu) + \mu^y(1+\mu) \pm \sqrt{\mathcal{D}_y(\mu)}\right],\end{aligned} \tag{19}$$

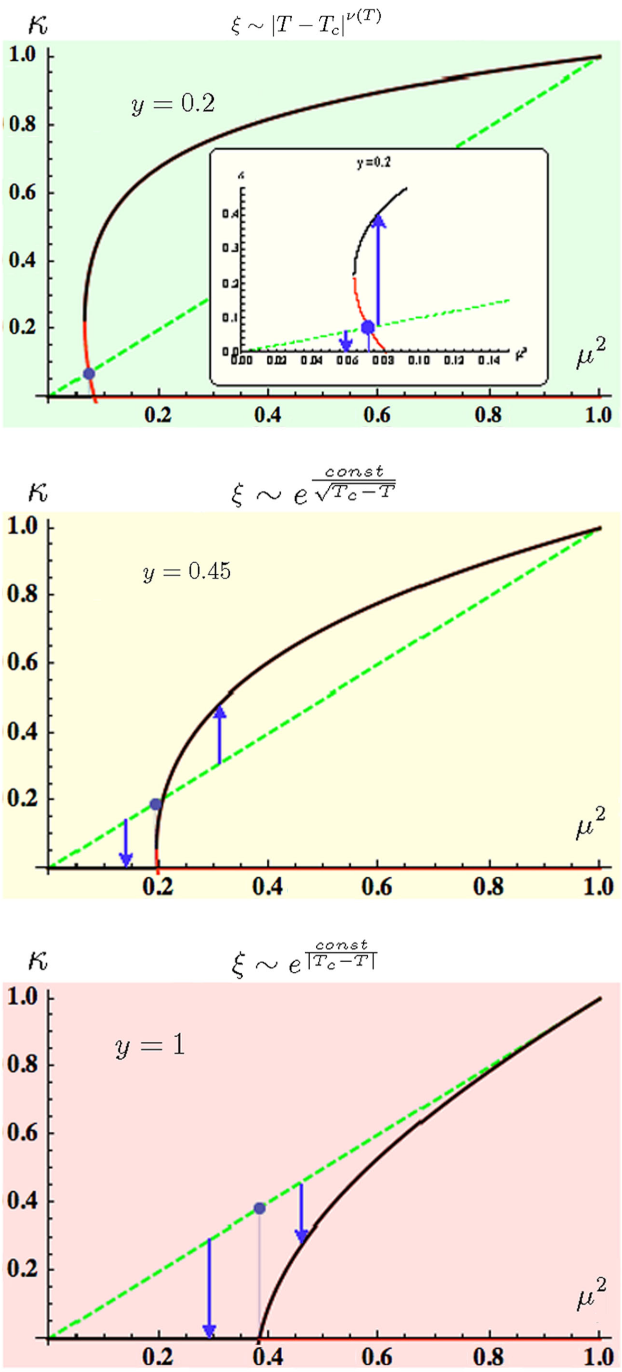

FIGURE 5 | Plot of κ^* in Eq. (19) as a function of μ^2 for y as defined in Eq. (12). The generic cases are represented by $y=0.2$ (top), $y=0.45$ (middle), and the case of HN5 discussed in Section 4 for $y=1$ (bottom), for which the branch-point (BP) has "sunset" out of the physical regime. In all cases, the dashed, green-shaded line indicates the initial conditions (IC) for the RG flow in Eqs. (13, 14), $\kappa^{(0)}=\mu^2$. For μ fixed, the RG flow *must* proceed vertically, either up or down, to the nearest stable line of fixed points. For low y (top), the IC crosses the unstable branch below BP (see enlargement in the inset) which then can *not* be reached by the flow. Once the IC cross above BP (middle), the flow *must* pass BP, unless BP sunsets (bottom). For each case, the generic divergence is indicated for the correlation length ξ at $T \to T_c$, as derived in Section 6.

abbreviating the discriminant

$$\mathcal{D}_y(\mu) = (1+\mu)\left[\mu^{2y}(1+\mu) - 4\left(1-\mu^{y}\right)(1-\mu)\right]. \tag{20}$$

For $y \to 0$, this solution degenerates into the high-temperature fixed point of HN3. But for any finite y, these lines of fixed points are non-trivial functions of μ, as depicted for $\kappa^*(\mu^2)$ in **Figure 5**. The dominant feature in these plots is the root-singularity in κ^* with a branch-point-separating the upper stable and lower unstable line of fixed points. Essentially, three distinct generic regimes can be discerned: (1) If the branch-point happens to lie below of the physical regime, we observe a phase transition without access to any unstable point (see bottom of **Figure 5**); a critical point akin to that for HN5 at $y=1$ analyzed in Section 4 arises. If the branch-point rises into the physical regime, here for $y < y_c = [\ln(3/2)/\ln 2] = 0.584963\ldots$, then depending on whether the initial conditions of the RG flow cross the critical line below or above the branch-point, we find (2) a transition seemingly of finite-order on intercepting the unstable lower branch (see top of **Figure 5**) for which the RG flow never accesses the branch-point singularity. If, in turn, the initial conditions cross above, (3) a BKT-like transition results because the RG flow now *must* pass the singularity (see middle of **Figure 5**), as we will show below.

6. GENERAL CLASSIFICATION OF CRITICAL REGIMES

Instead of the rather tedious analysis of the critical regimes resulting from Eq. (19) for a coupled set of variables (which can be found in Boettcher and Brunson, 2011), we rather take a step back and assess the larger picture here. It turns out to be easy to devise a simple theory[1] that reproduces all the previously found features in a generic way, thereby demonstrating the generality of this classification, not only accounting for other hierarchical networks (Andrade et al., 2005; Hinczewski and Berker, 2006; Hinczewski, 2007; Boettcher and Goncalves, 2008; Boettcher et al., 2008) but also for any physical system described by parameter-dependent renormalization group equations. Even systems on complex networks that have not been subjected to an RG treatment have been found to exhibit the peculiar infinite-order transitions found here (Dorogovtsev et al., 2008), and may eventually be related to this classification.

It is sufficient to consider the RG recursion for a single coupling, say, κ_n with some control parameter μ^2, defined as in Eq. (4), for instance. A conventional RG treatment (Goldenfeld, 1992; Plischke and Bergersen, 1994) for a system on a regular lattice leads to recursions that only depend on the evolving coupling itself. There, any parameter dependence, such as on the temperature for spin models or on the bond density for percolation problems, is limited to the initial conditions of the RG, which define the particular model being studied; they do not affect the properties near the fixed points. In contrast, inserting such a dependence influences the location of fixed points as well as the behavior near them. As the example displayed in **Figure 5** suggests, even the very fixed point that controls the dynamics may depend on the initial conditions, violating any conventional sense of universality. Yet, if we assume that the most elementary, generic fixed-point topology that deviates from the conventional picture is represented by a root branch-point[2], we can classify all observed critical phenomena into just a few regimes. To wit, we write

$$\kappa_{n+1} = \left[\kappa_b + 1 + f(\mu)\right]\kappa_n + 2\kappa_b\kappa_n^2 - \kappa_n^3. \tag{21}$$

This model of a generic RG recursion is cubic to ensure that, after extracting the trivial strong-coupling fixed point $\bar{\kappa} \equiv 0$[3], the remaining fixed-point equations produce a root branch-point,

$$\bar{\kappa}_{\pm} = \kappa_b \pm \sqrt{f(\mu)}, \tag{22}$$

revealing the undetermined constant κ_b as the coupling strength found at the branch-point, which we may choose freely to place the branch-point inside $(0 \le \kappa_b \le 1)$ or outside the physical regime. (Regarding Section 5, we could view $\kappa_b = \kappa_b(y)$ as the quantity that specifies the family of models considered.) The function $f(\mu)$ captures the minimal parameter dependence of the model, as expressed through the μ-dependence of both branches of the fixed points $\bar{\kappa}_{\pm}(\mu)$. Here, $f(\mu)$ is a monotone rising function on the physical interval $0 \le \mu \le 1$, which may contain a zero, $f(\mu_b) = 0$. Generically, it would have a simple Taylor expansion near μ_b, i.e., $f(\mu_b \pm \Delta\mu) \sim \pm\Delta\mu f'(\mu_c)$ for small $\Delta\mu$.

In light of **Figure 5**, the first two panels correspond to the case where both, κ_b and μ_b, are in the physical regime with a visible branch-point (although the RG recursions there are far more complicated); the last panel represents $\kappa_b \le 0$. The decisive difference between those first two panels is whether the location of the branch-point is above or below the line of initial conditions. Depending on the model, the line of initial conditions could be any monotone function of μ, possibly resulting in different critical behaviors in the way they pass by identical branch-points, but it is more convenient to imagine this line as a simple diagonal in the (μ^2, κ)-plane and move the branch-point instead. Within the physical regime, the lower fixed-point branch $\bar{\kappa}_{-}(\mu)$ is always unstable while the upper $\bar{\kappa}_{+}(\mu)$ remains stable, as a local analysis along each branch readily reveals. Stable and unstable fixed-point lines merge at the branch-point, where particularly interesting phenomena can arise.

In the case that long-range, hierarchical effects are weakest, as for the first panel in **Figure 5**, the branch-point is far on the right, and may even be outside to the right and/or above of the physical regime. Then, the initial conditions merely intersect the unstable branch $\bar{\kappa}_{-}(\mu)$ at some point μ_c. The RG flow (vertical blue arrows in **Figure 5**) for $0 \le \mu < \mu_c$ advances toward

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[1] Our approach is similar in spirit to Landau's description of a mean-field phase transition found in any text book on critical phenomena (Goldenfeld, 1992; Plischke and Bergersen, 1994).

[2] Of course, the simplest deviation from constant fixed points that are entirely independent of the parameter would be a linear dependence. However, such dependence can be subsumed into our model (by moving the branch cut far outside the physical domain, for instance).

[3] For the weak-coupling fixed point it is sufficient to require that $f(\mu)$ is chosen such that $\bar{\kappa}_{+}(\mu = 1) = 1$ in (Eq. 22).

strong coupling at $\bar{\kappa} \equiv 0$, while for $\mu_c < \mu \le 1$ it flows toward $\bar{\kappa}_+(\mu)$, making μ_c the critical point. Note that $\bar{\kappa}_+(\mu) \to 1$ only for $\mu \to 1$, reflecting the physical phenomenon of "patchiness" (Boettcher et al., 2009): hierarchical, long-range couplings enforce some semblance of order between otherwise uncorrelated (subextensive) patches of locally connected degrees of freedom even in the disordered regime; full disorder is often only reached at infinite temperature, dilution, etc. Near μ_c, all the critical dynamics of the system is then solely determined by the local properties of the unstable critical point $\bar{\kappa}_-(\mu_c)$ that has be (non-universally) selected by the particular system via the initial conditions. As in a conventional system, local analysis (Goldenfeld, 1992; Plischke and Bergersen, 1994) similar to Eq. (10) but near $\bar{\kappa}_-(\mu_c)$ with an Ansatz $\kappa_n \sim \bar{\kappa}_- + \epsilon_n$ for $\epsilon_n \ll 1$ on Eq. (21) provides, e.g., the scaling exponent for the divergence of the correlation length,

$$\xi \sim |\mu - \mu_c|^{-\nu}, \tag{23}$$

except that the exponent is non-universal, $\nu = \nu(\mu_c) = 1/\log_2(1 + 2\bar{\kappa}_-\sqrt{f})$ evaluated at the crossing point μ_c (Boettcher and Brunson, 2011). The RG flow in this case *never* passes sufficiently near the branch-point singularity to be affected.

The other extreme, when long-range couplings dominate, leads to a picture similar to the last panel of **Figure 5** but with $\kappa_b < 0$. No unstable fixed points can be reached for any choice of (physical) initial conditions. The RG flow always advances to the next best stable fixed point, either at strong coupling $\bar{\kappa} \equiv 0$ for $0 \le \mu < \mu_c$, or toward patchy order at $\bar{\kappa}_+(\mu)$ for $\mu_c < \mu \le 1$, making μ_c again the critical point. At μ_c, where both lines of stable fixed-points intersect, we find an exponentially divergent correlation length to signal the phase transition. The Ansatz $\kappa_n \sim \bar{\kappa}_+ + \epsilon_n$ for $\epsilon_n \ll 1$ on Eq. (21) provides

$$\epsilon_{n+1} \sim \left(1 - 2\bar{\kappa}_+\sqrt{f}\right)\epsilon_n. \tag{24}$$

Since $\bar{\kappa}_+(\mu_c + \Delta\mu) \sim \Delta\mu f'(\mu_c)/\sqrt{4f(\mu_c)}$ from expanding Eq. (22) near $\bar{\kappa}_+(\mu_c) = 0$, we get

$$\epsilon_n \sim \left[1 - \Delta\mu f'(\mu_c)\right]^n \epsilon_0 \sim e^{-n\Delta\mu f'(\mu_c)} = e^{-\frac{n}{n^*}}. \tag{25}$$

Thus, the correlation length diverges as

$$\xi(\mu) \sim 2^{n^*} = e^{\frac{const}{|\mu-\mu_c|}}, \qquad \mu \to \mu_c. \tag{26}$$

Again, the RG flow does not pass the branch-point singularity, as it is located outside (below) the physical domain.

Only in the intermediate regime, as represented by the middle panel of **Figure 5**, does the RG flow for some critical μ_c pass by the branch-point singularity, which then controls the critical behavior in a novel way. As before, for $0 \le \mu < \mu_c$ the flow reaches strong coupling at $\bar{\kappa} \equiv 0$ and patchy order at $\bar{\kappa}_+(\mu)$ for $\mu_c < \mu \le 1$. A local analysis of the flow just below the critical point, $\mu \sim \mu_c - \Delta\mu$, with $\kappa_n \sim \kappa_b(1 + \epsilon_n)$ for $\epsilon_n \ll 1$ on Eq. (21) yields

$$\epsilon_{n+1} - \epsilon_n = \frac{\Delta\epsilon_n}{\Delta n} \sim -\Delta\mu - \kappa_b^2\epsilon_n^2. \tag{27}$$

This relation exhibits a boundary layer, i.e., in the limit $\Delta\mu \to 0$ the solution drastically changes behavior. With the methods of Bender and Orszag (1978), we can transform into the "inner" boundary region by rescaling $\epsilon_n \to \eta\epsilon_n$ and $n \to \delta n$ applied to Eq. (27),

$$\frac{\Delta\epsilon_n}{\Delta n} \sim -\frac{\delta}{\eta}\Delta\mu - (\delta\eta)\,\kappa_b^2\epsilon_n^2, \tag{28}$$

which becomes balanced for $\delta \sim 1/\eta \sim 1/\sqrt{\Delta\mu}$. Accordingly, the characteristic width of the boundary layer scales with

$$n^* \sim \frac{1}{\sqrt{\mu_c - \mu}}, \tag{29}$$

which by Eq. (26) leads to the divergence in the correlation length characteristic of BKT,

$$\xi(\mu) \sim e^{\frac{const}{\sqrt{\mu_c-\mu}}}, \qquad \mu \to \mu_c^-. \tag{30}$$

Clearly, the physical origin of this singularity is not even remotely related to an actual BKT transition. In fact, instead of its rarity, confined to very particular lattice models, we may find it to be one of a few generic types of transition in networks.

7. CONCLUSION

We have analyzed the fixed-point structure of an Ising ferromagnet on a set of Hanoi networks with an exact real-space renormalization group. Using interpolating families of such networks, with the relative coupling strength between backbone and small-world bonds as the interpolation parameter, we reveal a number of regimes with distinct critical behaviors. While in each such regime the critical transition has non-universal features, the characteristics of the transition in each one has generic, robust features. For increasing strength, we observe that the divergence in the correlation length changes from a non-universal power-law $|x|^{-\nu}$, to a BKT-like essential singularity $e^{1/\sqrt{x}}$, then to a full singularity $e^{1/x}$, on approach to the critical point $x \sim |\mu_c - \mu| \to 0$. We trace the changes from one regime to the next in terms of the analytic structure of the RG flow. Finding an enumerable range of such characteristics suggest a possible classification of critical behavior of statistical models in networks generally, for which we propose a general description. Similar critical properties of the kind found here have also been observed in percolation (Berker et al., 2009; Boettcher et al., 2009; Nogawa and Hasegawa, 2009; Hasegawa et al., 2010), for example. The existence of entire regimes that exhibit essential singularities in the divergence of the correlations, as we have found here, might explain the surprising prevalence of typically quite rare BKT-like transitions in otherwise unrelated network models (Dorogovtsev et al., 2008). In the analysis of, say, social interaction networks, which have been found to have a hierarchical structure (Trusina et al., 2004), it is therefore essential to be aware of the novel phenomena describe here.

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The inert vs. the living state of matter: extended criticality, time geometry, anti-entropy – an overview

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The physical singularity of life phenomena is analyzed by means of comparison with the driving concepts of theories of the inert. We outline conceptual analogies, transferals of methodologies and theoretical instruments between physics and biology, in addition to indicating significant differences and sometimes logical dualities. In order to make biological phenomenalities intelligible, we introduce theoretical extensions to certain physical theories. In this synthetic paper, we summarize and propose a unified conceptual framework for the main conclusions drawn from work spanning a book and several articles, quoted throughout[1].

Keywords: criticality, biological time, anti-entropy, theoretical biology, symmetry, allometry, incompleteness

1. A DEFINITION OF LIFE?

Throughout the very old "physicalism/vitalism" debate, it has often been question of *defining* what is the phenomenon of life. A small but remarkable book by Schrödinger ("What is life?") contributed to reviving the debate in a way we find to be relevant, at least in its second part, and to which we refer in section 2. For the moment, let's precise how we approach this question:

> *Primo* An "ideal" definition of life phenomena seems out of the question: there is no *Platonic idea* of life to be grasped in a definite manner or with the maximal conceptual stability and invariance specific to mathematical notions (as there is with the definition or *idea* of the triangle...). It is rather a question of defining a few *operational notions* enabling to draw out concepts with which to work for a systemic approach in biology. Moreover, physics does not define "matter" otherwise than by means of an operative duality or contraposition (with respect to the concept of energy or to that of vacuum or of anti-matter, for example). Yet another, very rigorous, "provable impossibility to define the object of study" is presented in an appendix. Notice that Darwin's approach to evolution does neither use nor need a definition of life, but needs organisms.
> *Segundo* Any operational attempt, in our opinion, must be made with respect to the specific phenomenality of life phenomena: for example, it is possible that for any chosen finite list of "defining" properties of life, there would exist a sufficiently talented computer scientist able to create its virtual image to be rendered on a computer screen (it is quite simple to program an "autopoietic" system (Varela et al., 1974; Varela, 1989) or a formalized metabolic cycle in the manner of Rosen (1991) – see Mossio et al., 2009, for example). However, not only any human being, but also the most simple-minded of animals would recognize it as a series of non-living "virtual images" (which are typically detectable through identical iteration, as indirectly suggested by Turing's imitation game, see Longo, 2008).

It is rather a question of proposing a possibly robust intelligibility of a phenomenality in its constitutive history, while keeping in mind the fact that *any constitution is contingent* – both the constitution (evolution) of life and of our historical understanding of it. That is, life and our modest attempts to grasp it unfold over a material evolution (better still: over one of the *possible* evolutions, taking place on *this* Earth, in *these* ecosystems and with *this* physical matter and history). As for life, our point of view includes what biologists often express when they say that nothing can be understood in biology otherwise than in the light of evolution (Darwinian and in this world).

It should be clear that we do not discuss here how "life may be emerged from the inert," but rather we explore how to go from the current *theories* of inert to a sufficiently robust *theory* of the living. In particular, we hint here to an analysis of the physical singularity and of the specificity of the living object, by looking first at the properties we would want to have (or *not*) in any theory of the "living state of matter." It is indeed an *incomplete* (see appendix) attempt at providing a conceptual framework guiding more specific analyses. We thus present herein a brief synthesis, in an explicit play between physics and biology, which can only direct the interested reader toward the works presented in the references. In a methodological appendix, we will borrow from Mathematical

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[1] An activity enhanced by collaborations with Francis Bailly, deceased in 2009.

Logic an understanding of the role of incompleteness in "our theoretical endeavors toward knowledge" (to put it in H. Weyl's words) and of its relation to conceptual or formal "definitions," of life in particular.

2. FROM PHYSICS TO BIOLOGY BY THEORETICAL EXTENSIONS AND CONCEPTUAL DUALITIES

Various physical theories (classical, relativistic, quantum, thermodynamic) make the inert intelligible in a remarkable way. Significant incompatibilities exist (the relativistic and quantum fields are not unified; they are in fact incompatible). However, some major principles (of conceptual construction, see Bailly and Longo, 2011) confer a great unity to contemporary theoretical physics. The geodesic principle and its accompaniment by "symmetries" (Weyl, 1952; Van Fraassen, 1989; Bailly and Longo, 2011) enable to grasp, under a conceptually unitary perspective, a wide area of knowledge regarding the inert. Biology, having to date been less "theorized" and mathematized, can also progress in the construction of its theoretical frameworks by means of analogies, extensions, and differentiations regarding physical theories, even by means of conceptual dualities. Regarding dualities, we will mention below one which is, we believe, fundamental (and extensively addressed in other works, Frezza and Longo, 2010; Bailly and Longo, 2011; Longo and Montévil, 2011a): the *genericity* of physical objects (their theoretical and experimental invariance) and the *specificity* of their trajectories (their reconstruction by means of the geodesic principle) is inverted in the *specificity* (individuation and history) of the living object and the *genericity* of trajectories (evolutionary, ontogenetic: they are just "possibilities" within spaces – ecosystems – in co-constitution).

2.1. PHYSICAL ASPECTS

2.1.1. The exclusively physical

We exclude from our analyses those properties which come from physics (where they are often essential), but of which the transferal to biology is, from our point of view, misleading:

1. The *genericity* of objects (the theoretical and experimental invariance of physical objects – or symmetry by replacement) does not apply to biology: the living object is historical and individuated; it is not "interchangeable," in general or with the generality of physics, not theoretically nor empirically.
2. The *specificity* of trajectories (geodesics, in physics), because we exclude the prevalence of the geodesic principle (there is no "optimality") for ontogenetic and evolutive dynamics of "biological individualities" – cells, organisms, species (which we call, synthetically, "biolons"); in short, embryogenesis, development, and evolution are not optimal trajectories, but *possible* ones.
3. The *stability* of the reference system as such. Besides classical physics, also in general relativity and in the energy/geometry relationships in spacetime, the dimensions are set and do not vary during the phenomenal analysis. Instead, the space of observables in biology, of phenotypes for example, which can also be described by new "dimensions," is, itself, dynamically changing in an ecosystem. Using an informal analogy, we could say that the "*phase space*" (and the space of possibilities) of life phenomena is dynamically (co-)constituted (Longo et al., 2012). By a very informal analogy, observe that in relativity theory, *spacetime* is (co-)constituted by the energy/matter distribution, yet in stable dimensions and phase space – while in Longo et al. (2012) we claim that these change along biological processes, evolution in particular.

