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Fuzzy mathematical model and optimization in digital green innovation for industry 5.0

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Fuzzy mathematical model and optimization in digital green innovation for industry 5.0

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Editorial: Fuzzy mathematical model and optimization in digital green innovation for Industry 5.0

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Editorial on the Research Topic
Fuzzy mathematical model and optimization in digital green innovation for Industry 5.0

1 Introduction

Since the implementation of the Paris Agreement, countries worldwide have made significant efforts to promote green development and transition towards a low-carbon economy. Among the various sectors, the global manufacturing industry stands out as the largest contributor, responsible for approximately 30% of total carbon emissions (Huang and Zhang, 2023). The manufacturing industry plays a critical role in determining the success or failure of global climate change mitigation efforts through its actions aimed at reducing carbon emissions. As the world embraces the rapid advancement of artificial intelligence, its impact on the industrial landscape cannot be underestimated. The integration of intelligent manufacturing applications and the new generation of artificial intelligence is unlocking tremendous value from data, which serves as a core element of the digital economy. This convergence has the potential to revolutionize the way manufacturing processes are optimized and managed, leading to more sustainable and environmentally friendly practices (Singh et al., 2023). Digitization technology plays a pivotal role in empowering green manufacturing by providing a crucial pathway for reducing emissions within the manufacturing industry.

Digital technology plays a significant role in optimizing various aspects of the manufacturing process, including product design, production efficiency, material waste reduction, and energy emissions minimization. The integration of intelligent transformation and green upgrading emphasizes leveraging digital technology to enhance energy, resource, and environmental management within the manufacturing industry (Yin et al., 2022). This approach enables the manufacturing industry to explore new economic growth opportunities in the field of green and low-carbon initiatives. Green innovation, propelled by artificial intelligence and big data, not only facilitates the digital transformation and green upgrading of the manufacturing sector, but also brings forth various benefits, including environmental preservation, climate mitigation, and economic impact (Dong et al., 2023). Digital green innovation is the integration of information, computation, communication, and connectivity technologies to reshape digital resources and incorporate digital technology into green innovation. This integration leads to the development of new products, enhancements in production processes, changes in organizational structures, and the creation and modification of business models. All of these advancements have a profound positive impact on the environment (Yin and Yu, 2022).

The introduction of the artificial intelligence era has sparked considerable interest in the field of intelligent manufacturing, particularly in relation to fuzzy set theory. This interest is observed both in academic and industrial sectors. The components within the digital green innovation system of the manufacturing industry are intricately interconnected, giving rise to an integrated entity that is characterized by, complexity, non-linearity, and hysteresis (Mahmood et al., 2023; Singh et al., 2023). However, traditional precise mathematical methods have certain limitations when it comes to expressing this system in a fuzzy manner (Garg et al., 2022). Therefore, it is crucial to utilize a dynamic analysis tool that is capable of capturing these characteristics. Fuzzy set theory has emerged as a valuable tool in addressing this challenge and provides valuable insights into the complex landscape of digital green innovation management. Consequently, the application of fuzzy system models plays a pivotal role in addressing the challenges associated with digitization and green innovation management, thereby promoting the advancement of intelligent manufacturing and green manufacturing (Dong et al., 2022).

This Research Topic significantly contributes to the integration of fuzzy mathematics and digital green innovation within the context of Industry 5.0. The research focuses on two main areas: the use of fuzzy mathematics models in environmental economics and management, and the exploration of digital green innovation. In terms of environmental economics and management, the research examines the intersection between fuzzy mathematics-based tools such as artificial intelligence, big data, and the internet of things, and their impact on the green economy. The goal is to understand how these tools can be harnessed to promote sustainable practices and enhance environmental management. Regarding environmental management, the research centers on the application of fuzzy mathematics-based digital technologies in fostering green intelligent manufacturing through digital green innovation management. It highlights the critical role that fuzzy mathematics plays in facilitating the green development of the manufacturing industry. Through this research, we aim to the gap between fuzzy mathematics and digital green innovation, ultimately contributing to the advancement of Industry 5.0 and the promotion of sustainable practices in the manufacturing sector.

2 Overview of the Research Topic

The Research Topic reveals the international experiences in the application of fuzzy mathematics and digital green innovation management. Within Industry 5.0, the advancement of intelligent manufacturing and the transition to green manufacturing are facilitated by the implementation of digital green innovation technologies grounded in fuzzy mathematics theory. The Research Topic of papers pertaining to this research theme extensively examines and highlights the paramount significance of fuzzy mathematics and its derivative models in enabling digital and green innovations. These approaches are essential for effectively navigating the challenges and opportunities presented by the era of artificial intelligence and the imperative of carbon reduction.

In terms of environmental economics, Zhang et al. delved into the correlation between digital transformation and sustainable development within enterprises. It was posited that digital transformation could mitigate challenges such as knowledge circulation barriers, brain drain among technical personnel, industry monopolies, disadvantages faced by female board members, and the aging workforce, consequently injecting fresh momentum into the sustainable development of enterprises. Mingkai et al. examined the influencing factors impacting the harmonized development of green innovation and the digital economy using a fuzzy set qualitative comparative analysis method. The researchers found that a single influencing factor alone is not sufficient as a prerequisite for achieving a high level of coordination between green innovation and the digital economy. Consequently, the authors put forth four pathways to enhance the level of coordination between these two domains: human capital + research and development intensity + openness, excluding joint driving; research and development intensity + openness, excluding dual driving; human capital + government subsidy, excluding dual driving; and government subsidy direction. Zhao et al. revealed the influence of the digital economy on carbon emissions intensity. The authors assert the presence of a long-term equilibrium relationship between the two variables. Specifically, the digital economy exhibits a negative long-term impact on carbon emissions intensity, whereas carbon emissions intensity does not exert a negative impact on the digital economy. Ning et al. conducted an analysis of the influence of environmental liability insurance on green innovation within enterprises. Moreover, it examined the driving role of environmental liability insurance in fostering green innovation through the lens of green governance. Environmental liability insurance plays a crucial role in enhancing the capacity for green innovation by alleviating financing constraints and diminishing agency costs. Zhu et al. used the Entropy-CoCoSo method to assess the level of business environment for inter-provincial economic development. In the proposed Entropy-CoCoSo evaluation framework, the weight of each indicator is determined using the entropy method, and the comprehensive evaluation of the business environment for each province is achieved through a combination of compromise solution. Ashraf et al. proposed a complex intuitional fuzzy ELECTREE method to evaluate the impact of industrial pollution on the environment. This study focuses on four common criteria: air pollution, water pollution, land degradation, and waste generation. The proposed method is effective as it allows for the use of multiple criteria for multi-attribute decision-making and also allows us to visualize the ranking of members in a two-dimensional manner. Yang et al. developed a bipolar complex fuzzy set methodology to analyze the impact of various pollution on the environment and evaluate the most harmful types of pollutants. The authors believe that air pollution and water pollution are currently key factors affecting environmental safety.

In terms of environmental management, Hou et al. formulated the concept of digital green and proposed a novel Maclaurin

Symmetric Mean operator based on interaction operational laws for T-spherical fuzzy information to assess digital green technologies. The authors believe that the concept of digital green refers to the use of digital technologies, techniques, and procedures in the field of ecological or sustainable protection. Khan et al. developed an aggregation operator to solve the green supplier selection problem with q-rung orthopair fuzzy hypersoft information. Although fuzzy sets can handle incomplete information in various real-world problems, they cannot handle all types of uncertainty, such as incomplete data and uncertain data. The structure of q-rung orthopair fuzzy hypersoft sets is a very effective processing tool. Ashraf et al. extended the evaluation based on the distance from the average solution approach under Pythagorean fuzzy Z-numbers. In this study, Pythagorean fuzzy Z-number arithmetic aggregation operators are proposed to assist manufacturing producers while incorporating uncertainty and reliability. Junjie et al. revealed the impact mechanism of social entrepreneurship opportunity recognition and resource mobilization on growth performance. Social entrepreneurship, as a way for businesses to fulfill their social responsibilities, is becoming one of the important means to address social responsibility conflicts. The more ties in the entrepreneurial social network, the stronger the impact of social entrepreneurship opportunity recognition on the growth performance of business start-ups. Guang-lin and Tao investigated the correlation between management capability and the processes of digital transformation and green technology innovation. The findings indicate a notable and positive relationship between management capability and both green technology innovation and digital transformation. In addition, internal control was identified as a significant and positive moderating factor within this relationship. Sun screened the factors that influence green innovation risk within the global value chain. The characteristic root method of group decision-making was employed to establish an evaluation index system for assessing the risks associated with manufacturing green innovation. The identified factors include the global proportion of investment in green R&D personnel, the global proportion of investment in green R&D funding, the stability of global manufacturing green R&D, the challenges associated with international transfer of green technologies, and international protection of green technology patents.

Therefore, these articles successfully address the gap in digital green innovation management by utilizing the principles of fuzzy mathematics. They offer a comprehensive understanding of how digital green innovation is distributed within Industry 5.0's environmental economy and management in the age of artificial intelligence. Moreover, they present a holistic perspective on environmental economy and environmental management, highlighting the interconnectedness and interdependence of these facets.

3 Conclusion

The Research Topic introduces the concept of "digital green innovation", which refers to the use of digital technology in implementing green innovation practices. These papers offer valuable insights into the relationship between fuzzy mathematical theory and digital green development. The ultimate goal is to assist industries in adapting to the complexities of the AI era and the inevitable shift towards decarbonization. Furthermore, this research explores various avenues within the realms of digital innovation and green innovation, systematically expanding upon different directions. It investigates the reasons and phases of digital green innovation, aiming to shed light on the crucial aspects of this process. Moving forward, there is a pressing need for further exploration new industrial intelligent technologies that are applicable to large-scale green complex industrial systems. The development of such large-scale modeling technology is expected to have a transformative impact in tackling highly intricate problems within the domain of industrial intelligence and greening.

Author contributions

SY: Conceptualization, Writing–original draft, Writing–review and editing, Conceptualization, Writing–original draft, Writing–review and editing. DP: Investigation, Resources, Writing–review and editing, Investigation, Resources, Writing–review and editing. KU: Resources, Writing–review and editing, Resources, Writing–review and editing. HG: Resources, Writing–review and editing, Resources, Writing–review and editing.

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Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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Path and boundary of the influence of social entrepreneurial opportunity identification on the growth of commercial startups

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Social entrepreneurship, as a way for enterprises to fulfill social responsibility, is becoming one of the key means to reconcile social contradictions in China. In the field of social entrepreneurship as entrepreneurship research focus in the emerging background, such as type business start-ups face "vulnerable" new defects, social entrepreneurship, how to deal with scarce resources, social entrepreneurship resources come from and how to create such basic problems as what kind of results is still not well explain, related research is still in a state of "cumulative pieces". Therefore, this paper focuses on the social opportunity identification of commercial new ventures, and empirically analyzes the mechanism of social entrepreneurial opportunity identification and resource patchwork on their growth. The results demonstrate that finding social entrepreneurial opportunities is an important part of how resource collages can help social enterprises grow, and the entrepreneur's social network allows for the mediating effect of resource collage. The more sources of "strong relationships" in entrepreneurs' social networks, the stronger the impact of social entrepreneurial opportunity identification on the growth performance of commercial startups. Focusing on the field of corporate social entrepreneurship, our findings establish a complete chain of social entrepreneurship processes, from motivation to behavior to corporate sustainability. The findings confirm the mechanism by which social entrepreneurial opportunity identification and resource patchwork help to improve the growth performance of commercial new ventures, and also suggest that entrepreneurs' social networks can relieve entrepreneurs' dependence on external network construction to a certain extent.

KEYWORDS

growth of commercial startups, resource bricolage, social entrepreneurship opportunity identification, social network theory, social entrepreneurship

1 Introduction

Social entrepreneurship has attracted widespread attention and gradually evolved into a global phenomenon as an effective means to manage social problems, such as population aging and environmental pollution. An examination of entrepreneurial practices in recent years illustrates that enterprises' entrepreneurial behaviors are gradually beginning to focus on social issues and entrepreneurship beyond business activities, with startups being the driving force of the latter. Although the increasing importance of social entrepreneurship of startups is generally agreed upon in academic circles and the industry, startups often face challenges, such as resource constraints and teething problems, as their social entrepreneurship grows (Wang et al., 2019). The theoretical exploration of social entrepreneurship in commercial enterprises is still in the early stage and lacks empirical analysis. Liu et al. (2019) emphasized that the integration of resources and opportunities opened up a new way for future research, and made full use of individual social networks of entrepreneurs and transformed these relationships into close and stable ones, which would help realize the potential value of existing resources. Therefore, based on social opportunity identification theory, resource patchwork theory and social network theory, this study explores how social entrepreneurship opportunity identification propels the growth of new ventures to provide valuable insights for promoting and leading the sustainability of corporate social entrepreneurship.

Social entrepreneurship follows the logic of creating social value based on commercial means (Dees, 1998). Social entrepreneurship as a new form in the field of entrepreneurship, the purpose is to use innovative means to solve the social problems so as to realize the creation of economic value and social value, the study found that usually under the double malfunction of government and market, caused by a lack of common configuration and the reasons for the inefficient and produce social entrepreneurship opportunities (Shaker et al., 2008). Part of empirical research, points out that social entrepreneurship opportunity recognition effect on organizational performance has significant prediction model (Sebastian et al., 2020), such as the entrepreneurs of prosocial motives, this contains the mutual sharing of behavior prompted entrepreneurs and others on the perspective-taking, beneficial to individuals out of their own limitations, to promote the integration of view skills, so as to promote the creativity of the organization (Tu et al., 2020). However, a few studies have found that social entrepreneurial opportunity identification has no effect on the innovation growth of enterprises, and believe that the reason for this result is that social opportunity identification may affect the innovation growth of organizations through other ways (Pablo and Ricardo, 2021). In this regard, McDermott et al. (2018) attempted to further analyze the impact of social opportunity identification on organizational innovation growth by mobilizing the active participation of stakeholders and resource mobilization. Therefore, the primary purpose of this paper is to examine the impact of social opportunity identification on the realization of economic and social value of new ventures.

Some studies have demonstrated that resource bricolage is an essential path for resource innovation to create value because it solves the problem of resource constraint and advances corporate social innovation (Wang et al., 2019). Conversely, a few studies have also found that resource bricolage is inefficient and leads to repetition because it is limited by time and resources. Mainly, bricolage strategies used in certain situations do not work for all businesses, and too much bricolage can hurt startup' performance. However, resource bricolage is an essential method for firms to solve the problem of resource constraints. We must recognize the complicated nature of resource bricolage, inconsistency of the current research results, and that the theory of resource bricolage needs further attention. In short, the origination of firms' resources and their results are still in question. In this study, we seek to reveal how identifying social entrepreneurship opportunities affects the growth of startups using resource bricolage as a critical approach.

From the perspective of social network theory, entrepreneurs are the core personnel involved in social entrepreneurial activities. Their social networks are a key "way" to access the social assets, resources, and support needed for social entrepreneurship. However, the effect of entrepreneurial social networks on organizational performance is complex (Park and Luo, 2001). Existing theoretical research conclusions are not unified, which demonstrates that the social network theory of entrepreneurs is still insufficiently mature (Balagopal, 2011). Therefore, it may be necessary to re-examine and clarify the entrepreneur's social network in a specific context. Moreover, despite the growing interest in the social networks of entrepreneurs, little has been done to examine how these networks interact with social opportunity identification interactions to promote entrepreneurial firm growth. To this end, this study surveyed social entrepreneurs participating in the China Social Entrepreneurship Forum 2020. Held in an "online + offline" format in Beijing, China. The event brought together dual value creation-oriented entrepreneurs from all over China to provide methods and suggestions for the sustainable development of enterprises. We conducted a multi-stage questionnaire to investigate how social entrepreneurship opportunity identification can contribute to the growth of startups. By exploring the relationship between social networking and opportunity identification for entrepreneurs, we answer the following research question: How do startups achieve long-term growth performance *via* resource bricolage and entrepreneurs' social network in social entrepreneurship opportunity identification? Based on the empirical results, we propose a set of theoretical models involving social entrepreneurs' opportunity identification, resource bricolage, and social network to promote the growth of startups.

The theoretical contributions are as follows. First, the research on the identification mechanism of social entrepreneurial opportunities is deepened. Based on the theory of social entrepreneurial opportunity identification, this paper clarified the relationship between social entrepreneurial opportunity awareness and the growth of new ventures through literature review, and further verified the hypothesized relationship through empirical analysis. At the same time, it clarifies the social network of entrepreneurs that plays a reinforcing role in the process of social entrepreneurial opportunity identification, which has an important theoretical contribution to the research on social entrepreneurial opportunity identification. It deepens the theoretical research on social entrepreneurial opportunity identification from the theoretical level, and is also a response to previous research (Charles et al., 2020). Secondly, it enriches the research methods of social entrepreneurial opportunity identification. At present, the research on social entrepreneurship is mainly based on qualitative research methods, and the research results lack the support of empirical research. Moreover, the research on the identification of social entrepreneurship opportunities is in its initial stage, which also lacks the effective verification of empirical research results. In particular, previous studies on the growth of startups have shifted from focusing only on the economic benefit analysis to exploring the internal mechanism of social and economic value creation. The acknowledgment of the protection mechanism of social entrepreneurship opportunity identification and the motivation mechanism of entrepreneurs' social networks will shed new light on the understanding of the sustainable growth mechanism of commercial startups during the transitional period. Second, we provide theoretical support for innovative growth models that are not entirely about economic benefit and a theoretical reference for high-quality entrepreneurial practices and policymaking in emerging economies experiencing social and economic transitions.

The structure of this paper is as follows. First, we review the literature on social entrepreneurship opportunity identification and the growth of startups. Second, we look at the research on resource bricolage and entrepreneur social networks to build a theoretical model. Third, we discuss the methods and data used. Fourth, we discuss our results and summarize their theoretical and practical implications.

1.1 Theoretical background and research hypotheses

Social entrepreneurship is a new field (Gerometta et al., 2005). Existing theoretical studies are carried out in three dimensions: entrepreneurship content, process, and social empowerment participation (MacCallum et al., 2009). They are scattered across different disciplines, such as economics, sociology, and management (Gerometta, et al., 2005a). Social problems are the main source of opportunities for the development of enterprise entrepreneurship and innovation (Sagawa and Segal, 2000), and social entrepreneurship is a practical process in which the government and enterprises creatively integrate and utilize social resources to solve social problems or meet social needs in new ways or ways. Social entrepreneurship has the following basic characteristics: sociality of the goal, pluralism of the subject, and creativity of the method. The basic method is to creatively integrate and allocate superior resources of all parties individually or cooperatively, and generate new social technologies and methods through innovative ideas. The social goal is to solve social problems, meet social needs, maintain social order, and promote social progress. The basic conditions for implementation are universal values and replicable and diffusible patterns (Schwartz, 2012). The Bank of Boston, Bell Atlantic, General Electric, Wieppon, Wal-Mart, and others have turned social responsibility into social entrepreneurship, radically changing the role of enterprises in society (Saul, 2010).

Resource patchwork is an innovative way of applying resources that can help businesses survive and succeed. Based on the resource patchwork theory, scholars explain the phenomenon of entrepreneurs starting from scratch and the enterprise growth and expansion process from many perspectives. For example, new international ventures can improve their innovation performance and competitive position by using powerful resources. The flexible application of resource pooling can help enterprises improve their financial performance and growth rate in the early stage of their establishment (Li and Zhu, 2014). For example, Garud and Karne (2003) conducted field research on typical enterprises in the wind turbine industry in Denmark and concluded that entrepreneurs try their best to assemble financial resources to realize the purchase of newly established enterprises in the face of resource shortage.

The embeddedness of the social network of entrepreneurs is the prerequisite for obtaining complementary resources, and the research on social networks is particularly important in social entrepreneurship (Jack and Anderson, 2002). As entrepreneurs embedded in the network have heterogeneous resources, their network relationships and locations will affect the manner and efficiency of the resource flow. Therefore, the main research elements of social networks are relationship elements and structural elements. First, social network relationship elements, such as business networks, governments, and government support networks, are important resources in social entrepreneurship, and social entrepreneurs depend highly on them. Second, social entrepreneurship is a complex multi-stage dynamic evolutionary process, and the construction of social networks requires social entrepreneurs to establish good cooperative relationships with other organizations and individuals. In sum, social capital embedded in social networks and increasingly extensive social networks are the cornerstones for the further development of social enterprises. The key to the success of social entrepreneurship is the circular operation of social capital.

1.2 Social entrepreneurship opportunity identification and startups growth

The exact definition of social entrepreneurship is still debatable, with the most common principle in previous studies being the "broad concept of inclusiveness" (Choi and Majumdar, 2014). Some scholars have interpreted it based on an economic and social "dichotomy." For example, Dacin et al. (2010) argue that social entrepreneurship opportunity identification refers to the utilization of commercial methods to provide innovative solutions to social problems and balance economic and social values, and is, thus, "social," "innovative," and "market-oriented." argues that a "dichotomous" approach is difficult to result in consensus. Mair and Marti (2006) examine social entrepreneurship from the perspective of value creation and posit that opportunities identified to meet social needs facilitate the improvement of institutional systems. In this regard, social entrepreneurship opportunity identification is characterized by the overlap of a set of opportunities that solve social problems and a set of profitable business opportunities (Marcus et al., 2016). Regarding the research paradigm, entrepreneurship research goes beyond organization-level entrepreneurial processes to individual and team-level entrepreneurial cognition and decision-making behaviors. Opportunity identification and resource bricolage are core issues in entrepreneurship research. However, traditional strategic management and resource-based theories can hardly solve problems such as how social entrepreneurs identify opportunities when resources are scarce.

Consequently, scholars began to apply theories of opportunity discovery and creation to the field of social entrepreneurship and argue that the first step to social entrepreneurship is opportunity identification (McDermott et al., 2018). Halberstadt et al. (2021) also called for social entrepreneurship opportunity identification to receive the same attention as the business sector, especially regarding organizational performance. Although social entrepreneurship is prone to "initial weaknesses" that render breaking through resource constraints difficult, using empirical studies, McDermott et al. (2018) found that social entrepreneurship opportunities are associated with active stakeholder participation and resource mobilization. Thus, identifying social entrepreneurship opportunities for entrepreneurs is essential for integrating and utilizing resources and a prerequisite for obtaining external legitimacy.

This paper defines social entrepreneurship opportunity as the possibility of creatively integrating social assets and resources to meet social needs and create social values. Clarifying that social entrepreneurs are the pivot of entrepreneurship opportunity identification and are at the core of the network connecting startups and external stakeholders is essential. Entrepreneurs identifying social entrepreneurship opportunities is also a process of breaking resource constraints, ad hoc utilization, and improvisation; hence, identifying entrepreneurs identifying social entrepreneurship opportunities is directly related to enterprise growth. The development of startups has two aspects: economic performance and social performance. Startup growth is underpinned by specific social assets/ resource integration and the value created in social entrepreneurship (e.g., sociality, feasibility, profitability) that raises stakeholder expectations, which leads to the improvement of startup performance. First, sociality reflects the extent to which social entrepreneurs or startups focus on social value creation and not only business financial performance (Kraus et al., 2017). Sociality can reinforce the motivation of social entrepreneurs and stakeholders to achieve social goals. When the vision of social value creation connects stakeholders to inspiring plans, it may motivate stakeholders to consider socially responsible objectives and enable employees to work positively.

Similarly, high goal identification in high-performing organizations helps social entrepreneurs adopt the correct strategic direction and focus on corporate goals (Doherty et al., 2014). In addition, enhanced social purpose motivation can drive organizations to improve quality and efficiency and promote the creation of more social value (Ellsworth, 2002). Second, feasibility emphasizes the availability of social assets and social resources, which determines the feasibility of establishing market exchange relationships—no social enterprises and social entrepreneurship opportunities will exist without market exchange relationships (Hu et al., 2019). In managerial practice, feasibility identification meets the new need of stakeholders through the operability of entrepreneurial solutions, the novelty of ideas, and the practicability of product/service innovation; it helps the business grow. Third, profitability identification promotes the growth of startups in the following two ways: First, entrepreneurs develop new products through technological improvements to generate a differentiation advantage in the market; second, social startups build a vivid corporate image in the market, garner market reputation, and expand market share through product iteration and business model innovation. Based on the above, we propose the following hypothesis:

H1. Social entrepreneurship opportunity identification has a significant positive impact on the growth of startups.

1.3 Mediating role of resource bricolage

Access to resources is critical and forms a central element in identifying social entrepreneurship opportunities (Dorado, 2006; Hockerts, 2006; Murphy and Coombes, 2009; Virginie and Katia, 2021). Salunke et al. (2013) noted that entrepreneurs could gain a competitive edge by compounding existing resources. Entrepreneurs' bricolage behavior can be induced through strategic flexibility and relational learning, among others, to promote the growth of startups. Li and Zhu (2014) found that resource bricolage can effectively address the lack of resources for startups and promote inclusive business growth. Zhou et al. (2019) discussed the positive effect of resource bricolage on enterprises' innovation.

Similarly, Sunduramurthy et al. (2016) discovered that when entrepreneurial startups face severe resource constraints, resource bricolage enables social entrepreneurial firms to implement precision marketing strategies successfully, thus, driving business growth. By improving their resource bricolage skills, new social enterprises often get better at developing new ideas and becoming more competitive. There is a strong correlation between resource bricolage ability and enterprise growth. On the other hand, entrepreneurs with a greater ability to identify social enterprise opportunities have better perception and alertness, promoting resource bricolage development and use. In particular, when entrepreneurs pay attention to opportunities to create economic and social value, they obtain resources for cheap and use them quickly to solve problems as their businesses grow. They do this by "breaking resource constraints" through active searching, social exchange, and contract signing, among other things (Zahra et al., 2009). Second, entrepreneurs may creatively use existing resources to address business growth problems by "making do with what they have." Third, entrepreneurs creatively rebuild resources to promote growth per the enterprise's strategic intent and entrepreneurial goals (Baker and Nelson, 2005). When resources are not enough, resource bricolage that is new, flexible, and immediate can help businesses respond quickly to market needs at the lowest cost. By contrast, revitalizing redundant resources is difficult if the enterprise lacks creative bricolage. The bricolage can become an inefficient resource, thus complicating the realization of the entrepreneurial behavior. Based on the above, we propose the following hypothesis:

H2. Resource bricolage plays a mediating role between social entrepreneurship opportunity identification and the growth of startups.

1.4 Moderating role of entrepreneurs' social networks

Based on social network theory, network relationships are increasingly strategic, especially in early development, as social entrepreneurs often use social network resources to break the constraints of social entrepreneurial resources (Servantie and Rispal, 2018). In this study, we maintain that an entrepreneur's social network is a collection of relationships between the entrepreneur and external stakeholders within and outside the organization. These relationships contain resources that provide action opportunities for the growth of the startup, through which the entrepreneur obtains information, resources, services, and substantive support needed for social entrepreneurship opportunity identification and development. Whether entrepreneurs can leverage resources depends on the stability of the relationship between social entrepreneurs and stakeholders (Liu et al., 2019). Relationship stability implies the frequency of contact between entrepreneurs and stakeholders, which plays a vital role in the quantity and quality of resources acquired, ensures the effective transfer and acquisition of resources needed for the growth of startups. Recent studies have found that, compared with other aspects of social networks, trust can improve the frequency and quality of information sharing and exchange between entrepreneurs and stakeholders and lengthen the time spent talking. The emotional tool network significantly impacts how external resources are used (Sarkar, 2018). To this end, we focus on entrepreneurial social networks.