As discussed in Longo and Montévil (2011a), the genericity of physical objects and the specificity of their trajectory depend on the theoretical symmetries which allows to constitute them. In biology, our hypothesis is that the properly biological theoretical symmetries are unstable. This leads to a change of the theoretical status of biological objects with respect to physical situations. We will discuss this point further bellow.

2.1.2. Physical properties of the "transition" toward the living state

In the literature, we often find remarkable works concerning certain physical properties, sometimes transferred to the analyses of life phenomena, but which we will later consider in their exclusively biological form (i.e., that we only find in the living state; for example, critical transitions, which are pointwise in physics, will be "extended" in our approach, see below). In biology, we therefore *do not consider* them "as such," as they present themselves as components of the analysis of the inert, where they nevertheless provide a good starting point for reflections regarding life phenomena (see the references). For the moment, let's evoke them from a physical perspective ("as such") and stress that they partly pertain the biological theoretical vocabulary, but do not properly belong to it, in our view:

1. *Criticality* as such (in physics, present in phase transitions, as a mathematical point with respect to the control parameter) (Jensen, 1998; Zinn-Justin, 2007);
2. *Organizational closure* as such (present in physical chemistry: micelles, vesicles – whose structure is entirely organized along geodetic principles, in contrast to living organisms, see below);
3. *Passive plasticity* as such (present in changes of physical form or in phenomena of action/reaction/propagation in the manner of Turing, for example);
4. *Scaling properties* as such (present in numerous physical phenomena and namely in critical transitions, anomalous diffusion, etc.);
5. *Growth* phenomena as such (present in the growth of crystals, for example);
6. *Chirality* as such (present in the physics of particles or chemistry, for example);
7. Possibly negative variations of *entropy* (present in the passing from disorder to order, in critical transition for example),
8. The *dimensionality* of physical quantities (almost always present);
9. The *measurement* which is understood as an imprecision and as a result classical (except when fractal);
10. The *fractality* as such of certain objects and dynamics (present in a number of physical phenomena, but also in *organs* of plants and animals as forced by their role in the exchange of energy and matter) (Weibel, 1991);
11. The chemistry of *macromolecules* and of *in vitro* physical chemistry.

2.2. BIOLOGICAL ASPECTS

The contingent materiality of life phenomena includes, typically, the physical chemistry specific to biology, our first group of properties:

2.2.1. A few "physical" properties of life phenomena

1. The biological role of the *chirality* of molecules (amino acids, sugars) in the metabolism;
2. Various other physical *invariances* according to the level of organization (the chemical bases and geometric structure of DNA, relatively common to all living objects; the metabolic invariants, including the metabolism/mass/duration relationship, see below).

In addition to the above physical properties, which specifically (and only) manifest themselves in life phenomena, the following are certainly part of biological *theorization*:

1. Analysis in terms of *physico-chemical substrates* such as of molecular cascades that may be found only in cells;
2. The *mathematical extension* of certain physical laws including quantities that do not appear as such or in an operative way in physics (for example, our notion of anti-entropy in metabolic balances, mentioned below, which extend well-known balance equations in thermodynamics by a new observable).

2.2.2. The maintenance of biological organization

The setting of physiological activities (the functions of "orgons" – organelles, organs, populations, see Bailly et al., 1993; Bailly and Longo, 2011), is often accompanied by organizational closure which is accomplished by means of:

1. The *metabolism* and *physiological activities* (essential to integration and to regulation) which interact and, in fact, superimpose one another;
2. The coupling between *various levels of organization*, correlated in a causal manner, both "upward" and "downward," particularly by integration and regulation,
3. The *fractality* of orgons in their physiological functions (lung, vascular system, nervous system. . .intracellular structures);
4. The *scaling laws* (allometry describes temporality and metabolism in function of the adult biological mass) (Brown et al., 2002; West, 2006);
5. The importance of *pure numbers* (without physical dimensions) and of their *relative invariance (*total number of heartbeats, respirations. . .which are on average constant for mammals, and even among important groups of less studied species as for internal rhythms).

We will attempt to conceptually frame these properties of the living state of matter by means of relatively new concepts, including that of *extended critical transition* in 1, as locus and framework for these phenomena which we merely enumerated above.

2.2.3. The relationship to the environment

To these functions, we must add the relationship to the environment that is not only dynamic, but adaptive *and* (or *because*) cognitive (as are protentional activities). Moreover, the dynamic is also situated at the level of the reference space (relevant parameters and observables), as an organism co-modifies its own environment:

1. *Adaptive plasticity* at all levels of organization, in the interaction with an environment;
2. The cognitive, present as soon as there is life, resides, in particular, in the *capacity to discriminate* (the denumerable density of critical points within the zone of extended criticality mentioned below can represent this discriminatory capacity, by discontinuous passages (but without gaps) from one point to another);
3. The principle of *compatibility* (tendency to achieve all possibilities compatible with the given constraints), which justifies the genericity of evolutive and ontogenetic trajectories;
4. The *specificity* of the object and, as we were saying in 3, the *genericity* of trajectories (contrarily to physics);
5. The *changes in reference spaces,* which induces and enables, including in the number of relevant description dimensions (the "phase space" itself – relevant parameters and observables – changes over the course of the dynamics of life phenomena, as opposed to the physical frameworks, even quantum ones).

Again, most of this aspects are related to an instability of biological theoretical symmetries, associated here to the constitution by the biological object of the theoretically relevant environment.

2.2.4. Passage to analyses of the organism

Critical transitions are extensively discussed in the analysis of the passage, in particular, from the inert to the living (Kauffman, 1993). As such, they very well describe states of the inert that are quite interesting also for biology (see Binney et al., 1992; Mora and Bialek, 2011). In physics, though, "coherent structures" appear over pointwise transitions, and normally in a reversible way. We are, however, facing a living state of matter when criticality is *irreversible* and *endures* (till death). We deal with these issues by considering an organism as staying in a "continual" (ongoing) irreversible transition. Each mitosis, in a multicellular organism, yields an asymmetric bifurcation and the formation of a new coherence structure – new tissular matrix . . ., as components of a critical transition. In our approach, the interval of criticality is therefore extended in time and in all relevant control parameters (temperature, pressure. . .), see section 1. The key idea is that all the usual properties of critical phase transitions are preserved (the formation of coherence structures, diverging correlation lengths, symmetry changes . . .). Yet, while, in physics, these only apply in a topologically isolated point (at least this is the mathematical representation, where the renormalization methods apply, see section 1), we consider the "transition" to be defined on a non-trivial interval. This occurs when rhythms (point 1 below and section 3), protentional activity (point 2 and 3), and organization, as anti-entropy (point 3 and 4, see also 2) jointly appear.

We may then conceive (but this discussion is not our aim, here) that, at the origin of the extended criticality of life, there may have been particular critical transitions of the inert matter, a global transition suddenly superposing all the ones we are dealing with.

These may all be described as conceptual and material "bifurcations," with their organizational correlates: extension of criticality to an interval, by the formation of stabilizing membranes and of different levels of organization (as anti-entropy), bifurcation of the time dimension (autonomous rhythms). Yet, extended criticality is an ongoing phenomena for life, well beyond its origin. The five points below will be at the core of the synthesis in this text (and are described in detail in the articles included in the references). We therefore propose to organize these "bifurcations," which mark the (conceptual) passage from a state of the inert to the living state, as a *constitution of*:

1. The second temporal dimension, the *compactified* time of biological rhythms;
2. The *protention*, as a "proactive gesture" in the interaction with the ecosystem, present with even the simplest forms of life;
3. *Anti-entropy*, as the establishment and maintenance of organization (which is opposed to disorganization – in particular to the entropy produced by all irreversible processes);
4. The distinction in *several levels of organization*, at the core of the integration and regulation activity of any living unit (which may be conceptually unified as *orgons* – organelles, organs, populations – and *biolons* – cells, organisms, species).
5. An *instability of the theoretical symmetries* of the objects, which can be seen as a cascade of symmetry changes, in time, and leads to variability in the strong sense of changing theoretical symmetries.

In short, the intelligibility of life phenomena that we propose presupposes the existence ("somewhere," "at the origin of life") of correlated bifurcations whose understanding requires the addition of the new theoretical entities above. These are perfectly compatible with physical theorization, but they are not specific to it. In this sense, it is a matter of proposing compatible, but "strict" theoretical extensions of theories of the inert. Reduction may be a further step for the interested reductionist, who should prove that these theories are first conservative (in the sense of Logic), then only apparently "strict."

3. THE PHYSICAL SINGULARITY OF THE LIVING STATE OF MATTER

Let's now return, with a few more details, to these new objectivities relevant for the biological. In no way, by the concepts we hint to, we intend to define or characterize life. We just try to focus on some phenomenalities which seem particularly preeminent and try to treat them in a possibly conceptually robust fashion. The three points below briefly outline the work developed over several articles, to which we refer when appropriate.

3.1. EXTENDED CRITICALITY

The biological interest of physical theories of criticality is due first to the fact that, in physics, *critical phase transitions* are processes of change of state where, by the sudden change (a singularity w. r. to a control parameter), the global structure is involved in the behavior of its elements: the local situation depends upon (is correlated to) the global situation. Mathematically, this may be expressed by the fact that the correlation length formally tends toward infinity (the case with second order transitions, such as para-/ferromagnetic transition); physically, this means that the determination is global and not local. In other words, a critical transition is related to a change of phase and to the appearing of critical behaviors of some magnitudes of the system's states – magnetization, density, for example – or of some of its particular characteristics – such as correlation length. It is likely to appear at equilibrium (null fluxes) or far from equilibrium (non-null fluxes). In the first case, the physico-mathematical aspects are rather well-understood (renormalization as for the mathematics (Binney et al., 1992), thermodynamics for the bridge between microscopic and macroscopic description), while, in the second case, we are far from having theories as satisfactory.

Some specific cases, without particular emphasis on the far from equilibrium situation, have been extensively developed and publicized by Bak, Kauffman, and others (see Bak et al., 1988; Kauffman, 1993; Nykter et al., 2008). The sand hips, whose criticality reduces to the angle of formation of avalanches in all scales, percolation (see Bak et al., 1988; Laguës and Lesne, 2003) or even the formation of a snowflake are interesting examples. The perspective assumed is, in part, complementary to Prigogine's: it is not fluctuations within a weakly ordered situation that matter in the formation of coherence structures, but the "order that stems from chaos" (Kauffman, 1993). Yet, in both cases potential correlations are suddenly made possible by a change in one or more control parameters for a specific (pointwise) value of this parameter. For example, the forces attracting water molecules toward each other, as ice, are potentially there: the passage below a precise temperature, as decreasing Brownian motion, at a certain value of pressure and humidity, allows these forces to apply and, thus, the formation of a snow flakes, typically.

The critical transitions must be also understood as sudden symmetry changes (symmetry breakings and formation of new symmetries), and a transition between two different macroscopic physical objects (two different states), with a conservation of the symmetries of the components. The specific, local and global symmetry breakings give the variety of organized forms and their regularities (the new symmetries) as these transitions are (very) sensitive to fluctuations in the vicinity of criticality. In physics, the pointwise nature of the "critical value" of the control parameter is an essential mathematical issue, as for the treatment by the relevant mathematics of "renormalization" in theories of criticality (see Binney et al., 1992).

Along the lines of the physical approaches to criticality, but within the frame of far from equilibrium thermodynamics, we consider living systems as "coherent structures" in a continual (extended) critical transition. The permanent state of transition is maintained, at each level of organization, by the integration/regulation activities of the organism, that is by its global coherent structure.

In short, in recent work (Bailly and Longo, 2008; Longo and Montévil, 2011a), on the grounds of early ideas in Bailly (1991), we propose to analyze the organization of living matter as "extended critical transitions." These transitions are extended in spacetime and with respect to all pertinent control parameters (pressure, temperature, etc.), their unity being ensured through global causal relations between levels of organization (and their integration/regulation). More precisely, our main theoretical paradigm is provided by the analysis of critical phase transitions, as this peculiar form of critical state presents some particularly interesting aspects for the biological frame: the formation of extended (mathematically diverging) correlation lengths and coherence structures, the divergence of some observables with respect to the control parameter(s) and the change of symmetries associated to potentially swift organizational changes. However, the "coherent critical structures" which are the main focus of our work cannot be reduced to existing physical approaches, since phase transitions, in physics, are treated as "singular events," corresponding to a specific well-defined value of the control parameter, just one (critical!) point as we said. Whereas our claim is that in the case of living systems, these coherent critical transitions are "extended" and maintained in such a way that they persist in the many dimensional space of analysis, while preserving all the physical properties mentioned above (diverging correlation lengths, new coherence structures, symmetry changes . . .). A living object is not only a dynamics or a process, in the various possible senses analyzed by physical theories, but it is permanent *critical transition*: it is always on the border of a change, of symmetries changes in particular, as analyzed in Longo and Montévil (2011a). One then has an extended, permanently reconstructed and transforming *global* organization in an interaction with *local* structures, as the global/local interplay is proper to critical transitions (by singularities).

So far, our analysis, in the papers quoted above, has been largely conceptual, since, by the loss of the mathematics of renormalization, there seem to be little known Mathematical Physics that applies to this physically singular, far from equilibrium situation. The second major conceptual and technical difficulty is also clearly the instability of the symmetries involved. The question is then how to objectivize them, since, in contradiction with the physical situations they do not seem to be theoretically determined to be in a specific, pre-given set.

3.2. ANTI-ENTROPY

In (Bailly and Longo, 2009) our systemic perspective for biological complexity, both in phylogenesis and ontogenesis, is developed by an analysis of organization in terms of "anti-entropy," a notion we defined and which conceptually differs from the common use of "negative entropy." Note that both the formation and maintenance of organization (a permanent reconstruction of the coherent structure) go in the opposite direction of entropy increase. This is also Schrödinger's concern in the second part of his 1944 book. He considers the possible decrease of entropy by the construction of "order from order," that he informally calls negative entropy. In our approach, anti-entropy is mathematically presented as a new observable, it is therefore not just entropy with a negative sign (negative entropy, as more rigorously presented in Shannon's Work and in Brillouin, 1956). Typically, when summed up, equal entropy and negative entropy give 0: in our approach, entropy and anti-entropy are found simultaneously only in the non-discrete critical interval of the living state of matter. A purely conceptual analogy may be done with anti-matter in Quantum Physics: this is a new observable, relative to new particles, whose properties (charge, energy) have opposite sign. Along our wild analogy, matter and anti-matter never give 0, but a new energy state: the double energy production as gamma rays, when they encounter in a (pointwise!) singularity. Again, entropy and anti-entropy coexist in an organism, as extended zone (interval) of criticality.

To this purpose, we introduced two principles ("existence and maintenance of anti-entropy"), in addition to the thermodynamic ones, which are (mathematically) compatible with traditional principles but which have no meaning with regard to inert matter. The idea is that anti-entropy represents the key property of an organism, even a unicellular one, to be describable by several levels of organization (also an eukaryotic cell possesses organelles, say), regulating, integrating each other – they are parts that functionally integrate a whole, and the whole regulates them. This corresponds to the formation and maintenance of a global coherence structure, in correspondence to its extended criticality: organization increases, along embryogenesis say, and is maintained, by contrasting the ongoing entropy production due to all irreversible processes. No extended criticality nor its key property of coherence would be possible without anti-entropy production: (always renewed) organization (expresses and) allows the maintenance of the extended critical transition.

In Bailly and Longo (2009), we applied the notion of anti-entropy to an analysis of Gould's work on the complexification of life along evolution (Gould, 1997). We thus extended a traditional balance equation for the metabolism to the new notion as specified by the principles above. This equation is inspired by Gibbs' analysis of free energy, which is hinted as a possible tool for the analysis of biological organization in a footnote in Schrödinger (1944). We examined far from equilibrium systems and we focused in particular on the production of global entropy associated to the irreversible character of the processes. In the 2009 paper, a close analysis of anti-entropy has been performed from the perspective of a diffusion equation of biomass over phenotypic complexity along evolution. That is, we could reconstruct, on the grounds of general principles, Gould's complexity curve of biomass over complexity in evolution (Gould, 1997). Moreover, a quantitative evaluation of phenotypic complexity in embryogenesis is proposed, in relation to some empirical data (*Caenorhabditis elegans*). Once more, Quantum Mechanics indirectly inspired our mathematical approach: we borrowed Schrödinger's operatorial approach in his famous equation but in a classical framework. Classically, that equation may be understood as a diffusion equation (as a matter of fact, we used real coefficients instead of complex ones, thus outside of the mathematical framework of quantum theories).

3.3. BIOLOGICAL TIME

The usual physical (linear) representation of time is insufficient, in our view, for the understanding of some phenomena of life. An extended form of present seems more adequate for the understanding of memory, since this is an essential component of learning, for the purposes of future action (based on "protention," as pre-conscious expectation). In particular, while memory, as retention, is treated in some physical theories (relaxation phenomena), protention seems outside the scope of Physics. We then suggested some simple functional representation of biological retention and protention (Longo and Montévil, 2011b; see also Varela, 1999).

Similarly, the role of biological rhythms do not seem to have any counterpart in mathematical formalization of physical clocks, which are based on frequencies along the usual (possibly thermodynamically) time. In Bailly et al. (2011) a two-dimensional manifold as a "mathematical frame" for accommodating autonomous biological rhythms is presented: the second dimension is "compactified," that is, it is a circular fiber orthogonal to the oriented representation of physical time. The addition of a new (compactified) dimension for biological time is justified by the peculiar dimensional status of *internal* biological rhythms. Life is temporally scanned by both external (physical) rhythms (Circadian, typically), which are frequencies, and internal ones (metabolism, respiration, cardiac rhythms). These are pure numbers, not frequencies: they become average frequencies and produce the time of life span, when used as coefficient in scaling laws (Schmidt-Nielsen, 1984; West et al., 1997; Savage et al., 2004). These rhythms have also singular behaviors (multi-scale variations) with respect to the physical time, which can be visualized in our framework. In contradiction with physical situations, the scaling, however, does not seem to be associated to a stable exponent. These two peculiar features (pure numbers and fractal-like time series) are the main evidences of the autonomy of our compactified time with respect to the physical time.

The two new aspects of biological time allowed us to introduce the abstract notion of "biological inertia," as a component of the conceptual time analysis of extended criticality. Note that our approach of protention and retention is, for now, focused on local aspects of biological time, and should therefore be completed to accommodate the long range correlations observed experimentally (see Grigolini et al., 2009). Indeed, this kind correlations is relevant for both aspects of biological time, and fits in the conceptual framework of extended criticality.

Another aspect of biological time, introduced in Longo and Montévil (2011a), is the time constituted by the cascade of symmetry changes which takes place in extended critical transitions. In other terms, this time is defined by the ubiquitous organizational transformations occurring in biological matter. Here, time corresponds therefore to the *historicity* of biological objects and to the process of biological individuation (both ontogenetic and phylogenetic). Indeed, time is no longer the parameter of trajectories in the phase space since the latter is unstable, therefore the temporality defined by the changes of phase space has an original nature.

4. CONCLUSION

Broadly speaking, except for the consideration in terms of extended criticality and symmetry changes, the laws which we propose, while addressing these particular observables and quantities, specific to life phenomena, constitute a simple *extension* of existing physical laws: they preserve the same formal mathematical structure and, if we set the value of the considered observables or parameters to 0 (protention second temporal dimension, value of anti-entropy), they return theories of inert. Our theoretical propositions are thus compatible, although irreducible, to "existing physical theories." That is, they are reducible to these laws *only* if, but *as soon as* we are outside of the extended critical zone having its own temporality and its anti-entropy, or as soon as these specific quantities go to 0.

In conclusion, the peculiar phenomenality of life deserves some new observables (extended critical transition, biological organization, proper time, in our attempts). The point is the pertinence, if any, of these treatments, "*per se*." Those who claim that all these concepts should be reduced to physical (existing?) theories are welcome to try. But they should first look at the history of Physics itself, where novel theoretical frames are marked by the invention of new concepts and new perspectives. Their pertinence had to be judged "as such," not on the grounds of their reducibility to existing, thus "safe," explanatory grounds[2]. Note, however, that extended critical transitions, in association with ubiquitous symmetry changes, may, however, lead to more radical methodological changes, as associated to the specificity of objects and genericity of trajectories.

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[2] In other terms, should reduction or unification be performed, the first question is: what does one want to reduce?

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APPENDIX

INTERFACES OF INCOMPLETENESS

Do we need to have a definition of life, in order to construct robust theories of the living state of matter?

Let's now answer to this question by an analogy with a frame where it may be dealt with the highest rigor: Mathematical Logic.

Is the concept of integer (thus "standard" or finite) number captured (defined, characterized) by the (formal) theory of numbers? Frege (1984) believed so, as the absolute concept of number was, in his view, fully characterized by Peano-Dedekind theory. In modern logical terms, we can say that, for Frege, Peano Arithmetic (PA) was "categorical." That is, PA was believed to have just one model, up to isomorphisms: the standard model of integers (the one which the reader learned about in elementary school, with 0, though, and formal induction). Thus, the theory was also meant to uniquely define of "what a number is."

This turned out to be blatantly wrong. Löwenheim and Skolem (1915–1920) proved that PA has infinitely many non-isomorphic models. Moreover, a simple theorem ("compactness") showed that no predicate, definable in PA, may isolate all and exactly all the standard integers (see Marker, 2002). In short, any predicate valid on infinitely many standard integers, must hold also for (infinitely many) non-standard integers (which cannot be considered properly "finite") – this is known as the "overspill lemma." Gödel's incompleteness theorem reinforced these negative properties: PA is *incomplete* or, equivalently, it has lots of logically non-equivalent models, a much stronger property than *non-categoricity*. A fortiori, there is no hope to characterize in a finitistic way the concept of standard integer number. One has to add an axiom of infinity (Set Theory) or proper second order quantification in order to do so, and these are infinitary or impredicative formal frames.

Yet, everybody considers PA as the "natural" theory of numbers: it beautifully singles out the main relevant, and very robust, properties of numbers (0, successor, induction), even though it *cannot define what a number is.* In analogy to the impossibility of physics to define its own object of study, physical matter, as we mentioned at the beginning, we have here another example of sound theoretical frame, which cannot define, within itself, its own object of study, the natural number object. We do not see a way to get out from the language of physics or of biology as Mathematical Logic can do: what would ever correspond to an axiom of infinity or to higher order quantification? Perhaps: ..."take the point of view (and the language) of God"?

We encourage thus the reader to pursue his/her theoretical work in biology without the anguishing search for a *definition* of life. And with the clear perspective of the intrinsic incompleteness of all our theoretical endeavors (Longo, 2011): we can just hope to explicitly grasp and organize by theories some fragments of reality, whatever this word may mean. Let's try to do it toward the best of our knowledge, in a sufficiently broad and robust way, and in full theoretical and empirical freedom, without necessarily feeling stuck either to existing theories nor always searching for the "ultimate (complete?) Theory" nor the "ultimate reduction."

Longo's downloadable papers in http://www.di.ens.fr/users/longo/

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Criticality in large-scale brain fMRI dynamics unveiled by a novel point process analysis

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Functional magnetic resonance imaging (fMRI) techniques have contributed significantly to our understanding of brain function. Current methods are based on the analysis of *gradual and continuous* changes in the brain blood oxygenated level dependent (BOLD) signal. Departing from that approach, recent work has shown that equivalent results can be obtained by inspecting only the relatively large amplitude BOLD signal peaks, suggesting that relevant information can be condensed in *discrete* events. This idea is further explored here to demonstrate how brain dynamics at resting state can be captured just by the timing and location of such events, i.e., in terms of a spatiotemporal point process. The method allows, for the first time, to define a theoretical framework in terms of an order and control parameter derived from fMRI data, where the dynamical regime can be interpreted as one corresponding to a system close to the critical point of a second order phase transition. The analysis demonstrates that the resting brain spends most of the time near the critical point of such transition and exhibits avalanches of activity ruled by the same dynamical and statistical properties described previously for neuronal events at smaller scales. Given the demonstrated functional relevance of the resting state brain dynamics, its representation as a discrete process might facilitate large-scale analysis of brain function both in health and disease.

Keywords: fMRI, criticality, brain dynamics, point processes

1. INTRODUCTION

Important efforts to understand brain function, both in health and disease, are concentrated in the analysis of large-scale spatiotemporal patterns of brain activity available from fMRI techniques (Greicius et al., 2003; Beckmann and Smith, 2004; Beckmann et al., 2005; Raichle, 2006; Fox and Raichle, 2007; Smith et al., 2009), allowing for instance the unraveling of the functional connectivity between all possible brain regions, as is done under the Connectome project (Sporns et al., 2005; Sporns, 2011)[1]. At the same time, similar efforts are dedicated to place brain phenomenology in the context of statistical physics theory (Chialvo, 2010; Rolls and Deco, 2010; Sporns, 2010; Steyn-Rose and Steyn-Rose, 2010). Novel techniques of analysis are needed because of the increasing difficulty in managing extremely large data sets, generated by advances in imaging technology continuously improving temporal and spatial resolution.

Recent work has shown that important features of brain functional connectivity at rest can be computed from the relatively large amplitude BOLD fluctuations (Tagliazucchi et al., 2010a,b) after the signal crosses some amplitude threshold. Here we pursue further the same general idea of data reduction. In particular we are interested in a method often used to study the structure and properties of attractors of dynamical systems, which consists in the introduction of a Poincaré section. By definition, this approach decreases the dimension of the phase space and consequently the size of the data sets, facilitating in this way further numerical investigations. In general, there exist two possibilities: the first one is to analyze the set of points which are the coordinates of the successive intersections of the secant Poincaré plane by the phase space trajectories. The second possibility is to study the series of time intervals between the consecutive intersections. The resulting time intervals constitute a so-called point process (Cox and Isham, 1980), a construction useful in many areas of science, including neuroscience. It has been shown that under certain conditions the most important statistical features of the dynamical regime can be condensed into a point process (Packard et al., 1980; Roux et al., 1980; Takens, 1980; Roux and Swinney, 1981; Grassberger and Procaccia, 1983; Castro and Sauer, 1997).

The motivation to attempt a similar approach in fMRI data is strengthened by the observation that, in response to neuronal activation, the BOLD signal often repeats a stereotypical pattern (Friston et al., 1995, 1998; Aguirre et al., 1998; Tagliazucchi et al., 2010a,b). This feature suggests that it should be possible to

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[1] http://www.humanconnectomeproject.org/

compress the data sets using the temporal marks of a Poincaré section of the BOLD signal. This is the hypothesis explored here, which implies that, in principle, the entire brain resting state functional connectivity can be reconstructed solely on the basis of the time and location of the BOLD signal threshold crossings. Besides its practical importance for fMRI signal processing, this approach may provide further clues on the dynamical organization of the resting state brain activity.

The paper is organized as follows: the results section starts with the definition of the point process, as well as its connection with deconvolution techniques. This is followed by the replication of the fMRI brain resting state networks (RSN) maps using the point process. As further validation, the method's ability to evaluate functional connectivity changes is demonstrated for a motor task and for a pathological condition. The spatiotemporal statistics are then considered, revealing novel aspects of the brain dynamics which are scale-invariant, consistent with that shown for other systems at the critical state (Bak, 1996; Jensen, 1998; Chialvo, 2010; Expert et al., 2011). The paper closes with a discussion on the new questions raised by the current analysis. For readers' convenience, the methods are described at the end of the paper.

2. RESULTS

The fMRI dataset is reduced to a spatiotemporal point process by normalizing each BOLD signal by its own SD, and subsequently selecting the time points at which the signal crosses a given threshold (1 SD in this case) from below, as it is shown in the example of **Figure 1**. Notice that, despite the fact that in resting data there are not explicit inputs, the average BOLD signal around the extracted points (**Figure 1B**, termed rBeta function in Tagliazucchi et al., 2010b) still resembles the hemodynamic response function (HRF) evoked by an stimulus (Friston et al., 1995, 1998). The relation between the point process and the underlying HRF is exposed by the deconvolution of the BOLD signal with either the HRF (with default parameters) or the rBeta function (Tagliazucchi et al., 2010b) extracted from the time series in **Figure 1**. In both cases, as shown in **Figure 1C**, the peaks of the de-convolved BOLD signal coincide, on a great majority, with the timing of the point process in **Figure 1A**. At this point a remark is needed concerning the impulse-like signals in **Figure 1C**. They result from the deconvolution of the BOLD signal with a function similar to the HRF, and from a physiological viewpoint it can be conjectured that they constitute short-lived events triggering the relatively slow (up to 20 s) BOLD response. Notice, however, that the bulk of the present results is independent of the precise nature of these impulse-like signals. They serve to illustrate that a different and already established mathematical method (which is also amenable to a clear physiological interpretation) leads to similar inter-event timings than those derived from the Poincaré section. Therefore, these results show that important information is compressed in the timing and spatial location of the extracted points. For the parameter used here, from each voxel BOLD time series (240 samples) on the average only 15 ± 3 points are threshold crossings (about one point every 40 s) which corresponds to near 94% reduction of the data (additional details, including the robustness to changes in threshold, are discussed in the Materials and Methods Section).

2.1. RESTING STATE NETWORKS AND ACTIVATION MAPS CAN BE DERIVED FROM A FEW POINTS

Despite the very large data reduction, we found that the information content of the few remaining points is very high. As a proof of principle, we first used the point process to calculate the spatial location of six well known RSN maps. These maps describe the major independent components of brain spontaneous activity, and as such they can be used as a relevant benchmark. We used the point process to obtain the RSN maps and compare them with maps computed from the full BOLD signal using a well established method (probabilistic independent component analysis – PICA; Beckmann et al., 2005). This is done by calculating in six RSNs the rate of points co-occurrence (up to 2

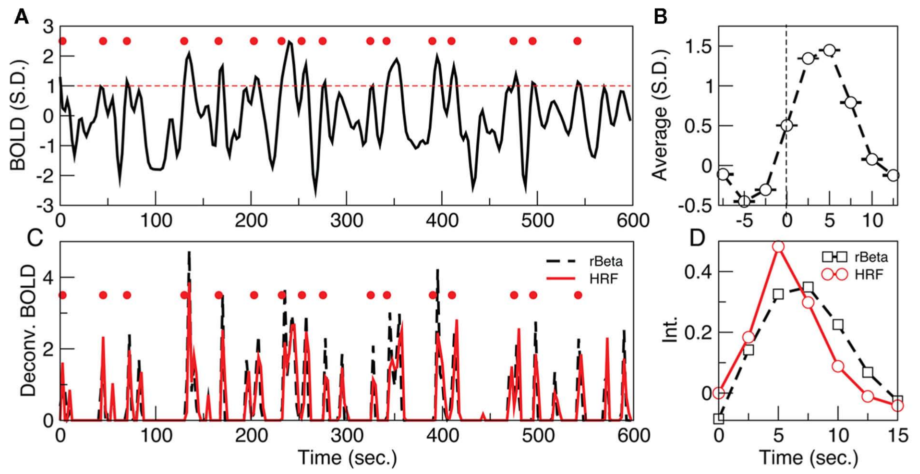

FIGURE 1 | (A) Example of a point process (filled circles) extracted from the normalized BOLD signal. Each point corresponds to a threshold (dashed line at 1 SD) crossing from below. **(B)** Average BOLD signal (from all voxels of one subject) triggered at each threshold crossing. **(C)** The peaks of the de-convolved BOLD signal, using either the hemodynamic response function (HRF) or the rBeta function (Tagliazucchi et al., 2010b) depicted in **(D)**, coincide on a great majority with the timing of the points shown in **(A)**.

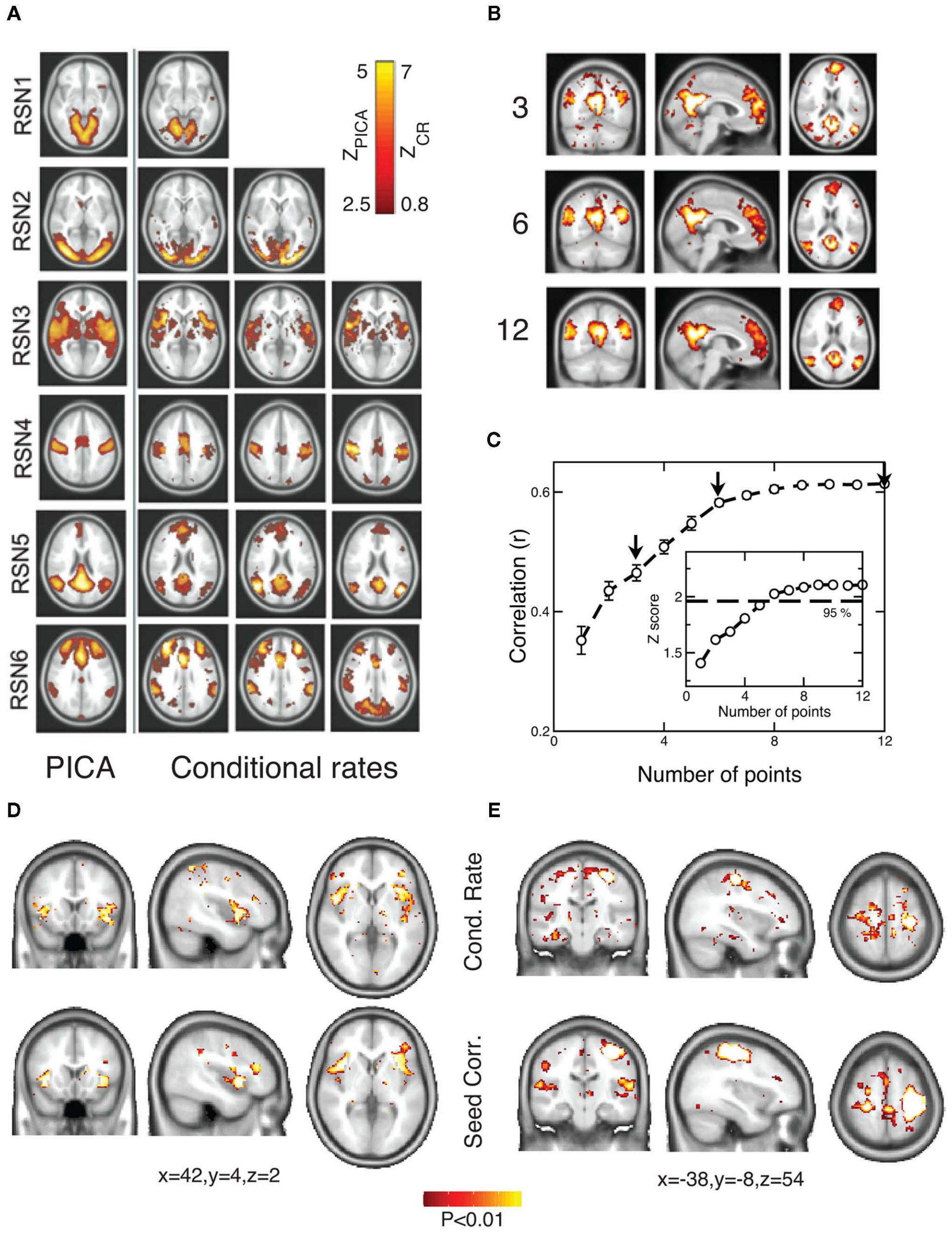

FIGURE 2 | RSN maps constructed with the point process compare very well with standard PICA of the raw continuous data. (A) PICA spatial maps (left column) and rate of points conditional to activity at a given seed (rightmost three columns, each one corresponds to a different seed). (Slice z coordinates are −12, 0, 0, 36, 20, 26 for RSN 1–6; for seed coordinates see **Table 1**). Scales for PICA (Z_{PICA}) and conditional rate (Z_{CR}) calculations are depicted in the inset. **(B)** Conditional rate maps constructed using 3, 6, and 12 events of the point process at the ANGL seed (averaged for ten subjects. Slice coordinates are x = −4, y = −60, z = 18). **(C)** Correlation between RSN5 (the default-mode network, DMN) PICA-derived map and the point process-derived conditional rate maps, as a function of the number of points used. Arrows denote the examples of **(B)**. Z scores (number of points as degrees of freedom) with the line of 95% confidence are plotted in the inset. **(D)** The point process is able to track the statistical differences between the functional connectivity maps of a group of chronic back pain patients and healthy controls already reported in (Tagliazucchi et al., 2010a). The conditional rate of points (top) reproduces well the standard seed correlation approach (bottom) derived from the same data. **(E)** The functional connectivity maps during a finger tapping task constructed from the conditional rate of points (top) compare well with the seed correlation maps derived from the same data (Tagliazucchi et al., 2010b).

time units later in this case) between representative sites ("seeds") and all other brain voxels and presented as maps in **Figure 2A–C** (see Materials and Methods for a detailed explanation of the computation). The seeds locations were selected according with previous work (see coordinates in **Table 1** of Materials and Methods Section). The similarities between our conditional rate maps

Table 1 | MNI coordinates for the seeds used in Figure 2.

RSN	Seed 1	Seed 2	Seed 3
RSN1	(2, −82, 20)		
RSN2	(30, −86, 16)	(−30, −86, 16)	
RSN3	(2, 6, 48)	(−54, 2, −8)	(58, 2, −4)
RSN4	(−2, −14, 48)	(−38, −14, 52)	(50, −14, 52)
RSN5	(2, 54, −8)	(50, −66, 28)	(−38, −70, 28)
RSN6	(34, 46, 20)	(−34, 42, 20)	(10, −42, 48)

and the respective PICA maps (rightmost three columns and left column of **Figure 2A** respectively) is already obvious to the naked eye and confirmed by the correlation plotted in **Figure 2C**. The calculation shows that despite using less than 6% of the raw fMRI information, about 5 points (on average) are enough to obtain RSN maps that are highly correlated (95% confidence) with those obtained using PICA of the full BOLD signals. Similar good performance can be demonstrated in tracking physio-pathological changes of brain activation. This is presented in **Figure 2D** which shows the statistical differences in functional connectivity between a group of chronic back pain (CBP) patients and healthy controls already reported in Tagliazucchi et al., 2010a; comparison with seed correlation based in the DMN, increased correlation with bilateral insula in CBP). Finally, the data analysis from a finger tapping task (Tagliazucchi et al., 2010b) demonstrates also the merits of the current approach when compared with a seed correlation based in primary motor cortex contralateral to the tapping hand (**Figure 2E**).

2.2. A PHASE TRANSITION IN THE DYNAMICS OF THE ACTIVE CLUSTERS

The results in the previous section show that the point process can efficiently compress the information needed to reproduce the underlying brain activity in a way comparable with conventional methods such as seed correlation and independent component analysis. Importantly, while the former methods represent averages over the entire data sets, the point process, by construction, compresses, and *preserves* the temporal information. This potential advantage, unique of the current approach, may provide additional clues on brain dynamics. This is explored here by compiling the statistics and dynamics of clusters of points both in space and time. Clusters are groups of contiguous voxels with signal above the threshold at a given time, identified by a scanning algorithm in each fMRI volume (see Materials and Methods for details). **Figure 3A** shows examples of clusters (in this case non-consecutive in time) depicted with different colors. Typically (**Figure 3B** top) the number of clusters at any given time varies only an order of magnitude around the mean (∼50). In contrast, the size of the largest active cluster fluctuates widely, spanning more than four orders of magnitude.

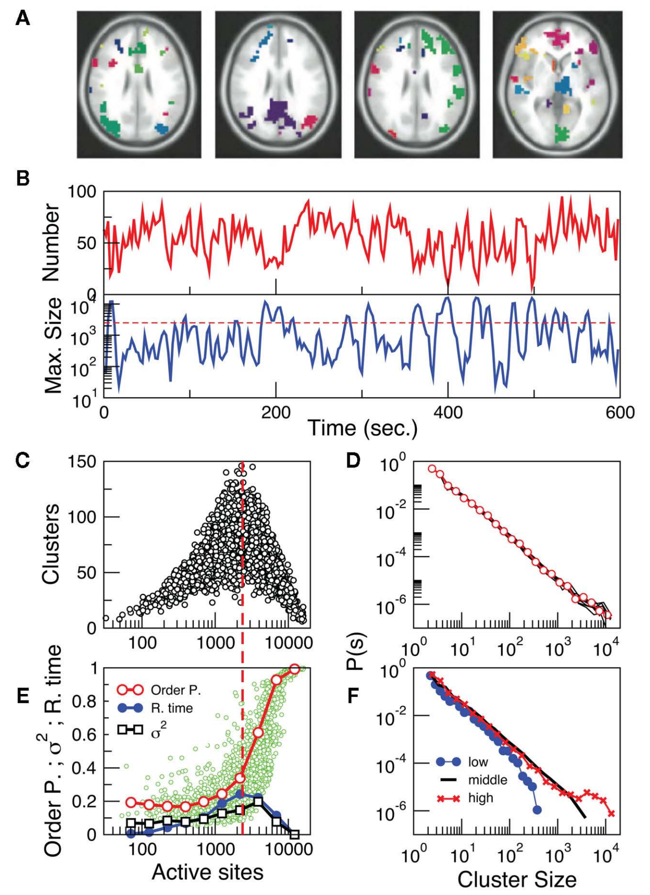

FIGURE 3 | The level of brain activity continuously fluctuates above and below a phase transition. (A) Examples of co-activated clusters of neighbor voxels (clusters are 3D structures, thus seemingly disconnected clusters may have the same color in a 2D slice). **(B)** Example of the temporal evolution of the number of clusters and its maximum size (in units of voxels) in one individual. **(C)** Instantaneous relation between the number of clusters vs. the number of active sites (i.e., voxels above the threshold) showing a positive/negative correlation depending whether activity is below/above a critical value [∼2500 voxels, indicated by the dashed line here and in **(B)**]. **(D)** The cluster size distribution follows a power law spanning four orders of magnitude. Individual statistics for each of the ten subjects are plotted with lines and the average with symbols. **(E)** The order parameter, defined here as the (normalized) size of the largest cluster is plotted as a function of the number of active sites (isolated data points denoted by dots, averages plotted with circles joined by lines). The calculation of the residence time density distribution (R. time, filled circles) indicates that the brain spends relatively more time near the transition point (which corresponds to about 0.4 of the largest giant cluster observed). Notice that the peak of the R. Time in this panel coincides with the peak of the number of clusters in **(C)**. Note also that the variance of the order parameter (squares) increases as expected for a phase transition. **(F)** The computation of the cluster size distribution calculated for three ranges of activity (low: 0–800; middle: 800–5000; and high >5000) reveals the same scale invariance plotted in **(D)** for relatively small clusters, but shows changes in the cut-off for large clusters.

The analysis reveals four novel dynamical aspects of the cluster variability which hardly could have been uncovered with previous methods. (1) At any given time, the number of clusters and the total activity (i.e., the number of active voxels) follows a non-linear relation resembling that of percolation (Stauffer and Aharony, 1992). At a critical level of global activity (∼2500 voxels, dashed horizontal line in **Figure 3B**, vertical in **Figure 3C**) the number of clusters reaches a maximum (∼100–150), together with its variability. (2)

The correlation between the number of active sites (an index of total activity) and the number of clusters reverses above a critical level of activity, a feature already described in other complex systems in which some increasing density competes with limited capacity (Stauffer and Aharony, 1992; Bak, 1996). (3) The rate at which the very large clusters (i.e., those above the dashed line in 3B) occurs (∼ one every 30–50 s) corresponds to the low frequency range at which RSN are typically detected using PICA (Beckmann and Smith, 2004; Beckmann et al., 2005). (4) The distribution of cluster sizes (**Figure 3D**) reveals a scale-free distribution (whose cut-off depends on the activity level, see **Figure 3F**).

These four features remind of other complex systems undergoing an order-disorder phase transition (Bak, 1996; Jensen, 1998; Tsang and Tsang, 1999; Chialvo, 2010; Tagliazucchi and Chialvo, 2011) thus suggesting further exploration. Following standard techniques in statistical physics, two parameters were defined and computed from the same data plotted in **Figure 3C**. To represent the degree of order (i.e., the order parameter), the size of the *largest* cluster (normalized by the number of active sites) in the entire brain was computed and plotted as a function of the number of active points (i.e., the control parameter). This was done for all time steps and plotted in **Figure 3E** (small circles). We avoided the use of the branching ratio (Beggs and Plenz, 2003) as a control parameter because its estimation from the data is less than straightforward. It cannot be computed for each fMRI volume as required here and only converges to a stable quantity for relatively long time series. In addition, it requires the *ad hoc* definition of the number of bins and a suitable bin-width for its computation (Beggs and Plenz, 2003), therefore making its use cumbersome for the spatiotemporal resolution of the present study. On the other hand, the parameter used here (i.e., global level of activity) is computed in a straightforward manner, converges relatively fast, requires no fine tuning of parameters and has clear analogies to control parameters of well studied models of order-disorder transitions (the clearest example being percolation; Stauffer and Aharony, 1992).

Several key features are worth to mention, all highly suggestive of a phase transition: First, there is sharp increase in the average order parameter (empty circles), accompanied by an increase of its variability (empty squares). Second, the transition coincides with the peak in the function discussed in **Figure 3C**, which accounts for the number (not the size) of the clusters. Finally, a calculation of the relative frequency of the number of active sites was performed (i.e., residence times, filled circles) showing that the brain spends, on the average, more time near the transition than in the highly ordered or the highly disordered states. This is a remarkable support for earlier conjectures suggesting that the brain at large-scale works at criticality (Bak, 1996; Chialvo, 2010; Expert et al., 2011; Tagliazucchi and Chialvo, 2011).