Within the framework of social network theory, the social entrepreneurial network is an important variable that affects how social entrepreneurship opportunity identification affects resource bricolage. The accumulation of resource bricolage ability in social entrepreneurship positively correlates with the strength of social entrepreneurs' network relationships (Liu et al., 2019). First is the effect of resource patchwork quantity on network relationship strength on entrepreneurial performance. We introduce an R&D model to illustrate the resource patchwork effect.

Assume that the piecework resource demand function is linear, $Q = 1 - P$, and the initial marginal cost of production of the entrepreneur's firm is c, and assume that $nc < 1$; the level of research i effort chosen by each entrepreneur is $s_i \in s = (0, \bar{c})$. The cooperation between the entrepreneur i and network actors is bilateral, allowing the entrepreneur to share research on behalf of the firm to reduce costs. The marginal costs incurred by entrepreneurs s dealing with the aggregation of research efforts and network members g are (j being i network close partners) $c_i(s|g) = (\bar{c} - s_i + \sum_{j \in N_{i(g)}} s_j$. It is also assumed that the research effort requires a cost, $Z(s_i) = as_i^2$, and this cost is $a > 0$, if a is large enough. Subsequently, the profit function is concave with respect to the network actor's own effort. Simultaneously, given $c = c_1, c_2, c_3 \ldots\ldots c_n$, the firm's choice of output promotes profit maximization. Additionally, it is assumed that network actors compete with output in the market, where the output selected by enterprises is $q = q_1, q_2, q_3 \ldots .. q_n$, and the total output is $Q = \sum_{j \in N} q_i$. Therefore, the economic profit obtained by the firm i, from the close cooperation network of entrepreneurs g, $\pi(s|g) = [1 - q_i(g) - \sum_{j \neq 1} q_i(g) - c_i(g)] q_i(g)] - as_i^2(g)$ is $q_i \frac{1 - nc_{i+} \sum_{j \neq 1} c_i}{n+1}$, and the equilibrium output of the firm can also be obtained as $q_i \frac{1 - nc_{i+} \sum_{j \neq 1} c_i}{n+1}$. Based on Goyal and Moraga-Gonzalez (2001) derivation, research effort s can be directly used to express the economic benefits of enterprises in the social network g of entrepreneurs i when they face the research set:

$$\pi(s|g) = \frac{\left[1 - \bar{c} + s_i(n - \eta_i) + \sum_{j \in N_i(g)} s_i[n - \eta_i(g)] - \sum_{l \in N/\{i\} \cup} s_l[1 + \eta_i(g)]^2\right.}{(n+1)^2} - as_i^2(g),$$

where, l is any other network actor. The payoff function shows the positive bound externality effect of closely related actions and the negative bound externality effect of non-closely related actions in the network. The actions of closely related actions are strategic complementarity, whereas the actions of non-closely related actions are strategic substitution. Thus, close cooperation of network actors can produce a series of out-of-bounds benefits.

Second is the influence of contact frequency (times) on network relationship strength on enterprise performance. According to the above conclusion and hypothesis, the problem of contact times, k, is further discussed. Assume that the number of interactions between entrepreneurs and network actors is k, which is also taken as a parameter. Thus, in a social network g^k with a number of is k, the revenue function of the corresponding enterprise is:

$$\pi(s|g^k) = \frac{\left[1 - \bar{c} + s_i(n - k) + \sum_{j \in N_i(g)} s_i[n - \eta_i(g)] - \sum_{l \notin N_i(g) \cup \{i\}} s_l(k+1)\right]^2}{(n+1)^2} - as_i^2.$$

Furthermore, to realize the symmetry of research effort in the network of degree k, the equilibrium effort level function $s_i^*(g^k) = \frac{(1-\bar{c})(n-k)}{a(n+1)^2 - (n-k)(k+1)}$ can be substituted into the above revenue function. According to Goyal and Moraga-Gonzalez (2001), the following expression can be obtained:

$$\Pi_i^*(g^k) = \frac{(1 - \bar{c})^2 a\left[a(n+1)^2 - (n-k)\right]}{\left[a(n+1)^2 - (n-k)(k+1)\right]^2}$$

We find that the profits in the network of degree $n - 2$ are greater than in the network of degree $n - 1$. In other words, when the profits change in degree, they are not monotonic. They get the maximum value at some intermediate degree; that is, with the increase in the number of contacts, the profits rise first and then fall.

In this section, we deduce the role of the contingent effect of relationship strength from the perspective of relationship scenarios such as cost and time, environmental elements, and cultural norms of social networks. Entrepreneurs can increase their ability to mobilize resources by investing more time and energy in stakeholders to improve communication quality. Resource mobilization in social enterprises differs from that in commercial enterprises. A strong network relationship can enhance the legitimacy of social entrepreneurship, which raises the utilization of resources held by stakeholders. The mutually beneficial symbiotic relationship thus formed also increases stakeholders' acceptance and recognition of social entrepreneurial ventures. Regarding environmental elements, the discovery view suggests that the process of developing social entrepreneurship is full of ecological uncertainties. Environments with high trust enable entrepreneurs to effectively exchange ideas with stakeholders, leverage partners' core competencies to serve the startup's benefit, and improve business performance (Wu and Liu, 2017).

Conversely, the environment is often reflected in the degree of marketization of the area where the business activities are carried out (Marquis et al., 2013). For example, market-oriented regions can influence social entrepreneurs' overall quality of resources (Phillips et al., 2013). In terms of cultural norms, a study by Xu et al. (2021) on the impact of the proportion of Buddhist entrepreneurs on social entrepreneurship based in some regions revealed that the ratio of entrepreneurs believing in Buddhism was positively correlated with the level of prosocial behaviors such as philanthropy. Moreover, they were more likely to establish corporate Buddhist values derived from the Four Immeasurable Minds in less developed regions. These values could encourage philanthropic behaviors and social entrepreneurial activities.

Therefore, we propose the following hypotheses:

H3. Entrepreneurs' social networks positively moderate the relationship between social entrepreneurship opportunity identification and resource bricolage.

H4. Entrepreneurs' social networks positively moderate the mediating relationship of social entrepreneurship opportunity identification that affects business growth through resource bricolage.

Explained in detail, the more robust an entrepreneur's network, the more significant the role of social entrepreneurship opportunity identification in affecting business growth through resource bricolage.

The model constructed in this paper is illustrated in Figure 1.

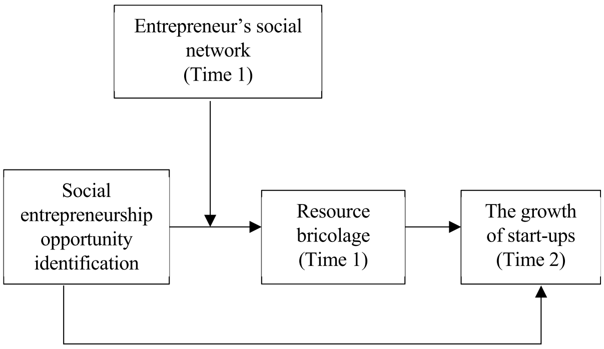

FIGURE 1
Research model.

2 Research design

2.1 Data sources

This study's research data are split into two groups based on the study by Zahra and Wright (2016). The first is nonprofit organizations and businesses that use new business models to improve their social services, and the second is for-profit businesses that meet social needs to improve their competitive advantage and profitability. Organizations and businesses in the Yangtze River Delta and Pearl River Delta were surveyed for this study. To minimize homologation bias, we used an anonymous questionnaire. The questionnaires were distributed in two periods: The first stage (Time 1) was from September to December 2020, and the survey covered background information, social networks, opportunity identification, and resource bricolage of commercial startups. We obtained a list of commercial entrepreneurial organizations with the help of the organizing committee of "The China Social Entrepreneurs Annual Meeting and Social Enterprise Week." We identified 442 new startups that met the definition provided in this article. To increase the reliability and authenticity of the questionnaire, the researchers called the entrepreneurs mentioned above in advance to inform them of the purpose, anonymity, and things to be noted during the study. 264 entrepreneurs were surveyed, and 231 valid questionnaires were collected. The survey's second phase (Time 2) lasted from January to March 2021. The survey sought information on the growth level of enterprises, and the respondents were the entrepreneurs who filled out the valid questionnaires at the first stage. After screening for questionnaires with highly irregular answers and missing data, we recovered 177 valid questionnaires; thus, the recovery rate was 76.62%. Before data analysis, we conducted an independent sample t-test on the questionnaires recovered, employing different techniques, and found no significant difference among them.

2.2 Measurement of variables

Based on the above research hypotheses, we examined the variables of opportunity identification, resource bricolage, social network, and business growth of social entrepreneurs. The questionnaire was scored on a 5-point Likert scale (where "1" means strongly agree and "5" means strongly disagree). This study's design of the measurement scale follows standard practices within the academic community.

2.2.1 Startups' growth performance

Using multidimensional indicators to measure organizational performance is of great significance. The existing growth performance is measured by the increase in "quantity" and "quality" of innovation, including metrics such as solvency, profitability, operational capacity, and growth capacity. However, startups have three growth paths to prioritize economic value, social value, and equal emphasis on economic and social value. Given the above and the commercial attributes of the enterprise, following Pless (2012), we designed a questionnaire that contained seven categories, including "sustained sales growth" and "the increase in the number of beneficiaries," aimed at measuring the growth of startups from the perspectives of economic and social value.

2.2.2 Social entrepreneurship opportunity identification

Based on the implications of social entrepreneurship opportunities, we focus on overall social objectives and treat economic stability as a prerequisite for sustainable success. We used the scale developed by Murphy et al. (1996). We prepared nine questions in the questionnaire, including "My enterprise's new projects can provide products or services that are in short supply in society."

2.2.3 Resource bricolage

Baker and Nelson (2005) indicate that companies can solve problems and find opportunities if they act quickly and make the most of their resources, including physical materials, human resources, skills, and market and institutional systems. Based on the above conceptual framework, we draw on the measurement scale designed by Senyard et al. (2014); the questionnaire consisted of eight questions, including "My company is more innovative than others in using resources at hand."

2.2.4 Entrepreneur's social network

We draw on studies by Yang (1994) and Qiao and Lu (2014) to measure entrepreneurs' social networks through four questions, including "I have close ties with potential or existing suppliers and manufacturers."

TABLE 1 Variable reliability test.

Variables	Measurement indicators	Factor load	C.R. value	AVE	Variables	Measurement indicators	Factor load	C.R. value	Variables
OI	OI1	0.84	0.85	0.72	SN	SN1	0.89	0.71	0.63
	OI2	0.90				SN2	0.81		
	OI3	0.60				SN3	0.85		
	OI4	0.78				SN4	0.75		
	OI5	0.90				SN5	0.86		
	OI6	0.86				SN6	0.82		
	OI7	0.65							
	OI8	0.89							
	OI9	0.84							
RB	RB1	0.64	0.75	0.74	SG	SG1	0.61	0.87	0.81
	RB2	0.61				SG 2	0.75		
	RB3	0.70				SG 3	0.69		
	RB4	0.66				SG 4	0.88		
	RB5	0.82				SG 5	0.78		
	RB6	0.60				SG 6	0.80		
	RB7	0.73				SG 7	0.72		
	RB8	0.66							

Notes: (1) N = 177; (2) OI, opportunity identification; RB, resource bricolage; SN, social network; SG, start-up growth.

TABLE 2 Variable validity tests.

Model	χ2	df	CFI	RMSEA	SRMR
Four-factor model (OI, RB, SN, SG)	853.68	371	0.91	0.10	0.07
Three-factor model (OI + RB, SN, SG)	1101.63	374	0.83	0.14	0.09
Two-factor model (OI + RB, SN + SG)	1771.73	376	0.76	0.15	0.10
One-factor model (OI + RB + SN + SG)	2896.80	377	0.45	0.19	0.13

Notes: (1) N = 177; (2) OI, opportunity identification; RB, resource bricolage; SN, social network; SG, start-up growth.

The background variables, such as gender, age, and education, were control variables.

2.2.5 Reliability and validity test

SPSS 20.0 and MPLUS 7.0 software were used in factor analysis to test the reliability and validity of the above measurement scales. The results are presented in Table 1 and Table 2. Table 1 shows that the combined reliability (C.R. values) of all factors is greater than 0.7 and the average variance extracted (AVE) is greater than 0.5, indicating high internal consistency in the measurement scales according to Hair et al. (2009). Table 2 displays the model fit indices and compares the four-factor measurement model constructed by opportunity identification (OI), resource bricolage (RB), entrepreneur social network (SN), and startup growth (SG) with the three alternative models involving three-factor, two-factor, and one-factor models. The result is that the four-factor model is a better fit for the actual data ($\chi2$ = 853.68; df = 371; CFI = 0.93; RMSEA = 0.10; SRMR = 0.07), and shows better discriminant validity.

2.3 Empirical analysis

2.3.1 Common method bias

We adopted Harman's one-way test to check for common method bias. The exploratory factor analysis was conducted for all the questions on the measurement scale of social opportunity identification, resource bricolage, entrepreneur's social network, and startup growth. The results demonstrated that when the data were not rotated, all the factors with eigenvalues greater than one explained 72.76%, and the unrotated factor explained 32.78% of the data lower than the critical value of 40%. Therefore, we concluded that common method bias did not exist and proceeded with subsequent analyses.

TABLE 3 Means, standard deviation, and correlation of the main variables.

	1	2	3	4	5	6	7
1. Gender	1						
2. Age	0.030	1					
3. Education	0.066	−0.199**	1				
4. OI	0.038	−0.045	−0.083	1			
5. RB	0.054	−0.093	−0.191*	0.581**	1		
6. SG	0.064	−0.147	0.100	0.488**	0.276**	1	
7. SN	0.150*	0.189*	−0.255**	0.309**	0.185*	0.249**	1
Average value	1.5	1.86	2.21	3.80	3.82	3.80	3.94
Standard deviation	0.46	0.77	0.56	0.78	0.56	0.65	0.88

Notes: (1) N = 177, * means $p < 0.05$; ** means $p < 0.01$. (2) OI, opportunity identification; RB, resource bricolage; SN, social network; SG, start-up growth.

TABLE 4 Test of mediating effect.

Variable	Start–up growth				Resource bricolage	
	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Gender	−0.032	−0.048	−0.095	−0.054	−0.094	0.031
Age	0.154**	0.189**	0.089	0.150**	0.085	0.042
Education	0.091	−0.020	0.001	−0.112	−0.076	0.075
OI		0.577***		−0.164	0.639***	0.465***
RB			0.581***	0.758***		
SN						−0.408
OI*SN						0.523**
Model statistics						
R2	0.146	0.181	0.337	0.352	0.417	0.49
R2 adjusted	0.142	0.167	0.333	0.337	0.409	0.475
F	4.142**	12.749***	89.14**	165.822***	50.164***	21.874***
△R2	—	0.035	0.279	0.015	—	0.29
VIF (max)	1.506	1.954	2.538	2.538	1.785	2.285

Notes: (1) N = 177, * means $p < 0.05$; ** means $p < 0.01$; *** means $p < 0.001$. (2) OI, opportunity identification; RB, resource bricolage; SN, social network; SG, start-up growth.

2.3.2 Descriptive statistics and correlation analysis

Before testing the hypotheses, descriptive statistical and correlation analyses were performed on the above variables, and the results are presented in Table 3. Social entrepreneurship opportunity identification is positively correlated with resource bricolage ($r = 0.581$, $p < 0.01$), and social opportunity identification is positively correlated with startup growth ($r = 0.488$, $p < 0.01$). Resource bricolage is positively correlated with startup growth ($r = 0.276$, $p < 0.01$), indicating that the relationship between the variables is consistent with previous theoretical hypotheses. However, regression is needed to verify the hypothesis.

2.3.3 Hypothesis testing

We adopted multiple regression models to test the hypotheses, as shown in Table 4. The regression models are expressed as follows: $Y_{SG} = \beta_{0+}\beta_1 OI_+\beta_2 RB_+\beta_3 SN+\beta_4 SN\bullet\ OI_+\varepsilon$. OI is opportunity identification abbreviations, RB is resource bricolage abbreviations, SN is social network abbreviations, and SG is startup growth abbreviations. The maximum VIF value for each model is 2.538, indicating no severe multicollinearity issues. Nonetheless, the independent and moderating variables are centered before the analysis of interaction effects. Model 1 presents the effect of the control variables on the growth of startups. After the independent variable of entrepreneurship opportunity identification is added, Model 2 shows that entrepreneurship opportunity identification significantly and

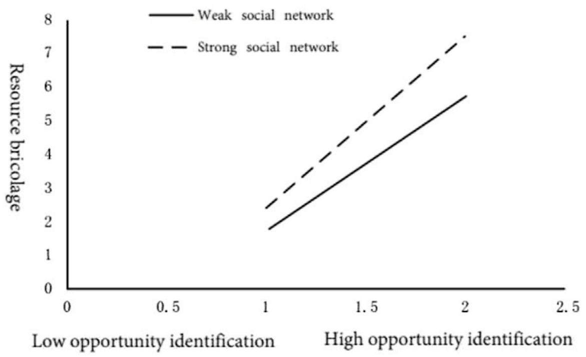

FIGURE 2
Moderating role of entrepreneur's social network between social opportunity identification and resource bricolage.

positively affects the growth of startups (β = 0.577, p < 0.001); hence, H1 is true.

We used the causal steps approach proposed by Baron and Kenny (1986) to test the mediating effect. Model 5 shows that entrepreneurship opportunity identification positively affects resource bricolage (β = 0.639, p < 0.001). Model 3 indicates that the effect of resource bricolage on startup growth is significant (β = 0.581, p < 0.001). We also compared Models 2 and 4 after adding opportunity identification and resource bricolage for regression. We found that the effect of entrepreneurship opportunity identification on startup growth becomes insignificant (β = −0.164), while the positive effect of resource bricolage on startup growth becomes more significant (β = 0.758, p < 0.001). Therefore, resource bricolage fully mediates between entrepreneurship opportunity identification and startup growth, implying that H2 is verified.

To test the moderating effect analysis, we added the control variables, independent variables, moderating variables, and product terms to the regression equation, with resource bricolage as the dependent variable. Model 6 shows that the moderating effect of social networks between resource bricolage and entrepreneurship opportunity identification is significant (β = 0.523, p < 0.01). It suggests that the stronger the entrepreneur's social network, the more significant the relationship between entrepreneurship opportunity identification and resource bricolage. Therefore, H3 is true. To visualize this moderating effect, we draw different effects of entrepreneurship opportunity identification on resource bricolage at different levels of the entrepreneur social network. With one standard deviation above the mean and one standard below the mean as the benchmark, the interaction effect diagram is illustrated in Figure 2.

In H4, we hypothesize that an entrepreneur's social network can indirectly moderate entrepreneurship opportunity identification and growth of startups through resource bricolage. We use Model 4 of the process macro program for conditional process modeling to test this first-stage moderating model with mediation. Table 5 shows the results based on 5,000 bootstrap samples. The findings show that when the entrepreneur has a strong social network, the indirect effect of entrepreneurship opportunity identification affecting startup growth through resource bricolage is 0.336 [Boot 95% CI = (0.235, 0.421)]. When the entrepreneur has a weak social network, the indirect effect of entrepreneurship opportunity identification through resource bricolage on the growth of a social enterprise is 0.279 [Boot 95% CI = (0.153, 0.407)]; the difference between groups is 0.057 [Boot 95% CI = (0.225, 0.388)], reaching the level of significance. In addition, the index calculated by process shows that the determining index of the moderating effect of an entrepreneur's social network on entrepreneurship opportunity identification that indirectly affects the growth of startups is 0.202 [Boot 95% CI = (0.001, 0.083)], with a confidence interval excluding 0. The above results prove the existence of a moderating effect with mediation, and H4 is verified.

3 Conclusion and implications

The opportunity process of social entrepreneurship is cooperative and open rather than closed and individual action. The operating subjects of enterprise social entrepreneurship coexist with multiple factors. Only with the participation of multiple factors and cooperative innovation can the effectiveness and prospect of social entrepreneurship practice be fundamentally improved and the social value of enterprises be realized. Previous studies on social entrepreneurship focus more on the growth of social enterprises, whereas this study focuses on the growth of commercial, entrepreneurial organizations. In this study, we recruited 177 social entrepreneurs as survey subjects and used multiple regression analysis to explore how social entrepreneurship opportunity identification impacts enterprise growth. The findings demonstrated that identifying social entrepreneurship opportunities significantly fosters the expansion of startups.

Social entrepreneurship opportunity identification boosts startups' growth performance by improving resource bricolage. The entrepreneur's social network, which serves as a resource channel, significantly contributes to the mediating effect of resource bricolage, meaning that the more "strong relationship" resources there are in the social network of entrepreneurs, the more influential the impact of social entrepreneurship opportunity identification on business growth performance.

Social entrepreneurship opportunity identification has a significant positive impact on the growth of startups. Although the impact of entrepreneurship opportunity identification on organizational development has been

TABLE 5 Analysis of moderating effect with mediation.

	Opportunity identification → Resource bricolage → Startup growth				
	Grouping of moderating variables	**Indirect effects**	**BootSE**	**Boot LLCI**	**BootULCI**
Conditional indirect effect	Strong social network (+1SD)	0.336	0.052	0.235	0.421
	Weak social network (−1SD)	0.279	0.063	0.153	0.407
	Difference between groups	0.057	0.051	0.225	0.388
Moderation with mediation	Determining Index	Index	BootSE	BootLLCI	BootULCI
		0.202	0.016	0.198	0.247

Notes: (1) 5,000 samplings. (2) LLCI, low level confidence interval; ULCI , up level confidence interval; SE = standard error.

examined in past research, some studies have also explored the relationship between entrepreneurial orientation and business performance by using startups as research subjects. However, prior studies have two shortcomings. First, they did not examine corporate social entrepreneurship from the perspective of economic and social value creation, and thus, no consensus has been reached on the definition of social entrepreneurship opportunities. Second, they lack empirical tests on social entrepreneurship opportunity identification. Therefore, we explored the underlying logic of social entrepreneurship opportunities from economic and social value perspectives and solved the problem of insufficient "cumulative fragmentation" found in previous studies. In addition, social entrepreneurship opportunity identification's impact on the growth of startups responds to what Halberstadt et al. (2021) advocates, deepens the mechanism of social entrepreneurship opportunity identification, and provides new ideas for the relationship between social entrepreneurship opportunities and the high-quality growth of startups.

Second, regarding the origination of resources for social entrepreneurship and the results created, Dwivedi and Weerawardena (2018), as well as Liu et al. (2021), explored the influence of entrepreneurs' social networks on resource bricolage besides opportunity identification. Based on the social network theory, this study supports Baker and Nelson's (2005) assertion that resource bricolage is the process of constructing environmental resources. It also adds to the body of knowledge regarding the link between social entrepreneurial resources and business growth, broadening the definition of social bricolage. The impact of opportunity identification on the growth of startups was finally examined from the perspective of entrepreneurs' social networks; earlier research (Zhao and Tian, 2021) used entrepreneurs' identity as a boundary condition while ignoring social networks, a vital social entrepreneurial resource. Numerous studies have confirmed that entrepreneurs' social networks are an essential condition for the development of startups. Pan and Li (2014) found that the emotional connection of entrepreneurs in entrepreneurship contributes to acquiring entrepreneurial resources. In this study, we developed the boundary conditions of social entrepreneurship opportunity identification for the growth performance of startups, realized the integration of entrepreneurs' social networks and opportunity identification, and enriched the theoretical implications of social entrepreneurship and social networks.

3.1 Entrepreneurial implications

During the initial phase of social entrepreneurship, pursuing economic profits and long-term strategic relationships with informal groups, NGOs, local associations, educational and research institutions, and using one's hybrid identity to create social values is critical. Social entrepreneurship can be difficult initially. Still, the results of this study show that connections such as relatives, marital relatives, clan members, compatriots, friends, classmates, comrades, former subordinates, and former leaders can be utilized and transformed into close and stable relationships. Social entrepreneurship can provide more significant and better social networks, information access, and a greater amount of donations or financial support and help avoid legal and social problems as well as risks of failure arising from the unreasonable integration of external resources. Entrepreneurs need to fully grasp the essence of the social aspect of social entrepreneurship to comprehensively promote the quality and efficiency of startups, explore new approaches to social innovation, and promote high-quality development of social entrepreneurship.

3.2 Limitations and future directions

This study has some shortcomings that need to be improved in future studies. First, although we collected data across a certain period, future research can reveal the process of how social entrepreneurship opportunity identification impacts business growth performance by adopting a tracking method and dynamic simulation. Second, this study focuses on the social network of entrepreneurs and the social entrepreneurship opportunity identification effects on the growth performance of startups. Although the study reveals the startup chain based on the social bricolage theory, whether the startup chain exists in other contexts such as social entrepreneurship and stakeholder involvement and their differences is unclear. Future research may consider cross-case comparative studies among various forms of social entrepreneurship to identify different management strategies in various contexts and examine the boundary roles of social entrepreneurship and stakeholder involvement.

Data availability statement

The original contributions presented in the study are included in the article/Supplementary Material, further inquiries can be directed to the corresponding author.

Ethics statement

Written informed consent was obtained from the individual(s) for the publication of any potentially identifiable images or data included in this article.

Author contributions

All authors listed have made a substantial, direct, and intellectual contribution to the work and approved it for publication.

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Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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The impact of digital transformation on corporate sustainability- new evidence from Chinese listed companies

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As Industry 4.0 is seen as the core industrial stage for achieving sustainable development, more and more scholars are exploring the practical effects of Industry 4.0. This paper evaluates the impact of digital transformation on business sustainability, explores whether digital transformation breaks down perceptions, and examines the mechanisms by which it works. First, we measured the digital transformation of each firm using textual analysis. and found that the coefficient of digital transformation is 0.006 on corporate sustainability at the 1% significant level. Secondly, we found that digital transformation eases knowledge flow barriers and makes knowledge more accessible to firms. Firms with higher digital transformation attract more skilled people, which can create talent barriers. Digital transformation can exacerbate firms' industry monopolies, while increasing the proportion of boardroom women and the inclusion of older members sends positive signals to outsiders. Finally, we find that low costs, high labor productivity, high innovation and low cost of sales are important channels for digital transformation. In addition, digital transformation increases the management costs of firms.