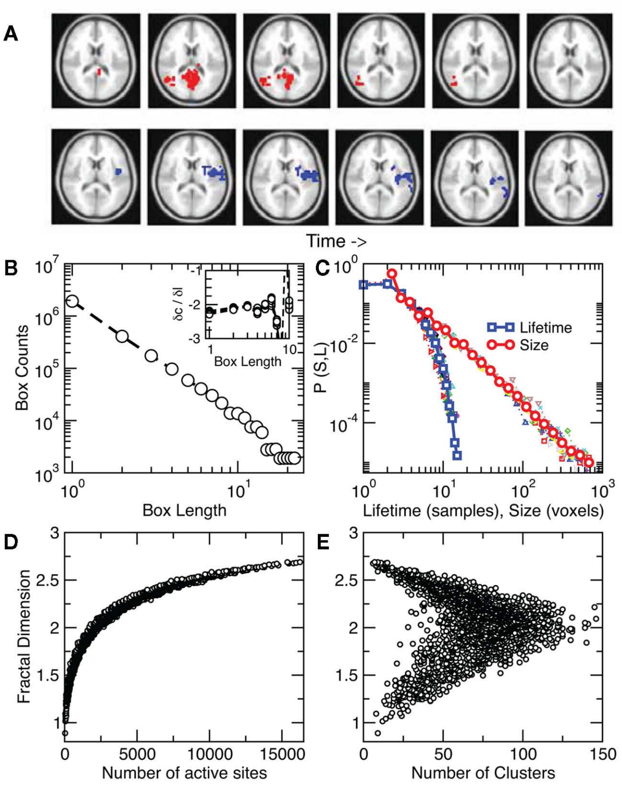

FIGURE 4 | Clusters spread throughout the brain as scale-free avalanches. (A) Two examples of avalanches, one triggered from the visual cortex (top) and another from insular cortex (bottom). Note that only a partial 2D slice is depicted here since avalanches evolve in 3D. **(B)** Average cluster fractal dimension $D \sim 2.15 \pm 0.02$ estimated by the slope of the counts vs. length plot. Inset: derivatives between points in the main plot for each subject. **(C)** Avalanche size and lifetime distribution function computed from about 8000 avalanches in each of 10 subjects (individual subjects with smaller symbols). While avalanche size follows a power law, their lifetimes density decreases faster as found previously for neuronal avalanches (Beggs and Plenz, 2003). The average cluster fractal dimension is plotted as a function of the number of active sites in **(D)** and of the number of clusters in **(E)**.

2.3. ACTIVITY SPREAD IS SCALE-FREE

The identification of a phase transition in the resting brain suggested additional work to characterize its properties, including a quantification of the dynamical properties of cluster spatial evolution. As shown in the example of **Figure 4A** an activated cluster can appear, grow to achieve a maximum size and then disappear (or translate or divide into sub-clusters). The present approach allows the study of two properties of the process. For each cluster we first measured a static space filling property, the average fractal dimension D. This is shown in **Figure 4B** which illustrates that $D \sim 2.15 \pm 0.02$. While the fractal dimension D departs from this value for the highly ordered and disordered regimes (**Figures 4D,E**), the residence time distribution computed in **Figure 3E** indicates that most of the time the level of activity is around the critical value, thus on average $D \sim 2$. Second, we looked at the dynamics of the cluster propagation, which was found to happen in bursts. The statistics in **Figure 4C** shows that avalanches could last up to 30 s. with sizes up to 10^3 and have no preferred scale, a behavior very similar to that of neuronal avalanches described previously in smaller scales (Beggs and Plenz, 2003; Petermann et al., 2009; Chialvo, 2010).

3. DISCUSSION

As far as we know, this is the first attempt to describe large-scale brain fMRI dynamics as a point process and the first to uncover a phase transition in the dynamics of the active clusters, with scale-free avalanching events in the whole human cortex. Regarding the point process analysis, the only previous report we are aware of (Vedel Jensen and Thorarinsdottir, 2007) dealt with the reverse situation: how to model the continuous fMRI signal starting from a spatiotemporal point process.

3.1. WHY FEW POINTS SUFFICE?

At first sight, the continuous nature of the fMRI BOLD signal, imposed by the nature of the neurovascular coupling itself (Friston et al., 1995, 1998), might have hindered the introduction of point process methods for its analysis. The situation is analogous to that of continuous rhythmic activity arising in scalp EEG due to predominant frequencies in the spiking activity and subthreshold oscillations which underlie the generation of discrete action potentials (Traub et al., 1989; Steriade et al., 1993; Contreras and Steriade, 1995). We have shown that the application of HRF deconvolution gives a way to invert the process and find the train of impulse-like signals (of whatever origin) which closely resembles the point process. The fact that the majority of the points coincide with the peaks of the BOLD HRF-de-convoluted signal (i.e., **Figure 1C**) reinforces the view that upward going BOLD signals are non-linear events where the crossing times preserve the most relevant information. This is in line with recent findings of all-or-none "coherence potentials" macroscopically propagating in the monkey cortex, as observed in local field potential recordings (Thiagarajan et al., 2010). Therefore it seems reasonable the conjecture that, at this level of coarse graining, we are dealing with all-or-none intermittent avalanching events which involve short and long range cortical co-activations.

Another remark needs to be made concerning the HRF: while it is true that extensive work established the fundamental details of the brain's BOLD HRF responding to a *well defined (single or repetitive) stimulus,* less is known about the BOLD response under the non-stationary conditions of resting state, in other words, the nature of the *resting state HRF* remains unknown. A theoretical formalism for the neuro-BOLD coupling at rest, as far as we know, has not been attempted but probably deserves to be considered in the future. Such studies should clarify up to what extent the HRF function obtained from stimuli spaced by relatively long intervals can predict the temporal evolution of the BOLD signal measured during resting state.

3.2. SCALE-INVARIANT BRAIN DYNAMICS IS MADE UP OF AVALANCHES

The reduction of the fMRI BOLD signal to discrete events not only allows for the identification of well-described resting state networks as shown in **Figure 2**, but also reveals that large-scale brain activity organizes in avalanches of activity with power law size distributions. The point process approach allowed for the first time to identify explicitly the order and control parameters and to define the state of the resting brain as a fluctuation around a phase transition. The analysis shows not only that activity spreads as scale-free avalanches resembling those seen in smaller scales (Beggs and Plenz, 2003) but – and importantly – that the brain spends most of the time at a level of activity which corresponds to the critical point. These new findings add to the previous observations that the correlation function of fMRI BOLD signals exhibits fractal properties (Expert et al., 2011) and that the correlation length of the activity measured with fMRI diverges as predicted by the theory of phase transitions (Fraiman and Chialvo, 2010), supporting the hypothesis that brain dynamics operates at a critical point of a second order phase transition (Bak, 1996; Chialvo, 2010; Tagliazucchi and Chialvo, 2011).

In connection with previous experiments, one must emphasize that for a non-equilibrium system in a critical state avalanches are observed at a wide range of temporal and spatial scales. Observations at smaller scales (Beggs and Plenz, 2003) show a clear cut-off of the avalanche distribution at the size of the electrode array used for the recordings (signaling that the experimental technique is unable to sample larger events) as well as a distortion of the distribution caused by sub-sampling effects (Priesemann et al., 2009). Due to very good spatial resolution and whole brain coverage fMRI allows to overcome these issues in the macroscopic domain ($\approx$1 mm). The observation of identically distributed avalanches at this level is direct evidence that the brain spatiotemporal dynamics is scale-free, as expected for a critical system. The present work also suggests the study of intermediate scales accessible by means of other experimental techniques to give further support for or against this hypothesis.

The observation that large-scale brain dynamics can be traced as discrete scale-free avalanches of activity raises the question of the physiologically relevance encoded in the timing of these large-scale events, already suggested by observations at smaller scales and computational models (Kinouchi and Copelli, 2006; Shew et al., 2009, 2011; de Arcangelis and Herrmann, 2010). For instance, although relatively rare, avalanches in the tail of the power law distribution emerge from a local origin and propagate as far as the length of the entire cortex, suggesting a role in the binding processes of far apart cortical regions. It would be interesting to investigate whether total or partial disruption of these large events, as well as alterations in the balance between activation and segregation into clusters are correlated with pathological conditions and with the level of awareness of the subject. Additionally, the non-linear relation between activated cortical tissue and number of clusters exhibits an optimal point, in which the level of brain activity is segregated into the maximum number of spatially isolated activations. We can hypothesize that this result is relevant to the solution of the integration/segregation dilemma long advocated by Tononi et al., 1994; Sporns, 2010) as the fundamental conundrum that the healthy cortex needs to be executing at any given time. If our hypothesis is true, we can predict, together with integration/segregation theories of consciousness (Tononi et al., 1994; Tononi, 2004), that a displacement of the optimal point should be observed for brain states of diminished conscious content such as deep sleep, anesthesia, or coma (Lee et al., 2009).

3.3. SYNCHRONY DOES NOT ALWAYS IMPLIES ORDER

A special place in the discussion should be dedicated to analyze the similarities and differences between the definition of the order parameter and phase transition used here and the concept of synchrony widely used in previous studies. To place this point in context, it is appropriate to recall the earlier studies a decade ago, by Varela and colleagues (Rodriguez et al., 1999) investigating the brain electrical activity of subjects viewing ambiguous visual stimuli (perceived either as faces or as meaningless shapes). They were able to show for the first time that "only face perception induces a long-distance pattern of synchronization, corresponding to the moment of perception itself and to the ensuing motor response. A period of strong desynchronization marks the transition between the moment of perception and the motor response" (Rodriguez et al., 1999). These results lead to the authors to suggest that "this desynchronization reflects a process of active uncoupling of the underlying neural ensembles that is necessary to proceed from one cognitive state to another" (Rodriguez et al., 1999). A number of papers followed Varela et al. idea (Rodriguez et al., 1999) that synchrony is physiologically relevant. At the fMRI level the timing and length of these epochs of synchrony were used recently to infer the presence of criticality (Kitzbichler et al., 2009). The present results indicate that while order (as defined here) implies always synchrony, the reverse is not always true, since space is not manifest in the definition of synchrony and then one can have a very synchronic but (spatially) disordered pattern of brain activity. Since our results indicate that the brain at rest spends most of its time in a mix of order and disorder, it would be very interesting in future studies to relate Varela's synchrony-asynchrony concept with the current ideas of proximity to a order-disorder transition.

3.4. *AD HOC* NOISE VS. NON-EQUILIBRIUM DYNAMICS

Attempts to construct biologically realistic equilibrium models of brain networks require as a main ingredient the introduction of (sometimes finely tuned) noise (Deco et al., 2009; Rolls and Deco, 2010). In this type of models, without the external noise the dynamics are stuck in a stable equilibrium state, thus noise must be introduced *ad hoc* to allow sufficient variability in the dynamical behavior of the system. One should be very careful, however, not to over-emphasize the biological relevance of a construct needed to overcome the shortcomings of a restricted class of models. Statistical physics results tell us that dynamical fluctuations around stationary states are small except near critical points (Prigogine, 1962).On the contrary, a non-equilibrium system undergoing criticality does not need the introduction of noise: variability is self-generated by the collective dynamics which spontaneously fluctuate near the critical point (for further discussion, see (Tagliazucchi and Chialvo, 2011)). Coincidently, the present results show that the spatiotemporal organization of the resting brain dynamics achieves maximum variability (i.e., **Figures 3C,E**) at a particular level of activation, and the analysis of the order and control parameters reveals that the origin of such variability can, in fact, be traced to a phase transition. Furthermore, the level of activity spends the largest amount of time around such transition. Then, these results point out that a different class of models is needed: one that emphasizes non-equilibrium self-generated variability over *ad hoc* introduced noise of uncertain origin.

3.5. RELATION WITH OTHER SCALES

The present results gathered in a large-scale domain can be also analyzed at the light of earlier observations of transient states at faster time scales, in which the scalp EEG is reduced to a certain number of stereotypical topographical maps (i.e., EEG microstates) (Koenig et al., 2002) and non-stationarities which define discrete segments of electrical activity are observed (Kaplan et al., 2005). Both descriptions of electrical scalp activity have also been shown to exhibit properties consistent with critical dynamics (Allegrini et al., 2010; Van De Ville et al., 2010). Further multimodal imaging studies could link these observations together in the context of discrete avalanches of neural activity propagating through the cortex and determine their functional relevance for health and disease (Greicius et al., 2004). Also, future work on the analysis of large-scale spontaneous fMRI signals as a train of activations should take advantage of the fact that the temporal information is not completely discarded (as in a straightforward correlation analysis) but kept in the timing of the events. The point process extracted from the BOLD signal can thus provide valuable information on the transient co-activations (or co-participation in an avalanche) of different brain regions. This measure can then be of value if correlated with the aforementioned EEG measures of instantaneous synchronization, as well with spontaneous index of perception or task performance.

3.6. LIMITATIONS OF THE APPROACH

The most obvious limitations of the point process approach stem from the spatiotemporal resolution of the fMRI recordings (i.e., TR and voxels dimensions) as well as the time constant of the BOLD HRF. Because of these limitations it is in principle impossible to distinguish two points in the process which are spaced by less than a characteristic time, as well as to detect very small clusters or avalanches whose size is smaller than the voxel dimensions. Therefore, it is impossible to guarantee that all points and clusters are included in the statistical analysis. However, since those which may be left out have (by definition) the smallest contributions, results are unlikely to be affected by this limitation. Another possible drawback of the method is that, while it yields more information than other methods such as linear correlation, there is a free parameter (threshold) to select. Nevertheless, in the Materials and Methods section we show that results are robust against different threshold choices.

3.7. SUMMING UP

Overall, the results show that the location and timing of the largest BOLD fluctuations define a spatial point process containing substantial information of the underlying brain dynamics. Despite the very large data reduction (>94%), the approach was validated by the favorable comparison of the conditional rate maps of avalanching activity with those constructed with the full fMRI BOLD signals using PICA as well by comparison with two distinct pathophysiological conditions (resting state in CBP patients and a finger tapping task). In addition to uncover new dynamical properties for the activated clusters, the method exposed scale-invariant features conjectured in the past (Chialvo, 2010) which are identical to those seen at smaller scales (Beggs and Plenz, 2003; Petermann et al., 2009; Chialvo, 2010). For the first time, the order and control parameters have been derived from human fMRI data allowing the identification of a phase transition and the demonstration that the resting brain spends most of the time near criticality. Beyond its potential value for fMRI signal processing, the ability of the present approach to capture relevant spatiotemporal brain dynamics underscoring non-linear aspects of the BOLD signal deserves further exploration.

4. MATERIALS AND METHODS

4.1. fMRI DATA ACQUISITION AND PREPROCESSING

Data was obtained, after informed consent, from ten right-handed healthy volunteers (9 female, 1 male; mean age = 49, SD = 12), during 10 min, requested to keep their eyes closed and to avoid falling asleep. The study was approved by the Clinical Research Ethics Committee of the University of the Balearic Islands (Palma de Mallorca, Spain). fMRI data acquisition was performed with a GE Medical Systems Signa HDx 3 T scanner using echo-planar sequences, 240 volumes were acquired with a TR of 2500 ms, TE of 35 ms, and 90° flip angle. Thirty-six slices of 64 × 64 dimensions were obtained with a field of view of 200 mm and slice thickness of 3 mm. Structural images consisted of a T1-weighted scans of 176 × 512 × 512 voxels, with a TR of 7176 ms, TE of 3150 ms, flip angle of 12°, FOV 240 mm and slice thickness of 1 mm. Preprocessing of BOLD signal was performed using FMRIB Expert Analysis Tool (Jezzard et al., 2001)[2], including motion correction using MCFLIRT, slice-timing correction using Fourier-space time series phase-shifting, non-brain removal using BET and spatial smoothing using a Gaussian kernel of full-width-half-maximum 5 mm. Brain images were normalized to standard space with FLIRT using the MNI 152 template and resampled to 4 mm × 4 mm × 4 mm resolution. Resting functional data was filtered with a zero lag finite impulse response band pass filter (0.01–0.1 Hz; Cordes et al., 2000, 2001). fMRI data used for the **Figures 2D,E**, as well as the preprocessing steps, were the same than in (Tagliazucchi et al., 2010a) and (Tagliazucchi et al., 2010b). Melodic was used for the PICA calculation of RSN (Beckmann and Smith, 2004) in **Figure 2** as well as for denoising motion artifacts.

4.2. DEFINITION OF THE POINT PROCESS

The point process is defined by the sequences of time points at which the BOLD signal crosses a given threshold from below. Formally, the problem is defined in an autonomous system as

$$\dot{\vec{x}} = f(\vec{x}), \tag{1}$$

where the dot denotes time derivative and $\vec{x} \in \mathbf{R}^d$. Let $y = h(\vec{x})$ be a scalar observable function (such as the BOLD signal, for instance) and consider the plot of y versus t. The times at which $y(t)$ upward (or downward) crosses some predetermined threshold $y = y_c$ determine a sequences of time points $\{t_k\}_{k=1,N}$ which defines the so-called point process (see **Figure 5**).

The Poincaré section of any given dynamical system reduces a d-dimensional continuous time description into an associated (d-1)-dimensional discrete map by finding the intersections of trajectories in phase space with a surface S transverse to the flow. If $\vec{x}_k \in S$ denotes the k^{th} intersection of the trajectories with S, a Poincaré map is defined as

$$\vec{x}_{k+1} = P(\vec{x}_k). \tag{2}$$

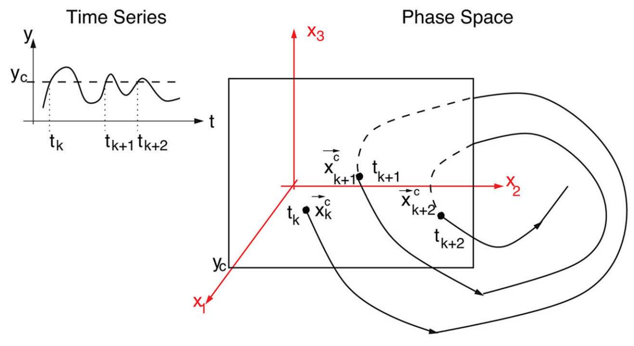

FIGURE 5 | Example of a Poincaré section defined by successive intersections of the trajectories in phase space with the plane denoted as $y = y_c$. The trajectory in phase space intersects the Poincaré section in space coordinates $\vec{x}^c_k(t_k)$, which then can be used as a map of the underlying dynamical process. Alternatively, a map can be defined by the sequence of crossing times $\{t_k\}_{k=1,N}$ if the conditions mentioned in the text are fulfilled.

A sequence of crossing times may be taken as a Poincaré section as representative as the typical phase space coordinates, when certain conditions are satisfied: Let $\vec{\gamma}(\vec{x}_0, t)$ a solution of Eq.(1) in an open interval $I =]t_0, T[$ and let $|\dot{\vec{\gamma}}(\vec{x}_0, t)| \neq 0$ for all $t \in I$. In terms of the underlying dynamical system, this means that the dynamics is not in a fixed-point or equilibrium of the system (as one can assume for the BOLD signal and neural dynamics in general). Under this condition, the arc-length, defined as $s(\vec{x}_0, t) = \int_{t_0}^{t} |\dot{\vec{\gamma}}(\vec{x}_0, t')| dt'$, is a suitable observable in the sense of embedding theory (Hegger and Kantz, 1997) and it is possible to reconstruct the attractor of the system by measuring line segments $\tilde{s}(t_{i+1}, t_i) = s(\vec{x}_0, t_{i+1}) - s(\vec{x}_0, t_i)$.

The derivative of s with respect to t, $\frac{ds}{dt} = |\dot{\vec{\gamma}}(\vec{x}_0, t)| = |\vec{f}(\vec{x})|$,, allows us to rewrite equation 1 as a set of differential equations in $s : d\vec{x}/ds = \vec{f}/|\vec{f}|$ and this gives the possibility to use the time t as an usual variable, which is no longer the independent parameter but an usual coordinate as the variable $\vec{x}$. Thus, the embedding theorems also apply to time or properly defined time sequences and it is possible to reconstruct the attractor (i.e., the full properties of the underlying dynamical system) from this sequence of times (i.e., the point process), as discussed in Hegger and Kantz (1997).

4.3. DECONVOLUTION PROCESS

The fMRI BOLD signal was de-convoluted using the `deconvlucy.m` function from Matlab[3]. For the Hemodynamic Response Function (HRF) standard parameters were those provided in the SPM8 package[4]. The deconvolution function follows the Lucy-Richardson algorithm (Richardson, 1972; Lucy, 1974) which converges to the maximum likelihood estimate of the de-convoluted process assuming a Poissonian source of noise. The results of the deconvolution, as in the example presented in **Figure 1C**, are impulse-like signals, whose underlying neural

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[2] http://www.fmrib.ox.ac.uk/fsl

[3] http://www.mathworks.com

[4] http://www.fil.ion.ucl.ac.uk/spm/

mechanisms are beyond the scope of the present work. Note that the deconvolution of all voxel's time series is in principle possible, however its numerical implementation is several orders of magnitude less efficient than the simple thresholding used here.

4.4. CONDITIONAL RATE MAPS

To construct the conditional rates reported in **Figure 2** the point process is defined at a seed location and at the targets throughout the entire brain. **Figure 6** illustrates the basic procedure. The BOLD signal is extracted from a seed region (top trace) and the points (arrows and vertical dashed lines) are defined by the crossings at 1 SD (horizontal dashed lines). Every time the signal at a target region crosses the threshold (asterisks) up to 2 time steps later than in the seed, the rate at the target is increased in one unit. This rate is normalized by the number of points in the seed. The top panel shows the location of the seed and of the two example targets, as well as the resulting average conditional rates maps (left) and DMN obtained from PICA (right). Medium panels show the BOLD signal at the seed and at the two example target regions. A similar procedure was used in (Tagliazucchi et al., 2010b) where the resting BOLD event triggered averages (rBeta) were calculated at similar seed and target regions. **Table 1** contains the seed coordinates used to reproduce the RSNs.