KEYWORDS

digital transformation, textual analysis, knowledge flow barriers, talent barriers, boardroom women, older members

1 Introduction

The concept of Industry 4.0 was firstly introduced in 2011 (Ustundag and Cevikcan, 2017). Industry 4.0 is considered as a new industrial scenario that combines various high technologies through internet of things (IoT) technologies, culminating in an integrated platform that can serve both businesses and consumers, in order to achieve sustainable development (Frank et al., 2019). This new industrial scenario is disrupting traditional corporate business models as well as industry structures (Dregger et al., 2016). However, the transformation process of companies turning into the Industry 4.0 Connected Smart Enterprise from the industrial stage can not succeed easily, and digital transformation is considered a intergrade (Dalenogare et al., 2018). As a new industrial reform, digital transformation is seen as an important stage in achieving corporate sustainability (Kamble et al., 2018; Feroz et al., 2021). According to the Global Environmental Performance Report published by Yale University in 2020 (Chen, 2022), China ranked 120th in terms of environmental performance in the world (out of 180 countries) with a score of 37.3. As one main emitter of pollution, China attaches great importance to the environmental management ability of Chinese enterprises. Therefore, it would be highly valuable to further explore the relationship between digital transformation and corporate sustainability.

The existing literature on digital transformation and sustainability mainly focuses on the literature analysis methods (Feroz et al., 2021), macro-level sustainability (Bieser and Hilty, 2018), industry-level sustainability (Hrustek, 2020) but rarely pays attention to the firm-level sustainability. For example, taking Chinese listed manufacturing firms in 2010–2020 as the objects, Guo and Xu (2021) found that digital transformation is positively related to process-based business performance and non-linearly related to profit-oriented financial performance. Wen et al. (2022) found that digitalization could help companies to innovate and achieve sustainable growth based on data from Chinese listed manufacturing companies. This paper fills a gap in the existing literature by exploring the relationship between digital transformation and corporate sustainability through quantitative analysis. Furthermore, the application of digital technologies such as big data, robotics, the IoT, blockchain, and smart technologies can lead to disruptive changes in competitive momentum, industry structures, innovation approaches, and board structures (Porter and Heppelmann, 2014; Appio et al., 2021). For example, Guo et al. (2022) explored that digital transformation can alleviate industry monopolies based on market competitiveness and firm scale. A missing part in estimating how digitalization can disrupt perceptions is studying innovation approaches and board structures. Digital transformation may affect different stages of the innovation process in a complex and causally ambiguous way (Barrett et al., 2015), providing greater scalability for corporate innovation (Iansiti and Lakhani, 2020). In addition, digital transformation could lead to disruptive changes in the board (Wolfe, 2020), making promotion and vetting mechanisms more transparent and more inclusive to different ages (Noor et al., 2016). Existing literature rarely examines the impact of digital transformation on innovation stages and board structures, so it is of meaning to explore how digital transformation changes innovation approaches and board structures.

Based on data from listed Chinese manufacturing companies in 2010–2019, a double-effects fixed model is used to explore whether digital transformation contributes to the sustainable development. Second, as an unknown corporate strategy, digital transformation varies across different types of firms, and we classify firms into several types in terms of innovation and corporate governance, such as the speed of knowledge flow, technical staff, competitiveness, the proportion of female directors and the average age of board members. Thirdly, considering that digital transformation is disrupting traditional business models, the potential pathways through which digital transformation affects the sustainability were explored in terms of changes in production and distribution patterns, innovation capabilities and firm labor productivity, respectively. Through this study, the following questions are to be answered: 1) Does digital transformation contribute to corporate sustainability? 2) If so, which types of companies are more sensitive to digital transformation? 3) And through which pathways does digital transformation affect corporate sustainability?

The main contributions of this study are as follows: 1) Unlike existing studies that separately discuss the impact of digital transformation on firm performance, innovation or organizational structure. This paper comprehensively measures corporate sustainability from the perspectives of environmental management, environmental control, human resources disclosure, product and consumer disclosure and community involvement. It fills the gap in existing research on the impact of digital transformation on corporate environmental governance. 2) It was found that the attitudes of different types of enterprises towards digital transformation strategies are inconsistent. Namely, digital transformation can alleviate knowledge and skill barriers between firms and mitigate the gap between monopolistic and non-monopolistic firms. In addition, digital transformation is more friendly to women and older directors in the board. This provides strong evidence for the adoption of digital transformation by firms under Industry 4.0 and reduces concerns about the uncertainty of digital transformation in developing countries such as China. 3) As mentioned earlier, digital transformation is disrupting traditional business models. Due to the availability and validity of data, it is difficult to keep track of changes in the production and sales patterns. Therefore, both models were measured in terms of cost of production and cost of sales, based on the fact that changes in production and sales models are ultimately reflected in costs (Hobbs, 2020). Moreover, the potential pathways of the impact of digital transformation on corporate sustainability were explored, offering new perspectives in order for business managers to achieve both corporate environmental and economic performance under the Industry 4.0 process.

The rest of the paper is organized as follows. Section 2 provides the theoretical analysis, Section 3 provides the research methodology and data, while Section 4 provides the results and discussion. Section 5 presents the conclusions and policy implications.

2 Theoretical analysis

2.1 Sustainability and digital transformation measurement

The concept of sustainable development originates from the sustainable forest management (Grober, 2007), which aims to manage forests for the purpose of sustainable yield. Over the years, the concept of sustainable development has evolved to focus more on ecological, economic, political, and cultural objectives (James, 2014). The existing literature focuses on measuring sustainable development at the macro level, such as at the national level (Vachon and Mao, 2008), the provincial level (Yang and Ding, 2018), and the municipal level (Lam and Yap., 2019). Studies on corporate sustainability mainly use ESG ratings as a proxy variable (Jia and Li, 2020) or divide corporate sustainability activities into sub-themes (Gray et al., 1995; Katmon et al., 2019). For example, Branco and Rodrigues (2008) categorized corporate sustainability activities into 23 items and divided them into several categories such as environmental, human resources, products, customers, and community involvement. Considering that ESG ratings in China are new and imperfectly developed, this paper refers to the sustainability evaluation system proposed by Zaid et al. (2020), combined with corporate disclosure in China, categorizes it into categories including environmental management, environmental regulation, environmental governance, human resources, consumer and community involvement.

Digital transformation is a complex process and one variable is difficult to proxy for it (Matt et al., 2015). Some scholars have analyzed only single technology, for example, robotics, the Internet of Things (IoT), blockchain, and big data (Ballestar et al., 2021; Chin et al., 2021). For example, Chin et al. (2021) explored the impact of blockchain on innovation. Other scholars decomposed digitalization into multiple technologies (Ballestar et al., 2021) or used the proportion of R&D investment and innovation output as a proxy variable for digital transformation (Jafari-Sadeghi et al., 2021). This paper uses textual analysis to measure digital transformation for firms disclose more information in the form of text.

2.2 Digital transformation and corporate sustainability

Digital transformation is seen as a key way to achieve sustainable development (Andriushchenko et al., 2020). Digital technology achieves sustainable development by reducing costs, improving management efficiency and improving labor productivity. Existing research on digital transformation has focused on employment, labor (Ballestar et al., 2021), productivity (Guo et al., 2022), and innovation (Abdalla and Nakagawa, 2021), while has rarely explored corporate sustainability. In a recent study, Jiao and Sun (2021) used urban data and found that the digital economy can improve economic development and employment to achieve sustainable urban development. Zuo et al. (2021) found that digital transformation had a significant increase in the sustainable efficiency of technology investment in the banking sector. It is worth noting that all these studies focus on the macro level, so it is meaningful to explore the impact of digital transformation on sustainability at the firm level to fill the gaps in existing literature.

2.3 Digital transformation disrupts perceptions

Digital transformation is disrupting perceptions (Rasiwala and Kohli, 2021). Digital transformation may disrupt existing organisational rules and structures (Philip and Gavrilova Aguilar., 2022), changing traditional competitive patterns and business models (Nadkarni and Prügl, 2021; Vaska et al., 2021; Chen and Hao, 2022). Some studies have found that digital transformation can strengthen the market position of monopolies (Soto Setzke et al., 2021) and weaken the differences brought about by firm scale (Hu, 2022). This paper focuses more on innovation structures and changes in the organizational structure of firms, such as knowledge flows, technical staff, and board structures. Based on organizational capability theory, strategic organizational activities require external environmental resources and the ability to deal with the external environment. Digital transformation can accelerate the flow of information, knowledge (Hao and Zhang, 2021). On one hand, senior digital technologies can convert data into digital resources in symbolic formats for communication; on the other hand, digital technologies can reduce the cost of storing knowledge (Gomber et al., 2018; Chen, 2022). Technical staff are one of the key factors influencing innovation in firms, which is rarely considered in existing researches on digital transformation. The digital construction of a company brings more than just efficiency, it also requires a high level of technical skills among employees (Trenerry et al., 2021). Does this give rise to a talent monopoly in companies? It is worth thinking about.

Board diversity is one of the most important factors influencing corporate sustainability (Zaid et al., 2020), and in the same way, digital transformation is changing the organisational structure and board diversity of firms (Gfrerer et al., 2021). On one hand, emerging technologies promote a culture of gender equality, and digital technologies can increase 'information centralisation' and public access to information (Antonio and Tuffley, 2014). On the other hand, digital technologies can exacerbate gender dichotomies (Dragiewicz et al., 2018) and widen perceptions between men and women, such as health issues and risk awareness (Kohlrausch and Weber, 2021). Age diversity is also an important part of board diversity.

Some studies suggest that younger directors are more sensitive to emerging technologies and can respond positively to changes in the external environment (Hsu et al., 2019). However, it is worth noting that digital transformation entails unpredictable risks (Verhoef et al., 2021), which can be mitigated by older directors' relatively rich managerial experience and resource relationships net (Chen, 2011). Gender and age equality are essential links, and it is worth thinking about how digital transformation, as an important means of achieving sustainable development, can affect gender and age equality. Although available data does not allow us to capture the age and gender of a company's employees, changes in board structure can influence overall corporate decision-making results and send different signals to the society (Certo, 2003; Mallin and Michelon, 2011). By looking at the impact of digital transformation on board gender and age members, this paper explores whether it promotes gender and age equalisation, which is relevant for achieving gender equality and addressing aging issues.

3 Data and model setting

3.1 Data

This paper uses Chinese A-share listed manufacturing companies from 2009 to 2019 as a sample, using corporate financial data from the China Securities Market and Accounting Research Database (CSMAR) and the WinGo Text Analytics Database (wingodata.com). The sample study year of this paper starts from 2009, as key data are disclosed from 2009 onwards. Considering that ST&ST* firms are often in a state of poor operating income, these extreme values can affect the regression results. Therefore, we present these enterprises. In addition, to make up for some missing data, we supplemented them with the wind database and manually reviewed annual reports. We ended up with 16,159 firm-year observations.

3.1.1 Dependent variables

We refer to existing studies on corporate sustainability, some of which use ESG indexes to measure corporate sustainability (Drempetic et al., 2020; Jia and Li, 2020), while others construct sustainability evaluation systems through disclosure information of companies (Madaleno and Vieira, 2020; Zaid et al., 2020). As the ESG index of Chinese listed manufacturing companies is seriously missing, this paper adopts the approach of Zaid et al. (2020) and constructs a new corporate sustainability evaluation system by combining the information disclosed by Chinese listed companies (Appendix Table A1). In this study, we use a dichotomous approach to score corporate sustainability. Then, according to Zaid et al. (2020), the sustainability index for each firm is calculated as follows:

$$CS_{it} = \frac{\sum_{i=1}^{26} X_{it}}{26} \tag{1}$$

Where CS_{it} is corporate i's sustainability score in year t 26 is the maximum number of items for every firm. And $X_{it} = 1$ if the item is disclosed in t year, otherwise $X_{it} = 1$.

3.1.2 Independent variables

As it is difficult to measure the digital transformation of enterprises through quantitative analysis, this paper uses textual analysis to measure the digital transformation (Guo et al., 2022; Zhai et al., 2022). Unlike Zhai et al. (2022), who examined only five keywords related to digital transformation (digitalization, artificial intelligence, big data, Internet of Things, cloud computing). This paper constructs nine keywords to describe the digital transformation, namely, big data, informatization, intelligence, robotics, Internet of Things, blockchain, automation, digitalization, cloud computing. We count the number of occurrences of these nine keywords and then, the logarithm of the number of keywords +1 to measure the digital transformation (Zhai et al., 2022).

3.1.3 Mediation variables

We analyze the mechanism of the corporate sustainability in terms of three aspects: costs, labor productivity, and innovation. In this paper, the ratio of production cost to total assets (CoA) is used to measure the change of firm selling expense (Khalifaturofi'ah, 2018). We measure labor productivity in terms of a firm's value added per unit of labor (VAL) (Tang, 2017). This data is taken from the CSMAR database on the topic of enterprise EVA (economic value added), which is equal to operating profit - income tax expense + [interest expense (non-financial institutions) + asset impairment losses + R&D expenses] * (1 - corporate income tax rate) + increase in deferred income tax liabilities - increase in deferred income tax assets. And use the logarithm of the R&D investment (RD) to measure a firm innovation (Santacreu and Zhu, 2018).

3.1.4 Control variables

With reference to previous empirical studies on firms (Liu et al., 2020), the following control variables are introduced in this paper. These variables include ratio of net fixed assets to total assets (Fix), return on assets (ROA), logarithm of total employees (Size), logarithm of firm age (Age) and ratio of total liabilities to total assets (Lev).

3.2 Model setting

To test the impact of digital transformation on the corporate sustainability, the baseline model is constructed as follows:

TABLE 1 The distribution of the mean values by keyword.

Variable	BD	IT	Int	Rob	IoT	Block	Auto	Dig	CC
2009	0	0.95	1.24	0.24	0.09	0	1.54	0.232	0.01
2010	0	1.27	2.02	0.24	0.23	0	1.83	0.20	0.07
2011	0.001	1.71	2.81	0.22	0.38	0	1.88	0.23	0.13
2012	0.02	1.83	2.88	0.28	0.33	0	2.04	0.25	0.11
2013	0.13	1.99	3.52	0.52	0.29	0	2.34	0.25	0.11
2014	0.33	2.29	4.77	0.70	0.5	0	2.37	0.26	0.23
2015	0.80	2.63	7.89	1.44	0.66	0.001	3.11	0.40	0.34
2016	1.05	3.07	10.39	1.78	0.89	0.02	3.41	0.50	0.43
2017	1.19	3.39	13.22	1.92	1.13	0.08	3.92	0.69	0.39
2018	1.36	3.89	16.99	2.51	1.65	0.09	4.63	1.00	0.45
2019	1.42	3.67	17.07	2.27	1.73	0.15	4.17	1.46	0.50

Note: BD, Big data; IT, information technology; Int, intelligence; Rob, robotics; IoT, Internet of Things; Block, blockchain; Auto, automation; Dig, digitalization; CC, cloud computing.

TABLE 2 Descriptive statistics.

Variables	Obs	Mean	SD	Min	Max
CS	16,159	0.312	0.252	0	1.154
DT	16,159	1.959	1.379	0	6.864
CoA	16,157	0.618	0.451	0	16.749
VAL	16,147	17.650	0.002	17.640	18.451
Pat	16,159	2.694	1.617	0	6.709
Age	16,159	2.695	0.423	0	3.951
Size	16,153	7.709	1.170	0.693	12.342
ROA	16,159	0.042	0.324	-0.239	0.201
Lev	16,159	0.409	0.556	0.007	42.857
Fix	16,159	0.237	0.260	0	27.907

Note: In the case of the "VAL" variable, there was a case of having a negative value, in which case the log value was taken after applying the method ($VAL^{*} = VAL + |\min\ (VAL)| + 1$) proposed by Wicklin (2011).

$$CS_{i,t} = \alpha_0 + \beta DT_{i,t} + \sum_{i=1}^{5} \gamma_i control + \alpha_i + \delta_t + \epsilon_{i,t} \qquad (2)$$

Where CS_{it} is a variable for corporate i's sustainability in year t. The variable of interest is DT_{it} is the logarithm of the number of keywords +1 to measure the digital transformation. α_i and δ_t are vectors of firm and year dummy variables that account for firm and year fixed effects to remove the interference of un-observables in the regression results. The $\epsilon_{i,t}$ is the error term.

4 Empirical results and analysis

4.1 Frequency distribution and descriptive statistics

Table 1 is the distribution of the mean values by keyword over the years. It can show that the digital conversion is gradually deepening, and it is worth noting that the frequency of each keyword has increased substantially since 2012. One possible explanation is that since 2012 the Chinese government has issued a smart city pilot policy, referring to the use of various information technologies to improve resource use efficiency (Fan et al., 2021).

Table 2 provides descriptive statistical analyses for all variables, which present the mean, standard deviation, and minimum and maximum values for the variables. In the sample data, the mean, minimum and maximum values for corporate sustainability are 0.312, 0, and 1.154, indicating a large variation between corporate sustainability. The mean, minimum and maximum values for digital conversion are 1.959, 0, and 6.864, indicating that most firms are at a low digital stage. The mean, minimum and maximum values for the other variables show that there is a large variation between firms. In addition, we used the variance inflation factor (VIF) to test multicollinearity between the variables. The test results show that the VIF values for all variables are less than 10, with an average VIF value of 1.22, indicating that the impact of multicollinearity is negligible on the interpretation of the regression results.

4.2 Analysis of regression results

Table 3, column 1) and 2) provide the regression results between digital transformation and corporate sustainability without controlling individual effects and with controlling individual effects, respectively. Digital transformation has effects on firms, which may reinforce advantages or moderate unbalances based on their firm characteristics. We set up several dummy variables to test whether digital transformation mitigates knowledge flow barriers, technician outflow, industry monopolies, boardroom female disadvantage, and boardroom advanced age. Column 3) is a dummy variable for knowledge flow barriers, namely, KFB = 1 if the number of patent citations

TABLE 3 Regression results.

Variables	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	BR		KFB	TM	HHI	SEX	H-Mage	L-Mage
	CS	CS	CS	CS	CS	CS	CS	CS
DT_KFB	—	—	0.006***	—	—	—	—	—
	—	—	(0.002)	—	—	—	—	—
DT_TM	—	—	—	0.002*	—	—	—	—
	—	—	—	(0.001)	—	—	—	—
DT_HHI	—	—	—	—	0.002*	—	—	—
	—	—	—	—	(0.001)	—	—	—
DT_SEX	—	—	—	—	—	0.006***	—	—
	—	—	—	—	—	(0.002)	—	—
DT	0.009***	0.006***	—	—	—	—	0.010***	0.005**
	(0.001)	(0.002)	—	—	—	—	(0.003)	(0.003)
Age	0.026***	0.039***	0.039***	0.039***	0.040***	0.039***	0.083***	0.024
	(0.005)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.022)	(0.015)
Size	0.078***	0.027***	0.027***	0.027***	0.028***	0.027***	0.031***	0.022***
	(0.002)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.004)	(0.004)
ROA	0.015***	0.006	0.006	0.006	0.006	0.006	0.027*	0.002
	(0.006)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.015)	(0.005)
Lev	0.002	0.001	0.001	0.001	0.002	0.001	0.008	0.0003
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.017)	(0.004)
Fix	0.190***	0.014	0.014	0.012	0.011	0.014	0.017	0.0004
	(0.013)	(0.015)	(0.015)	(0.015)	(0.015)	(0.015)	(0.024)	(0.022)
C	-0.391***	-0.181***	-0.181***	-0.178***	-0.188***	-0.181***	-0.301***	-0.139***
	(0.016)	(0.030)	(0.023)	(0.030)	(0.030)	(0.030)	(0.060)	(0.039)
Firm	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Obs	16,153	16,153	16,153	16,153	16,153	16,153	8,224	7,929
R-sq	0.274	0.753	0.753	0.750	0.753	0.753	0.797	0.748

Note: *** indicates significance at the $p < 0.01$, ** indicates significance at the $p < 0.05$, * indicates significance at the $p < 0.1$.

is greater than the median, otherwise, KFB = 0. The greater the number of patent citations, the lower the barriers to knowledge flow between firms. Column 4) is a dummy variable for technician flow. TM = 1 means that technicians are flowing in, TM = 0 means that technicians are flowing out. Column 5) is a dummy variable for industry competitiveness. We calculated the industry-level Herfindahl-Hirschman index (HHI) by operating income. HHI = 1, if this is greater than the median, which indicates that the firm is in a monopoly position. Structural changes in the board can affect strategic decisions throughout the firm (Sidhu et al., 2021). Column 6) is a dummy variable for the proportion of boardroom women. SEX = 1 when the proportion of boardroom women is greater than the median, otherwise, SEX = 0, where the proportion of boardroom women is [0, 66.67]. Column 7) and 8) represent dummy variables for the boardroom mean age. Mage = 1 if the mean age is greater than the median, otherwise, Mage = 0.

The impact of digital transformation on corporate sustainability is consistent whether individual effects are controlled or not. Taking the results in column 2) as an example, the coefficient of digital transformation is 0.006 on corporate sustainability at the 1% significant level. This complements the findings of Sheina et al. (2019) who found that smart city building can significantly improve urban sustainability. We have extended the study to the firm level to enrich the theoretical foundation.

The results in column 3) show that the coefficient of the cross-term between digital transformation and KFB is 0.006 at the 1% significant level, which means that one standard error (1.379) increase in DT improves CS of firms with high patent citations by 0.827% (1.379 × 0.006) than that of firms with low citations. Namely, digital transformation can enhance knowledge exchange between firms and thus alleviate the knowledge flow barriers (Thompson, 2006). One possible explanation is that digital transformation can help firms to have access easier and timely to external innovation information (Moi and Cabiddu, 2021), or strengthen the links between organizations to collaborative innovation (Castagna et al., 2020). This is similar to the findings of Abdalla and Nakagawa (2021), who found that digital transformation can enhance innovation collaboration between firms and alleviate knowledge flow barriers.

The results in column 4) show that the coefficient of the cross-term between digital transformation and TM is 0.002 at the 10% significant level, which means that one standard error increase in DT improves CS of firms with talent inflow by 0.276% than that of firms with talent outflow. In other words, digital transformation may create a talent flow barrier, and the higher the digital transformation of a company, the more it attracts talent inflows. One possible explanation is that technical talent needs a better working environment (Perry, 2001), and the changes brought about by digital transformation meet this requirement (Okrepilov et al., 2019). This is contrary to the findings of Ballestar et al. (2021), who found that digital transformation had a significant negative impact on employment. In contrast, this paper finds that digital transformation can attract an influx of skilled talent, bridging the gap in existing research.

The results in column 5) show that the coefficient of the cross-term between digital transformation and HHI is 0.002 at the 10% significant level, namely, one standard error increase in DT improves CS of firms with high HHI by 0.276% than that of firms with low HHI. This means that firms with high HHI I have a unique market advantage, and that digital transformation further strengthens this monopoly. This is consistent with the findings of Guo et al. (2022), who found that digital transformation exacerbates the monopoly position of firms.

The results in column 6) show that the coefficient of the cross-term between digital transformation and SEX is 0.006 at the 1% significant level, namely, one standard error increase in DT improves CS of firms with more boardroom female by 0.827% than that of firms with low boardroom female. This implies that firms with high digital transformation can attract more professional women to their boards. On the one hand, digital transformation requires flexible thinking (Verhoef et al., 2021) and female directors can moderate communication within, between organizations and between organizations and individuals (Pucheta-Martínez et al., 2019). On the other hand, digital transformation can make promotion and performance appraisal mechanisms more transparent and standardized, potentially reducing women's disadvantages (Chen et al., 2021). This may send positive signals to society, encouraging more women to join (Terjesen et al., 2016) and thus improve the corporate image (Reguera-Alvarado et al., 2017). This is in line with the findings of Chen and Hao (2022), who found that digital transformation can significantly boost women's career development. This refutes role congruity theory of prejudice toward female leaders (Eagly and Karau, 2002), where women are less likely to be leaders and less likely to succeed in leadership positions. Instead, this paper argues that digital transformation can mitigate workplace gender discrimination and that an increase in boardroom women will promote corporate sustainability.

The results in columns 7) and 8) show that the coefficient of the digital transformation is 0.010 for firms with high boardroom age and 0.005 for firms with high boardroom age at the 1% and 5% significant level, namely, the digital transformation can accommodate more older board members. This refutes the management signaling hypothesis (Arioglu, 2021), where younger directors, driven by reputational concerns, want to demonstrate their value to the market and are thus more active and adventurous. One possible explanation is that older directors use their connections to have easier access to key resources (e.g., IT or raw materials) that help firms to digitize quickly (Miller and Triana, 2009). Or it could be that younger directors increase board risk (Berger et al., 2014), while older directors are more mature and stable (Ali et al., 2014), which could hedge some of the risks that may be involved in the digital transformation of a firm. This finding is very significant in that the changing boardroom age under digital transformation may send a positive signal to society to attract more skilled older employees to join and thus alleviate the employment difficulties associated with ageing.

4.3 Analysis of mechanisms

Digital technologies have a transformative or disruptive impact on companies (Boulton, 2020). Integrating different processes and resources to improve quality and efficiency through IoT, cloud services, big data and analytics (Frank et al., 2019). Based on the characteristics of digital transformation, we explain the mechanisms of its effects through the perspectives of cost (Zeng and Lei, 2021), labor productivity (Ballestar et al., 2021), and R&D investment (Guo et al., 2022). Namely, lower costs, higher labor productivity, and more innovative outputs can contribute to corporate sustainable growth. Matt et al. (2015) and Kraus et al. (2021) mention that digital transformation may affect firms' sales patterns and management expense, however, the existing literature has not verified this, so we test these two paths of action. We use the ratio of management expense to total assets (MoA) measures the change in a firm's management expense (Setyawati, 2017), such as technology transfer fees, software, hardware, and training cost. The ratio of selling expenses in total assets (SoA) measures the change in a firm's selling expenses (Jiraporn and Chintrakarn, 2013) in Table 4.

The regression results in columns 1), 2) and 3) find that digital transformation can reduce production costs and increase labor productivity and innovation output. This is consistent with the findings of Zeng and Lei (2021), Wen et al. (2022). Digital transformation can reduce unnecessary costs in corporate management and optimize internal resource mobilization. Secondly, digital transformation implies that firms use more and more intelligent technologies to improve work efficiency. Finally,

TABLE 4 Analysis of mechanisms.

Variables	(1)	(2)	(3)	(4)	(5)
	CoA	VAL	RD	MoA	SoA
DT	-0.015***	0.0001**	0.033**	0.0008*	-0.0016***
	(0.003)	(0.0002)	(0.009)	(0.0004)	(0.0003)
Age	0.162***	-0.0006**	-0.488***	0.007**	0.004**
	(0.021)	(0.0002)	(0.060)	(0.003)	(0.002)
Size	0.019***	0.0002***	0.650***	-0.016***	0.004***
	(0.005)	(0.00003)	(0.016)	(0.0007)	(0.0005)
ROA	0.072***	0.0008***	-0.450***	-0.005***	0.005***
	(0.010)	(0.0001)	(0.048)	(0.001)	(0.001)
Lev	0.027***	0.0002*	-0.518***	0.007***	-0.0002
	(0.005)	(0.00004)	(0.028)	(0.0007)	(0.0005)
Fix	0.138***	-0.004**	-0.465***	0.043***	0.009***
	(0.029)	(0.0002)	(0.088)	(0.004)	(0.003)
C	0.073	17.651**	12.699***	0.176***	0.009
	(0.056)	(0.0004)	(0.165)	(0.008)	(0.006)
Firm&Year	Yes	Yes	Yes	Yes	Yes
Obs	16,151	16,147	16,153	16,151	16,147
R-sq	0.862	0.476	0.840	0.725	0.860

Note: *** indicates significance at the $p < 0.01$, ** indicates significance at the $p < 0.05$, * indicates significance at the $p < 0.1$.

digital transformation can help companies to access external R&D information more easily and attract technical talent.