4.5. CLUSTERS AND AVALANCHES

Spatial clusters of activated voxels were identified using an algorithm implemented in MATLAB, based on the detection of connected components in a co-activated first neighbors graph. Clusters' fractal dimension was calculated using a standard box-counting algorithm. Avalanches were defined (similar as in sandpile models, and others (Bak, 1996; Jensen, 1998)) as starting with the isolated activation (i.e., not by any of its neighbors) of a previously inactive voxel (or group of voxels), continuing while at least one contiguous voxel is active in the next time step and otherwise ends. The avalanche tracking algorithm implemented in this work

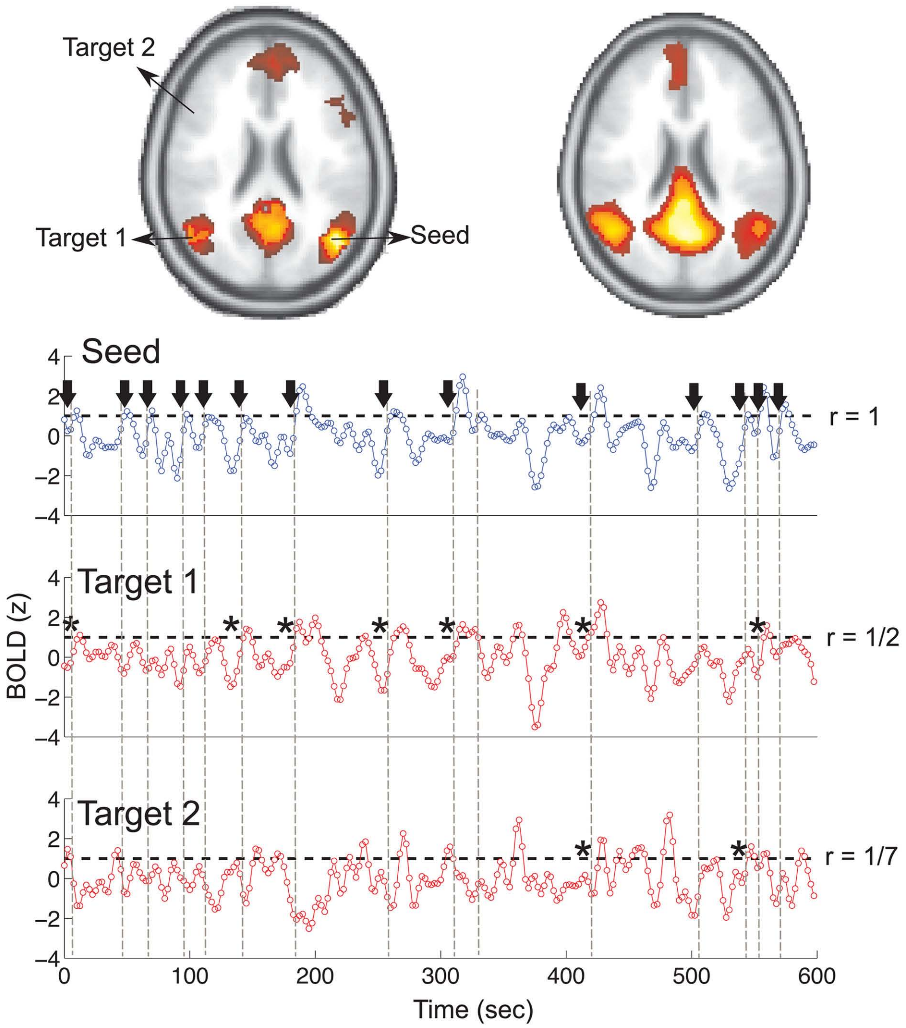

FIGURE 6 | Illustration of the basic procedure to calculate the conditional rate maps presented in Figure 2. The r values on the right side of the traces correspond to the conditional rates between the 14 events at the seed and those at the two targets (1/2 and 1/7 in this example).

uses as a criteria for avalanche membership the non-empty intersection with a previously identified cluster of the avalanche at a previous time. This is able to resolve shrinking and expanding of clusters, translation, and division, whenever there is spatial overlap at subsequent times.

4.6. CLUSTER DETECTION ALGORITHM

To detect contiguous clusters of activated voxels (defined as those crossing the threshold), for each time step, the problem was reduced to the detection of connected components in a suitably defined graph or network. More precisely, for each volume, a graph was constructed having each voxel as a node, and two nodes connected with a link if they were both activated (BOLD signal above 1 SD) and also first neighbors in the spatial sense. The connected components of this graph correspond to clusters of contiguous activated voxels isolated from other similarly defined clusters.

4.7. AVALANCHE DETECTION ALGORITHM

In simple terms an avalanche starts with the activation of a previously inactive voxel, follows while in the next time step at least one contiguous voxels is active and otherwise ends. The avalanche detection algorithm is based on the connected cluster decomposition. Clusters are followed during different volumes, belonging to the same avalanche if they have spatial intersection during consecutive times. Formally, the algorithm is as follows: Let C_i^t be the i-th cluster at time *t*. We consider a cluster i_0 starting an avalanche at time t_0 if for all *j*, $C_j^{t_0-1} \cap C_{i_0}^{t_0} = \emptyset$(i.e., no clusters were present in that region of the brain at the previous time step). An *id* is assigned to this avalanche and the same *id* is assigned to all clusters intersecting this cluster at the following time, this is all clusters *i* such that $C_{i_0}^{t_0} \cap C_i^{t_0+1} \neq \emptyset$. The same procedure is applied recursively to all clusters satisfying the former condition until no more intersections are found. When this happens, all clusters labeled with this *id* constitute the avalanche.

ROBUSTNESS AGAINST THRESHOLD CHANGES

In this work, the only free parameter used in the definition of the point process is the threshold. In this sense, it is important to know how the main spatiotemporal statistical properties of the point process dynamics, namely cluster size distributions, avalanche size and duration distributions depend on threshold values. **Figure 7**

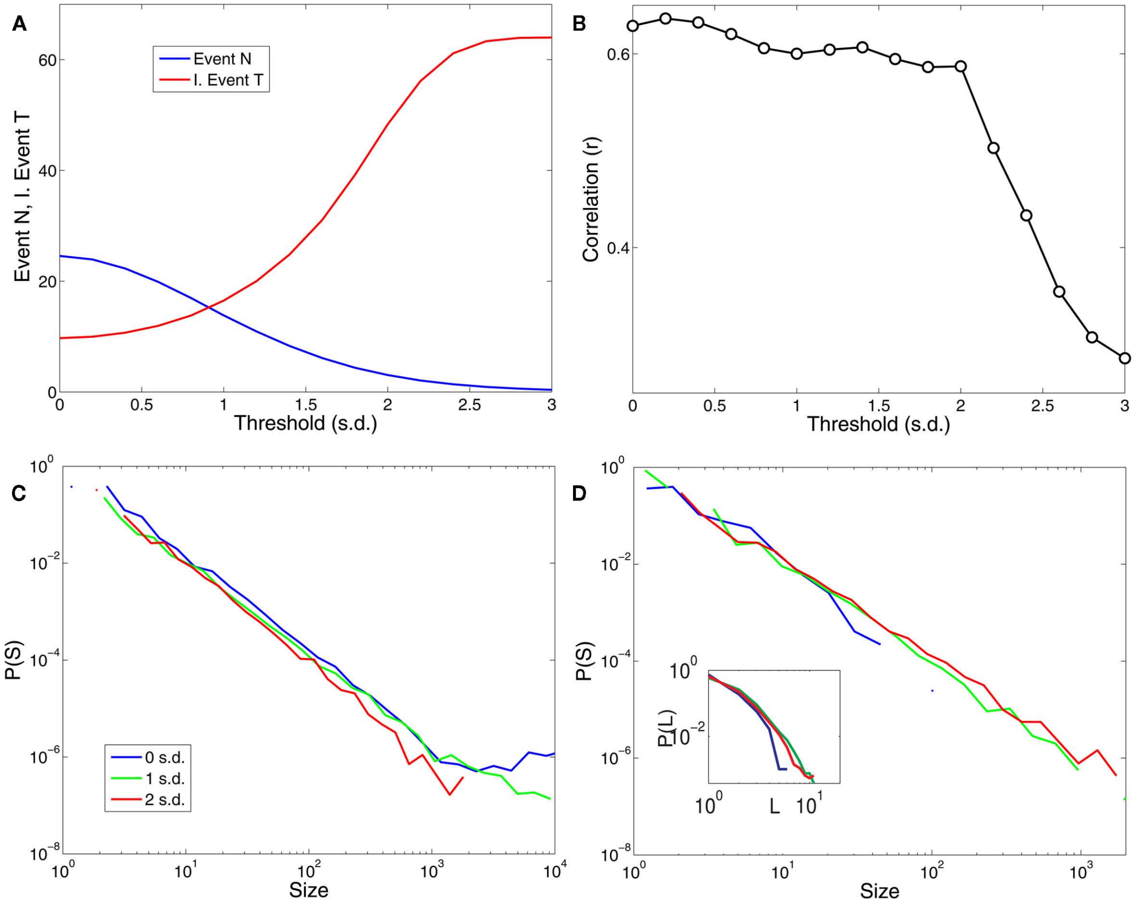

FIGURE 7 | The results are robust to changes in the threshold over a reasonable range. (A) Shows the dependence of the number of points and the average inter-event time (expressed in units of samples or scanning volumes) for a range of threshold values (in units of SD). **(B)** Illustrates the dependence of the correlations plotted in **Figure 2C** (correlations with PICA DMN) with respect to the threshold values. **(C)** Shows the distribution of cluster sizes and **(D)** the avalanche sizes and avalanche durations (inset) for three different thresholds values (0, 1, and 2 SD).

shows that these results are robust against changes in threshold over a wide range of choices.

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From self-organized to extended criticality

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We address the issue of criticality that is attracting the attention of an increasing number of neurophysiologists. Our main purpose is to establish the specific nature of some dynamical processes that although physically different, are usually termed as "critical," and we focus on those characterized by the cooperative interaction of many units. We notice that the term "criticality" has been adopted to denote both noise-induced phase transitions and Self-Organized Criticality (SOC) with no clear connection with the traditional phase transitions, namely the transformation of a thermodynamic system from one state of matter to another. We notice the recent attractive proposal of extended criticality advocated by Bailly and Longo, which is realized through a wide set of critical points rather than emerging as a singularity from a unique value of the control parameter. We study a set of cooperatively firing neurons and we show that for an extended set of interaction couplings the system exhibits a form of temporal complexity similar to that emerging at criticality from ordinary phase transitions. This extended criticality regime is characterized by three main properties: (i) In the ideal limiting case of infinitely large time period, temporal complexity corresponds to Mittag-Leffler complexity; (ii) For large values of the interaction coupling the periodic nature of the process becomes predominant while maintaining to some extent, in the intermediate time asymptotic region, the signature of complexity; (iii) Focusing our attention on firing neuron avalanches, we find two of the popular SOC properties, namely the power indexes 2 and 1.5 respectively for time length and for the intensity of the avalanches. We derive the main conclusion that SOC emerges from extended criticality, thereby explaining the experimental observation of Plenz and Beggs: avalanches occur in time with surprisingly regularity, in apparent conflict with the temporal complexity of physical critical points.

Keywords: SOC, extended criticality, neural networks, cooperation

1. INTRODUCTION

Bridging psychology and neurophysiology is a challenging issue of the twenty-first century whose origin traces back to the middle nineteenth century. In fact, as pointed out by Kinouchi and Copelli (2006) the work of Weber and Fechner in the middle of the nineteenth century, on how physical stimuli turn into psychological sensation is a fundamental mind-brain problem, which has influenced the foundation of psychology by James (Hawkins, 2011). Kinouchi and Copelli (2006) show that synchronization and global oscillations allow the coding of information spanning several orders of magnitude and claim that this mechanism could provide a microscopic neural basis for psychophysical laws. The basic idea behind the work of Kinouchi and Copelli (2006) is that in biological complex systems the optimal information processing is found near phase transitions, and that the efficiency of biologically relevant processes is optimized at criticality. Criticality of phase transition is one of the most important achievements of the twentieth century physics, thereby implying a transition from the middle nineteenth century to the second half of the twentieth century.

Chialvo (2010) points out that, although understanding the brain is still certainly a challenge (Chialvo, 2010), there are spatiotemporal brain data with which to confront theories, provided that criticality is taken into due account. Criticality is a concept that is attracting a fastly increasing number of researchers in the field of Complexity, with applications ranging from neurophysiology (Levina et al., 2009) to stock-price fluctuations (Kiyono et al., 2006). The term criticality, however, remains not quite clearly defined. In addition to the classical second-order phase transition, where temperature is the control parameter, the term criticality has been used to denote processes as different as noise-induced phase transitions (Van den Broeck et al., 1994) and Self-Organized Criticality (SOC), whereas the identification of the occurrence of criticality through the observation of time series is considered to be a challenging task requiring special techniques (Varotsos et al., 2011) holding true for both the 2D Ising model and SOC.

More recently, it has been pointed out (Bailly and Longo, 2011) that moving from physical to biological systems may require an extension of the concept of criticality, namely, the adoption of

Extended Criticality (EC). These authors point out that in physics phase transitions are treated as "singular events," corresponding to a specific well-defined value of the control parameter. When this singular event occurs, the fluctuations are enhanced and their correlation length becomes as large as the system's size (Stanley, 1999). In biology, these "coherent critical structures" are "extended" and organized in such a way that they persist in space and time. Living matter organizes itself in a sequence of phase transitions, beyond the punctual edge of chaos that seems to be shared by all forms of phase transitions in physics, from noise-induced to self-organized criticality.

Where does the transition from *physical* to *extended* criticality occur? Is it possible that complex physical systems may show signs of EC? In the last few years the researchers of the Pisa lab of professor Leone Fronzoni (Fronzoni and Allegrini, 2009; Silvestri et al., 2009; Allegrini et al., 2010) have studied the weak turbulence on liquid crystals and have found that as an effect of the cooperative interaction between defects, annihilation, and creation of coherent structures take place through the occurrence of *crucial* events, namely, the distance between two consecutive events, denoted by the symbol τ, does not have memory of the earlier distances and has a distribution density $\psi(\tau)$ with the form $\psi(\tau) \propto 1/\tau^{\mu}$ and $1 < \mu < 3$. These authors defined the control parameter $\epsilon = V/V_0 - 1$, where V_0 is a threshold voltage at which convective instability emerges, and found that the phase transition to temporal complexity, rather than being restricted to a single value of ϵ is extended to a wide interval of values of ϵ.

In this article we study the statistical properties of a model for neuron dynamics recently proposed by Geneston and Grigolini (2011). We shall show that this model generates experimental avalanches of the same kind as those revealed by the experimental work of Beggs and Plenz (2003, 2004), interpreted by Chialvo (2010) as a SOC property, thereby suggesting that this model may produce SOC criticality.

The time distance between two consecutive avalanches is described by a waiting time distribution density $\psi(\tau)$ that has to be compared to the distribution density of time distances between two consecutive liquid crystal crucial events. We shall focus our attention on the function $\Psi(\tau)$ defined by

$$\Psi(\tau) \equiv \int_{\tau}^{\infty} ds \psi(s), \tag{1}$$

namely, the probability that no new avalanche occurs up to a time distance τ from an earlier avalanche.

We shall find properties similar to those experimentally observed through the liquid crystals of the lab of professor Fronzoni. The neurons of the model interact through a cooperation parameter K generating different values of a complexity parameter α, which is related to the temporal complexity of the liquid crystals through $\mu = 1 + \alpha$. The parameter α is frequently adopted to denote the Mittag-Leffler (ML) relaxation process. As pointed out by Metzler and Klafter (2002), the important property of the ML function is the establishment of a bridge between stretched exponential and inverse-power law relaxation. To point out the importance of ML complexity, let us notice that the temporal complexity of liquid crystals is described by a distribution density of inter-event times

$$\psi(\tau) = (\mu - 1)\frac{T^{\mu-1}}{(\tau + T)^{\mu}}, \tag{2}$$

and by the corresponding survival probability

$$\Psi(\tau) = \left(\frac{T}{\tau + T}\right)^{\mu-1}. \tag{3}$$

In equation 2 the parameter T is introduced for the main purpose of eliminating the unphysical short-time divergencies and ensuring the proper normalization condition, thereby making it possible for us to focus on the long-time regime. Thus, the short-time regime remains essentially unknown and T only plays the limited role of effective parameter. In other words, T defines the time scale of the microscopic regime where the collective properties generated by the interaction between the system's units do not appear yet. In the case of ML complexity, on the contrary, the short-time relaxation is described by the stretched exponential $\exp(-(\lambda t)^{\alpha})$, with $\alpha < 1$. The term short-time scale is not appropriate to describe the stretched exponential portion of the ML relaxation. In fact, the deviation from the ordinary exponential relaxation mirrors collective properties, closely related to the temporal complexity emerging at later times, which is the inverse-power law $1/\tau^{\alpha}$. The term of intermediate asymptotics coined by Barenblatt (1996) is more appropriate to denote the time scale of the stretched exponential relaxation. At the intermediate asymptotics level scaling already appears, in a form that is not yet affected by physical limitations such as boundary conditions. In the case of the model discussed in this article the emergence of the inverse-power law $1/\tau^{\alpha}$ is accompanied by the first signs of periodicity that for large values of the cooperation parameter K yields an abrupt truncation of the inverse-power law regime.

The ML complexity settles the controversy between the advocates of complexity as being characterized by relaxation departing from the usual exponential form through a stretched exponential structure and the advocates of inverse-power laws as the most significant signature of complexity. In fact, the long-time regime of the ML relaxation is described by a power law with index $\alpha = \mu - 1$ that makes the far asymptotic time regime of the ML relaxation identical to that of the survival probability of equation 3. Thus, even if statistical inaccuracy, time series finiteness, and other possible reasons may make it difficult to reveal the inverse-power law nature of the long-time regime, the temporal complexity of the ML relaxation is of the same nature as the temporal complexity of the form of equation 3.

These interesting properties generate the question of which is the form of criticality emerging from the neuron cooperation. We shall come back to this interesting problems in the last Section of this article.

2. MATERIALS AND METHODS

2.1. MITTAG-LEFFLER TEMPORAL COMPLEXITY

As mentioned in the Introduction, the temporal complexity emerging from the extended criticality of our model of interacting neurons is given by the Mittag-Leffler (ML) function. A convenient way to illustrate this form of temporal complexity rests on the use of Laplace representation. In this article we adopt for the Laplace transform of the function $f(\tau)$ the notation

$$\hat{f}(u) \equiv \int_0^\infty d\tau f(\tau) \exp(-u\tau). \tag{4}$$

Using this notation, if we assign to $\Psi(\tau)$ the ML form, we have

$$\hat{\Psi}(u) = \frac{1}{u + \lambda^\alpha u^{1-\alpha}}. \tag{5}$$

Via inverse Laplace transformation we get

$$\Psi(t) = E_\alpha\left(-(\lambda t)^\alpha\right), \tag{6}$$

where $E_\alpha(z)$ is the ML function expressed with respect to time. The series expansion of $E_\alpha(z)$ is

$$E_\alpha(z) = \sum_{n=0}^{\infty} \frac{(-z)^n}{\Gamma(1+\alpha n)}. \tag{7}$$

It is known that for $\alpha > 1$, this function can become oscillatory. We shall restrict our use of the form of temporal complexity to $0 < \alpha \leq 1$, where the ML function is positive and strictly monotonically decreasing.

It is important to stress, as already mentioned in the Introduction, that in the short-time regime $\tau < 1/\lambda$ the ML function has the form of the stretched exponential function

$$\Psi(\tau) = \exp\left(-(\lambda\tau)^\alpha\right) \tag{8}$$

It is important to reiterate an observation made in the earlier work by Geneston and Grigolini (2011). This has to do with the long-time truncation of the ML survival probability. Usually, the long-time truncation of inverse-power laws is thought to be a consequence of the statistical inaccuracy due to the finite sizes of the systems under study. For instance, Failla et al. (2004) and Bianco et al. (2007) adopted the form

$$\hat{\Psi}(u) = \frac{1}{u + \lambda^\alpha (u + \Gamma_t)^{1-\alpha}}, \tag{9}$$

with $\alpha < 1$. In the case $\Gamma_t = 0$ this is the Laplace transform of the ML function of equation 5. However, the experimental observation has forced these authors to introduce the parameter $\Gamma_t > 0$. The experimental results are explained by setting $1/\Gamma_t$ of the order of $1/\lambda$ and both much larger than the unit time step: in this case equation 5 generates a function $\Psi(t)$ indistinguishable from a stretched exponential function. In conclusion, in the earlier work the ML complexity is violated in the large time limit by environmental fluctuations that make the system fall in the exponential regime. In the case of this article, on the contrary, as already pointed out by Geneston and Grigolini (2011), the breakdown of the ML condition is due to the same cooperative effect as that making the ML function emerge in the first place.

In this article we observe quakes, namely events corresponding to one or more neurons firing. The physical meaning of the survival probability $\Psi(\tau)$ is defined by referring ourselves to the occurrence of a quake as time origin. Given that a quake occurs at the time origin, what is the probability that no new quake occurs up to time τ? The probability of no quake occurrence is the survival probability $\Psi(\tau)$. In the absence of cooperation

$$\Psi(\tau) = \exp(-G\tau). \tag{10}$$

In this case $\alpha = 1$ and $\lambda = G$. The quantity corresponding to the magnetization of the Ising model is

$$m = \lambda^\alpha - G. \tag{11}$$

At criticality $\lambda^\alpha = G$ begins departing from the vanishing value. In practice, we observe numerically the quantity

$$g(K) = \lambda(K)^{\alpha(K)}, \tag{12}$$

and we use it to establish the critical value of K at which phase-transition occurs.

2.2. STOCHASTIC VERSION OF THE MODEL OF MIROLLO AND STROGATZ

This is a model originally proposed by Mirollo and Strogatz (1990) that Geneston and Grigolini (2011) generalized by adding a stochastic fluctuation. Therefore we shall refer to it as Stochastic Mirollo and Strogatz (SMS) model.

The SMS model is based on the dynamics of N_{tot} neurons. In the absence of cooperation, the dynamics of each neuron are described by the simplest model of a leaky integration-and-fire neuron:

$$x(t+1) = (1-\gamma)\, x(t) + S + \sigma\xi(t), \tag{13}$$

where the natural number t denotes the discrete time, and the leakage factor γ is taken $\gamma \ll 1$ so as to make the integer time virtually continuous when $\gamma t \approx 1$. The variable x moves from the initial condition $x = 0$, therefore the term $S > 0$, favoring its motion toward threshold $x = 1$ describes the average neural activity of afferent connections (coming from outside the system of interest); $\xi(t)$ is a random variable getting either the value of 1 or -1, with equal probability, with no memory of the earlier values, and the quantity σ is the noise intensity. The role of the term $\sigma\xi$ is that of describing the fluctuations of the afferent neural activity. At the threshold, the neuron fires and resets back to the initial value $x = 0$.