The regression results in columns 4) and 5) find that digital transformation can increase overheads and reduce selling expenses. This validates Setyawati (2017) conjecture that high management costs may hinder the digital transformation of firms and Mattila et al. (2021) opinion that digital transformation would change the sales model to reduce the costs under the traditional sales model. Firstly, digital transformation is seen as the process of transforming a firm from its previous industrial phase to a connected and intelligent industry 4.0 firm, which can be affected by high software costs, hardware costs, and maintenance costs. Secondly, digital transformation involves the creation of special channels among companies, suppliers, and consumers, such as the Internet of Things and blockchain, which can significantly reduce the selling cost, all of which can contribute to corporate sustainability (Wen et al., 2022).

4.4 Selection bias and endogeneity issues

In general, a firm's choice of geographical location is not random and many factors can influence the choice of firm location, which leads to the problem of selection bias (Cader and Leatherman, 2011). In this paper, we choose the Heckman two-stage model (Heckman and Vytlacil, 1998) to address the endogeneity issues. Second, there is little literature exploring the endogeneity of digital transformation and to mitigate potential endogeneity, we use an instrumental variables approach. We use the number of Internet users in each province to indicate the technological environment in which a firm exists. This data from the statistical yearbooks of each province. Finally, considering the possibility of reverse causality, we used the benchmark natural experiment quasi natural experiment of Broadband China, Broadband China was promulgated in 2014 to promote the digitalization of enterprises and cities through Internet infrastructure development. The Broadband China strategy was promulgated for pilot cities in 2014, 2015 and 2016 respectively, and we use this strategy as a natural experimental group (Zhao et al., 2022) in Table 5.

TABLE 5 Selection bias and endogeneity issues.

Variables	(1)	(2)	(3)	(4)
	DT	CS	CS	CS
DT	—	0.006***	0.109**	0.023***
	—	(0.002)	(0.051)	(0.008)
Age	-0.368***	0.067***	0.031*	0.063***
	(0.029)	(0.016)	(0.018)	(0.019)
Size	0.175***	0.018***	0.0006	0.017***
	(0.010)	(0.004)	(0.008)	(0.004)
ROA	-0.096	0.013***	4.29e-05	-0.010
	(0.086)	(0.005)	(0.006)	(0.011)
Lev	-0.258***	0.020**	-0.0006	-0.018*
	(0.050)	(0.008)	(0.003)	(0.010)
Fix	-1.931***	0.110***	0.037	-0.038
	(0.081)	(0.041)	(0.036)	(0.025)
Lambda	—	-0.076**	—	—
	—	(0.030)	—	—
C	-0.827***	0.002	—	-0.111**
	(0.107)	(0.034)	—	(0.051)
LM statistic	—	—	19.958***	—
	—	—	(0.000)	—
Weak identification test	—	—	19.975	—
Firm&Year	Yes	Yes	Yes	Yes
Obs	16,153	16,153	10,022	5,678
R-squared	—	0.753	0.138	0.761

Note: *** indicates significance at the $p < 0.01$, ** indicates significance at the $p < 0.05$, * indicates significance at the $p < 0.1$.

Zhang et al. (2020) mentioned that PSM(Propensity Score Matching) and DID (Differences-in-Differences) methods handle the reverse causal relationship. Zhang et al. (2020) showed that the PSM approach (propensity score matching) can match treatment and control groups by pre-policy city characteristics, handling the reverse causal relationship. We matched two sub-groups using nearest neighbor matching using pre-policy corporate characteristics. This was eventually

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TABLE 6 Robustness checks.

Variables	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	BR	KFB	TM	HHI	SEX	H-Mage	L-Mage
	CS	CS	CS	CS	CS	CS	CS
BAP_KFB	—	0.001*	—	—	—	—	—
	—	(0.001)	—	—	—	—	—
BAP_TM	—	—	0.0003*	—	—	—	—
	—	—	(0.000)	—	—	—	—
BAP_HHI	—	—	—	0.002**	—	—	—
	—	—	—	(0.001)	—	—	—
BAP_SEX	—	—	—	—	0.013*	—	—
	—	—	—	—	(0.010)	—	—
BAP	0.013***	—	—	—	—	0.002**	0.001*
	(0.010)	—	—	—	—	(0.017)	(0.015)
Age	0.039***	0.038***	0.038***	0.041***	0.039***	0.043***	0.059**
	(0.013)	(0.013)	(0.013)	(0.013)	(0.013)	(0.017)	(0.026)
Size	0.026***	0.026***	0.026***	0.026***	0.026***	0.020***	0.033***
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.004)	(0.005)
ROA	0.000	0.000	0.000	0.000	0.000	-0.014	0.025
	(0.009)	(0.009)	(0.009)	(0.009)	(0.009)	(0.011)	(0.016)
Lev	-0.008	-0.008	-0.008	-0.008	-0.008	-0.020*	0.018
	(0.008)	(0.008)	(0.008)	(0.008)	(0.008)	(0.010)	(0.021)
Fix	-0.006	-0.005	-0.005	-0.006	-0.006	-0.018	-0.028
	(0.018)	(0.018)	(0.018)	(0.018)	(0.018)	(0.026)	(0.030)
C	-0.228***	-0.136***	-0.141***	-0.158***	-0.228***	-0.133	-0.228*
	(0.079)	(0.036)	(0.035)	(0.035)	(0.079)	(0.124)	(0.123)
Firm&Year	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Obs	11,278	11,278	11,278	11,278	11,278	5,635	5,643
R-sq	0.746	0.746	0.746	0.746	0.746	0.737	0.788

Note: *** indicates significance at the $p < 0.01$, ** indicates significance at the $p < 0.05$, * indicates significance at the $p < 0.1$.

reduced from 16,159 firm-year observations in the original sample to 5,678 firm-year observations. Appendix Table A2 shows a significant reduction in the standard deviation between the treatment and control groups after matching. This implies that selection bias was eliminated through the PSM. According to Appendix Figure A1, after matching, the PSM probability densities of the experimental and control groups were close, and the matching effect in this paper was good.

Columns 1) and 2) are the regression results of the Heckman two-stage model. The Lambda coefficient in column 2) is statistically significant, which indicates the presence of selection bias. The regression results for DT in column 2) are consistent with the previous regression results. The LM statistics used to test whether the selected IV and endogenous variables are correlated in the lower stratum of column 3) are all significant at the 1% level, indicating that the under-identification hypothesis is rejected. In addition, the weak identification test indicates that the weak instrumentality problem can be eliminated. The above statistics indicate that our choice of instrumental variables is valid. The regression results show that digital transformation can contribute to corporate sustainability. This means that the above regression results remain valid after the endogeneity problem has been overcome. In addition, in the Appendix Table A1, A2, we provide the results of the PSM approach test. addressing reverse causality, we found that broadband China can contribute to corporate sustainable development using the PSM-DID approach in column (4). This is consistent with our results.

4.5 Robustness checks

To test the robustness of the results, we used the logarithm of the internet broadband access port (BAP) as a proxy variable for digital transformation (Lee et al., 2022). The results in Table 6 show consistency with the previous results. Therefore, it can be confirmed that our test results are robust.

5 Conclusion and policy implications

This paper explores the relationship between digital transformation and corporate sustainability based on listed Chinese manufacturing companies in 2010–2019. The corporate sustainability were measured from the perspectives of environmental management, environmental control, human resources disclosure, product and consumer disclosure, and community involvement. Moreover, the firm digital transformation was measured by using textual analysis. Firstly, it was found that digital transformation contributes to corporate sustainability. Secondly, based on the fact that digital transformation is disrupting traditional organizational structures and business models, it was found that digital transformation can mitigate the disadvantages of knowledge flow barriers, technician staff exodus, industry monopolies, female board disadvantages and advanced board directors' aging issues, providing new impetus for corporate sustainability. Finally, the mechanisms of digital transformation were explored in terms of production cost, labor productivity, innovation, and cost of management and sales. It was found that low production costs, high labor productivity, high innovation and low cost of sales are important channels between digital transformation and corporate sustainability.

This paper provides an empirical basis and unique theoretical insights to promote Industry 4.0 construction and corporate sustainable development in emerging developing countries. Based on the findings, the following recommendations are proposed. For governments policy makers: Governments shall increase the construction of digital technology infrastructure such as the Internet, blockchain and big data to accelerate the Industry 4.0 process. Secondly, the government shall recognize that different types of companies have different attitudes towards digital transformation strategies, and take an institutional perspective to break down regional market barriers, as well as the barriers of mobility of knowledge and technology talents, achieving the sustainable development. A healthy knowledge protection system should be established to ensure a smooth flow of knowledge. Encourage corporate R&D collaboration to counteract the negative effects of talent shortages and provide some policy support for companies that are disadvantaged in the market to follow the principle of equality in a market economy. For business managers: Firstly, business managers should recognize the benefits of digital transformation for sustainable development and actively adopt a digital transformation strategy, for example, set up a digital office to effectively guide the digital transformation of the company. Secondly, they should understand that digital transformation is disrupting traditional views, blurring traditional business boundaries and facilitating the flow of knowledge, skilled people, female employees, senior staff and other elements. Finally, despite the increased costs of software, equipment, etc., digital transformation has a positive impact on production costs, sales costs and innovation. Business managers should establish a sound system of resource utilization, production chains and supply chains to realize the maximum effect of digital transformation.

There are also a few limitations to this paper. Firstly, due to the limitations of the study data, we were unable to capture the dynamics and long-term effects of digital transformation. Secondly, we have only explored the changes to traditional business models as a result of digital transformation, but ignored firm and regional heterogeneity, for example, small, medium and large enterprises, state-owned and non-state-owned enterprises, polluting and non-polluting enterprises, coastal and non-coastal enterprises, etc. Thirdly, corporate management characteristics are also important factors influencing digital transformation strategies. For example, overseas background of the board, educational background of the board, and foreign directors on the board. Therefore, we will keep exploring the relationship between digital transformation and corporate sustainability in depth in our future research.

Data availability statement

The data analyzed in this study is subject to the following licenses/restrictions: The datasets generated during and/or analysed during the current study are available in the CSMAR (China Stock Market and Accounting Research Database). Requests to access these datasets should be directed to https://www.gtarsc.com/.

Author contributions

PC, YH, and CZ wrote, edited and revised the text, created and edited figures and tables. PC and CZ contributed analysis and figures and edited. YH revised the manuscript. All authors contributed to the tables, wrote portions of the text, and edited the manuscript.

Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

Publisher's note

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Appendix A:

TABLE A1 Corporate sustainability disclosure items.

Category	NO	Items
Environmental management	1	Environmental goals
	2	Environmental management system
	3	Special environmental action
	4	Environmental incident response mechanism
	5	Three-simultaneous system
	6	Environment and sustainable development
Environmental control	7	Exhaust gas abatement control
	8	Wastewater reduction control
	9	Dust, fume control
	10	Solid waste utilization and disposal
	11	Noise, light pollution, radiation control
	12	Cleaner production facilities
Human resources disclosure	13	Shareholder rights protection
	14	Creditor rights protection
	15	Employee rights protection
	16	Education and training
	17	Number of employees
	18	safety production
Product and consumer disclosure	19	Supplier rights protection
	20	Product quality information
	21	Customer rights protection
	22	Consumer complaints and satisfaction
Community involvement disclosure	23	Charitable donations and activities
	24	Sponsoring conferences, seminars or exhibits
	25	Public relations and social welfare
	26	Social responsibility system building and improvement measures

TABLE A2 Balancing test.

Variable	Sample	Mean		Bias (%)	Reduct |bias|(%)	t-test	
		Treated	Control			t	p>|t|
Age	Unmatched	2.638	2.607	7.9	23.8	1.38	0.167
	Matched	2.638	2.615	6.0		0.78	0.435
Size	Unmatched	7.704	7.837	-11.6	-7.8	-2.02	0.044
	Matched	7.673	7.817	-12.5		-1.62	0.106
ROA	Unmatched	0.048	0.040	4.3	38.4	0.68	0.499
	Matched	0.035	0.042	-2.6		-1.17	0.243
Lev	Unmatched	0.398	0.419	-9.4	15.8	-1.63	0.103
	Matched	0.399	0.416	-7.9		-1.06	0.288
Fix	Unmatched	0.236	0.273	-26.4	78.6	-4.67	0.000
	Matched	0.254	0.262	-5.7		-0.79	0.427

Note: All covariates were measured at pre-treatment period.

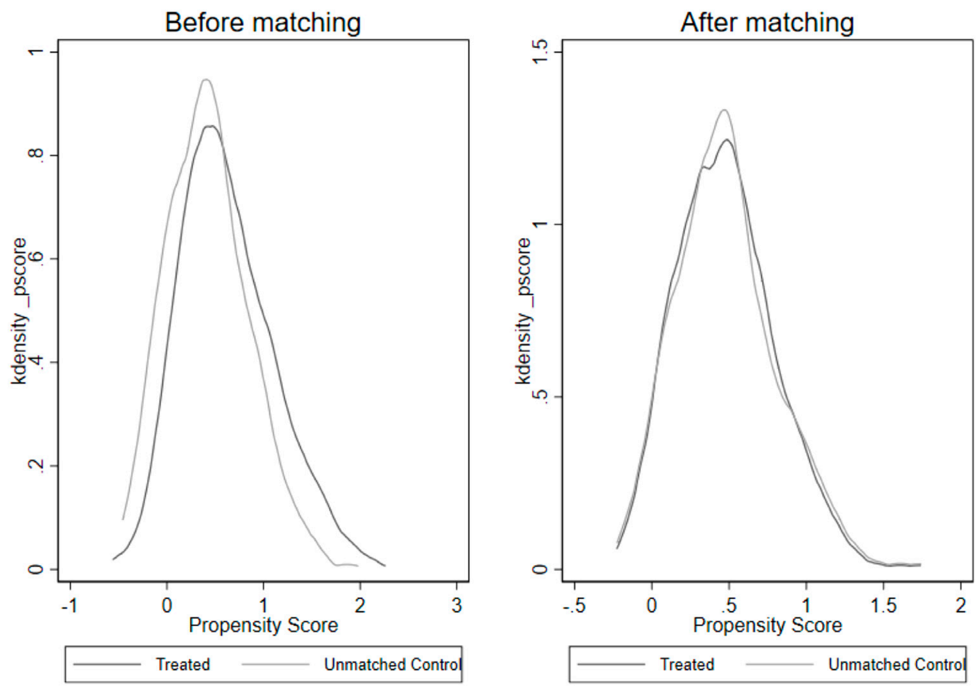

FIGURE A1
Propensity score of the treated, matched and unmatched controls.

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Analyzing the effect of different types of pollution with bipolar complex fuzzy power Bonferroni mean operators

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When any amount of harmful materials (any substance or any type of energy) is introduced into the climate at a rate quicker than it very well may be scattered or securely put away, then pollution occurs. These harmful materials are known as pollutants which can be natural and can also be manmade such as trash generated by factories. These harmful materials harm the quality of land, air, and water and cause various types of pollution, which affects the environment. In this article, we analyze the effect of various types of pollution on the environment and evaluate the most harmful type of pollution through an illustrative example by employing power Bonferroni mean (BM) operators in the setting of the bipolar complex fuzzy set (BCFS), like bipolar complex fuzzy (BCF) power BM (BCFPBM), BCF weighted power BM (BCFWPBM), BCF power geometric BM (BCFPGBM), and BCF weighted power geometric BM (BCFWPGBM) operators and a decision-making (DM) procedure created on these operators in the environment of the BCFS which are introduced in this article. Furthermore, we illustrate that the introduced operators and a DM procedure in the environment of the BCFS are more effective and have a wide model and advantages than certain prevailing works.

KEYWORDS

Bonferroni mean operators, bipolar complex fuzzy set, pollution, environment, decision-making procedure

Introduction

According to Pure Earth (an environmental organization), over 200 million people from all over the world are affected by toxic pollution. In certain parts of the world where pollution is high, women give birth to babies with defects, children have lost 30 to 40 IQ points, and the average life might be around 45 years due to various kinds of diseases. Various things beneficial to people cause pollution, such as generating electricity from burning coal, cars discharging through exhaust pipes, sewage and garbage produced by homes and industries, and utilizing pesticides and herbicides. Finding out which pollutants have greater effects on the environment is a genuine life DM issue.

Zadeh (1965) introduced the model of the fuzzy set (FS) for addressing abstruse and ambiguous information in genuine life issues. Before the model of the FS, there was a model of a crisp set in which every element has a satisfying grade (SG), either 0 or 1. In the FS, every element has a satisfying grade either 0 or 1 or any value between them. Thus, the FS is a generalization of the crisp set, enlarging the range from the set of 0 and 1 to the interval [0, 1]. Scholars have employed the FS in various fields, such as DM (De, 2020), economy (Rytova and Gutman, 2019), and medicine (Choudhury et al., 2021). Kwon et al. (2022) combined the AHP and FS theory. Gulistan et al. (2022) described a new fuzzy decision support system. Jiang (2022) presented an improved algorithm for the FS. Chen and Tian (2022) developed the FS QCA approach. Scholars have often modified the FS and defined additional models because of the lack of the ability of the FS to address more complicated information such as the complex interval-valued Pythagorean fuzzy set (Ali et al., 2021), distance measures based on picture FS (PFS), rough set (Sahu et al., 2021), interval-valued PFS (Ashraf et al., 2022), and DM approach based on the integration of the fuzzy CoCoSO method (Narang et al., 2022).

Zhang (1994) introduced one of the modifications of the FS called the bipolar FS (BFS). In the BFS, every element has a positive satisfying grade (PSG) between 0 and 1 and a negative satisfying grade between −1 and 0. Singh (2022) interpreted bipolar fuzzy (BF) concept reduction by granular-based weighted entropy. Pandey et al. (2022) described BF centrality measures. The graphs in the environment of the BF set were studied by various authors, including Poulik and Ghorai (2020), Wan et al. (2022), and Bharathi and Felixia (2022). Gong and Hua (2022) initiated fuzzy edge connectivity in BF networks. The DM technique for BF sets was developed by Yamini et al. (2022). The multi-criteria DM (MCDM) technique for BF was created by Rajalakshmi and Mary (2022) and Garai et al. (2022). Riaz et al. (2022) defined sine trigonometric (ST) aggregation operators (AOs) for BF sets. Jana et al. (2019) introduced BF Dombi AOs (DAOs), and Wei et al. (2018) introduced BF Hamacher AOs (HAOs).

Ramot et al. (2002) developed another modification of the FS called the complex FS (CFS) because the FS cannot handle data involving a second dimension or extra fuzzy information. In the CFS, every element has a satisfying grade in $\{z \in \mathbb{C} \mid z \leq 1\}$, i.e., a unit disc of a complex plane with SG in the polar form, i.e., $re^{i\theta}$. Tamir et al. (2011) have provided another version of the CFS by considering the range of the satisfying grade as a unit square in a complex plane instead of a unit circle, i.e., every element of the CFS has a satisfying grade in a unit square and is in the structure of the Cartesian form, i.e., $\alpha + \mathfrak{i}\,\beta$. Akram et al. (2021) presented a competition graph in the environment of the CFS. Zeeshan and Khan (2022) developed a DM technique based on the CFS. Akram and Bashir (2021) described ordered, weighted, quadratic, and averaging operators based on the CFS. Hu et al. (2019) proposed CF power AOs. Mahmood and Ur Rehman (2022a) provided one of the most modified models of the FS because the FS lacks the ability to tackle negative aspects of human beings, as well as information involving the second dimension, and called it a bipolar complex fuzzy set (BCFS). The BCFS also modified the model of the BFS and CFS as the BFS lacks the ability to tackle the data containing the second dimension, and the CFS lacks the ability to tackle the negative aspects of human beings. In the BCFS, every element has the PSG in the first quadrant and NSG in the third quadrant of a unit square in a complex plane. Ur Rehman and Mahmood (2022) defined generalized dice similarity measures (SM) for the bipolar complex fuzzy (BCF) set.

To get a more accurate alternative method, we have to consider the given assessment information and the relationship between them. The power average (PA) and power-ordered weighted average operators were first established by Yager (2001), which incorporate variable weights. Xu and Yager (2009) used power geometric operators, and Garg et al. (2021) established PA operators in the setting of the T-spherical FS. There is another apparatus called the BM operator defined by Bonferroni (1950), which considers the interrelationship of inputs. It has been utilized in numerous fields: the geometric BM (GBM) was used by Zhu et al. (2012), the BM operator was utilized in the intuitionistic FS (IFS) by Xu and Yager (2010), and picture fuzzy BM operators were investigated by Ateş and Akay (2020). Liu and Li (2017) developed PBM operators for the interval-valued IFS (IVFS). Furthermore, PA has the ability to ignore the effect of the smallest or largest information by taking various weights, and the BM operator has the ability to consider the interrelationship of each input argument. However, in DM issues and circumstances, the PA and DM operators lack the ability to tackle the data in the model of the BCFS.

The BCFS can easily portray fuzzy data or information containing positive and negative aspects, as well as extra fuzzy information (second dimension), and is very handy in dealing with complicated DM issues and genuine life issues. The model of the BCFS is wider and richer than certain prevailing conceptions like FS, BFS, and CFS, and numerous scholars have defined various AOs in the setting of the BCFS. Mahmood et al. (2021) have defined BCF HAOs, and Mahmood and Ur Rehman (2022b) have defined DAOs for the BCF set. Mahmood et al. (2022a) initiated AOs, and Rehman et al. (2022) presented Frank AOs in the environment of the BCF set. The BM operators based on BCFS were defined by Mahmood et al. (2022b). However, these defined AOs have their drawbacks, and these operations have one common drawback; it is that these operators cannot provide a fair and unbiased weight vector to the attributes because in these operators, the decision expert provides weight vectors to each attribute on his/her own choice. In the literature, there is no such structure or operator which can overcome this dilemma. Thus, this research gap created various questions such as

- How do we get fair and unbiased decisions if the data are in the structure of the BCFS?

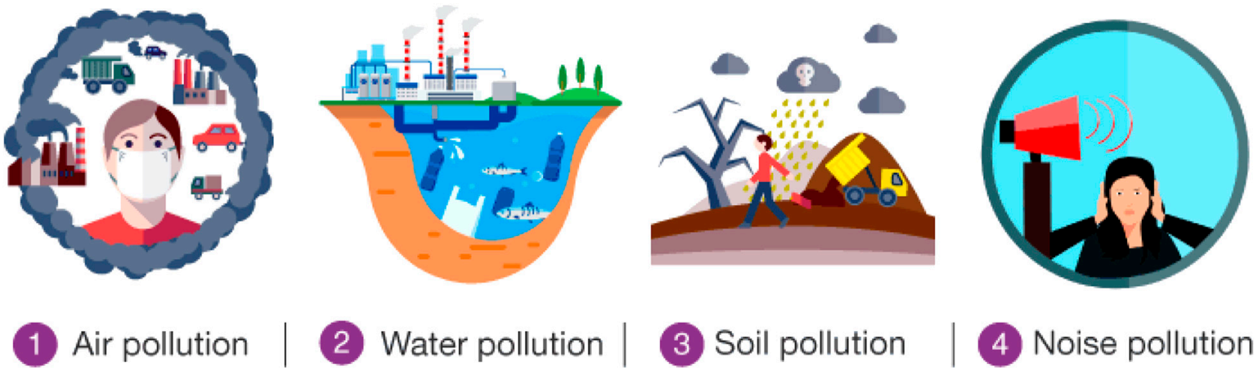

FIGURE 1
Major types of pollution.

- How do we make decisions if the weight vector is not given and the information is in the setting of the BCFS?
- How do we get the advantages of the combination of PA and BM operators in the environment of the BCFS?

Up until now, no one has answered these questions; thus, in this study, we answer these questions by combining PA, geometric BM, and BM operators in the environment of the BCFS to introduce BCFPBM, BCFWPBM, BCFPGBM, and BCFWPGBM operators. Furthermore, we describe what we are achieving from this study.

- The structure of the BCFS is one of the most advanced and widely used structures, and up until now, no one has defined these operators in the environment of the BCFS.
- Over time, the information became more complicated and tricky and almost impossible to handle with ordinary mathematical models and operators. This study will help handle complicated and tricky data and provide a fair decision. This study is the requirement of time.
- PA, GBM, and BM operators have their advantages; this study provides us with all these advantages together.
- To show the practical advantage of this study, we investigate a genuine-life numerical example related to the environment in this study.
- By taking the arbitrary weight vector, the introduced operators would be reduced to the BM operators, as defined by Mahmood et al. (2022b). Thus, the defined operators are the modification of BM operators, as defined by Mahmood et al. (2022b).

In the *Background* section, we review the basic concept of pollution and its types, as well as the idea of the BCFS and its properties. In the *BCF power Bonferroni mean operators* section, we introduce power BM operators in the setting of BCF sets to define BCFPBM, BCFWPBM, BCFPGBM, and BCFWPGBM operators. In the *Application* section, we introduce a DM procedure for solving genuine life issues and then present a numerical example related to pollution and its types. In the *Comparison* section, we show that the introduced PBM operators (BCFPBM, BCFWPBM, BCFPGBM, and BCFWPGBM) and a DM procedure in the environment of BCFS are more effective than other prevailing operators. The *Conclusion* section contains the concluding remarks of this article.

Background

The word pollution originates from the Latin word polluere, which essentially implies contamination. Thus, in a simple way, pollution is something that pollutes the climate. According to Pure Earth (an environmental organization), over 200 million people from all over the world are affected by toxic pollution. Various beneficial things can cause pollution, such as generating electricity from burning coal causes pollution in the air, cars discharging pollutants through exhaust pipes, the sewage and garbage produced by homes and industries cause water and land pollution, and the use of pesticides to kill insects and weeds causes water pollution , which is harmful to wildlife. These pollutants harm the quality of land, air, and water. Various kinds of pollution can be caused by natural events and also by artificial activities. The four major types of pollution are

1) Air pollution,
2) Water pollution,
3) Soil pollution, and
4) Noise pollution.

Figure 1 portrays the types of pollution.

1) Air pollution: There is an exact synthetic structure of the air that we breathe. Its vast majority comprises oxygen, dormant gases, nitrogen, and water fume. When the pollutants or things are introduced in the air, which are not normally there, they cause air pollution. A usual kind of air pollution takes place when people emit particles of burning fuels into the air. This sort of pollution seems to be excess, having a large number of minuscule particles, drifting in the air. One more typical sort of air pollution is caused by harmful gases, like nitrogen oxides, sulfur dioxide, chemical fumes, and carbon monoxide. When these gases are introduced in the air, they can be a part of the chemical reactions and produce smog and acid rain. Over two million people die every year because of air pollution. The impact of air pollution on human well-being can fluctuate generally contingent upon the contamination. Assuming the poison is exceptionally harmful, the consequences for well-being can be far and wide extreme. For instance, the leakage of methyl isocyanate gas in Bhopal, India, in 1984, killed more than two thousand people, and in north, two hundred thousand people experienced respiratory issues.
2) Water pollution: When chemicals or other harmful substances such as sewage, fertilizers, metals, mercury, and pesticides are introduced into the water, then water pollution occurs. All over the world, 44 percent of streams, 64 percent of lakes, and 30 percent of the bay are not hygienic for fishing and swimming as per the Environmental Protection Agency (EPA). According to the EPA, the usual pollutants in the USA are mercury, nitrogen, bacteria, and phosphorus.
3) Soil pollution: Household and industrial garbage and waste cause soil pollution. According to the EPA, USA generated over 258 million tons of waste and garbage in 2014. More than half, i.e., 136 million tons, of the waste was assembled in landfills, and merely 34% was composted. The effects of land pollution are not that noticeable, but its implication can easily be observed. Oil spills, industrial accidents, acid rain, improper waste disposal, mining activities, etc., are a few usual reasons for soil pollution.
4) Noise pollution: despite the fact that people cannot smell or see noise pollution, it influences the climate. When various sounds reach the harmful level, for example, sound coming from industries, jets, etc., it causes noise pollution. It is obvious that there are immediate connections between well-being and noise, causing hearing loss, increase in the blood pressure, speech interference, and pressure-related ailments. For instance, according to the WHO, thousands of people die every year because of noise pollution. Ships cause underwater noise pollution. Similarly Noise pollution influence wild species to impart stronger, which can reduce their life expectancy.

Definition 1: (Mahmood and Ur Rehman, 2022b) The theory is modeled as follows.

$$\mathcal{M}_{BCFS} = \{(\xi, (\Upsilon^{P}_{\mathcal{M}_{BCFS}}(\xi), \Upsilon^{N}_{\mathcal{M}_{BCFS}}(\xi)))|\xi \in \mathrm{E}_{\xi}\} = \left\{\left(\xi, \begin{pmatrix} \Upsilon^{RP}_{\mathcal{M}_{BCFS}}(\xi) + \mathrm{i}\,\Upsilon^{IP}_{\mathcal{M}_{BCFS}}(\xi), \\ \Upsilon^{RN}_{\mathcal{M}_{BCFS}}(\xi) + \mathrm{i}\,\Upsilon^{IN}_{\mathcal{M}_{BCFS}}(\xi) \end{pmatrix}\right)\middle|\xi \in \mathrm{E}_{\xi}\right\} \tag{1}$$

is known as the BCFS, where $\Upsilon^{P}_{\mathcal{M}_{BCFS}}(\xi)$ is known as a positive satisfying grade and $\Upsilon^{N}_{\mathcal{M}_{BCFS}}(\xi)$ is known as a negative satisfying grade and $\Upsilon^{RP}_{\mathcal{M}_{BCFS}}(\xi), \Upsilon^{IP}_{\mathcal{M}_{BCFS}}(\xi) \in [0, 1]$ and $\Upsilon^{RN}_{\mathcal{M}_{BCFS}}(\xi), \Upsilon^{IN}_{\mathcal{M}_{BCFS}}(\xi) \in [-1, 0]$. A set $\mathcal{M}_{BCFS} = (\Upsilon^{P}_{\mathcal{M}_{BCFS}}, \Upsilon^{N}_{\mathcal{M}_{BCFS}}) = (\Upsilon^{RP}_{\mathcal{M}_{BCFS}} + \mathrm{i}\,\Upsilon^{IP}_{\mathcal{M}_{BCFS}}, \Upsilon^{RN}_{\mathcal{M}_{BCFS}} + \mathrm{i}\,\Upsilon^{IN}_{\mathcal{M}_{BCFS}})$ is identified as the BCF number (BCFN).

Definition 2: (Mahmood and Ur Rehman, 2022b) For a BCFN $\mathcal{M}_{BCFS} = (\Upsilon^{P}_{\mathcal{M}_{BCFS}}, \Upsilon^{N}_{\mathcal{M}_{BCFS}}) = (\Upsilon^{RP}_{\mathcal{M}_{BCFS}} + \mathrm{i}\,\Upsilon^{IP}_{\mathcal{M}_{BCFS}}, \Upsilon^{RN}_{\mathcal{M}_{BCFS}} + \mathrm{i}\,\Upsilon^{IN}_{\mathcal{M}_{BCFS}})$, the score value (SV) and accuracy value (AV) would be obtained as follows:

$$\mathcal{S}(\mathcal{M}_{BCFS}) = \frac{1}{4}(2 + \Upsilon^{RP}_{\mathcal{M}_{BCFS}} + \Upsilon^{IP}_{\mathcal{M}_{BCFS}} + \Upsilon^{RN}_{\mathcal{M}_{BCFS}} + \Upsilon^{IN}_{\mathcal{M}_{BCFS}}),\ \mathcal{S}(\mathcal{M}_{BCFS}) \in [0, 1], \tag{2}$$

$$\mathcal{H}(\mathcal{M}_{BCFS}) = \frac{\Upsilon^{RP}_{\mathcal{M}_{BCFS}} + \Upsilon^{IP}_{\mathcal{M}_{BCFS}} + \Upsilon^{RN}_{\mathcal{M}_{BCFS}} + \Upsilon^{IN}_{\mathcal{M}_{BCFS}}}{4},\ \mathcal{H}(\mathcal{M}_{BCFS}) \in [0, 1]. \tag{3}$$

Definition 3: (Mahmood and Ur Rehman, 2022b) If we have two BCFNs $\mathcal{M}_{BCFS-1} = (\Upsilon^{P}_{\mathcal{M}_{BCFS-1}}, \Upsilon^{N}_{\mathcal{M}_{BCFS-1}}) = (\Upsilon^{RP}_{\mathcal{M}_{BCFS-1}} + \mathrm{i}\,\Upsilon^{IP}_{\mathcal{M}_{BCFS-1}}, \Upsilon^{RN}_{\mathcal{M}_{BCFS-1}} + \mathrm{i}\,\Upsilon^{IN}_{\mathcal{M}_{BCFS-1}})$ and $\mathcal{M}_{BCFS-2} = (\Upsilon^{P}_{\mathcal{M}_{BCFS-2}}, \Upsilon^{N}_{\mathcal{M}_{BCFS-2}}) = (\Upsilon^{RP}_{\mathcal{M}_{BCFS-2}} + \mathrm{i}\,\Upsilon^{IP}_{\mathcal{M}_{BCFS-2}}, \Upsilon^{RN}_{\mathcal{M}_{BCFS-2}} + \mathrm{i}\,\Upsilon^{IN}_{\mathcal{M}_{BCFS-2}})$, with $\sigma \geq 0$, then

1) $\mathcal{M}_{BCFS-1} \oplus \mathcal{M}_{BCFS-2} = \begin{pmatrix} \Upsilon^{RP}_{\mathcal{M}_{BCFS-1}} + \Upsilon^{RP}_{\mathcal{M}_{BCFS-2}} - \Upsilon^{RP}_{\mathcal{M}_{BCFS-1}}\Upsilon^{RP}_{\mathcal{M}_{BCFS-2}} \\ +\mathrm{i}\,(\Upsilon^{IP}_{\mathcal{M}_{BCFS-1}} + \Upsilon^{IP}_{\mathcal{M}_{BCFS-2}} - \Upsilon^{IP}_{\mathcal{M}_{BCFS-1}}\Upsilon^{IP}_{\mathcal{M}_{BCFS-2}}), \\ -(\Upsilon^{RN}_{\mathcal{M}_{BCFS-1}}\Upsilon^{RN}_{\mathcal{M}_{BCFS-2}}) + \mathrm{i}\,(-(\Upsilon^{IN}_{\mathcal{M}_{BCFS-1}}\Upsilon^{IN}_{\mathcal{M}_{BCFS-2}})) \end{pmatrix}$,
2) $\mathcal{M}_{BCFS-1} \otimes \mathcal{M}_{BCFS-2} = \begin{pmatrix} \Upsilon^{RP}_{\mathcal{M}_{BCFS-1}}\Upsilon^{RP}_{\mathcal{M}_{BCFS-2}} + \mathrm{i}\,\Upsilon^{IP}_{\mathcal{M}_{BCFS-1}}\Upsilon^{IP}_{\mathcal{M}_{BCFS-2}}, \\ \Upsilon^{RN}_{\mathcal{M}_{BCFS-1}} + \Upsilon^{RN}_{\mathcal{M}_{BCFS-2}} + \Upsilon^{RN}_{\mathcal{M}_{BCFS-1}}\Upsilon^{RN}_{\mathcal{M}_{BCFS-2}} \\ \mathrm{i}\,(\Upsilon^{IN}_{\mathcal{M}_{BCFS-1}} + \Upsilon^{IN}_{\mathcal{M}_{BCFS-2}} + \Upsilon^{IN}_{\mathcal{M}_{BCFS-1}}\Upsilon^{IN}_{\mathcal{M}_{BCFS-2}}) \end{pmatrix}$,
3) $\sigma\mathcal{M}_{BCFS-1} = \begin{pmatrix} 1 - (1 - \Upsilon^{RP}_{\mathcal{M}_{BCFS-1}})^{\sigma} + \mathrm{i}\,(1 - (1 - \Upsilon^{IP}_{\mathcal{M}_{BCFS-1}})^{\sigma}), \\ -|\Upsilon^{RN}_{\mathcal{M}_{BCFS-1}}|^{\sigma} + \mathrm{i}\,(-|\Upsilon^{IN}_{\mathcal{M}_{BCFS-1}}|^{\sigma}) \end{pmatrix}$,
4) $\mathcal{M}^{\sigma}_{BCFS-1} = \begin{pmatrix} (\Upsilon^{RP}_{\mathcal{M}_{BCFS-1}})^{\sigma} + \mathrm{i}\,(\Upsilon^{IP}_{\mathcal{M}_{BCFS-1}})^{\sigma}, \\ -1 + (1 + \Upsilon^{RN}_{\mathcal{M}_{BCFS-1}})^{\sigma} + \mathrm{i}\,(-1 + (1 + \Upsilon^{IN}_{\mathcal{M}_{BCFS-1}})^{\sigma}) \end{pmatrix}$.

TABLE 1 Decision matrix containing BCFNs is provided by an expert.

	$\mathcal{A}_{b-1}$	$\mathcal{A}_{b-2}$	$\mathcal{A}_{b-3}$	$\mathcal{A}_{b-4}$
$\mathcal{M}_{BCFS-1}$	$\begin{pmatrix} 0.69 + \mathfrak{i}\,0.18, \\ -0.2 - \mathfrak{i}\,0.93 \end{pmatrix}$	$\begin{pmatrix} 0.82 + \mathfrak{i}\,0.76, \\ -0.28 - \mathfrak{i}\,0.67 \end{pmatrix}$	$\begin{pmatrix} 0.19 + \mathfrak{i}\,0.18, \\ -0.49 - \mathfrak{i}\,054. \end{pmatrix}$	$\begin{pmatrix} 0.87 + \mathfrak{i}\,0.37, \\ -0.43 - \mathfrak{i}\,0.61 \end{pmatrix}$
$\mathcal{M}_{BCFS-2}$	$\begin{pmatrix} 0.57 + \mathfrak{i}\,0.5, \\ -0.29 - \mathfrak{i}\,063. \end{pmatrix}$	$\begin{pmatrix} 0.83 + \mathfrak{i}\,0.66, \\ -0.38 - \mathfrak{i}\,0.66 \end{pmatrix}$	$\begin{pmatrix} 0.38 + \mathfrak{i}\,0.71, \\ -0.7 - \mathfrak{i}\,0.56 \end{pmatrix}$	$\begin{pmatrix} 0.88 + \mathfrak{i}\,0.19, \\ -0.44 - \mathfrak{i}\,028. \end{pmatrix}$
$\mathcal{M}_{BCFS-3}$	$\begin{pmatrix} 0.29 + \mathfrak{i}\,0.5, \\ -0.10 - \mathfrak{i}\,0.39 \end{pmatrix}$	$\begin{pmatrix} 0.84 + \mathfrak{i}\,0.56, \\ -0.48 - \mathfrak{i}\,0.65 \end{pmatrix}$	$\begin{pmatrix} 0.6 + \mathfrak{i}\,0.28, \\ -0.8 - \mathfrak{i}\,0.92 \end{pmatrix}$	$\begin{pmatrix} 0.89 + \mathfrak{i}\,0.57, \\ -0.54 - \mathfrak{i}\,0.72 \end{pmatrix}$
$\mathcal{M}_{BCFS-4}$	$\begin{pmatrix} 0.9 + \mathfrak{i}\,0.72, \\ -0.3 - \mathfrak{i}\,0.6 \end{pmatrix}$	$\begin{pmatrix} 0.85 + \mathfrak{i}\,0.46, \\ -0.58 - \mathfrak{i}\,0.64 \end{pmatrix}$	$\begin{pmatrix} 0.6 + \mathfrak{i}\,0.18, \\ -0.5 - \mathfrak{i}\,0.91 \end{pmatrix}$	$\begin{pmatrix} 0.71 + \mathfrak{i}\,0.61, \\ -0.29 - \mathfrak{i}\,0.5 \end{pmatrix}$

Definition 4: (He et al., 2014) Take a group of real numbers $\mathcal{M}_{R-\mathfrak{p}} = (\mathcal{M}_{R-1}, \mathcal{M}_{R-2}, \mathcal{M}_{R-3}, \ldots, \mathcal{M}_{R-\eta})$, with $u, v > 0$, then the power BM operator is inspected as

$$PBM^{u,v}(\mathcal{M}_{R-1}, \mathcal{M}_{R-2}, \mathcal{M}_{R-3}, \ldots, \mathcal{M}_{R-\eta}) = \left(\frac{1}{\eta^2 - \eta} \sum_{\substack{\mathfrak{p}=1, \mathfrak{q}=1 \\ \mathfrak{p} \neq \mathfrak{q}}}^{\eta} \left(\left(\frac{\eta(\mathcal{T}(\mathcal{M}_{R-\mathfrak{p}})+1)}{\sum_{l=1}^{\eta}(\mathcal{T}(\mathcal{M}_{R-l})+1)} \mathcal{M}_{BCFS-\mathfrak{p}} \right)^{u} \otimes \left(\frac{\eta(\mathcal{T}(\mathcal{M}_{R-\mathfrak{q}})+1)}{\sum_{l=1}^{\eta}(\mathcal{T}(\mathcal{M}_{R-l})+1)} \mathcal{M}_{BCFS-\mathfrak{q}} \right)^{v} \right) \right)^{1/u+v}. \quad (4)$$

Definition 5: (He et al., 2014) Take a group of real numbers $\mathcal{M}_{R-\mathfrak{p}} = (\mathcal{M}_{R-1}, \mathcal{M}_{R-2}, \mathcal{M}_{R-3}, \ldots, \mathcal{M}_{R-\eta})$, with $u, v > 0$, then the power GBM operator is inspected as

$$PGBM^{u,v}(\mathcal{M}_{R-1}, \mathcal{M}_{R-2}, \ldots, \mathcal{M}_{R-\eta}) = \frac{1}{u+v} \left(\prod_{\substack{\mathfrak{p}=1, \mathfrak{q}=1 \\ \mathfrak{p} \neq \mathfrak{q}}}^{\eta} \left(u(\mathcal{M}_{R-\mathfrak{p}})^{\frac{\eta(\mathcal{T}(\mathcal{M}_{R-\mathfrak{p}})+1)}{\sum_{l=1}^{\eta}(\mathcal{T}(\mathcal{M}_{R-l})+1)}} \oplus v(\mathcal{M}_{R-\mathfrak{q}})^{\frac{\eta(\mathcal{T}(\mathcal{M}_{R-\mathfrak{q}})+1)}{\sum_{l=1}^{\eta}(\mathcal{T}(\mathcal{M}_{R-l})+1)}} \right) \right)^{1/\eta^2-\eta}. \quad (5)$$

In Def (4) and (5), $\mathcal{T}(\mathcal{M}_{R-\mathfrak{p}}) = \sum_{\substack{\mathfrak{q}=1 \\ \mathfrak{q} \neq \mathfrak{p}}}^{\eta} Sup(\mathcal{M}_{R-\mathfrak{p}}, \mathcal{M}_{R-\mathfrak{q}})$ is the support to $\mathcal{M}_{R-\mathfrak{p}}$ from $\mathcal{M}_{R-\mathfrak{q}}$ with these axioms

1) $Sup(\mathcal{M}_{R-\mathfrak{p}}, \mathcal{M}_{R-\mathfrak{q}}) \in [0, 1]$,
2) $Sup(\mathcal{M}_{R-\mathfrak{p}}, \mathcal{M}_{R-\mathfrak{q}}) = Sup(\mathcal{M}_{R-\mathfrak{q}}, \mathcal{M}_{R-\mathfrak{p}})$,
3) $Sup(\mathcal{M}_{R-\mathfrak{p}}, \mathcal{M}_{R-\mathfrak{q}}) \geq Sup(\mathcal{M}_{R-r}, \mathcal{M}_{R-s})$ if $\mathfrak{D}(\mathcal{M}_{R-\mathfrak{p}}, \mathcal{M}_{R-\mathfrak{p}}) < \mathfrak{D}(\mathcal{M}_{R-r}, \mathcal{M}_{R-s})$, where $\mathfrak{D}$ is any distance measured.

BCF power Bonferroni mean operators

In this section of this article, we introduce power BM operators in the setting of BCF sets to define BCFPBM, BCFWPBM, BCFPGBM, and BCFWPGBM operators.

Definition 6: Take a group of BCFNs $\mathcal{M}_{BCFS-\mathfrak{p}} = (\Upsilon^{P}_{\mathcal{M}_{BCFS-\mathfrak{p}}}, \Upsilon^{N}_{\mathcal{M}_{BCFS-\mathfrak{p}}}) = (\Upsilon^{RP}_{\mathcal{M}_{BCFS-\mathfrak{p}}} + \mathfrak{i}\,\Upsilon^{IP}_{\mathcal{M}_{BCFS-\mathfrak{p}}}, \Upsilon^{RN}_{\mathcal{M}_{BCFS-\mathfrak{p}}} + \mathfrak{i}\,\Upsilon^{IN}_{\mathcal{M}_{BCFS-\mathfrak{p}}})$, $\mathfrak{p} = \mathfrak{p}, 2, \ldots, \eta$, then the BCFPBM operator is inspected as

$$BCFPBM^{u,v}(\mathcal{M}_{BCFS-1}, \mathcal{M}_{BCFS-2}, \ldots, \mathcal{M}_{BCFS-\eta}) = \left(\frac{1}{\eta^2 - \eta} \sum_{\substack{\mathfrak{p}=1, \mathfrak{q}=1 \\ \mathfrak{p} \neq \mathfrak{q}}}^{\eta} \left(\left(\frac{\eta(\mathcal{T}(\mathcal{M}_{BCFS-\mathfrak{p}})+1)}{\sum_{l=1}^{\eta}(\mathcal{T}(\mathcal{M}_{BCFS-l})+1)} \mathcal{M}_{BCFS-\mathfrak{p}} \right)^{u} \otimes \left(\frac{\eta(\mathcal{T}(\mathcal{M}_{BCFS-\mathfrak{q}})+1)}{\sum_{l=1}^{\eta}(\mathcal{T}(\mathcal{M}_{BCFS-l})+1)} \mathcal{M}_{BCFS-\mathfrak{q}} \right)^{v} \right) \right)^{1/u+v}, \quad (6)$$

where $u, v > 0$ and $\mathcal{T}(\mathcal{M}_{BCFS-\mathfrak{p}}) = \sum_{\substack{\mathfrak{q}=1 \\ \mathfrak{q} \neq \mathfrak{p}}}^{\mathfrak{n}} Sup(\mathcal{M}_{BCFS-\mathfrak{p}}, \mathcal{M}_{BCFS-\mathfrak{q}})$ are the support to $\mathcal{M}_{BCFS-\mathfrak{p}}$ from $\mathcal{M}_{BCFS-\mathfrak{q}}$ with the following axioms

1) $Sup(\mathcal{M}_{BCFS-\mathfrak{p}}, \mathcal{M}_{BCFS-\mathfrak{q}}) \in [0, 1]$,
2) $Sup(\mathcal{M}_{BCFS-\mathfrak{p}}, \mathcal{M}_{BCFS-\mathfrak{q}}) = Sup(\mathcal{M}_{BCFS-\mathfrak{q}}, \mathcal{M}_{BCFS-\mathfrak{p}})$,
3) $Sup(\mathcal{M}_{BCFS-\mathfrak{p}}, \mathcal{M}_{BCFS-\mathfrak{q}}) \geq Sup(\mathcal{M}_{BCFS-r}, \mathcal{M}_{BCFS-s})$ if $\mathfrak{D}(\mathcal{M}_{BCFS-\mathfrak{p}}, \mathcal{M}_{BCFS-\mathfrak{p}}) < \mathfrak{D}(\mathcal{M}_{BCFS-r}, \mathcal{M}_{BCFS-s})$, where $\mathfrak{D}$ is the distance measured among BCFSs.

Theorem 1: The aggregated outcome from Eq. 6 based on the group of BCFNs $\mathcal{M}_{BCFS-\mathfrak{p}} = (\Upsilon^{P}_{\mathcal{M}_{BCFS-\mathfrak{p}}}, \Upsilon^{N}_{\mathcal{M}_{BCFS-\mathfrak{p}}}) = (\Upsilon^{RP}_{\mathcal{M}_{BCFS-\mathfrak{p}}} + \mathfrak{i}\,\Upsilon^{IP}_{\mathcal{M}_{BCFS-\mathfrak{p}}}, \Upsilon^{RN}_{\mathcal{M}_{BCFS-\mathfrak{p}}} + \mathfrak{i}\,\Upsilon^{IN}_{\mathcal{M}_{BCFS-\mathfrak{p}}})$, $\mathfrak{p} = \mathfrak{p}, 2, \ldots, \mathfrak{n}$ is a BCFN and inspected as

$$BCFPBM^{u,v}(\mathcal{M}_{BCFS-1}, \mathcal{M}_{BCFS-2}, \ldots, \mathcal{M}_{BCFS-\mathfrak{n}}) = \begin{pmatrix} \left(1 - \left(\prod_{\substack{\mathfrak{p}=1, \mathfrak{q}=1 \\ \mathfrak{p} \neq \mathfrak{q}}}^{\mathfrak{n}} \left(1 - \begin{pmatrix} \left(1 - \left(1 - \Upsilon^{RP}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right)^{\frac{\mathfrak{n}(\mathcal{T}(\mathcal{M}_{BCFS-\mathfrak{p}})+1)}{\sum_{l=1}^{\mathfrak{n}}(\mathcal{T}(\mathcal{M}_{BCFS-l})+1)}}\right)^{u} \\ \times \left(1 - \left(1 - \Upsilon^{RP}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\frac{\mathfrak{n}(\mathcal{T}(\mathcal{M}_{BCFS-\mathfrak{q}})+1)}{\sum_{l=1}^{\mathfrak{n}}(\mathcal{T}(\mathcal{M}_{BCFS-l})+1)}}\right)^{v} \end{pmatrix}\right)\right)^{1/\mathfrak{n}^2-\mathfrak{n}}\right)^{1/u+v} \\ +\mathfrak{i}\left(1 - \left(\prod_{\substack{\mathfrak{p}=1, \mathfrak{q}=1 \\ \mathfrak{p} \neq \mathfrak{q}}}^{\mathfrak{n}} \left(1 - \begin{pmatrix} \left(1 - \left(1 - \Upsilon^{IP}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right)^{\frac{\mathfrak{n}(\mathcal{T}(\mathcal{M}_{BCFS-\mathfrak{p}})+1)}{\sum_{l=1}^{\mathfrak{n}}(\mathcal{T}(\mathcal{M}_{BCFS-l})+1)}}\right)^{u} \\ \times \left(1 - \left(1 - \Upsilon^{IP}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\frac{\mathfrak{n}(\mathcal{T}(\mathcal{M}_{BCFS-\mathfrak{q}})+1)}{\sum_{l=1}^{\mathfrak{n}}(\mathcal{T}(\mathcal{M}_{BCFS-l})+1)}}\right)^{v} \end{pmatrix}\right)\right)^{1/\mathfrak{n}^2-\mathfrak{n}}\right)^{1/u+v}, \\ -1 + \left(1 - \left|-\prod_{\substack{\mathfrak{p}=1, \mathfrak{q}=1 \\ \mathfrak{p} \neq \mathfrak{q}}}^{\mathfrak{n}} \begin{pmatrix} -1 + \left(1 - \left|\Upsilon^{RN}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right|^{\frac{\mathfrak{n}(\mathcal{T}(\mathcal{M}_{BCFS-\mathfrak{p}})+1)}{\sum_{l=1}^{\mathfrak{n}}(\mathcal{T}(\mathcal{M}_{BCFS-l})+1)}}\right)^{u} \\ \times \left(1 - \left|\Upsilon^{RN}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right|^{\frac{\mathfrak{n}(\mathcal{T}(\mathcal{M}_{BCFS-\mathfrak{q}})+1)}{\sum_{l=1}^{\mathfrak{n}}(\mathcal{T}(\mathcal{M}_{BCFS-l})+1)}}\right)^{v} \end{pmatrix}\right|^{1/\mathfrak{n}^2-\mathfrak{n}}\right)^{1/u+v} \\ +\mathfrak{i}\left(-1 + \left(1 - \left|-\prod_{\substack{\mathfrak{p}=1, \mathfrak{q}=1 \\ \mathfrak{p} \neq \mathfrak{q}}}^{\mathfrak{n}} \begin{pmatrix} -1 + \left(1 - \left|\Upsilon^{IN}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right|^{\frac{\mathfrak{n}(\mathcal{T}(\mathcal{M}_{BCFS-\mathfrak{p}})+1)}{\sum_{l=1}^{\mathfrak{n}}(\mathcal{T}(\mathcal{M}_{BCFS-l})+1)}}\right)^{u} \\ \times \left(1 - \left|\Upsilon^{IN}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right|^{\frac{\mathfrak{n}(\mathcal{T}(\mathcal{M}_{BCFS-\mathfrak{q}})+1)}{\sum_{l=1}^{\mathfrak{n}}(\mathcal{T}(\mathcal{M}_{BCFS-l})+1)}}\right)^{v} \end{pmatrix}\right|^{1/\mathfrak{n}^2-\mathfrak{n}}\right)^{1/u+v}\right) \end{pmatrix}. \quad (7)$$

Proof: Suppose that $\mathfrak{F}_{\mathfrak{p}} = \frac{\mathfrak{n}(\mathcal{T}(\mathcal{M}_{BCFS-\mathfrak{p}})+1)}{\sum_{l=1}^{\mathfrak{n}}(\mathcal{T}(\mathcal{M}_{BCFS-l})+1)} (\mathfrak{p} = 1, 2, \ldots, \mathfrak{n})$, then Eq. 7 becomes

$$BCFPBM^{u,v}(\mathcal{M}_{BCFS-1}, \mathcal{M}_{BCFS-2}, \ldots, \mathcal{M}_{BCFS-\mathfrak{n}}) = \left(\frac{1}{\mathfrak{n}^2 - \mathfrak{n}} \sum_{\substack{\mathfrak{p}=1, \mathfrak{q}=1 \\ \mathfrak{p} \neq \mathfrak{q}}}^{\mathfrak{n}} \begin{pmatrix} (\mathfrak{F}_{\mathfrak{p}}\mathcal{M}_{BCFS-\mathfrak{p}})^{u} \\ \otimes (\mathfrak{F}_{\mathfrak{q}}\mathcal{M}_{BCFS-\mathfrak{q}})^{v} \end{pmatrix}\right)^{1/u+v}. \quad (8)$$