When $\sigma = 0$, it is straightforward to prove that the time necessary for the neuron to reach the threshold is given by

$$T_{MS} = \frac{1}{\gamma} \ln\left(\frac{1}{1 - \frac{\gamma}{S}}\right). \tag{14}$$

We assume, with Mirollo and Strogatz (1990), that the neurons cooperate, namely, that when one neuron fires all the other neurons make a step ahead by the quantity $K < 1$ which is the cooperation strength. This condition is equivalent to assume that each neuron interacts with all the other neurons, and it will be referred to as All-To-All coupling condition. This model is very close to the one used by Levina et al. (2010). These authors consider both inhibition and excitation links between neurons. The model of this paper can be derived from that of Levina et al. (2010) by eliminating the inhibition links. Notice that All-To-All condition (or almost All-To-All) and the absence of inhibitory control are here considered as a simplifying approximation. However, they may also be of importance in describing specific pathological conditions, like the Morvan's syndrome (Liguori et al., 2001).

Mirollo and Strogatz (1990) found the exact solution of this problem when $\sigma = 0$ and proved that, after a transient, all the neurons fire at the same time, with time period T_{MS} of equation 14, the MS time.

As an effect of noise, it may take a time either shorter or longer than T_{MS} for a neuron to fire. Furthermore, the time distance between two consecutive firings may be much shorter than T_{MS}. When $K = 0$, even if we assign to all the neurons the same initial condition, $x = 0$, due to stochastic fluctuations the neurons fire at different times so that the system eventually reaches a steady state with a constant firing rate G given by

$$G = \frac{N_{tot}}{\langle \tau \rangle}, \tag{15}$$

where $\langle \tau \rangle$ denotes the mean time between two consecutive firings of the single neuron. For $\sigma \ll 1$, $\langle \tau \rangle \approx T_{MS}$. From the condition of constant rate G we immediately derive the Poisson survival probability

$$\Psi(\tau) = \exp(-G\tau). \tag{16}$$

We shall see hereby that this heuristic argument agrees very well with numerical results. For the numerical simulation we select the condition

$$G \ll 1 \ll N_{tot} \ll T_{MS}, \tag{17}$$

yielding

$$\frac{1}{G} \approx \frac{T_{MS}}{N_{tot}} \ll T_{MS}, \tag{18}$$

thereby realizing the time scale of interest for this paper, which is much shorter than the MS time. It is evident that the condition of non-interacting neurons fits equation 5 with $\alpha = 1$ and

$$\lambda(K = 0) = G. \tag{19}$$

For our numerical calculations we adopt the values $\sigma = 0.001$, $N_{tot} = 100$, $\gamma = 0.0001$, and $S = 0.00019$. We have assessed numerically that $\langle \tau \rangle = 7431$ thereby yielding, thanks to equation 15, $G = 0.0135$, and $T_{MS} = 7472.14$. Throughout the paper the time units are expressed in terms of the number of the discrete time steps of the numerical model. The connection with physical times can in principle be established via the connection of T_{MS} with the periodic rhythm under study. However, the scope of the paper is to explore at a theoretical level a possible interplay between criticality and periodicity in cooperative systems. We therefore believe it is premature to adopt at this stage the usage of physical units (e.g., seconds).

Finally, it is important to point out that for larger values of N_{tot} the same properties are observed as long as the condition (17) is applied.

3. RESULTS

We begin the illustration of the numerical results with **Figure 1**, which shows the number of firing per unit of time moving from an initial condition where all the neurons are assigned a random value x in the interval (0, 1).

We see that for small values of K the number of firings is almost constant in time. Increasing K has the effect of generating a more complex time structure, which in the limiting case of very large values of K is expected to fit the periodicity of the original model of Mirollo and Strogatz.

We are in the presence of a situation more complex than that corresponding to the ordinary renewal condition (Turalska et al., 2011). When the ordinary criticality condition applies, namely the phase-transition generated by a single value of the control parameter is realized, organizational collapses occur (Vanni et al., 2011), and they are proved to be renewal events (Turalska et al., 2011). In the case of the SMS model, as already suggested by **Figure 1**, the emergence of criticality is accompanied by signs of periodicity that become predominant at large values of the control parameter K.

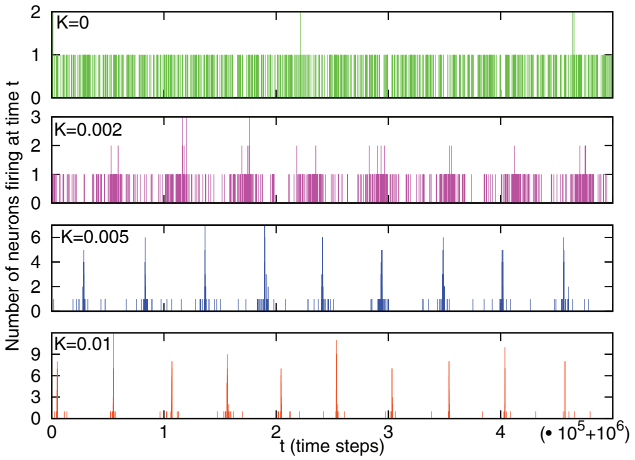

FIGURE 1 | This figure shows the number of firings moving from initial conditions x_i taken randomly in the interval (0,1). A very large time (one million time steps) is waited so that memory of initial conditions is completely lost. Different panels refer to different values of the cooperation parameter K. We observe a change in behavior from no coupling ($K = 0$, upper panel) to larger values of K. For $K = 0$, for each time, only one or, rarely, two neurons are firing, and the firing times are homogeneous, following a Poisson statistics. For $K = 0.002$ (second panel) we see some sporadic events with three neurons firing, tending to take place in periodical patterns; non-Poisson "holes" of silence, with large duration variability, emerge in between. For $K = 0.005$ (third panel) we see events with higher numbers of neurons concurrently firing, and longer silence durations. For $K = 0.01$ (lower panel) the system is almost periodic, and silence durations can be as long as the distance between major peaks.

Let us now move to discuss the statistical properties of firing avalanches. First of all we study the time distance between two consecutive avalanches. We are interested in the time asymptotic properties of the survival probability $\Psi(\tau)$. For this reason we do not have to define the minimal distance between two consecutive avalanches, as we shall have to do to study the avalanche time duration. We limit ourselves to record the time distance between two consecutive firings, regardless of the number of neurons contributing to them. This allows us to define $\psi(\tau)$. Then, using equation 1 we determine the survival probability $\Psi(\tau)$, illustrated in **Figure 2**. **Figure 2** is helpful to evaluate with theoretical arguments the surprising mixture of time complexity and time periodicity that is suggested by **Figure 1**. We see, in fact, that increasing K has the effect of turning the Poisson statistics of the condition with no cooperation (see the insert) into a form of exact periodicity, as shown by the fifth curve (the upper one, denoted by $K = 0.1$) of this Figure, where the abrupt jump to 0 of the survival probability corresponds to all the neurons firing together with the time period of equation 14.

Figure 3 illustrates the fitting procedure adopted to prove the emergence of the cooperation-induced ML complexity. At variance with the earlier analysis of Geneston and Grigolini (2011), we fitted directly the numerical Laplace transforms of the experimental survival probabilities with the Mittag-Leffler function in the Laplace representation, given by equations 6 and 7. For this purpose we selected a suitable interval for u. From the parameters α and λ obtained via the fitting procedure, the stretched exponential (8) has been visually superimposed to the survival probability data in the time representation, in the early regime, and a slope $\alpha + 1$ has been superimposed to an intermediate regime, before the cutoff, as shown in the inset of **Figure 3**. As a result, the quality of fit has been excellent for all values of coupling parameter K, with very slight deviations only at the critical value $K = 0.002$.

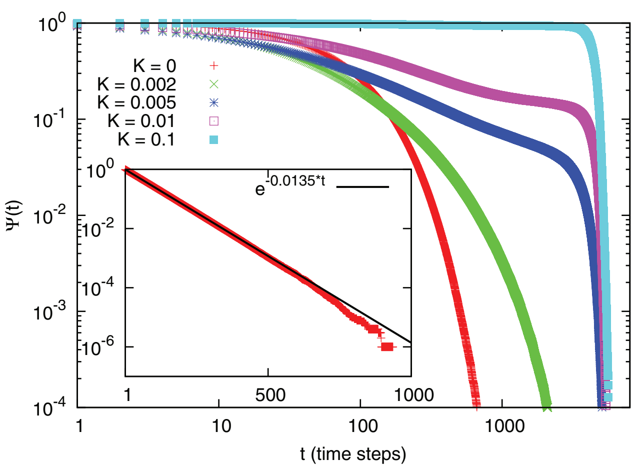

FIGURE 2 | Survival probability for different values of *K*. As we move from $K = 0$ (lower curve) to higher values of K we see the passage from Poisson behavior (shown in the insert as a straight line in semi-log scale) to a step-like function at the highest vale of $K (K = 0.1)$ here reported, signaling complete periodicity. For intermediate values complex non-Poisson decays are visible, followed by abrupt cutoffs, due to periodicity.

It is possible to see the emergence of a clear trend, illustrated in **Figure 4**. The temporal complexity parameter α at $K = 0$ has the value $\alpha = 1$, corresponding to $\mu = 2$, which is, in fact, the boundary with the ergodic regime. The inverse-power law component drops to zero, and the stretched exponential function becomes an ordinary exponential function. As soon as we switch on the cooperation parameter, temporal complexity emerges with $\alpha < 1$ and $\mu < 2$. While for $K < 0.002$ α slowly decreases with increasing K, at $K \approx 0.002$ it undergoes a fast drop and remains close to $\alpha \approx 0.6$ for an extended range. The parameter λ shows a steep increase at $K = 0.002$, reaches a maximum around $k = 0.00275$ and decreases for larger values of K. The complexity indicator λ^α increases slowly from the value of G at $K = 0$ to larger values at about $K = 0.002$ where it undergoes a faster increase that Geneston and Grigolini (2011) interpreted as a form of ordinary transition rounded by finite size effects. Actually this parameter does not saturate for increasing values of K, since for values larger than $K \approx 0.0035$ it tends to slowly decrease. On the basis of the fitting procedure of this article we are inclined to interpret all this as a form of extended criticality rather than an ordinary criticality corresponding to the single value $K \approx 0.002$.

Finally, **Figures 5–7** refer to the neural avalanches that have been the focus of the research work of Beggs and Plenz (2004). Chialvo (2010) considers the avalanches experimentally revealed by Beggs and Plenz to be a striking manifestation of SOC (Zapperi et al., 1995; De Arcangelis et al., 2006). **Figure 5** shows the intensity of avalanches as a function of the cooperation parameter K. Some caution has to be exerted in defining the avalanche: Due to finite statistics and discrete time, it happens that actual avalanches

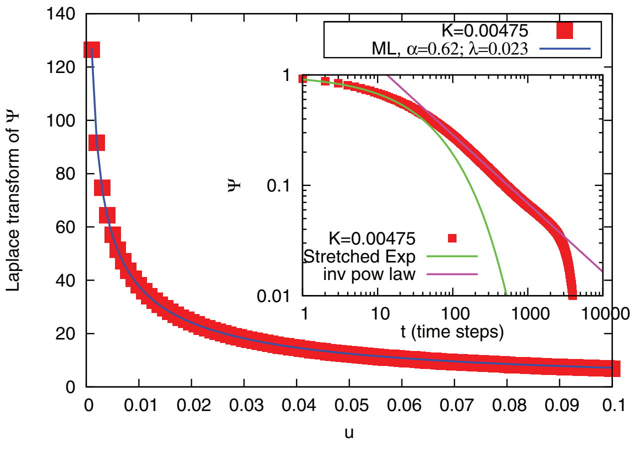

FIGURE 3 | Illustration of the fitting procedure adopted to prove the cooperation-induced emergence of the ML complexity. In the main figure the numerical Laplace transform of the Survival probability is plotted, for $K = 0.00475$ (squares). The solid line represent the fitted ML function, in the Laplace representation. The resulting fitting parameters are in this example $\alpha = 0.62$ and $\lambda = 0.023$. In the insert we plot the same survival probability, but this time in the time domain. A stretched exponential $\exp[-(\lambda t)^\alpha]$ is superimposed in the shorter time regime, called *intermediate asymptotic regime*; an inverse-power law *constant*/t^α is superimposed in the far asymptotic regime. We notice the good agreement in both the Laplace and the time representation.

may present "holes," namely times within the avalanche where no neuron is firing. This makes it difficult to operatively define an avalanche starting from the data. As in the original work of Beggs and Plenz (2004) we defined a tolerance time Δt: Avalanches are thus defined as patterns of neuron firing separated from one another of at least Δt time steps. In other words, the starting of an avalanche is the first time step following a silence period lasting at least Δt time steps. Results not herein reported show that the analysis of avalanche size, duration, and mean shape depend on the choice of Δt. We choose $\Delta t = 5$ time steps, because we saw that numerically the inverse-power law indexes of the probability densities for avalanche size and duration would not change with further increasing that value, for all values of K herein reported.

We see that in the case of no coupling the avalanche's intensity falls down exponentially upon increasing their intensity. With increasing the cooperation parameter K there is an accumulation of curves around an inverse-power law with index $\zeta = 1.5$ as K overcomes the critical value. With further increase of K the inverse-power law does not change, but a peak becomes predominant at large times. This is a manifestation of the tendency for the system to recover the periodicity of the model of Mirollo and Strogatz.

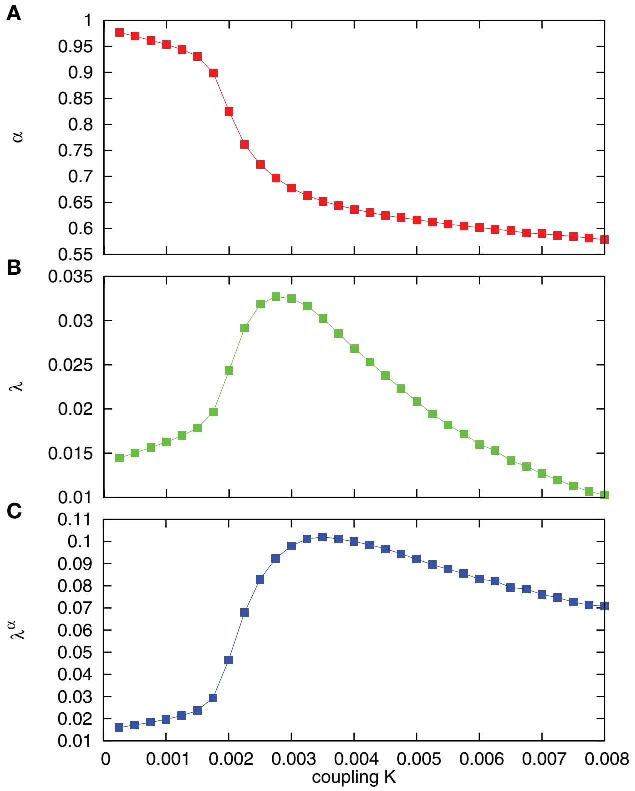

FIGURE 4 | (A) Complexity fractal index α as a function of the cooperation parameter K. Notice the change in the steepness of descent near $K = 0.002$. **(B)** Temporal parameter λ reaches a maximum for $K = 0.00275$. Notice again a change in the steepness near $K = 0.002$. **(C)** The order parameter λ^{α}, as a function K, shows a behavior similar to second-order phase transition, but with rounded increase at $K = 0.002$ and slow decay after a maximum.

The analysis of the distribution of time duration of the avalanches generates, as shown in **Figure 6**, results similar to those concerning the intensity distribution, thereby suggesting a correspondence between intensity and duration.

It is interesting to notice that the theoretical value of $\zeta = 1.5$ for avalanche sizes and $\nu = 2$ for avalanche durations experimentally observed by Beggs and Plenz (2003) are realized in our model, even in a region that is strongly influenced by periodicity. We remind that although these SOC critical signatures are conserved in this broad region, the fractal statistics of inter-avalanche times, corresponding to **Figure 4**, change. In particular, the fractal

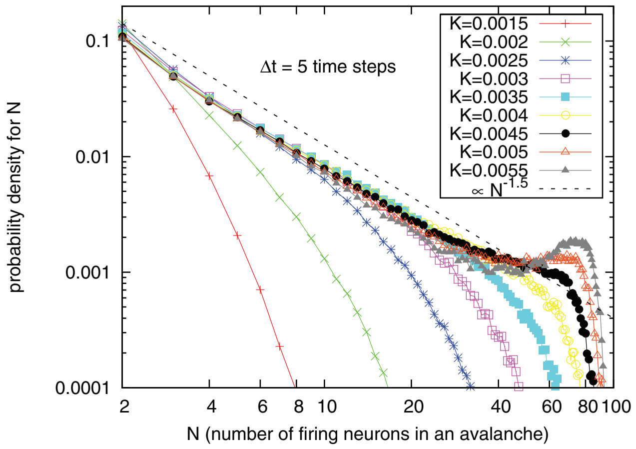

FIGURE 5 | Probability density for *N*, the number of neurons within an avalanche, as a function of *N*, for different values of cooperation parameter *K*. The dashed line is a guide to the eye, signaling an inverse-power law decay $\propto N^{-1.5}$. For $K > 0.002$ there is an accumulation of curves on a straight line parallel to the eye guide.

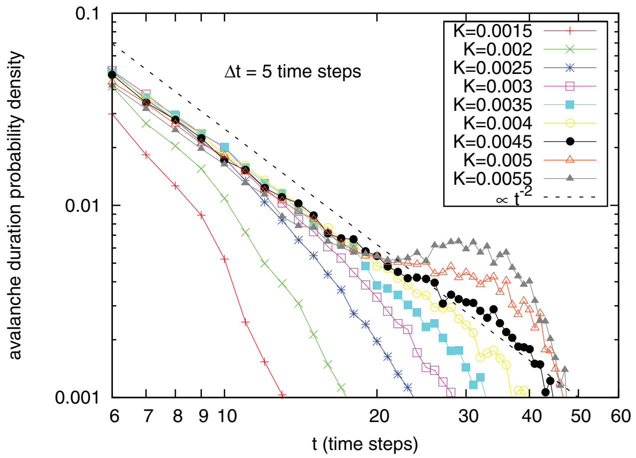

FIGURE 6 | Probability density for avalanche durations *t*, namely the time difference between an avalanche starting and a silence period of at least $\Delta t = 5$ time steps, for different values of cooperation parameter *K*. The dashed line is a guide to the eye, signaling an inverse-power law decay $\propto t^{-2}$. Again, for $K > 0.002$ there is an accumulation of curves on a straight line parallel to the eye guide.

index α continuously varies from the value $\alpha = 1$, for independent neurons, to smaller values, of the order of 1/2 for large choices of cooperation parameter *K*. All this thereby suggests a significant departure from the condition of temporal complexity corresponding to the conventional criticality (Turalska et al., 2011), corresponding in turn to a single value of the control parameter.

We have also adopted the procedure of Levina et al. (2007) to determine the avalanche's shape. This is described by **Figure 7**. We have considered the times at which avalanches begin as the time origin. Then, for any time larger than Δt we have evaluated the mean number of firing neurons, as an average over all the avalanches. We have recovered an interesting effect found by Levina et al. (2007). In the correspondence of a cooperation parameter so large as to make periodicity predominant, $K = 0.007$, we see that the decay of the avalanche's shape is not monotonic, and that the avalanche's intensity increases, reaches a maximum, and it decays again. This is a consequence of the avalanche's periodicity (Beggs and Plenz, 2004). Adopting a Gibbs' ensemble representation, we can state that in this regime the events signaling that an avalanche begins can be interpreted as precursors of big avalanches occurring later.

4. DISCUSSION

4.1. CRITICALITY OF NEURAL NETWORKS

The SMS model generates neural avalanches: for many values of *K* it yields the value of $\zeta = 1.5$ for avalanche size and $\nu = 2$ for avalanche durations, in accordance with the experimental observations by Beggs and Plenz (2003). According to Chialvo (2010) these are typical SOC signatures, thereby generating the impression that the SMS model may be a form of SOC. However, Aschwanden (2011) has recently pointed out that there are "three necessary and (perhaps) sufficient criteria" to establish if a system is a SOC system. These are: Statistical Independence, Non-linear Coherent Growth, and Random Duration of Rise Time. We think that none of these properties is obeyed by the SMS model of this paper. Therefore we are inclined to believe that its criticality is not a SOC. This makes us conclude that the SOC theoretical framework is not the only one explaining the main properties of neural avalanches.

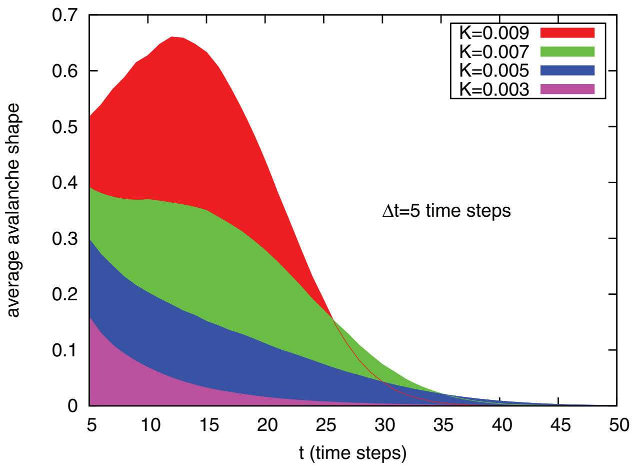

FIGURE 7 | Avalanche mean shape, i.e. average number of firing neurons after *t* time steps form the avalanche starting, for different values of *K*.

The next question to answer is as to whether the cooperation-induced temporal complexity is of the same kind as that produced by the phase-transition processes. Interesting examples of this form of criticality are given by the recent papers of Turalska et al. (2011) and Vanni et al. (2011). These authors show that at phase transition a metastable consensus condition occurs, with abrupt organizational collapses. The survival probability, namely the probability that no organizational collapse occurs, has the inverse-power law structure of equation 3. This property is limited to a singular value of the cooperation parameter *K*. With the SMS model, as clearly shown by **Figure 4**, changing *K* has the effect of changing temporal complexity. We are therefore inclined to believe that the SMS model is a form of extended criticality, whose temporal complexity in the intermediate asymptotic condition is characterized by stretched exponential relaxation.

In the far asymptotic regime the fat tail of the survival probability $\Psi(\tau)$ is truncated by a periodicity-induced fast drop, thereby breaking the renewal ML complexity (Metzler and Klafter, 2002). Is this a biological property compatible with the concept of EC? According to Bailly and Longo, EC signals the transition from physics to biology, and the SMS model is a model introduced for the specific purpose of interpreting neural dynamics. Although it may be questionable as all models that are currently proposed to shed light into complexity, the SMS model generates a form of periodicity that agrees qualitatively with the experimental observation (Beggs and Plenz, 2004). Furthermore, we have to keep in mind the natural periodicity of biological systems, evident in the concept itself of the ubiquitous "biological clock" (Winfree, 2000), and this does in fact support the view that periodicity reinforces rather than weakens the biological nature of a model. These arguments strengthen our conviction that the SMS model is compatible with the concept of extended criticality of Bailey and Longo.