First, determine $\mathfrak{F}_{\mathfrak{p}}\mathcal{M}_{BCFS-\mathfrak{p}}$ and $\mathfrak{F}_{\mathfrak{q}}\mathcal{M}_{BCFS-\mathfrak{q}}$ as follows:

$$\mathfrak{F}_{\mathfrak{p}}\mathcal{M}_{BCFS-\mathfrak{p}} = \Big(1 - \left(1 - \Upsilon^{RP}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right)^{\mathfrak{F}_{\mathfrak{p}}} + \mathfrak{i}\Big(1 - \left(1 - \Upsilon^{IP}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right)^{\mathfrak{F}_{\mathfrak{p}}}\Big), -\left|\Upsilon^{RN}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right|^{\mathfrak{F}_{\mathfrak{p}}} + \mathfrak{i}\Big(-\left|\Upsilon^{IN}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right|^{\mathfrak{F}_{\mathfrak{p}}}\Big)\Big),$$

$$\mathfrak{F}_{\mathfrak{q}}\mathcal{M}_{BCFS-\mathfrak{q}} = \Big(1 - \left(1 - \Upsilon^{RP}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\mathfrak{F}_{\mathfrak{q}}} + \mathfrak{i}\Big(1 - \left(1 - \Upsilon^{IP}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\mathfrak{F}_{\mathfrak{q}}}\Big), -\left|\Upsilon^{RN}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right|^{\mathfrak{F}_{\mathfrak{q}}} + \mathfrak{i}\Big(-\left|\Upsilon^{IN}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right|^{\mathfrak{F}_{\mathfrak{q}}}\Big)\Big).$$

Now,

$$(\mathfrak{F}_{\mathfrak{p}}\mathcal{M}_{BCFS-\mathfrak{p}})^{u} = \begin{pmatrix} \left(1 - \left(1 - \Upsilon^{RP}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right)^{\mathfrak{F}_{\mathfrak{p}}}\right)^{u} + \mathfrak{i}\left(1 - \left(1 - \Upsilon^{IP}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right)^{\mathfrak{F}_{\mathfrak{p}}}\right)^{u}, \\ -1 + \left(1 - \left|\Upsilon^{RN}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right|^{\mathfrak{F}_{\mathfrak{p}}}\right)^{u} + \mathfrak{i}\left(-1 + \left(1 - \left|\Upsilon^{IN}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right|^{\mathfrak{F}_{\mathfrak{p}}}\right)^{u}\right) \end{pmatrix},$$

$$(\mathfrak{F}_{\mathfrak{q}}\mathcal{M}_{BCFS-\mathfrak{q}})^{v} = \begin{pmatrix} \left(1 - \left(1 - \Upsilon^{RP}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\mathfrak{F}_{\mathfrak{q}}}\right)^{v} + \mathfrak{i}\left(1 - \left(1 - \Upsilon^{IP}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\mathfrak{F}_{\mathfrak{q}}}\right)^{v}, \\ -1 + \left(1 - \left|\Upsilon^{RN}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right|^{\mathfrak{F}_{\mathfrak{q}}}\right)^{v} + \mathfrak{i}\left(-1 + \left(1 - \left|\Upsilon^{IN}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right|^{\mathfrak{F}_{\mathfrak{q}}}\right)^{v}\right) \end{pmatrix}.$$

Now, determine $(\mathfrak{F}_{\mathfrak{p}}\mathcal{M}_{BCFS-\mathfrak{p}})^{u} \otimes (\mathfrak{F}_{\mathfrak{q}}\mathcal{M}_{BCFS-\mathfrak{q}})^{v}$ and get:

$$\begin{aligned}
&(\mathfrak{F}_{\mathfrak{p}}\mathcal{M}_{BCFS-\mathfrak{p}})^{u} \otimes (\mathfrak{F}_{\mathfrak{q}}\mathcal{M}_{BCFS-\mathfrak{q}})^{v} \\
&= \begin{pmatrix} \left(1 - \left(1 - \Upsilon^{RP}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right)^{\mathfrak{F}_{\mathfrak{p}}}\right)^{u} + \mathfrak{i}\left(1 - \left(1 - \Upsilon^{IP}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right)^{\mathfrak{F}_{\mathfrak{p}}}\right)^{u}, \\ -1 + \left(1 - \left|\Upsilon^{RN}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right|^{\mathfrak{F}_{\mathfrak{p}}}\right)^{u} + \mathfrak{i}\left(-1 + \left(1 - \left|\Upsilon^{IN}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right|^{\mathfrak{F}_{\mathfrak{p}}}\right)^{u}\right) \end{pmatrix} \\
&\otimes \begin{pmatrix} \left(1 - \left(1 - \Upsilon^{RP}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\mathfrak{F}_{\mathfrak{q}}}\right)^{v} + \mathfrak{i}\left(1 - \left(1 - \Upsilon^{IP}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\mathfrak{F}_{\mathfrak{q}}}\right)^{v}, \\ -1 + \left(1 - \left|\Upsilon^{RN}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right|^{\mathfrak{F}_{\mathfrak{q}}}\right)^{v} + \mathfrak{i}\left(-1 + \left(1 - \left|\Upsilon^{IN}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right|^{\mathfrak{F}_{\mathfrak{q}}}\right)^{v}\right) \end{pmatrix} \\
&= \begin{pmatrix} \left(1 - \left(1 - \Upsilon^{RP}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right)^{\mathfrak{F}_{\mathfrak{p}}}\right)^{u} \times \left(1 - \left(1 - \Upsilon^{RP}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\mathfrak{F}_{\mathfrak{q}}}\right)^{v} \\ +\mathfrak{i}\left(\left(1 - \left(1 - \Upsilon^{IP}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right)^{\mathfrak{F}_{\mathfrak{p}}}\right)^{u} \times \left(1 - \left(1 - \Upsilon^{IP}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\mathfrak{F}_{\mathfrak{q}}}\right)^{v}\right), \\ \left(-1 + \left(1 - \left|\Upsilon^{RN}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right|^{\mathfrak{F}_{\mathfrak{p}}}\right)^{u}\right) + \left(-1 + \left(1 - \left|\Upsilon^{RN}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right|^{\mathfrak{F}_{\mathfrak{q}}}\right)^{v}\right) + \\ \left(-1 + \left(1 - \left|\Upsilon^{RN}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right|^{\mathfrak{F}_{\mathfrak{p}}}\right)^{u}\right) \times \left(-1 + \left(1 - \left|\Upsilon^{RN}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right|^{\mathfrak{F}_{\mathfrak{q}}}\right)^{v}\right) \\ +\mathfrak{i}\begin{pmatrix} \left(-1 + \left(1 - \left|\Upsilon^{IN}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right|^{\mathfrak{F}_{\mathfrak{p}}}\right)^{u}\right) + \left(-1 + \left(1 - \left|\Upsilon^{IN}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right|^{\mathfrak{F}_{\mathfrak{q}}}\right)^{v}\right) + \\ \left(-1 + \left(1 - \left|\Upsilon^{IN}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right|^{\mathfrak{F}_{\mathfrak{p}}}\right)^{u}\right) \times \left(-1 + \left(1 - \left|\Upsilon^{IN}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right|^{\mathfrak{F}_{\mathfrak{q}}}\right)^{v}\right) \end{pmatrix} \end{pmatrix} \\
&= \begin{pmatrix} \left(1 - \left(1 - \Upsilon^{RP}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right)^{\mathfrak{F}_{\mathfrak{p}}}\right)^{u} \times \left(1 - \left(1 - \Upsilon^{RP}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\mathfrak{F}_{\mathfrak{q}}}\right)^{v} \\ +\mathfrak{i}\left(\left(1 - \left(1 - \Upsilon^{IP}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right)^{\mathfrak{F}_{\mathfrak{p}}}\right)^{u} \times \left(1 - \left(1 - \Upsilon^{IP}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\mathfrak{F}_{\mathfrak{q}}}\right)^{v}\right), \\ -1 + \begin{pmatrix} 1 + \left(-1 + \left(1 - \left|\Upsilon^{RN}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right|^{\mathfrak{F}_{\mathfrak{p}}}\right)^{u}\right) + \left(-1 + \left(1 - \left|\Upsilon^{RN}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right|^{\mathfrak{F}_{\mathfrak{q}}}\right)^{v}\right) \\ \left(-1 + \left(1 - \left|\Upsilon^{RN}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right|^{\mathfrak{F}_{\mathfrak{p}}}\right)^{u}\right) \times \left(-1 + \left(1 - \left|\Upsilon^{RN}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right|^{\mathfrak{F}_{\mathfrak{q}}}\right)^{v}\right) \end{pmatrix} \\ +\mathfrak{i}\left(-1 + \begin{pmatrix} 1 + \left(-1 + \left(1 - \left|\Upsilon^{IN}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right|^{\mathfrak{F}_{\mathfrak{p}}}\right)^{u}\right) + \left(-1 + \left(1 - \left|\Upsilon^{IN}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right|^{\mathfrak{F}_{\mathfrak{q}}}\right)^{v}\right) \\ \left(-1 + \left(1 - \left|\Upsilon^{IN}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right|^{\mathfrak{F}_{\mathfrak{p}}}\right)^{u}\right) \times \left(-1 + \left(1 - \left|\Upsilon^{IN}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right|^{\mathfrak{F}_{\mathfrak{q}}}\right)^{v}\right) \end{pmatrix}\right) \end{pmatrix} \\
&= \begin{pmatrix} \left(1 - \left(1 - \Upsilon^{RP}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right)^{\mathfrak{F}_{\mathfrak{p}}}\right)^{u} \times \left(1 - \left(1 - \Upsilon^{RP}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\mathfrak{F}_{\mathfrak{q}}}\right)^{v} \\ +\mathfrak{i}\left(\left(1 - \left(1 - \Upsilon^{IP}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right)^{\mathfrak{F}_{\mathfrak{p}}}\right)^{u} \times \left(1 - \left(1 - \Upsilon^{IP}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\mathfrak{F}_{\mathfrak{q}}}\right)^{v}\right), \\ -1 + \begin{pmatrix} \left(-1 + \left(1 - \left|\Upsilon^{RN}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right|^{\mathfrak{F}_{\mathfrak{p}}}\right)^{u}\right)\left(1 + \left(-1 + \left(1 - \left|\Upsilon^{RN}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right|^{\mathfrak{F}_{\mathfrak{q}}}\right)^{v}\right)\right) + \\ 1\left(1 + \left(-1 + \left(1 - \left|\Upsilon^{RN}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right|^{\mathfrak{F}_{\mathfrak{q}}}\right)^{v}\right)\right) \end{pmatrix} \\ +\mathfrak{i}\left(-1 + \begin{pmatrix} \left(-1 + \left(1 - \left|\Upsilon^{IN}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right|^{\mathfrak{F}_{\mathfrak{p}}}\right)^{u}\right)\left(1 + \left(-1 + \left(1 - \left|\Upsilon^{IN}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right|^{\mathfrak{F}_{\mathfrak{q}}}\right)^{v}\right)\right) + \\ 1\left(1 + \left(-1 + \left(1 - \left|\Upsilon^{IN}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right|^{\mathfrak{F}_{\mathfrak{q}}}\right)^{v}\right)\right) \end{pmatrix}\right) \end{pmatrix} \\
&= \begin{pmatrix} \left(1 - \left(1 - \Upsilon^{RP}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right)^{\mathfrak{F}_{\mathfrak{p}}}\right)^{u} \times \left(1 - \left(1 - \Upsilon^{RP}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\mathfrak{F}_{\mathfrak{q}}}\right)^{v} \\ +\mathfrak{i}\left(\left(1 - \left(1 - \Upsilon^{IP}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right)^{\mathfrak{F}_{\mathfrak{p}}}\right)^{u} \times \left(1 - \left(1 - \Upsilon^{IP}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\mathfrak{F}_{\mathfrak{q}}}\right)^{v}\right), \\ -1 + \left(1 + \left(-1 + \left(1 - \left|\Upsilon^{RN}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right|^{\mathfrak{F}_{\mathfrak{p}}}\right)^{u}\right) \times \left(1 + \left(-1 + \left(1 - \left|\Upsilon^{RN}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right|^{\mathfrak{F}_{\mathfrak{q}}}\right)^{v}\right)\right)\right) \\ +\mathfrak{i}\left(-1 + \left(1 + \left(-1 + \left(1 - \left|\Upsilon^{IN}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right|^{\mathfrak{F}_{\mathfrak{p}}}\right)^{u}\right) \times \left(1 + \left(-1 + \left(1 - \left|\Upsilon^{IN}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right|^{\mathfrak{F}_{\mathfrak{q}}}\right)^{v}\right)\right)\right)\right) \end{pmatrix} \\
&= \begin{pmatrix} \left(1 - \left(1 - \Upsilon^{RP}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right)^{\mathfrak{F}_{\mathfrak{p}}}\right)^{u} \times \left(1 - \left(1 - \Upsilon^{RP}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\mathfrak{F}_{\mathfrak{q}}}\right)^{v} \\ +\mathfrak{i}\left(\left(1 - \left(1 - \Upsilon^{IP}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right)^{\mathfrak{F}_{\mathfrak{p}}}\right)^{u} \times \left(1 - \left(1 - \Upsilon^{IP}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\mathfrak{F}_{\mathfrak{q}}}\right)^{v}\right), \\ -1 + \left(1 - \left|\Upsilon^{RN}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right|^{\mathfrak{F}_{\mathfrak{p}}}\right)^{u} \times \left(1 - \left|\Upsilon^{RN}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right|^{\mathfrak{F}_{\mathfrak{q}}}\right)^{v} \\ +\mathfrak{i}\left(-1 + \left(1 - \left|\Upsilon^{IN}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right|^{\mathfrak{F}_{\mathfrak{p}}}\right)^{u} \times \left(1 - \left|\Upsilon^{IN}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right|^{\mathfrak{F}_{\mathfrak{q}}}\right)^{v}\right) \end{pmatrix}.
\end{aligned}$$

Next, determine $\sum_{\substack{\mathfrak{p}=1, \mathfrak{q}=1 \\ \mathfrak{p} \neq \mathfrak{q}}}^{\mathfrak{n}} (\mathfrak{F}_{\mathfrak{p}}\mathcal{M}_{BCFS-\mathfrak{p}})^{u} \otimes (\mathfrak{F}_{\mathfrak{q}}\mathcal{M}_{BCFS-\mathfrak{q}})^{v}$

TABLE 2 Aggregated values of information are described in Table 1.

Method	$\mathcal{M}_{BCFS-1}$	$\mathcal{M}_{BCFS-2}$	$\mathcal{M}_{BCFS-3}$	$\mathcal{M}_{BCFS-4}$
BCFPBM	$\begin{pmatrix} 0.544+\mathbf{i}\,0.55, \\ -0.177-\mathbf{i}\,0.18 \end{pmatrix}$	$\begin{pmatrix} 0.581+\mathbf{i}\,0.392, \\ -0.182-\mathbf{i}\,0.171 \end{pmatrix}$	$\begin{pmatrix} 0.606+\mathbf{i}\,0.446, \\ -0.185-\mathbf{i}\,0.169 \end{pmatrix}$	$\begin{pmatrix} 0.642+\mathbf{i}\,0.413, \\ -0.159-\mathbf{i}\,0.186 \end{pmatrix}$
BCFWPBM	$\begin{pmatrix} 0.359+\mathbf{i}\,0.359, \\ -0.187-\mathbf{i}\,0.187 \end{pmatrix}$	$\begin{pmatrix} 0.393+\mathbf{i}\,0.232, \\ -0.188-\mathbf{i}\,0.185 \end{pmatrix}$	$\begin{pmatrix} 0.417+\mathbf{i}\,0.268, \\ -0.184-\mathbf{i}\,0.189 \end{pmatrix}$	$\begin{pmatrix} 0.453+\mathbf{i}\,0.245, \\ -0.189-\mathbf{i}\,0.181 \end{pmatrix}$
BCFPGBM	$\begin{pmatrix} 0.442+\mathbf{i}\,0.551, \\ -0.549-\mathbf{i}\,0.545 \end{pmatrix}$	$\begin{pmatrix} 0.511+\mathbf{i}\,0.381, \\ -0.578-\mathbf{i}\,0.443 \end{pmatrix}$	$\begin{pmatrix} 0.548+\mathbf{i}\,0.45, \\ -0.619-\mathbf{i}\,0.418 \end{pmatrix}$	$\begin{pmatrix} 0.614+\mathbf{i}\,0.416, \\ -0.628-\mathbf{i}\,0.35 \end{pmatrix}$
BCFWPGBM	$\begin{pmatrix} 0.645+\mathbf{i}\,0.713, \\ -0.386-\mathbf{i}\,0.352 \end{pmatrix}$	$\begin{pmatrix} 0.71+\mathbf{i}\,0.627, \\ -0.408-\mathbf{i}\,0.258 \end{pmatrix}$	$\begin{pmatrix} 0.739+\mathbf{i}\,0.627, \\ -0.438-\mathbf{i}\,0.241 \end{pmatrix}$	$\begin{pmatrix} 0.785+\mathbf{i}\,0.595, \\ -0.444-\mathbf{i}\,0.191 \end{pmatrix}$

$$\sum_{\substack{þ=1,\mathfrak{q}=1\\ þ\neq\mathfrak{q}}}^{ŋ} (\mathfrak{F}_þ \mathcal{M}_{BCFS-þ})^{u} \otimes (\mathfrak{F}_\mathfrak{q} \mathcal{M}_{BCFS-\mathfrak{q}})^{v}$$

$$= \begin{pmatrix} 1-\prod_{\substack{þ=1,\mathfrak{q}=1\\ þ\neq\mathfrak{q}}}^{ŋ}\left(1-\left(\left(1-\left(1-\Upsilon^{RP}_{\mathcal{M}_{BCFS-þ}}\right)^{\mathfrak{F}_þ}\right)^{u}\times\left(1-\left(1-\Upsilon^{RP}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\mathfrak{F}_\mathfrak{q}}\right)^{v}\right)\right) \\ +\mathbf{i}\left(1-\prod_{\substack{þ=1,\mathfrak{q}=1\\ þ\neq\mathfrak{q}}}^{ŋ}\left(1-\left(\left(1-\left(1-\Upsilon^{IP}_{\mathcal{M}_{BCFS-þ}}\right)^{\mathfrak{F}_þ}\right)^{u}\times\left(1-\left(1-\Upsilon^{IP}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\mathfrak{F}_\mathfrak{q}}\right)^{v}\right)\right)\right), \\ -\prod_{\substack{þ=1,\mathfrak{q}=1\\ þ\neq\mathfrak{q}}}^{ŋ}\left(-1+\left(1-\left|\Upsilon^{RN}_{\mathcal{M}_{BCFS-þ}}\right|^{\mathfrak{F}_þ}\right)^{u}\times\left(1-\left|\Upsilon^{RN}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right|^{\mathfrak{F}_\mathfrak{q}}\right)^{v}\right) \\ +\mathbf{i}\left(-\prod_{\substack{þ=1,\mathfrak{q}=1\\ þ\neq\mathfrak{q}}}^{ŋ}\left(-1+\left(1-\left|\Upsilon^{IN}_{\mathcal{M}_{BCFS-þ}}\right|^{\mathfrak{F}_þ}\right)^{u}\times\left(1-\left|\Upsilon^{IN}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right|^{\mathfrak{F}_\mathfrak{q}}\right)^{v}\right)\right) \end{pmatrix}.$$

Now,

$$\frac{1}{ŋ^2-ŋ}\sum_{\substack{þ=1,\mathfrak{q}=1\\ þ\neq\mathfrak{q}}}^{ŋ} (\mathfrak{F}_þ \mathcal{M}_{BCFS-þ})^{u} \otimes (\mathfrak{F}_\mathfrak{q} \mathcal{M}_{BCFS-\mathfrak{q}})^{v}$$

$$= \begin{pmatrix} 1-\left(\prod_{\substack{þ=1,\mathfrak{q}=1\\ þ\neq\mathfrak{q}}}^{ŋ}\left(1-\left(\left(1-\left(1-\Upsilon^{RP}_{\mathcal{M}_{BCFS-þ}}\right)^{\mathfrak{F}_þ}\right)^{u}\times\left(1-\left(1-\Upsilon^{RP}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\mathfrak{F}_\mathfrak{q}}\right)^{v}\right)\right)\right)^{1/ŋ^2-ŋ} \\ +\mathbf{i}\left(1-\left(\prod_{\substack{þ=1,\mathfrak{q}=1\\ þ\neq\mathfrak{q}}}^{ŋ}\left(1-\left(\left(1-\left(1-\Upsilon^{IP}_{\mathcal{M}_{BCFS-þ}}\right)^{\mathfrak{F}_þ}\right)^{u}\times\left(1-\left(1-\Upsilon^{IP}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\mathfrak{F}_\mathfrak{q}}\right)^{v}\right)\right)\right)^{1/ŋ^2-ŋ}\right), \\ -\left|-\prod_{\substack{þ=1,\mathfrak{q}=1\\ þ\neq\mathfrak{q}}}^{ŋ}\left(-1+\left(1-\left|\Upsilon^{RN}_{\mathcal{M}_{BCFS-þ}}\right|^{\mathfrak{F}_þ}\right)^{u}\times\left(1-\left|\Upsilon^{RN}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right|^{\mathfrak{F}_\mathfrak{q}}\right)^{v}\right)\right|^{1/ŋ^2-ŋ} \\ +\mathbf{i}\left(-\left|-\prod_{\substack{þ=1,\mathfrak{q}=1\\ þ\neq\mathfrak{q}}}^{ŋ}\left(-1+\left(1-\left|\Upsilon^{IN}_{\mathcal{M}_{BCFS-þ}}\right|^{\mathfrak{F}_þ}\right)^{u}\times\left(1-\left|\Upsilon^{IN}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right|^{\mathfrak{F}_\mathfrak{q}}\right)^{v}\right)\right|^{1/ŋ^2-ŋ}\right) \end{pmatrix}.$$

Finally, determine $\left(\frac{1}{ŋ^2-ŋ}\sum_{\substack{þ=1,\mathfrak{q}=1\\ þ\neq\mathfrak{q}}}^{ŋ} (\mathfrak{F}_þ \mathcal{M}_{BCFS-þ})^{u} \otimes (\mathfrak{F}_\mathfrak{q} \mathcal{M}_{BCFS-\mathfrak{q}})^{v}\right)^{1/u+v}$ as

$$\left(\frac{1}{ŋ^2-ŋ}\sum_{\substack{þ=1,\mathfrak{q}=1\\ þ\neq\mathfrak{q}}}^{ŋ} (\mathfrak{F}_þ \mathcal{M}_{BCFS-þ})^{u} \otimes (\mathfrak{F}_\mathfrak{q} \mathcal{M}_{BCFS-\mathfrak{q}})^{v}\right)^{1/u+v}$$

$$= \begin{pmatrix} \left(1-\left(\prod_{\substack{þ=1,\mathfrak{q}=1\\ þ\neq\mathfrak{q}}}^{ŋ}\left(1-\left(\left(1-\left(1-\Upsilon^{RP}_{\mathcal{M}_{BCFS-þ}}\right)^{\mathfrak{F}_þ}\right)^{u}\times\left(1-\left(1-\Upsilon^{RP}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\mathfrak{F}_\mathfrak{q}}\right)^{v}\right)\right)\right)^{1/ŋ^2-ŋ}\right)^{1/u+v} \\ +\mathbf{i}\left(1-\left(\prod_{\substack{þ=1,\mathfrak{q}=1\\ þ\neq\mathfrak{q}}}^{ŋ}\left(1-\left(\left(1-\left(1-\Upsilon^{IP}_{\mathcal{M}_{BCFS-þ}}\right)^{\mathfrak{F}_þ}\right)^{u}\times\left(1-\left(1-\Upsilon^{IP}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\mathfrak{F}_\mathfrak{q}}\right)^{v}\right)\right)\right)^{1/ŋ^2-ŋ}\right)^{1/u+v}, \\ -1+\left(1-\left|-\prod_{\substack{þ=1,\mathfrak{q}=1\\ þ\neq\mathfrak{q}}}^{ŋ}\left(-1+\left(1-\left|\Upsilon^{RN}_{\mathcal{M}_{BCFS-þ}}\right|^{\mathfrak{F}_þ}\right)^{u}\times\left(1-\left|\Upsilon^{RN}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right|^{\mathfrak{F}_\mathfrak{q}}\right)^{v}\right)\right|^{1/ŋ^2-ŋ}\right)^{1/u+v} \\ +\mathbf{i}\left(-1+\left(1-\left|-\prod_{\substack{þ=1,\mathfrak{q}=1\\ þ\neq\mathfrak{q}}}^{ŋ}\left(-1+\left(1-\left|\Upsilon^{IN}_{\mathcal{M}_{BCFS-þ}}\right|^{\mathfrak{F}_þ}\right)^{u}\times\left(1-\left|\Upsilon^{IN}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right|^{\mathfrak{F}_\mathfrak{q}}\right)^{v}\right)\right|^{1/ŋ^2-ŋ}\right)^{1/u+v}\right) \end{pmatrix}. \quad (9)$$

Now, replace $\mathfrak{F}_þ = \frac{ŋ(\Upsilon(\mathcal{M}_{BCFS-þ})+1)}{\sum_{l=1}^{ŋ}(\Upsilon(\mathcal{M}_{BCFS-l})+1)}$ in Eq. 9, then

$$\left(\frac{1}{ŋ^2-ŋ}\sum_{\substack{þ=1,\mathfrak{q}=1\\ þ\neq\mathfrak{q}}}^{ŋ}\begin{pmatrix}\left(\frac{ŋ(\Upsilon(\mathcal{M}_{BCFS-þ})+1)}{\sum_{l=1}^{ŋ}(\Upsilon(\mathcal{M}_{BCFS-l})+1)}\mathcal{M}_{BCFS-þ}\right)^{u} \\ \otimes\left(\frac{ŋ(\Upsilon(\mathcal{M}_{BCFS-\mathfrak{q}})+1)}{\sum_{l=1}^{ŋ}(\Upsilon(\mathcal{M}_{BCFS-l})+1)}\mathcal{M}_{BCFS-\mathfrak{q}}\right)^{v}\end{pmatrix}\right)^{1/u+v}$$

$$= \begin{pmatrix} \left(1-\left(\prod_{\substack{þ=1,\mathfrak{q}=1\\ þ\neq\mathfrak{q}}}^{ŋ}\left(1-\begin{pmatrix}\left(1-\left(1-\Upsilon^{RP}_{\mathcal{M}_{BCFS-þ}}\right)^{\frac{ŋ(\Upsilon(\mathcal{M}_{BCFS-þ})+1)}{\sum_{l=1}^{ŋ}(\Upsilon(\mathcal{M}_{BCFS-l})+1)}}\right)^{u} \\ \times\left(1-\left(1-\Upsilon^{RP}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\frac{ŋ(\Upsilon(\mathcal{M}_{BCFS-\mathfrak{q}})+1)}{\sum_{l=1}^{ŋ}(\Upsilon(\mathcal{M}_{BCFS-l})+1)}}\right)^{v}\end{pmatrix}\right)\right)^{1/ŋ^2-ŋ}\right)^{1/u+v} \\ +\mathbf{i}\left(1-\left(\prod_{\substack{þ=1,\mathfrak{q}=1\\ þ\neq\mathfrak{q}}}^{ŋ}\left(1-\begin{pmatrix}\left(1-\left(1-\Upsilon^{IP}_{\mathcal{M}_{BCFS-þ}}\right)^{\frac{ŋ(\Upsilon(\mathcal{M}_{BCFS-þ})+1)}{\sum_{l=1}^{ŋ}(\Upsilon(\mathcal{M}_{BCFS-l})+1)}}\right)^{u} \\ \times\left(1-\left(1-\Upsilon^{IP}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\frac{ŋ(\Upsilon(\mathcal{M}_{BCFS-\mathfrak{q}})+1)}{\sum_{l=1}^{ŋ}(\Upsilon(\mathcal{M}_{BCFS-l})+1)}}\right)^{v}\end{pmatrix}\right)\right)^{1/ŋ^2-ŋ}\right)^{1/u+v}, \\ -1+\left(1-\left|-\prod_{\substack{þ=1,\mathfrak{q}=1\\ þ\neq\mathfrak{q}}}^{ŋ}\begin{pmatrix}-1+\left(1-\left|\Upsilon^{RN}_{\mathcal{M}_{BCFS-þ}}\right|^{\frac{ŋ(\Upsilon(\mathcal{M}_{BCFS-þ})+1)}{\sum_{l=1}^{ŋ}(\Upsilon(\mathcal{M}_{BCFS-l})+1)}}\right)^{u} \\ \times\left(1-\left|\Upsilon^{RN}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right|^{\frac{ŋ(\Upsilon(\mathcal{M}_{BCFS-\mathfrak{q}})+1)}{\sum_{l=1}^{ŋ}(\Upsilon(\mathcal{M}_{BCFS-l})+1)}}\right)^{v}\end{pmatrix}\right|^{1/ŋ^2-ŋ}\right)^{1/u+v} \\ +\mathbf{i}\left(-1+\left(1-\left|-\prod_{\substack{þ=1,\mathfrak{q}=1\\ þ\neq\mathfrak{q}}}^{ŋ}\begin{pmatrix}-1+\left(1-\left|\Upsilon^{IN}_{\mathcal{M}_{BCFS-þ}}\right|^{\frac{ŋ(\Upsilon(\mathcal{M}_{BCFS-þ})+1)}{\sum_{l=1}^{ŋ}(\Upsilon(\mathcal{M}_{BCFS-l})+1)}}\right)^{u} \\ \times\left(1-\left|\Upsilon^{IN}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right|^{\frac{ŋ(\Upsilon(\mathcal{M}_{BCFS-\mathfrak{q}})+1)}{\sum_{l=1}^{ŋ}(\Upsilon(\mathcal{M}_{BCFS-l})+1)}}\right)^{v}\end{pmatrix}\right|^{1/ŋ^2-ŋ}\right)^{1/u+v}\right) \end{pmatrix}$$

$$= BCFPBM^{u,v}\left(\mathcal{M}_{BCFS-1}, \mathcal{M}_{BCFS-2}, \ldots, \mathcal{M}_{BCFS-ŋ}\right).$$

The BCFPBM operator holds the following axioms.

1) Idempotency: Let $\mathcal{M}_{BCFS-þ} = \mathcal{M}_{BCFS} = (\Upsilon^{P}_{\mathcal{M}_{BCFS}}, \Upsilon^{N}_{\mathcal{M}_{BCFS}}) = (\Upsilon^{RP}_{\mathcal{M}_{BCFS}} + \mathbf{i}\,\Upsilon^{IP}_{\mathcal{M}_{BCFS}}, \Upsilon^{RN}_{\mathcal{M}_{BCFS}} + \mathbf{i}\,\Upsilon^{IN}_{\mathcal{M}_{BCFS}})$, $þ = 1, 2, .., ŋ$. Then,

TABLE 3 SVs of the aggregated values of Table 2 and their ranking.

Method	$\mathcal{S}(\mathcal{M}_{BCFS-1})$	$\mathcal{S}(\mathcal{M}_{BCFS-2})$	$\mathcal{S}(\mathcal{M}_{BCFS-3})$	$\mathcal{S}(\mathcal{M}_{BCFS-4})$	*Ranking*
BCFPBM	0.863	0.832	0.852	0.85	$\mathcal{M}_{BCFS-1} > \mathcal{M}_{BCFS-3} > \mathcal{M}_{BCFS-4} > \mathcal{M}_{BCFS-2}$
BCFWPBM	0.773	0.749	0.765	0.767	$\mathcal{M}_{BCFS-1} > \mathcal{M}_{BCFS-4} > \mathcal{M}_{BCFS-3} > \mathcal{M}_{BCFS-2}$
BCFPGBM	0.475	0.468	0.49	0.513	$\mathcal{M}_{BCFS-4} > \mathcal{M}_{BCFS-3} > \mathcal{M}_{BCFS-1} > \mathcal{M}_{BCFS-2}$
BCFWPGBM	0.655	0.648	0.672	0.686	$\mathcal{M}_{BCFS-4} > \mathcal{M}_{BCFS-3} > \mathcal{M}_{BCFS-1} > \mathcal{M}_{BCFS-2}$

$$BCFPBM^{u,v}(\mathcal{M}_{BCFS-1}, \mathcal{M}_{BCFS-2}, \ldots, \mathcal{M}_{BCFS-ŋ}) = \mathcal{M}_{BCFS}.$$

2) Commutativity: Let $\mathcal{M}_{BCFS-ƥ} = (\Upsilon^{P}_{\mathcal{M}_{BCFS-ƥ}}, \Upsilon^{N}_{\mathcal{M}_{BCFS-ƥ}}) = (\Upsilon^{RP}_{\mathcal{M}_{BCFS-ƥ}} + \mathfrak{i}\,\Upsilon^{IP}_{\mathcal{M}_{BCFS-ƥ}}, \Upsilon^{RN}_{\mathcal{M}_{BCFS-ƥ}} + \mathfrak{i}\,\Upsilon^{IN}_{\mathcal{M}_{BCFS-ƥ}})$, $ƥ = ƥ, 2, \ldots,$ ŋ have the permutation $\mathcal{M}'_{BCFS-ƥ} = (\Upsilon'^{P}_{\mathcal{M}_{BCFS-ƥ}}, \Upsilon'^{N}_{\mathcal{M}_{BCFS-ƥ}}) = (\Upsilon'^{RP}_{\mathcal{M}_{BCFS-ƥ}} + \mathfrak{i}\,\Upsilon'^{IP}_{\mathcal{M}_{BCFS-ƥ}}, \Upsilon'^{RN}_{\mathcal{M}_{BCFS-ƥ}} + \mathfrak{i}\,\Upsilon'^{IN}_{\mathcal{M}_{BCFS-ƥ}})$, $ƥ = ƥ, 2, \ldots, ŋ$. Then,

$$BCFPBM^{u,v}(\mathcal{M}_{BCFS-1}, \mathcal{M}_{BCFS-2}, \ldots, \mathcal{M}_{BCFS-ŋ}) = BCFPBM^{u,v}\left(\mathcal{M}'_{BCFS-1}, \mathcal{M}'_{BCFS-2}, \ldots, \mathcal{M}'_{BCFS-ŋ}\right).$$

3) Boundedness: Let a group of BCFNs $\mathcal{M}_{BCFS-ƥ} = (\Upsilon^{P}_{\mathcal{M}_{BCFS-ƥ}}, \Upsilon^{N}_{\mathcal{M}_{BCFS-ƥ}}) = (\Upsilon^{RP}_{\mathcal{M}_{BCFS-ƥ}} + \mathfrak{i}\,\Upsilon^{IP}_{\mathcal{M}_{BCFS-ƥ}}, \Upsilon^{RN}_{\mathcal{M}_{BCFS-ƥ}} + \mathfrak{i}\,\Upsilon^{IN}_{\mathcal{M}_{BCFS-ƥ}})$, $ƥ = ƥ, 2, \ldots, ŋ$ and let $\mathcal{M}^{-}_{BCFS} = (\min_{ƥ}\{\Upsilon^{RP}_{\mathcal{M}_{BCFS-ƥ}}\} + \mathfrak{i}\min_{ƥ}\{\Upsilon^{IP}_{\mathcal{M}_{BCFS-ƥ}}\}, \max_{ƥ}\{\Upsilon^{RN}_{\mathcal{M}_{BCFS-ƥ}}\} + \mathfrak{i}\max_{ƥ}\{\Upsilon^{IN}_{\mathcal{M}_{BCFS-ƥ}}\})$ and $\mathcal{M}^{+}_{BCFS} = (\max_{ƥ}\{\Upsilon^{RP}_{\mathcal{M}_{BCFS-ƥ}}\} + \mathfrak{i}\max_{ƥ}\{\Upsilon^{IP}_{\mathcal{M}_{BCFS-ƥ}}\}, \min_{ƥ}\{\Upsilon^{RN}_{\mathcal{M}_{BCFS-ƥ}}\} + \mathfrak{i}\min_{ƥ}\{\Upsilon^{IN}_{\mathcal{M}_{BCFS-ƥ}}\})$. Then,

$$\mathcal{M}^{-}_{BCFS} \le BCFPBM^{u,v}(\mathcal{M}_{BCFS-1}, \mathcal{M}_{BCFS-2}, \ldots, \mathcal{M}_{BCFS-ŋ}) \le \mathcal{M}^{+}_{BCFS}.$$

Definition 7: Take a group of BCFNs $\mathcal{M}_{BCFS-ƥ} = (\Upsilon^{P}_{\mathcal{M}_{BCFS-ƥ}}, \Upsilon^{N}_{\mathcal{M}_{BCFS-ƥ}}) = (\Upsilon^{RP}_{\mathcal{M}_{BCFS-ƥ}} + \mathfrak{i}\,\Upsilon^{IP}_{\mathcal{M}_{BCFS-ƥ}}, \Upsilon^{RN}_{\mathcal{M}_{BCFS-ƥ}} + \mathfrak{i}\,\Upsilon^{IN}_{\mathcal{M}_{BCFS-ƥ}})$, $ƥ = ƥ, 2, \ldots, ŋ$, then the BCFWPBM operator is inspected as

$$BCFWPBM^{u,v}(\mathcal{M}_{BCFS-1}, \mathcal{M}_{BCFS-2}, \ldots, \mathcal{M}_{BCFS-ŋ}) = \left(\frac{1}{ŋ^2 - ŋ}\sum_{\substack{ƥ=1,\,ɋ=1\\ ƥ\neq ɋ}}^{ŋ}\left(\left(\frac{ŋ w_{\mathfrak{v}-ƥ}(\mathcal{T}(\mathcal{M}_{BCFS-ƥ})+1)}{\sum_{l=1}^{ŋ} w_{\mathfrak{v}-l}(\mathcal{T}(\mathcal{M}_{BCFS-l})+1)}\mathcal{M}_{BCFS-ƥ}\right)^{u} \otimes \left(\frac{ŋ w_{\mathfrak{v}-ɋ}(\mathcal{T}(\mathcal{M}_{BCFS-ɋ})+1)}{\sum_{l=1}^{ŋ} w_{\mathfrak{v}-l}(\mathcal{T}(\mathcal{M}_{BCFS-l})+1)}\mathcal{M}_{BCFS-ɋ}\right)^{v}\right)\right)^{1/u+v}, \quad (10)$$

where $u, v > 0$ and $\mathcal{T}(\mathcal{M}_{BCFS-ƥ}) = \sum_{\substack{ɋ=1\\ ɋ\neq ƥ}}^{ŋ} Sup(\mathcal{M}_{BCFS-ƥ}, \mathcal{M}_{BCFS-ɋ})$, and $w_{\mathfrak{v}} = (w_{\mathfrak{v}-1}, w_{\mathfrak{v}-2}, \ldots, w_{\mathfrak{v}-ŋ})$ is the weight vector (WV) with the property that $w_{\mathfrak{v}-ƥ} \in [0, 1]$ and $\sum_{ƥ=1}^{ŋ} w_{\mathfrak{v}-ƥ} = 1$.

Theorem 2: The aggregated outcome from Eq. 10 based on the group of BCFNs $\mathcal{M}_{BCFS-ƥ} = (\Upsilon^{P}_{\mathcal{M}_{BCFS-ƥ}}, \Upsilon^{N}_{\mathcal{M}_{BCFS-ƥ}}) = (\Upsilon^{RP}_{\mathcal{M}_{BCFS-ƥ}} + \mathfrak{i}\,\Upsilon^{IP}_{\mathcal{M}_{BCFS-ƥ}}, \Upsilon^{RN}_{\mathcal{M}_{BCFS-ƥ}} + \mathfrak{i}\,\Upsilon^{IN}_{\mathcal{M}_{BCFS-ƥ}})$, $ƥ = ƥ, 2, \ldots, ŋ$ is a BCFN and inspected as

$$BCFWPBM^{u,v}(\mathcal{M}_{BCFS-1}, \mathcal{M}_{BCFS-2}, \ldots, \mathcal{M}_{BCFS-ŋ})$$

$$= \left(\begin{array}{l}
\left(1-\left(\prod_{\substack{ƥ=1,\,ɋ=1\\ ƥ\neq ɋ}}^{ŋ}\left(1-\left(\begin{array}{l}\left(1-\left(1-\Upsilon^{RP}_{\mathcal{M}_{BCFS-ƥ}}\right)^{\frac{ŋ w_{\mathfrak{v}-ƥ}(\mathcal{T}(\mathcal{M}_{BCFS-ƥ})+1)}{\sum_{l=1}^{ŋ} w_{\mathfrak{v}-l}(\mathcal{T}(\mathcal{M}_{BCFS-l})+1)}}\right)^{u}\\ \times\left(1-\left(1-\Upsilon^{RP}_{\mathcal{M}_{BCFS-ɋ}}\right)^{\frac{ŋ w_{\mathfrak{v}-ɋ}(\mathcal{T}(\mathcal{M}_{BCFS-ɋ})+1)}{\sum_{l=1}^{ŋ} w_{\mathfrak{v}-l}(\mathcal{T}(\mathcal{M}_{BCFS-l})+1)}}\right)^{v}\end{array}\right)\right)\right)^{1/ŋ^2-ŋ}\right)^{1/u+v}\\
+\mathfrak{i}\left(1-\left(\prod_{\substack{ƥ=1,\,ɋ=1\\ ƥ\neq ɋ}}^{ŋ}\left(1-\left(\begin{array}{l}\left(1-\left(1-\Upsilon^{IP}_{\mathcal{M}_{BCFS-ƥ}}\right)^{\frac{ŋ w_{\mathfrak{v}-ƥ}(\mathcal{T}(\mathcal{M}_{BCFS-ƥ})+1)}{\sum_{l=1}^{ŋ} w_{\mathfrak{v}-l}(\mathcal{T}(\mathcal{M}_{BCFS-l})+1)}}\right)^{u}\\ \times\left(1-\left(1-\Upsilon^{IP}_{\mathcal{M}_{BCFS-ɋ}}\right)^{\frac{ŋ w_{\mathfrak{v}-ɋ}(\mathcal{T}(\mathcal{M}_{BCFS-ɋ})+1)}{\sum_{l=1}^{ŋ} w_{\mathfrak{v}-l}(\mathcal{T}(\mathcal{M}_{BCFS-l})+1)}}\right)^{v}\end{array}\right)\right)\right)^{1/ŋ^2-ŋ}\right)^{1/u+v},\\
-1+\left(1-\left|-\prod_{\substack{ƥ=1,\,ɋ=1\\ ƥ\neq ɋ}}^{ŋ}\left(\begin{array}{l}-1+\left(1-\left|\Upsilon^{RN}_{\mathcal{M}_{BCFS-ƥ}}\right|^{\frac{ŋ w_{\mathfrak{v}-ƥ}(\mathcal{T}(\mathcal{M}_{BCFS-ƥ})+1)}{\sum_{l=1}^{ŋ} w_{\mathfrak{v}-l}(\mathcal{T}(\mathcal{M}_{BCFS-l})+1)}}\right)^{u}\\ \times\left(1-\left|\Upsilon^{RN}_{\mathcal{M}_{BCFS-ɋ}}\right|^{\frac{ŋ w_{\mathfrak{v}-ɋ}(\mathcal{T}(\mathcal{M}_{BCFS-ɋ})+1)}{\sum_{l=1}^{ŋ} w_{\mathfrak{v}-l}(\mathcal{T}(\mathcal{M}_{BCFS-l})+1)}}\right)^{v}\end{array}\right)\right|^{1/ŋ^2-ŋ}\right)^{1/u+v}\\
+\mathfrak{i}\left(-1+\left(1-\left|-\prod_{\substack{ƥ=1,\,ɋ=1\\ ƥ\neq ɋ}}^{ŋ}\left(\begin{array}{l}-1+\left(1-\left|\Upsilon^{IN}_{\mathcal{M}_{BCFS-ƥ}}\right|^{\frac{ŋ w_{\mathfrak{v}-ƥ}(\mathcal{T}(\mathcal{M}_{BCFS-ƥ})+1)}{\sum_{l=1}^{ŋ} w_{\mathfrak{v}-l}(\mathcal{T}(\mathcal{M}_{BCFS-l})+1)}}\right)^{u}\\ \times\left(1-\left|\Upsilon^{IN}_{\mathcal{M}_{BCFS-ɋ}}\right|^{\frac{ŋ w_{\mathfrak{v}-ɋ}(\mathcal{T}(\mathcal{M}_{BCFS-ɋ})+1)}{\sum_{l=1}^{ŋ} w_{\mathfrak{v}-l}(\mathcal{T}(\mathcal{M}_{BCFS-l})+1)}}\right)^{v}\end{array}\right)\right|^{1/ŋ^2-ŋ}\right)^{1/u+v}\right)
\end{array}\right). \quad (11)$$

The BCFWPBM operator holds the following axioms.

1) *Idempotency*: Let $\mathcal{M}_{BCFS-ƥ} = \mathcal{M}_{BCFS} = (\Upsilon^{P}_{\mathcal{M}_{BCFS}}, \Upsilon^{N}_{\mathcal{M}_{BCFS}}) = (\Upsilon^{RP}_{\mathcal{M}_{BCFS}} + \mathfrak{i}\,\Upsilon^{IP}_{\mathcal{M}_{BCFS}}, \Upsilon^{RN}_{\mathcal{M}_{BCFS}} + \mathfrak{i}\,\Upsilon^{IN}_{\mathcal{M}_{BCFS}})$, $ƥ = 1, 2, .., ŋ$. Then,

$$BCFWPBM^{u,v}(\mathcal{M}_{BCFS-1}, \mathcal{M}_{BCFS-2}, \ldots, \mathcal{M}_{BCFS-ŋ}) = \mathcal{M}_{BCFS}.$$

2) Commutativity: *Let* $\mathcal{M}_{BCFS-ϸ} = (\Upsilon^{P}_{\mathcal{M}_{BCFS-ϸ}}, \Upsilon^{N}_{\mathcal{M}_{BCFS-ϸ}}) = (\Upsilon^{RP}_{\mathcal{M}_{BCFS-ϸ}} + \mathfrak{i}\,\Upsilon^{IP}_{\mathcal{M}_{BCFS-ϸ}}, \Upsilon^{RN}_{\mathcal{M}_{BCFS-ϸ}} + \mathfrak{i}\,\Upsilon^{IN}_{\mathcal{M}_{BCFS-ϸ}})$, $ϸ = ϸ, 2, \ldots, ŋ$ have the permutation $\mathcal{M}'_{BCFS-ϸ} = (\Upsilon'^{P}_{\mathcal{M}_{BCFS-ϸ}}, \Upsilon'^{N}_{\mathcal{M}_{BCFS-ϸ}}) = (\Upsilon'^{RP}_{\mathcal{M}_{BCFS-ϸ}} + \mathfrak{i}\,\Upsilon'^{IP}_{\mathcal{M}_{BCFS-ϸ}}, \Upsilon'^{RN}_{\mathcal{M}_{BCFS-ϸ}} + \mathfrak{i}\,\Upsilon'^{IN}_{\mathcal{M}_{BCFS-ϸ}})$, $ϸ = ϸ, 2, \ldots, ŋ$. Then,

$$BCFWPBM^{u,v}(\mathcal{M}_{BCFS-1}, \mathcal{M}_{BCFS-2}, \ldots, \mathcal{M}_{BCFS-ŋ}) = BCFWPBM^{u,v}(\mathcal{M}'_{BCFS-1}, \mathcal{M}'_{BCFS-2}, \ldots, \mathcal{M}'_{BCFS-ŋ}).$$

3) *Boundedness*: Let a group of BCFNs $\mathcal{M}_{BCFS-ϸ} = (\Upsilon^{P}_{\mathcal{M}_{BCFS-ϸ}}, \Upsilon^{N}_{\mathcal{M}_{BCFS-ϸ}}) = (\Upsilon^{RP}_{\mathcal{M}_{BCFS-ϸ}} + \mathfrak{i}\,\Upsilon^{IP}_{\mathcal{M}_{BCFS-ϸ}}, \Upsilon^{RN}_{\mathcal{M}_{BCFS-ϸ}} + \mathfrak{i}\,\Upsilon^{IN}_{\mathcal{M}_{BCFS-ϸ}})$, $ϸ = ϸ, 2, \ldots, ŋ$ and let $\mathcal{M}^{-}_{BCFS} = (\min_{ϸ}\{\Upsilon^{RP}_{\mathcal{M}_{BCFS-ϸ}}\} + \mathfrak{i}\min_{ϸ}\{\Upsilon^{IP}_{\mathcal{M}_{BCFS-ϸ}}\}, \max_{ϸ}\{\Upsilon^{RN}_{\mathcal{M}_{BCFS-ϸ}}\} + \mathfrak{i}\max_{ϸ}\{\Upsilon^{IN}_{\mathcal{M}_{BCFS-ϸ}}\})$ and $\mathcal{M}^{+}_{BCFS} = (\max_{ϸ}\{\Upsilon^{RP}_{\mathcal{M}_{BCFS-ϸ}}\} + \mathfrak{i}\max_{ϸ}\{\Upsilon^{IP}_{\mathcal{M}_{BCFS-ϸ}}\}, \min_{ϸ}\{\Upsilon^{RN}_{\mathcal{M}_{BCFS-ϸ}}\} + \mathfrak{i}\min_{ϸ}\{\Upsilon^{IN}_{\mathcal{M}_{BCFS-ϸ}}\})$. Then,

$$\mathcal{M}^{-}_{BCFS} \le BCFWPBM^{u,v}(\mathcal{M}_{BCFS-1}, \mathcal{M}_{BCFS-2}, \ldots, \mathcal{M}_{BCFS-ŋ}) \le \mathcal{M}^{+}_{BCFS}$$

Definition 8: Take a group of BCFNs $\mathcal{M}_{BCFS-ϸ} = (\Upsilon^{P}_{\mathcal{M}_{BCFS-ϸ}}, \Upsilon^{N}_{\mathcal{M}_{BCFS-ϸ}}) = (\Upsilon^{RP}_{\mathcal{M}_{BCFS-ϸ}} + \mathfrak{i}\,\Upsilon^{IP}_{\mathcal{M}_{BCFS-ϸ}}, \Upsilon^{RN}_{\mathcal{M}_{BCFS-ϸ}} + \mathfrak{i}\,\Upsilon^{IN}_{\mathcal{M}_{BCFS-ϸ}})$, $ϸ = ϸ, 2, \ldots, ŋ$, then the BCFPGBM operator is inspected as

$$BCFPGBM^{u,v}(\mathcal{M}_{BCFS-1}, \mathcal{M}_{BCFS-2}, \ldots, \mathcal{M}_{BCFS-ŋ}) = \frac{1}{u+v}\left(\prod_{\substack{ϸ=1, q=1\\ ϸ\neq q}}^{ŋ}\left(\begin{array}{c} u(\mathcal{M}_{BCFS-ϸ})^{\frac{ŋ(\mathcal{T}(\mathcal{M}_{BCFS-ϸ})+1)}{\sum_{l=1}^{ŋ}(\mathcal{T}(\mathcal{M}_{BCFS-l})+1)}} \\ \oplus\, v(\mathcal{M}_{BCFS-q})^{\frac{ŋ(\mathcal{T}(\mathcal{M}_{BCFS-q})+1)}{\sum_{l=1}^{ŋ}(\mathcal{T}(\mathcal{M}_{BCFS-l})+1)}}\end{array}\right)\right)^{1/ŋ^2-ŋ}, \tag{12}$$

where $u, v > 0$ and $\mathcal{T}(\mathcal{M}_{BCFS-ϸ}) = \sum_{\substack{q=1\\ q\neq ϸ}}^{ŋ} Sup(\mathcal{M}_{BCFS-ϸ}, \mathcal{M}_{BCFS-q})$ is the support to $\mathcal{M}_{BCFS-ϸ}$ from $\mathcal{M}_{BCFS-q}$ with these axioms:

1) $Sup(\mathcal{M}_{BCFS-ϸ}, \mathcal{M}_{BCFS-q}) \in [0, 1]$,
2) $Sup(\mathcal{M}_{BCFS-ϸ}, \mathcal{M}_{BCFS-q}) = Sup(\mathcal{M}_{BCFS-q}, \mathcal{M}_{BCFS-ϸ})$,
3) $Sup(\mathcal{M}_{BCFS-ϸ}, \mathcal{M}_{BCFS-q}) \ge Sup(\mathcal{M}_{BCFS-r}, \mathcal{M}_{BCFS-s})$ if $\mathfrak{D}(\mathcal{M}_{BCFS-ϸ}, \mathcal{M}_{BCFS-ϸ}) < \mathfrak{D}(\mathcal{M}_{BCFS-r}, \mathcal{M}_{BCFS-s})$, where $\mathfrak{D}$ is the distance measured among BCFSs.