We note that the theoretical interpretation of the experimental results on liquid crystals (Silvestri et al., 2009; Allegrini et al., 2010), although sharing with EC the condition that the control parameter is not a singular value, is based on the concept of renewal aging that implies the occurrence of quakes or organizational collapses (Vanni et al., 2011). These organizational collapses reset to zero the system's memory, thereby creating a condition incompatible with the temporal precision of activity patterns (Beggs and Plenz, 2004). We therefore expect that the SMS model of this paper should stimulate theoretical progress to extend the concept of temporal criticality so as to make it compatible with the complexity of the far asymptotics.

4.2. CRITICALITY AND THE BRAIN

The intriguing issue as to which form of criticality is obeyed by the brain has been debated in the earlier work by Allegrini et al. (2010). These authors found that the index ζ of the distribution of avalanches intensity is 1.92, and thus significantly different from the SOC prediction $\zeta = 1.5$. On the other hand, the statistical analysis made by Allegrini and co-workers seems to be more compatible with the standard concept of phase-transition induced criticality. These authors find that the temporal complexity of the events corresponding to the simultaneous occurrence of bursts in different channels is different from the temporal complexity of the single channels. The analysis of the single channels (Ignaccolo et al., 2010) discloses the emergence of a coherent behavior that may be related to the time periodicity addressed in this article, thereby leading to the conclusion that the global behavior of the brain may correspond to the phase-transition induced criticality, triggered by the cooperation among different brain components, while the dynamics of the single channels may correspond to a different kind of criticality, compatible with coherent dynamics. This form of criticality might be the extended criticality of Bailly and Longo (2011).

It is important to keep in mind that there is a deep difference between the brain, thought of as network of networks, and the dynamics of its components, namely neural networks that may be adequately described by the SMS model of this paper. The recent model proposed by Kello et al. (2011) is an attempt at understanding the global properties of the brain and the $1/f$ scaling of this model fits the observation of Allegrini et al. (2009b) who interpret the brain as a source of ideal $1/f$ noise.

4.3. COMPLEXITY MANAGEMENT: A CHALLENGE

A further issue generated by the result of this article is the extension of the Fluctuation Dissipation Theorem of first kind (Allegrini et al., 2009a) and of the Complexity Management (Aquino et al., 2010, 2011). To understand the conceptual difficulty of this important issue, we must recall that Fluctuation Dissipation Theorem in statistical physics has a long history, ranging from the 1905 Einstein paper on Brownian motion and Onsagers regression hypothesis to the more recent work of Kubo et al. (1985), whose linear response theory (LRT) is in fact considered by Lee (2007) to be the basic theoretical tool for the ergodic condition produced by Hamiltonian systems. In the case of the dynamics of the brain this traditional approach cannot be adopted, because of the lack of ergodicity condition, not to speak about the lack (or the impossibility) of a Hamiltonian treatment. The dynamical Fluctuation Dissipation Theorem proposed by Allegrini et al. (2007) is the first attempt at establishing a LRT for a system driven by renewal events violating the ergodic condition. The Complexity Management (or Complexity Matching) proposed by Aquino et al. (2011) rests on the surprising discovery that information can be transmitted by a complex network to another via an extremely weak coupling provided that a form of resonance exists between the complexity of the perturbed system and the complexity of the perturbing one.

The coexistence of temporal complexity and periodicity raises the question of whether the Fluctuation Dissipation Theorem and the related complexity management can be extended so as to deal with this more complex condition.

4.4. CONJECTURES AND PLAN FOR FUTURE RESEARCH WORK

One of the most important issues concerning the brain function is as to whether the brain does or does not compute. Werner made a strong warning against the adoption of the computer paradigm to explain the brain function (Werner, 2011). Neurons do not compute and function at meso- and macroscopic levels through a process of self-organization that makes significant their collective dynamics and irrelevant the behavior of single neurons as computational units. According to Werner the role of computer simulation is that of affording an epistemic access to their ontological states. This is an important warning that must be taken into a proper account. It seems to us that this warning may fit Varela's principles (Varela, 1979), in the form recently advocated by Mira and Delgado García (2007). These authors developed an "extended computation paradigm" whose main aim is to avoid contamination between the "External Observer Domain" and the "Own Domain." We freely interpret the External Observer Domain and the Own Domain of Mira and Delgado as the epistemic level and ontological levels of Werner, respectively.

We notice that the role of the neuron cooperation is also recognized by Soares et al. (2011) who propose the creation of coupled networks to make a significant progress toward the creation of intelligent systems.

The interaction between different neuron networks may account for the apparent conflict between the observation of coherence (Ignaccolo et al., 2010) and the observation of ordinary criticality at the level of global behavior of the brain (Allegrini et al., 2009b). A theoretical result based on the interaction between different neural networks would reinforce the perspective that to shed light into consciousness it is necessary to move from the action of the single neurons or group of neurons to the interaction and probably cooperative behavior of neural networks interpreted as complex systems by their own.

Research work in this direction is possible thanks to significant technological advances with neuron cultures done in the last 30 years (Gross et al., 1982). It is interesting to notice that there exists a gap between these experimental advances and the theoretical understanding. The results of this article suggest research directions that should fill this gap. We should move from the All-To-All condition to the adoption of a more proper topology. This is closely connected to the issue of how "information" is transferred from one to another network. As earlier stated, this requires the extension of Complexity Management to the case where temporal complexity and time periodicity coexist. Furthermore, although the assumption of extremely weak stimuli makes it possible the adoption of analytical treatments, it is necessary to go beyond it while maintaining the extremely useful suggestion that complexity matching may be the crucial ingredient for information transmission.

Finally, we want to stress that if the kind of criticality operating at the level of the model adopted in this article, is really a form of EC (Bailly and Longo, 2011) the arguments of these authors can be adopted to support our conviction that we are operating at a genuinely biological level.

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The conundrum of functional brain networks: small-world efficiency or fractal modularity

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The human brain has been studied at multiple scales, from neurons, circuits, areas with well-defined anatomical and functional boundaries, to large-scale functional networks which mediate coherent cognition. In a recent work, we addressed the problem of the hierarchical organization in the brain through network analysis. Our analysis identified functional brain modules of fractal structure that were inter-connected in a small-world topology. Here, we provide more details on the use of network science tools to elaborate on this behavior. We indicate the importance of using percolation theory to highlight the modular character of the functional brain network. These modules present a fractal, self-similar topology, identified through fractal network methods. When we lower the threshold of correlations to include weaker ties, the network as a whole assumes a small-world character. These weak ties are organized precisely as predicted by theory maximizing information transfer with minimal wiring costs.

Keywords: fractal networks, brain functional networks, small-world, modularity, percolation, fMRI

1. INTRODUCTION

The functional magnetic resonance imaging (fMRI) technique is a tool that has greatly improved our ability to probe brain activity. The method detects changes in blood oxygenation when areas of the brain are activated and consequently require increased blood flow. In this way, we can monitor what brain areas respond to different mental activities. The resulting datasets offer a three-dimensional image of the brain indicating the level of activation at various regimes.

Many methods have been applied to analyze fMRI data, ranging from statistics to signal processing techniques. Recently, the brain organization has been described as a complex network (Eguiluz et al., 2005; Sporns et al., 2005; Bullmore and Sporns, 2009). This approach can take various forms, such as physical connections between neurons or correlations in the activity between brain areas at a coarser level. In a recent work (Gallos et al., 2012) we used recent advances in fractal network theory to characterize the brain clusters structure, and studied one key problem of neuroscience, namely the integration of modular clusters in a larger scale. Here, we expand on those findings and describe the methodology in detail, focusing on the use of network theory in the study of fMRI data.

One of the main features of our sensations is its unitary nature. The brain can receive many concurrent stimuli. These have to be processed independently of each other, but at the same time they have to be integrated into a unified entity. This suggests that the modalities in the brain that process different characteristics have to act isolated for efficient computations, but they need also be sufficiently connected in order to perform coherent functions.

The notion of a complex network can be suitably adapted to address this scaling problem and study optimal information flow in modular networks. This representation of complicated interactions has offered new insight in many processes across different disciplines. A key feature of many such networks is their modular character, a topic which has attracted a lot of interest in the literature. Many algorithms have been proposed for the detection of modules, loosely defined as network areas well-connected within themselves but sparsely connected to the rest of the network. The detection and behavior of modules at different observation scales, though, remains a largely unexplored problem. Network analysis of functional (Eguiluz et al., 2005) and structural (Sporns et al., 2005) data has been used to characterize global connectivity and topological organization of the human brain (Bullmore and Sporns, 2009). Many of those studies indicate the small-world character (Watts and Strogatz, 1998) of brain networks, but the idea of a simple small-world structure can be contradictory to modular network.

In the present manuscript we implement a complex network analysis to understand the hierarchical organization of functional brain networks, and we study how we can explain the emergence of both small-world and modular features in the same network. We capitalize on a well known dual-task paradigm, the psychological refractory period, in which information from different sensory modalities (visual and auditory) has to be coherently routed to different motor effectors (in this experiment, the left and right-hand).

The combination of high-temporal resolution fMRI with novel network analysis tools allows the study of the module properties and their synergy toward accomplishing a cognitive task. A functional correlation network is derived from the fMRI phase information. We implement percolation and scaling analysis methods to uncover a highly modular functional operation and a network that is almost optimally connected for efficient information flow.

2. MATERIALS AND METHODS

2.1. EXPERIMENTAL DESIGN

We use time-resolved fMRI (Menon et al., 1998), based on analysis of the phase signal (Sigman et al., 2007). Time-resolved fMRI is capable of identifying the series of processing stages which unfold sequentially during the execution of a compound dual-task (Dux et al., 2006; Sigman and Dehaene, 2008).

The details of the experiments are described in Sigman et al. (2007), and are briefly reviewed here. Sixteen participants performed a dual-task paradigm: first a visual task of comparing an Arabic numeral (target T1) to a fixed reference, with a right-hand response and, second, an auditory task of judging the pitch of an auditory tone (target T2) with a left-hand response. The stimulus onset asynchrony (SOA) between T1 and T2 was varied between 0, 300, 900, and 1200 ms. In the course of this analysis we did not detect significant differences in the resulting patterns of different SOA conditions.

While subjects performed the dual-task, whole-brain fMRI images were recorded at a sampling time (TR) of 1.5 s, and subsequently the phase and amplitude of the hemodynamic response were computed (Sigman et al., 2007). This activated map exhibits phases consistently falling within the expected response latency for a task-induced activation. As expected for an experiment involving visual and auditory stimuli and bimanual responses, the responsive regions included bilateral visual occipito-temporal cortices, bilateral auditory cortices, motor, premotor and cerebellar cortices, and a large-scale bilateral parieto-frontal network (Sigman and Dehaene, 2008). In this study we try to understand the topology of the modular organization of this broad functional network during dual-task performance. For this purpose, we derived a large functional network of brain areas by measuring the phase correlations in these responses for all pairs of voxels. We then connected the highly correlated pairs which gave us the brain cluster network structure.

2.2. PHASE CORRELATIONS AND FUNCTIONAL BRAIN NETWORK

We use network theory concepts for the analysis of correlations between different brain areas, based on the temporal activation of these areas when a subject responds to external stimuli. We reconstruct the network topology of brain voxels, where a network link indicates a high correlation in the phase-space activity of the two connected voxels, and compare this structure with the corresponding topology of the voxel location in the brain.

The time evolution of the phase of all brain voxels over 440 s was recorded for each participant and each of the four SOA conditions, for a total of 64 measurements. For our analysis, we create a mask where we only keep voxels which were activated in more than 75% of the cases, i.e., in at least 48 instances.

We want to detect the correlation between the phases of two voxels i and j in the activated mask. The measure of correlation for vectors is the co-directionality, i.e., we need to calculate the angle between the two vectors. Therefore, the correlation c_{ij} between two vectors $\vec{r}_i$ and $\vec{r}_j$ is given, in general, by $c_{ij} \equiv \vec{r}_i \cdot \vec{r}_j / |\vec{r}_i||\vec{r}_j|$, which is equivalent to the cosine of the included angle, i.e., $c_{ij} = \cos(\theta)$, where θ is now the phase difference $a_i - a_j$. We average the correlation between any two voxels i and j in the activated mask over roughly 40 trials of each experiment. The resulting correlation p_{ij} between these two voxels is then given by

$$p_{ij} = \frac{1}{N} \sum_{t=1}^{N} cos\left[a_i(t) - a_j(t)\right], \tag{1}$$

where N is the number of trials for a given combination of subject and stimulus. We link two voxels if their correlation is larger than a threshold value p. The resulting network is a representation of functional relations among voxels for a specific subject and stimulus.

The topology of this network strongly depends on the value of p. The variation of p describes a percolation process. A large p-value enables isolated module identification, since only the strongest (i.e., more correlated) functional links between voxels are preserved. As p is lowered, these modules get progressively merged to larger entities and the emphasis is shifted toward large-scale properties of the spanning network.

The complex network representation (**Figure 1A**) reveals functional links between brain areas, but cannot directly reveal spatial correlations. Since voxels are embedded in space, we also study the topological features of spatial clusters in three-dimensions, where now voxels assume their known positions in the brain and links between them are transferred from the corresponding network (**Figure 1B**), i.e., they are assigned according to the degree of correlation between any two voxels, independently of the voxels proximity in real-space.

The above procedure yields a different network or spatial clusters for each subject. We study each of those networks and clusters separately and show that they all carry statistically similar properties. For efficiency purposes, we focus our attention to the case of the largest p_c value where three clusters, including at least 1000 voxels, emerge in each trial. The spread of the corresponding p_c values is small, demonstrating a similar behavior in the brain response of different subjects.

2.3. FRACTAL ANALYSIS

We analyze the resulting networks and the embedded three-dimensional clusters in terms of their fractal and modular properties. For the spatial representation, we characterize the fractality of a connected cluster through the standard Hausdorff dimension d_f. Starting from an arbitrary point in a cluster, d_f measures how the mass N_f (number of voxels in the same cluster) scales with the Euclidean distance r from this origin, i.e.:

$$N_f(r) \sim r^{d_f}. \tag{2}$$

The exponent d_f shows how densely the area is covered by a specific cluster.

The box-covering technique is used for the fractal analysis of the complex networks. A network (in our case each cluster) is first tiled with the minimum possible number of boxes, N_B, of a given size ℓ_B. A box is defined as a union of nodes, all of which are at a distance from each other smaller than a given threshold length, the box size ℓ_B (the distance between two nodes, ℓ, is defined as the number of links along the shortest path between those nodes in the functional brain network).

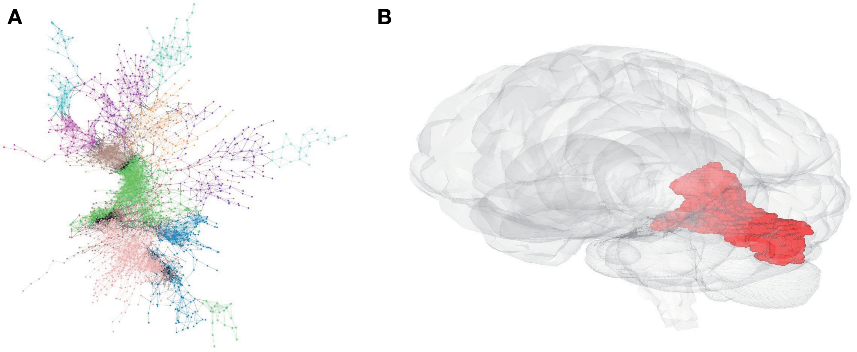

FIGURE 1 | (A) Network representation of a brain cluster, as found by the phase correlation between pairs of voxels. **(B)** The same cluster in real-space representation, where each voxel is now placed in its known location in the brain.

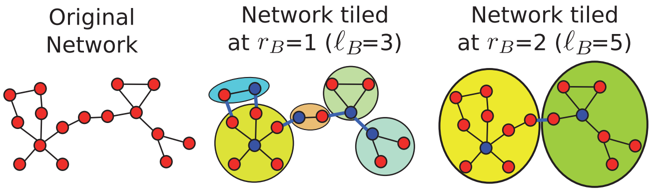

FIGURE 2 | Demonstration of the MEMB box-covering algorithm. For a given radius value, e.g., $r_B = 1$ in the center panel and $r_B = 2$ in the right panel, we cover the network with the smallest possible number of boxes. The diameter of the box, ℓ_B (i.e., the distance between any two nodes in a box) is defined as $\ell_B = 2r_B + 1$. First, we detect the smallest possible number of box origins (shown with blue color) that provide the maximum number of nodes (mass) in each box, according to an optimization algorithm described in Song et al. (2007). Then, we build the boxes through simultaneous burning from these center nodes, until the entire network is covered with boxes. For the calculation of modularity, we consider the boxes at each r_B value as separate modules. Then we calculate the ratio between the number of links within the modules (black links) and the number of links between modules (blue links).

The fractality (self-similarity) of the network is quantified in the power-law relation between the number of boxes needed to cover the network and the box size ℓ_B:

$$N_B(\ell_B) = N_0 \ell_B^{-d_B}, \tag{3}$$

where d_B is the fractal dimension (or box dimension) and N_0 is the number of nodes in the original network (Song et al., 2005a, 2006; Goh et al., 2006; Kim et al., 2007; Radicchi et al., 2008). Finite and small values of d_B show that the network has fractal features, where the covering boxes retain their connectivity scheme under different scales, and larger-scale boxes behave in a similar way as the original network.

The requirement that the number of boxes should be minimized poses an optimization problem which can be solved using a number of box-covering algorithms. The method that we implement here is called Maximum Excluded Mass Burning algorithm (MEMB), and the algorithm can be downloaded from http://lev.ccny.cuny.edu/hmakse/soft_data.html). The method is roughly explained in **Figure 2**. The detection of modules or boxes in our work follows from the application of this algorithm (Song et al., 2005a, 2007) at different length-scales.

The MEMB method starts by determining the minimum number of boxes of radius r_B required for a complete coverage of the network. This radius is the distance from a box "center," so that by definition all nodes in a box are within a distance from each other smaller than $\ell_B = 2r_B + 1$. The method detects the nodes that will act as the centers of the boxes, by calculating the mass around each node if it would act as a center. The node with maximum mass around it is selected as a center and we proceed iteratively to find the minimum number of such centers. Once these nodes have been determined, the boxes are built by including successive layers of nodes around the centers. The details of the method are reported in Song et al. (2007).

The resulting boxes are characterized by the proximity between all their nodes, at a given length-scale and the maximization of the mass associated with each module center. Thus, MEMB detects boxes that also tend to maximize modularity. Different values of the box diameter ℓ_B yield boxes of different size. These boxes are then identified as modules which at a smaller scale ℓ_B may be separated, but merge into larger entities as we increase ℓ_B. Thus, we can study the hierarchical character of modularity, i.e., modules of modules, and we can detect whether modularity is a feature of the network that remains scale-invariant.

For this, we can extend the box-covering concept to act as a community detection algorithm (Galvao et al., 2010). MEMB identifies modules of size ℓ_B, composed of highly connected brain areas. Typical modularity approaches do not place constraints on the size of the modules, but they focus on minimizing the number of inter-module links. The MEMB approach, though, has the additional advantage that modularity can be studied at different scales. The requirement of minimal number of modules to cover the network (N_B) guarantees that the partition of the network is such that each module contains the largest possible number of nodes and links inside the module with the constraint that the modules cannot exceed size ℓ_B. This optimized tiling process gives rise to modules with the fewest number of links connecting to other modules. This implies that the degree of modularity for a given ℓ_B value is maximized, and we can define a modularity measure, $\mathcal{M}$ through (Newman and Girvan, 2004; Guimerà and Amaral, 2005; Caldarelli and Vespignani, 2007; Gallos et al., 2007)

$$\mathcal{M}(\ell) \equiv \frac{1}{N_B} \sum_{i=1}^{N_B} \frac{L_i^{\text{in}}}{L_i^{\text{out}}}. \tag{4}$$

Here L_i^{in} and L_i^{out} represent the number of links that start in a given module i and end either within or outside i, respectively. Large values of $\mathcal{M}$ (i.e., $L_i^{\text{out}} \to 0$) correspond to a higher degree of modularity (Gallos et al., 2007).

The value of the modularity of the network $\mathcal{M}$ varies with ℓ_B, so that we can detect the dependence of modularity on different length-scales, or equivalently how the modules themselves are organized into larger modules that enhance the degree of modularity. In the case that the dependence has a power-law form, we can define a modularity exponent d_M, through the relation:

$$\mathcal{M}\left(\ell_B\right) \sim \ell_B^{d_M}. \tag{5}$$

3. RESULTS

3.1. PERCOLATION ANALYSIS REVEALS THE MODULAR STRUCTURE

We use percolation theory (Bunde and Havlin, 1996) to identify the functional clusters resulting from the correlation between the phases of two voxels. The percolation problem is a paradigm of critical phase transitions (Stanley, 1971; Vicsek, 1992) which can be used to identify the functional clusters in the brain network. In the simplest version of percolation, we can consider a lattice where each bond is absent with probability p or present with probability $1 - p$ (Bunde and Havlin, 1996). In lattices, it is well known that there exists a critical probability p_c, below which the largest cluster of connected bonds spans the whole length of the lattice, while for $p > p_c$ only small isolated clusters survive.

In the case of the functional brain network, the corresponding probability p for the existence of a link between any two voxels in the brain is based on the value of the phase correlation between them. For each participant, we calculated the mass of the largest cluster as a function of the percolation threshold p. As explained above, in a broad variety of systems in nature, the size of the largest cluster in a percolation process remains very small and increases abruptly through a phase transition, in which a single largest cluster spans the whole system (Bunde and Havlin, 1996). A single incipient cluster is expected to appear if the bonds in the network are occupied at random without correlations, i.e., when the probability to find an active bond is independent on the activity of all the other bonds in the network. For the functional brain network our results revealed a more complex picture.

We found that, for all participants in this study, the cluster size increased progressively with a series of sharp jumps (**Figure 3**) and not with a single jump as expected for the simpler picture of uncorrelated percolation. Moreover, in random percolation the second largest cluster has a strong peak around p_c and vanishes otherwise. In the brain network, the second largest cluster also increases through jumps of absorbing smaller clusters. This second cluster remains comparable in size with the largest cluster over a wider range of p. The evolution of these cluster sizes with p is a strong indication of strong correlations deviating from a random process.

We identified each of the jumps in the largest cluster as a single percolation transition focused on a region of the brain that is highly correlated and therefore represents a well-defined module (**Figure 3**). These sharp transitions are indicative of a marked modular structure in the network. They indicate that at any given p-value there are many isolated clusters in the brain network, which subsequently merge into the largest cluster as p decreases. This is a universal behavior observed in all participants, and allows the identification of functional modules, which we proceed to study next.

The clusters identified by percolation analysis at a given threshold p are functionally connected, but the nodes in such a cluster are not necessarily clustered in space. Thus, we first studied whether the percolation clusters had a consistent spatial projection. The p-values at which clusters appear varied across participants. To group the data, we measured, for each participant, the highest correlation p-value for which there were at least three clusters of 1000 voxels each. The topography of these clusters reflected coherent patterns across different individuals. In virtually all participants we observed a cluster covering the premotor, supplementary motor area (SMA) region, a cluster covering the medial part of the posterior parietal cortex (PPC) and a cluster covering the medial part of early retinotopic cortex (area V1), along the calcarine fissure.

We then measured the likelihood that a voxel may appear in a percolation cluster, by counting, for each voxel, the number of individuals for which it was included in one of the first three percolation clusters (**Figure 4**).

Clusters in the three main nodes, V1, SMA, PPC, are ubiquitously present in percolation clusters and, to a lesser extent, voxels in the motor cortex (along the central sulcus) slightly more predominantly on the left hemisphere.

This analysis demonstrated that the correlation networks obtained from each subject yielded percolation clusters with consistent topographic projections. Next we focus on our main aim; exploring the topology and scaling properties of the network modules using fractal network analysis.

3.2. FRACTAL ANALYSIS RESULTS

For each of the 16 participants and each of the 4 SOA conditions we calculated the resulting network through the phase correlation. Then, for each network, we estimated the percolation threshold that yields three clusters of at least 1000 voxels each. This results in a total of 192 clusters which were pooled together for the present analysis.

We applied the box-covering algorithm (Song et al., 2005a, 2007) to measure the fractal dimension d_B of these 192 clusters. The fractal dimension d_B was calculated separately for each cluster. The resulting network fractal dimensions were distributed in a relatively narrow range, with an average value $d_B = 1.9 \pm 0.1$ (**Figure 5A**).

The cluster structure can be also probed by its topological features when every node-voxel assumes its assigned location at the brain. Each cluster identified by the box-covering algorithm can be mapped to their anatomical projections, where two voxels are still connected according to their correlation but their distance is now defined by the Euclidean three-dimensional spatial distance

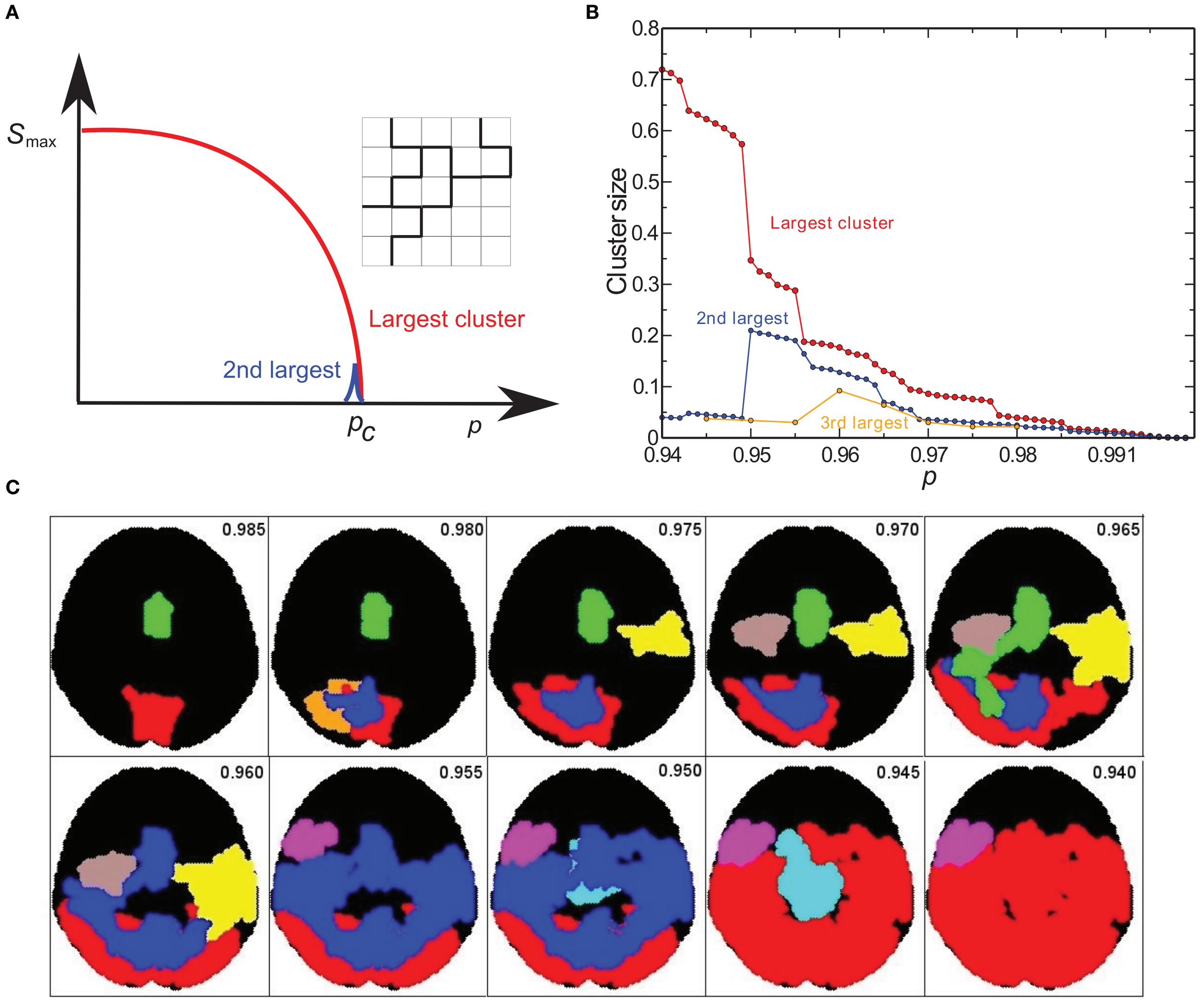

FIGURE 3 | (A) Bond percolation in a 2-dimensional lattice. We remove a random bond with probability p, and with probability $1 - p$ a bond remains (denoted by a solid black line). The lattice with solid black bonds is at the percolation transition $p = p_c$. **(B)** Evolution of modules at different thresholds. The size of the three largest clusters as a function of the correlation threshold p for a given subject. As we lower p the cluster size increases in jumps, and new clusters emerge, grow, and finally get absorbed by the largest cluster. This behavior is significantly different from the random percolation in **(A)**. **(C)** Cluster evolution. Brain clusters with more than 1000 voxels, as identified through correlation analysis for a given p-value.

r (**Figure 5B**). This mapping allows the use of the classical fractal dimension in real-space for the study of the structure of these functional clusters in the brain.

The method that we use to calculate the fractal dimension here is an alternative method to the one used in Gallos et al. (2012). There, d_f was calculated by measuring the number of nodes, N_C, in a cluster as a function of the cluster diameter. Here, for every cluster we start from a random point and open a circle of radius r and measure the number of nodes $N_f(r)$ in this circle. The dependence of $N_f(r)$ on r for this cluster gives its fractal dimension, and the process is repeated for all clusters. The scaling of the mass $N_f(r)$ (i.e., number of nodes in the cluster) included in a sphere with Euclidean radius r follows the power-law form of equation (2). The calculation of the individual Euclidean fractal dimensions yields an average of $d_f = 2.1 \pm 0.1$ (**Figure 5B**), which is similar for all clusters, and which is exactly the same as the one found in Gallos et al. (2012). The network fractal dimension of all clusters was systematically lower than the real-space fractal dimension, which was in the range 2–2.4.

It is possible that the difference between the fractal dimensions of individual clusters can be due to systematic variations, influenced by various factors. We performed a number of tests to identify the stability of these calculations. In **Figure 6A** we show a cross-plot for the exponents d_B and d_f, as calculated for each individual cluster. The value of d_B was systematically below d_f. From the same plot we deduce that the value of the percolation

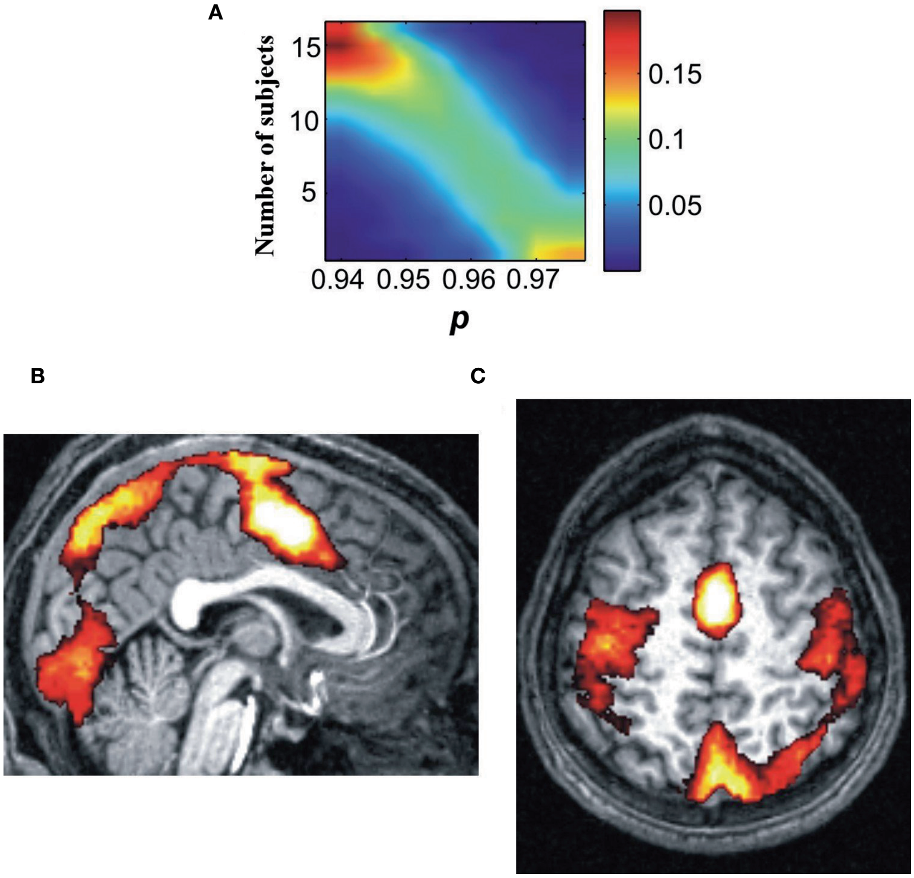

FIGURE 4 | The emerging clusters have consistent spatial projections. (A) The color denotes the fraction of the total number of voxels that appear to one of the three largest clusters in *N* subjects at a given percolation threshold *p*. As we reduce the threshold the peak shifts toward larger *N* values, i.e., the same voxels appear consistently in the largest clusters for all subjects. **(B,C)** Spatial distribution of the first percolation clusters (in subject counts). The two brain slices show for the highest p-values the shared voxels. White bleached regions correspond to voxels which are included in the first percolation cluster for all subjects. The SMA, a region involved in planning motor action is the only shared region for all subjects.

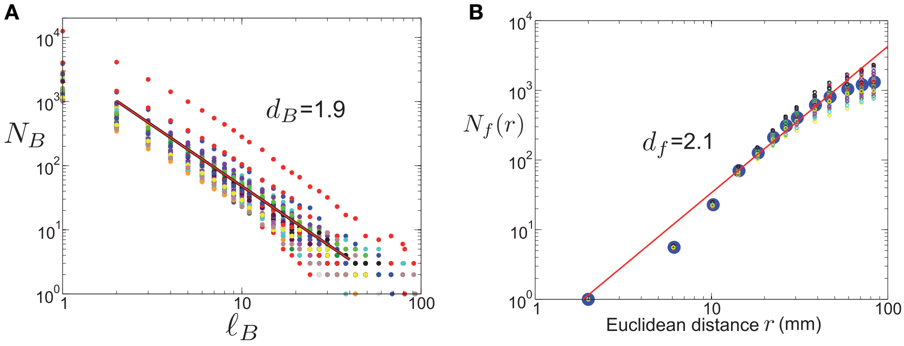

FIGURE 5 | (A) Fractal dimension d_B of the network clusters. The line is representative of the average dimension $d_B = 1.9$. **(B)** Fractal dimension d_f of the spatially embedded clusters. The large points represent the number of nodes $N_f(r)$ included within a fixed distance *r*, averaged over all clusters, while smaller points refer to individual clusters. The fitted line corresponds to the average dimension $d_f = 2.1$.

transition does not influence the fractal dimension, since the different p_c values of different clusters yield a uniform spreading of the fractal dimensions. It is also possible that the location of the brain clusters may have an effect on their fractal character. Our results do not provide any evidence toward this direction, either. In **Figure 6B** we plot the exponents d_B and d_f for each cluster as

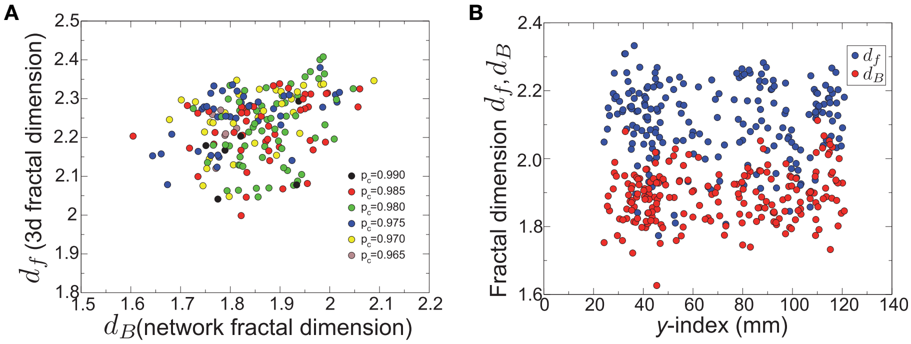

FIGURE 6 | Consistency of the fractal dimension calculations. (A) Cross-plot of d_f vs d_B for individual brain clusters. The colors correspond to the threshold values of p_c where the first percolation transition was identified. **(B)** The network fractal dimension, d_f (blue), and the three-dimensional fractal dimension, d_B (red) as a function of the location of each cluster. This location corresponds to the center of mass, and is expressed through the y-index, posterior to anterior.

a function of the y-coordinate of the cluster's center of mass, i.e., increasing y indexes corresponds to moving from the posterior to the anterior part of the brain. It is obvious that there is no systematic variation of the exponents in different locations. The above results emphasize the robustness of the fractal structure and indicate that we can consider the averages over all those structures to be representative of a typical brain module.

We can now characterize each single cluster, both at the functional level and at the topological level (i.e., the shape that the cluster assumes in the brain). Together, these results indicate that none of the clusters fill the 3D space densely; although the objects are embedded in three-dimensions their fractal dimension d_f is significantly smaller than 3. The network structure provides information on functional clusters, since it relates areas that are highly correlated independently of their physical proximity. Since the network fractal dimension d_B is even smaller than d_f, connections are fewer than one would expect through nearest-neighbor connections only. In simpler words, clusters do not form densely connected neighborhoods.

3.3. MODULAR STRUCTURE

In the Materials and Methods section we described how we can use the optimal MEMB coverage of the network with N_B nodes for a given ℓ_B value, in order to characterize the network modularity. Analysis of the modularity equation (4) in **Figure 7** reveals a monotonic increase of $\mathcal{M}(\ell_B)$ with a lack of a characteristic value of ℓ_B. Indeed, the data can be approximately fitted with a power-law functional form, equation (5), which is characterized by the modularity exponent d_M. We analyze the resulting networks of different subjects and we find that $d_M = 1.9 \pm 0.1$ is approximately constant over different individuals (**Figure 7**).

This value reveals a considerable degree of modularity in the entire system as evidenced by the network structure. For comparison, a random network has $d_M = 0$ and a uniform lattice has $d_M = 1$ (Gallos et al., 2007). The lack of a characteristic length-scale in the modularity shown in **Figure 7** suggests that the modules appear at all length-scales, i.e., modules are organized within larger modules in a self-similar way, so that the inter-connections between those clusters repeat the basic modular character of the entire brain network. Thus, the modular organization of the network remains statistically invariant when observed at different scales.

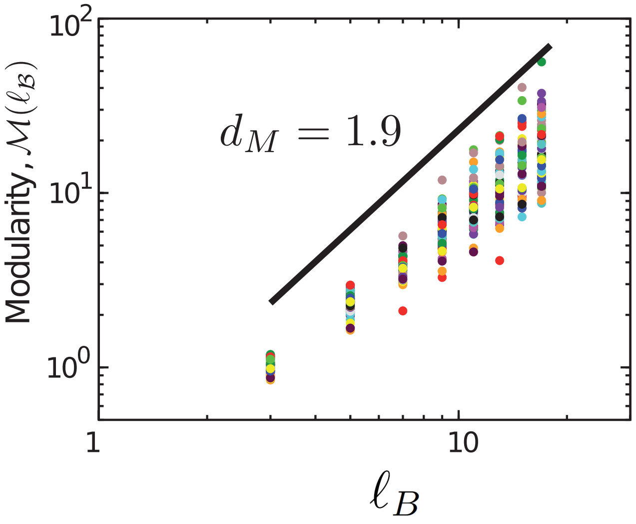

FIGURE 7 | Modularity as a function of ℓ_B for different clusters. The average value of the exponent is $d_M = 1.9$, shown by the solid line.

3.4. SHORT-CUT WIRING IS OPTIMIZED FOR EFFICIENT FLOW

A major advantage of the present analysis approach is that the analysis of the type of short-cuts present in the brain networks can convey a notion of optimal navigability in the network.

The addition of long-range links can turn the balance of a network structure toward either a self-similar structure with significant modularity but poor transfer or toward a small-world structure with very efficient flow at the cost of modularity (specialization). A small number of such short-cuts, quantified through renormalization group analysis (Rozenfeld et al., 2010), has been shown to provide the optimal trade-off between these two properties. In the case of the brain clusters the need for specialization/modularity is obvious, as also shown in the previous section, so it is important to understand how short-cuts influence the efficiency of signal transport in these structures.

In order to study how the modules that we recovered by the first percolation transition integrate at a larger scale, we also considered another percolation transition that corresponds to the emergence of a spanning cluster. We chose this transition as the correlation point where the largest cluster is equal to half of the total size. This global network connects practically all the smaller brain modules.

We probed the connectivity for this network, by analyzing the distance distribution of the links in the network, i.e., the Euclidean distance between any two voxels that are connected through their phase correlation (**Figure 8A**). We find an approximately power-law distribution (**Figure 8B**) of the form:

$$P(r) \sim r^{-\alpha}, \tag{6}$$

with a short-cut exponent $\alpha \approx 3.1$. The value of this exponent is very significant, since it approximately satisfies the scaling relation with the fractal dimension of the brain network:

$$\alpha = d_f + 1. \tag{7}$$

Such a scaling relation was recently (Li et al., 2010) found to optimize the transfer of information across a network with fractal dimension d_f when the short-cuts in the network are added with a cost constraining the number of total links. Thus, our scaling and modular analysis suggests that, taking into account the spatial restrictions, the functional behavior of the brain is optimally wired for facilitating efficient information transfer among different areas.

4. DISCUSSION

Our analysis revealed a fractal structure for the individual brain clusters. These clusters have a consistent topological behavior and are located at the areas that correspond to the expected brain responses. These modular structures present consistent fractal properties, both at the functional level and at a topological level. This indicates that the individual processing units that we recover do not have significant small-world properties. In contrast, when we include weaker correlations, the modules that appear at smaller scales are connected through long-range links. These short-cuts give a small-world character to the brain network as a whole, i.e., when studied at scales larger than an individual module.

The study of the distribution for these links suggests interestingly that they are optimizing transfer network properties, by also considering the wiring cost. In simpler terms, this topology does not minimize the global connectivity, simply to connect all the nodes; instead it minimizes the amount of wire required to achieve the goal of shrinking the network to a small-world.

The existence of modular organization of strong ties in a sea of weak ties is reminiscent of the structure found to bind dissimilar communities in social networks. Granovetter's (1973) work in social sciences proposes the existence of weak ties to cohese well-defined social groups into a large-scale social network. Such a two-scale structure has a large impact on the diffusion and influence of information across the entire social structure. Our observation of this two-layer organization in brain networks suggests that it may be a ubiquitous natural solution to the puzzle of information flow in highly modular structures.

Previous studies have found that wiring of neuronal networks at the cellular level is close to optimal (Song et al., 2005b). Specifically it is found that long-range connections do not minimize wiring but achieve network benefits. In agreement with this observation, at the mesoscopic scale explored here, we find an optimization which reduces wiring cost while maintaining network proximity. An intriguing element of our observation is that this minimization assumes that broadcasting and routing information are known to each node. How this may be achieved – what aspects of the neural code convey its own routing information – remains an open question in Neuroscience.

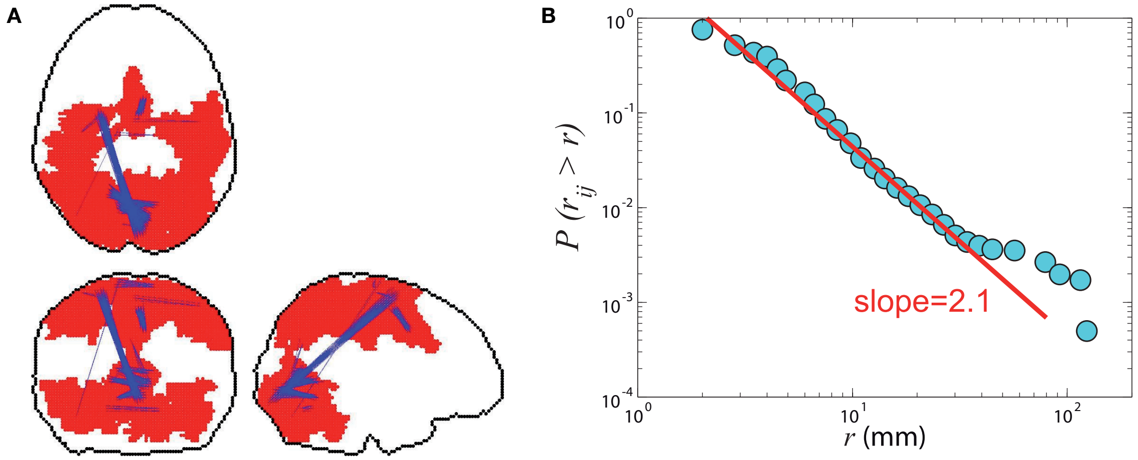

FIGURE 8 | (A) Real-space representation of the network at the second global percolation. The largest component is half the total mass. The blue lines highlight the longest links in Euclidean distance, which correspond to the weak ties. **(B)** Cumulative probability distribution $P(r_{ij} > r)$ of Euclidean distances r_{ij} between any two voxels that are directly connected in the correlation network. The straight line fitting yields an exponent $\alpha - 1 = 2.1 \pm 0.1$ indicating optimal information transfer with wiring cost minimization.

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