Theorem 3: The aggregated outcome from Eq. 12 based on the group of BCFNs $\mathcal{M}_{BCFS-ϸ} = (\Upsilon^{P}_{\mathcal{M}_{BCFS-ϸ}}, \Upsilon^{N}_{\mathcal{M}_{BCFS-ϸ}}) = (\Upsilon^{RP}_{\mathcal{M}_{BCFS-ϸ}} + \mathfrak{i}\,\Upsilon^{IP}_{\mathcal{M}_{BCFS-ϸ}}, \Upsilon^{RN}_{\mathcal{M}_{BCFS-ϸ}} + \mathfrak{i}\,\Upsilon^{IN}_{\mathcal{M}_{BCFS-ϸ}})$, $ϸ = ϸ, 2, \ldots, ŋ$ is a BCFN and inspected as

$$BCFPGBM^{u,v}(\mathcal{M}_{BCFS-1}, \mathcal{M}_{BCFS-2}, \ldots, \mathcal{M}_{BCFS-ŋ})$$

$$= \left(\begin{array}{c}
\left(1-\left(1-\left(\prod_{\substack{ϸ=1, q=1\\ ϸ\neq q}}^{ŋ}\left(1-\left(\begin{array}{c}\left(1-(\Upsilon^{RP}_{\mathcal{M}_{BCFS-ϸ}})^{\frac{ŋ(\mathcal{T}(\mathcal{M}_{BCFS-ϸ})+1)}{\sum_{l=1}^{ŋ}(\mathcal{T}(\mathcal{M}_{BCFS-l})+1)}}\right)^{u}\\ \times\left(1-(\Upsilon^{RP}_{\mathcal{M}_{BCFS-q}})^{\frac{ŋ(\mathcal{T}(\mathcal{M}_{BCFS-q})+1)}{\sum_{l=1}^{ŋ}(\mathcal{T}(\mathcal{M}_{BCFS-l})+1)}}\right)^{v}\end{array}\right)\right)\right)^{1/ŋ^2-ŋ}\right)^{1/u+v}\right)\\
+\mathfrak{i}\left(1-\left(1-\left(\prod_{\substack{ϸ=1, q=1\\ ϸ\neq q}}^{ŋ}\left(1-\left(\begin{array}{c}\left(1-(\Upsilon^{IP}_{\mathcal{M}_{BCFS-ϸ}})^{\frac{ŋ(\mathcal{T}(\mathcal{M}_{BCFS-ϸ})+1)}{\sum_{l=1}^{ŋ}(\mathcal{T}(\mathcal{M}_{BCFS-l})+1)}}\right)^{u}\\ \times\left(1-(\Upsilon^{IP}_{\mathcal{M}_{BCFS-q}})^{\frac{ŋ(\mathcal{T}(\mathcal{M}_{BCFS-q})+1)}{\sum_{l=1}^{ŋ}(\mathcal{T}(\mathcal{M}_{BCFS-l})+1)}}\right)^{v}\end{array}\right)\right)\right)^{1/ŋ^2-ŋ}\right)^{1/u+v}\right),\\
-\left|-1+\left(\prod_{\substack{ϸ=1, q=1\\ ϸ\neq q}}^{ŋ}\left(1-\left(\left(\begin{array}{c}-\left|-1+(1+\Upsilon^{RN}_{\mathcal{M}_{BCFS-ϸ}})^{\frac{ŋ(\mathcal{T}(\mathcal{M}_{BCFS-ϸ})+1)}{\sum_{l=1}^{ŋ}(\mathcal{T}(\mathcal{M}_{BCFS-l})+1)}}\right|^{u}\\ \times\left(-\left|-1+(1+\Upsilon^{RN}_{\mathcal{M}_{BCFS-q}})^{\frac{ŋ(\mathcal{T}(\mathcal{M}_{BCFS-q})+1)}{\sum_{l=1}^{ŋ}(\mathcal{T}(\mathcal{M}_{BCFS-l})+1)}}\right|^{v}\right)\end{array}\right)\right)\right)\right)^{1/ŋ^2-ŋ}\right|^{1/u+v}\\
+\mathfrak{i}\left(-\left|-1+\left(\prod_{\substack{ϸ=1, q=1\\ ϸ\neq q}}^{ŋ}\left(1-\left(\left(\begin{array}{c}-\left|-1+(1+\Upsilon^{IN}_{\mathcal{M}_{BCFS-ϸ}})^{\frac{ŋ(\mathcal{T}(\mathcal{M}_{BCFS-ϸ})+1)}{\sum_{l=1}^{ŋ}(\mathcal{T}(\mathcal{M}_{BCFS-l})+1)}}\right|^{u}\\ \times\left(-\left|-1+(1+\Upsilon^{IN}_{\mathcal{M}_{BCFS-q}})^{\frac{ŋ(\mathcal{T}(\mathcal{M}_{BCFS-q})+1)}{\sum_{l=1}^{ŋ}(\mathcal{T}(\mathcal{M}_{BCFS-l})+1)}}\right|^{v}\right)\end{array}\right)\right)\right)\right)^{1/ŋ^2-ŋ}\right|^{1/u+v}\right)
\end{array}\right). \tag{13}$$

Proof: Suppose that $\mathfrak{F}_{ϸ} = \frac{ŋ(\mathcal{T}(\mathcal{M}_{BCFS-ϸ})+1)}{\sum_{l=1}^{ŋ}(\mathcal{T}(\mathcal{M}_{BCFS-l})+1)}$ $(ϸ = 1, 2, \ldots, ŋ)$, then Eq. 13 becomes

$$BCFGPBM^{u,v}(\mathcal{M}_{BCFS-1}, \mathcal{M}_{BCFS-2}, \ldots, \mathcal{M}_{BCFS-ŋ}) = \frac{1}{u+v}\left(\prod_{\substack{ϸ=1, q=1\\ ϸ\neq q}}^{ŋ}\left(\begin{array}{c}u(\mathcal{M}_{BCFS-ϸ})^{\mathfrak{F}_{ϸ}}\\ \oplus\, v(\mathcal{M}_{BCFS-q})^{\mathfrak{F}_{q}}\end{array}\right)\right)^{1/ŋ^2-ŋ}. \tag{14}$$

First, determine $(\mathcal{M}_{BCFS-ϸ})^{\mathfrak{F}_{ϸ}}$ and $(\mathcal{M}_{BCFS-q})^{\mathfrak{F}_{q}}$ as follows:

$$(\mathcal{M}_{BCFS-ϸ})^{\mathfrak{F}_{ϸ}} = \left((\Upsilon^{RP}_{\mathcal{M}_{BCFS-ϸ}})^{\mathfrak{F}_{ϸ}} + \mathfrak{i}(\Upsilon^{IP}_{\mathcal{M}_{BCFS-ϸ}})^{\mathfrak{F}_{ϸ}}, -1+(1+\Upsilon^{RN}_{\mathcal{M}_{BCFS-ϸ}})^{\mathfrak{F}_{ϸ}} + \mathfrak{i}\left(-1+(1+\Upsilon^{IN}_{\mathcal{M}_{BCFS-ϸ}})^{\mathfrak{F}_{ϸ}}\right)\right),$$

$$(\mathcal{M}_{BCFS-q})^{\mathfrak{F}_{q}} = \left((\Upsilon^{RP}_{\mathcal{M}_{BCFS-q}})^{\mathfrak{F}_{q}} + \mathfrak{i}(\Upsilon^{IP}_{\mathcal{M}_{BCFS-q}})^{\mathfrak{F}_{q}}, -1+(1+\Upsilon^{RN}_{\mathcal{M}_{BCFS-q}})^{\mathfrak{F}_{q}} + \mathfrak{i}\left(-1+(1+\Upsilon^{IN}_{\mathcal{M}_{BCFS-q}})^{\mathfrak{F}_{q}}\right)\right).$$

Now,

$$u(\mathcal{M}_{BCFS-ϸ})^{\mathfrak{F}_{ϸ}} = \left(\begin{array}{c}1-\left(1-(\Upsilon^{RP}_{\mathcal{M}_{BCFS-ϸ}})^{\mathfrak{F}_{ϸ}}\right)^{u} + \mathfrak{i}\left(1-\left(1-(\Upsilon^{IP}_{\mathcal{M}_{BCFS-ϸ}})^{\mathfrak{F}_{ϸ}}\right)^{u}\right),\\ -\left|-1+(1+\Upsilon^{RN}_{\mathcal{M}_{BCFS-ϸ}})^{\mathfrak{F}_{ϸ}}\right|^{u} + \mathfrak{i}\left(-\left|-1+(1+\Upsilon^{IN}_{\mathcal{M}_{BCFS-ϸ}})^{\mathfrak{F}_{ϸ}}\right|^{u}\right)\end{array}\right),$$

$$v\left(\mathcal{M}_{BCFS-\mathfrak{q}}\right)^{\mathfrak{F}_{\mathfrak{q}}}=\begin{pmatrix}1-\left(1-\left(\Upsilon^{RP}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\mathfrak{F}_{\mathfrak{q}}}\right)^{v}+\mathfrak{i}\left(1-\left(1-\left(\Upsilon^{IP}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\mathfrak{F}_{\mathfrak{q}}}\right)^{v}\right),\\ -\left|-1+\left(1+\Upsilon^{RN}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\mathfrak{F}_{\mathfrak{q}}}\right|^{v}+\mathfrak{i}\left(-\left|-1+\left(1+\Upsilon^{IN}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\mathfrak{F}_{\mathfrak{q}}}\right|^{v}\right)\end{pmatrix}.$$

Now, determine $u\left(\mathcal{M}_{BCFS-\mathfrak{p}}\right)^{\mathfrak{F}_{\mathfrak{p}}}\oplus v\left(\mathcal{M}_{BCFS-\mathfrak{q}}\right)^{\mathfrak{F}_{\mathfrak{q}}}$ and get

$$u\left(\mathcal{M}_{BCFS-\mathfrak{p}}\right)^{\mathfrak{F}_{\mathfrak{p}}}\oplus v\left(\mathcal{M}_{BCFS-\mathfrak{q}}\right)^{\mathfrak{F}_{\mathfrak{q}}}$$

$$=\begin{pmatrix}1-\left(1-\left(\Upsilon^{RP}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right)^{\mathfrak{F}_{\mathfrak{p}}}\right)^{u}+\mathfrak{i}\left(1-\left(1-\left(\Upsilon^{IP}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right)^{\mathfrak{F}_{\mathfrak{p}}}\right)^{u}\right),\\ -\left|-1+\left(1+\Upsilon^{RN}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right)^{\mathfrak{F}_{\mathfrak{p}}}\right|^{u}+\mathfrak{i}\left(-\left|-1+\left(1+\Upsilon^{IN}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right)^{\mathfrak{F}_{\mathfrak{p}}}\right|^{u}\right)\end{pmatrix}$$

$$\otimes\begin{pmatrix}1-\left(1-\left(\Upsilon^{RP}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\mathfrak{F}_{\mathfrak{q}}}\right)^{v}+\mathfrak{i}\left(1-\left(1-\left(\Upsilon^{IP}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\mathfrak{F}_{\mathfrak{q}}}\right)^{v}\right),\\ -\left|-1+\left(1+\Upsilon^{RN}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\mathfrak{F}_{\mathfrak{q}}}\right|^{v}+\mathfrak{i}\left(-\left|-1+\left(1+\Upsilon^{IN}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\mathfrak{F}_{\mathfrak{q}}}\right|^{v}\right)\end{pmatrix}$$

$$=\begin{pmatrix}1-\left(1-\left(\Upsilon^{RP}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right)^{\mathfrak{F}_{\mathfrak{p}}}\right)^{u}+1-\left(1-\left(\Upsilon^{RP}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\mathfrak{F}_{\mathfrak{q}}}\right)^{v}\\ -\left(\left(1-\left(1-\left(\Upsilon^{RP}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right)^{\mathfrak{F}_{\mathfrak{p}}}\right)^{u}\right)\times\left(1-\left(1-\left(\Upsilon^{RP}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\mathfrak{F}_{\mathfrak{q}}}\right)^{v}\right)\right)\\ +\mathfrak{i}\begin{pmatrix}1-\left(1-\left(\Upsilon^{IP}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right)^{\mathfrak{F}_{\mathfrak{p}}}\right)^{u}+1-\left(1-\left(\Upsilon^{IP}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\mathfrak{F}_{\mathfrak{q}}}\right)^{v}\\ -\left(\left(1-\left(1-\left(\Upsilon^{IP}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right)^{\mathfrak{F}_{\mathfrak{p}}}\right)^{u}\right)\times\left(1-\left(1-\left(\Upsilon^{IP}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\mathfrak{F}_{\mathfrak{q}}}\right)^{v}\right)\right)\end{pmatrix},\\ -\left(\left(-\left|-1+\left(1+\Upsilon^{RN}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right)^{\mathfrak{F}_{\mathfrak{p}}}\right|^{u}\times\left(-\left|-1+\left(1+\Upsilon^{RN}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\mathfrak{F}_{\mathfrak{q}}}\right|^{v}\right)\right)\right)\\ +\mathfrak{i}\left(-\left(\left(-\left|-1+\left(1+\Upsilon^{IN}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right)^{\mathfrak{F}_{\mathfrak{p}}}\right|^{u}\times\left(-\left|-1+\left(1+\Upsilon^{IN}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\mathfrak{F}_{\mathfrak{q}}}\right|^{v}\right)\right)\right)\right)\end{pmatrix}$$

$$=\begin{pmatrix}1-\left(\left(1-\left(\Upsilon^{RP}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right)^{\mathfrak{F}_{\mathfrak{p}}}\right)^{u}\times\left(1-\left(\Upsilon^{RP}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\mathfrak{F}_{\mathfrak{q}}}\right)^{v}\right)\\ +\mathfrak{i}\left(1-\left(\left(1-\left(\Upsilon^{IP}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right)^{\mathfrak{F}_{\mathfrak{p}}}\right)^{u}\times\left(1-\left(\Upsilon^{IP}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\mathfrak{F}_{\mathfrak{q}}}\right)^{v}\right)\right),\\ -\left(\left(-\left|-1+\left(1+\Upsilon^{RN}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right)^{\mathfrak{F}_{\mathfrak{p}}}\right|^{u}\times\left(-\left|-1+\left(1+\Upsilon^{RN}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\mathfrak{F}_{\mathfrak{q}}}\right|^{v}\right)\right)\right)\\ +\mathfrak{i}\left(-\left(\left(-\left|-1+\left(1+\Upsilon^{IN}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right)^{\mathfrak{F}_{\mathfrak{p}}}\right|^{u}\times\left(-\left|-1+\left(1+\Upsilon^{IN}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\mathfrak{F}_{\mathfrak{q}}}\right|^{v}\right)\right)\right)\right)\end{pmatrix}.$$

Next, determine $\prod_{\substack{\mathfrak{p}=1,\mathfrak{q}=1\\ \mathfrak{p}\neq\mathfrak{q}}}^{\eta}\left(u\left(\mathcal{M}_{BCFS-\mathfrak{p}}\right)^{\mathfrak{F}_{\mathfrak{p}}}\oplus v\left(\mathcal{M}_{BCFS-\mathfrak{q}}\right)^{\mathfrak{F}_{\mathfrak{q}}}\right)$.

$$\prod_{\substack{\mathfrak{p}=1,\mathfrak{q}=1\\ \mathfrak{p}\neq\mathfrak{q}}}^{\eta}\left(u\left(\mathcal{M}_{BCFS-\mathfrak{p}}\right)^{\mathfrak{F}_{\mathfrak{p}}}\oplus v\left(\mathcal{M}_{BCFS-\mathfrak{q}}\right)^{\mathfrak{F}_{\mathfrak{q}}}\right)$$

$$=\begin{pmatrix}\prod_{\substack{\mathfrak{p}=1,\mathfrak{q}=1\\ \mathfrak{p}\neq\mathfrak{q}}}^{\eta}\left(1-\left(\left(1-\left(\Upsilon^{RP}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right)^{\mathfrak{F}_{\mathfrak{p}}}\right)^{u}\times\left(1-\left(\Upsilon^{RP}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\mathfrak{F}_{\mathfrak{q}}}\right)^{v}\right)\right)\\ +\mathfrak{i}\prod_{\substack{\mathfrak{p}=1,\mathfrak{q}=1\\ \mathfrak{p}\neq\mathfrak{q}}}^{\eta}\left(1-\left(\left(1-\left(\Upsilon^{IP}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right)^{\mathfrak{F}_{\mathfrak{p}}}\right)^{u}\times\left(1-\left(\Upsilon^{IP}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\mathfrak{F}_{\mathfrak{q}}}\right)^{v}\right)\right),\\ -1+\prod_{\substack{\mathfrak{p}=1,\mathfrak{q}=1\\ \mathfrak{p}\neq\mathfrak{q}}}^{\eta}\left(1-\left(\left(-\left|-1+\left(1+\Upsilon^{RN}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right)^{\mathfrak{F}_{\mathfrak{p}}}\right|^{u}\times\left(-\left|-1+\left(1+\Upsilon^{RN}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\mathfrak{F}_{\mathfrak{q}}}\right|^{v}\right)\right)\right)\right)\\ -1+\prod_{\substack{\mathfrak{p}=1,\mathfrak{q}=1\\ \mathfrak{p}\neq\mathfrak{q}}}^{\eta}\left(1-\left(\left(-\left|-1+\left(1+\Upsilon^{IN}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right)^{\mathfrak{F}_{\mathfrak{p}}}\right|^{u}\times\left(-\left|-1+\left(1+\Upsilon^{IN}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\mathfrak{F}_{\mathfrak{q}}}\right|^{v}\right)\right)\right)\right)\end{pmatrix}.$$

Now,

$$\left(\prod_{\substack{\mathfrak{p}=1,\mathfrak{q}=1\\ \mathfrak{p}\neq\mathfrak{q}}}^{\eta}\left(u\left(\mathcal{M}_{BCFS-\mathfrak{p}}\right)^{\mathfrak{F}_{\mathfrak{p}}}\oplus v\left(\mathcal{M}_{BCFS-\mathfrak{q}}\right)^{\mathfrak{F}_{\mathfrak{q}}}\right)\right)^{1/\eta^{2}-\eta}$$

$$=\begin{pmatrix}\left(\prod_{\substack{\mathfrak{p}=1,\mathfrak{q}=1\\ \mathfrak{p}\neq\mathfrak{q}}}^{\eta}\left(1-\left(\left(1-\left(\Upsilon^{RP}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right)^{\mathfrak{F}_{\mathfrak{p}}}\right)^{u}\times\left(1-\left(\Upsilon^{RP}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\mathfrak{F}_{\mathfrak{q}}}\right)^{v}\right)\right)\right)^{1/\eta^{2}-\eta}\\ +\mathfrak{i}\left(\prod_{\substack{\mathfrak{p}=1,\mathfrak{q}=1\\ \mathfrak{p}\neq\mathfrak{q}}}^{\eta}\left(1-\left(\left(1-\left(\Upsilon^{IP}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right)^{\mathfrak{F}_{\mathfrak{p}}}\right)^{u}\times\left(1-\left(\Upsilon^{IP}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\mathfrak{F}_{\mathfrak{q}}}\right)^{v}\right)\right)\right)^{1/\eta^{2}-\eta},\\ -1+\left(\prod_{\substack{\mathfrak{p}=1,\mathfrak{q}=1\\ \mathfrak{p}\neq\mathfrak{q}}}^{\eta}\left(1-\left(\left(-\left|-1+\left(1+\Upsilon^{RN}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right)^{\mathfrak{F}_{\mathfrak{p}}}\right|^{u}\times\left(-\left|-1+\left(1+\Upsilon^{RN}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\mathfrak{F}_{\mathfrak{q}}}\right|^{v}\right)\right)\right)\right)\right)^{1/\eta^{2}-\eta}\\ +\mathfrak{i}\left(-1+\left(\prod_{\substack{\mathfrak{p}=1,\mathfrak{q}=1\\ \mathfrak{p}\neq\mathfrak{q}}}^{\eta}\left(1-\left(\left(-\left|-1+\left(1+\Upsilon^{IN}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right)^{\mathfrak{F}_{\mathfrak{p}}}\right|^{u}\times\left(-\left|-1+\left(1+\Upsilon^{IN}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\mathfrak{F}_{\mathfrak{q}}}\right|^{v}\right)\right)\right)\right)\right)^{1/\eta^{2}-\eta}\right)\end{pmatrix}.$$

Finally, determine $\frac{1}{u+v}\left(\prod_{\substack{\mathfrak{p}=1,\mathfrak{q}=1\\ \mathfrak{p}\neq\mathfrak{q}}}^{\eta}\left(u\left(\mathcal{M}_{BCFS-\mathfrak{p}}\right)^{\mathfrak{F}_{\mathfrak{p}}}\oplus v\left(\mathcal{M}_{BCFS-\mathfrak{q}}\right)^{\mathfrak{F}_{\mathfrak{q}}}\right)\right)^{1/\eta^{2}-\eta}$ as follows:

$$\frac{1}{u+v}\left(\prod_{\substack{\mathfrak{p}=1,\mathfrak{q}=1\\ \mathfrak{p}\neq\mathfrak{q}}}^{\eta}\left(u\left(\mathcal{M}_{BCFS-\mathfrak{p}}\right)^{\mathfrak{F}_{\mathfrak{p}}}\oplus v\left(\mathcal{M}_{BCFS-\mathfrak{q}}\right)^{\mathfrak{F}_{\mathfrak{q}}}\right)\right)^{1/\eta^{2}-\eta}$$

$$=\begin{pmatrix}1-\left(1-\left(\prod_{\substack{\mathfrak{p}=1,\mathfrak{q}=1\\ \mathfrak{p}\neq\mathfrak{q}}}^{\eta}\left(1-\left(\begin{matrix}\left(1-\left(\Upsilon^{RP}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right)^{\mathfrak{F}_{\mathfrak{p}}}\right)^{u}\\ \times\left(1-\left(\Upsilon^{RP}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\mathfrak{F}_{\mathfrak{q}}}\right)^{v}\end{matrix}\right)\right)\right)^{1/\eta^{2}-\eta}\right)^{1/u+v}\\ +\mathfrak{i}\left(1-\left(1-\left(\prod_{\substack{\mathfrak{p}=1,\mathfrak{q}=1\\ \mathfrak{p}\neq\mathfrak{q}}}^{\eta}\left(1-\left(\begin{matrix}\left(1-\left(\Upsilon^{IP}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right)^{\mathfrak{F}_{\mathfrak{p}}}\right)^{u}\\ \times\left(1-\left(\Upsilon^{IP}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\mathfrak{F}_{\mathfrak{q}}}\right)^{v}\end{matrix}\right)\right)\right)^{1/\eta^{2}-\eta}\right)^{1/u+v}\right),\\ -\left|-1+\left(\prod_{\substack{\mathfrak{p}=1,\mathfrak{q}=1\\ \mathfrak{p}\neq\mathfrak{q}}}^{\eta}\left(1-\left(\left(\begin{matrix}-\left|-1+\left(1+\Upsilon^{RN}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right)^{\mathfrak{F}_{\mathfrak{p}}}\right|^{u}\\ \times\left(-\left|-1+\left(1+\Upsilon^{RN}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\mathfrak{F}_{\mathfrak{q}}}\right|^{v}\right)\end{matrix}\right)\right)\right)\right)^{1/\eta^{2}-\eta}\right|^{1/u+v}\\ +\mathfrak{i}\left(-\left|-1+\left(\prod_{\substack{\mathfrak{p}=1,\mathfrak{q}=1\\ \mathfrak{p}\neq\mathfrak{q}}}^{\eta}\left(1-\left(\left(\begin{matrix}-\left|-1+\left(1+\Upsilon^{IN}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right)^{\mathfrak{F}_{\mathfrak{p}}}\right|^{u}\\ \times\left(-\left|-1+\left(1+\Upsilon^{IN}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\mathfrak{F}_{\mathfrak{q}}}\right|^{v}\right)\end{matrix}\right)\right)\right)\right)^{1/\eta^{2}-\eta}\right|^{1/u+v}\right)\end{pmatrix}. \tag{15}$$

Now, replace $\mathfrak{F}_{\mathfrak{p}}=\frac{\eta\left(\Upsilon\left(\mathcal{M}_{BCFS-\mathfrak{p}}\right)+1\right)}{\sum_{l=1}^{\eta}\left(\Upsilon\left(\mathcal{M}_{BCFS-l}\right)+1\right)}$ in Eq. 15, then

$$\frac{1}{u+v}\left(\prod_{\substack{\mathfrak{p}=1,\mathfrak{q}=1\\ \mathfrak{p}\neq\mathfrak{q}}}^{\eta}\left(u\left(\mathcal{M}_{BCFS-\mathfrak{p}}\right)^{\mathfrak{F}_{\mathfrak{p}}}\oplus v\left(\mathcal{M}_{BCFS-\mathfrak{q}}\right)^{\mathfrak{F}_{\mathfrak{q}}}\right)\right)^{1/\eta^{2}-\eta}$$

$$=\begin{pmatrix}1-\left(1-\left(\prod_{\substack{\mathfrak{p}=1,\mathfrak{q}=1\\ \mathfrak{p}\neq\mathfrak{q}}}^{\eta}\left(1-\left(\begin{matrix}\left(1-\left(\Upsilon^{RP}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right)^{\frac{\eta\left(\Upsilon\left(\mathcal{M}_{BCFS-\mathfrak{p}}\right)+1\right)}{\sum_{l=1}^{\eta}\left(\Upsilon\left(\mathcal{M}_{BCFS-l}\right)+1\right)}}\right)^{u}\\ \times\left(1-\left(\Upsilon^{RP}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\frac{\eta\left(\Upsilon\left(\mathcal{M}_{BCFS-\mathfrak{q}}\right)+1\right)}{\sum_{l=1}^{\eta}\left(\Upsilon\left(\mathcal{M}_{BCFS-l}\right)+1\right)}}\right)^{v}\end{matrix}\right)\right)\right)^{1/\eta^{2}-\eta}\right)^{1/u+v}\\ +\mathfrak{i}\left(1-\left(1-\left(\prod_{\substack{\mathfrak{p}=1,\mathfrak{q}=1\\ \mathfrak{p}\neq\mathfrak{q}}}^{\eta}\left(1-\left(\begin{matrix}\left(1-\left(\Upsilon^{IP}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right)^{\frac{\eta\left(\Upsilon\left(\mathcal{M}_{BCFS-\mathfrak{p}}\right)+1\right)}{\sum_{l=1}^{\eta}\left(\Upsilon\left(\mathcal{M}_{BCFS-l}\right)+1\right)}}\right)^{u}\\ \times\left(1-\left(\Upsilon^{IP}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\frac{\eta\left(\Upsilon\left(\mathcal{M}_{BCFS-\mathfrak{q}}\right)+1\right)}{\sum_{l=1}^{\eta}\left(\Upsilon\left(\mathcal{M}_{BCFS-l}\right)+1\right)}}\right)^{v}\end{matrix}\right)\right)\right)^{1/\eta^{2}-\eta}\right)^{1/u+v}\right),\\ -\left|-1+\left(\prod_{\substack{\mathfrak{p}=1,\mathfrak{q}=1\\ \mathfrak{p}\neq\mathfrak{q}}}^{\eta}\left(1-\left(\left(\begin{matrix}-\left|-1+\left(1+\Upsilon^{RN}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right)^{\frac{\eta\left(\Upsilon\left(\mathcal{M}_{BCFS-\mathfrak{p}}\right)+1\right)}{\sum_{l=1}^{\eta}\left(\Upsilon\left(\mathcal{M}_{BCFS-l}\right)+1\right)}}\right|^{u}\\ \times\left(-\left|-1+\left(1+\Upsilon^{RN}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\frac{\eta\left(\Upsilon\left(\mathcal{M}_{BCFS-\mathfrak{q}}\right)+1\right)}{\sum_{l=1}^{\eta}\left(\Upsilon\left(\mathcal{M}_{BCFS-l}\right)+1\right)}}\right|^{v}\right)\end{matrix}\right)\right)\right)\right)^{1/\eta^{2}-\eta}\right|^{1/u+v}\\ +\mathfrak{i}\left(-\left|-1+\left(\prod_{\substack{\mathfrak{p}=1,\mathfrak{q}=1\\ \mathfrak{p}\neq\mathfrak{q}}}^{\eta}\left(1-\left(\left(\begin{matrix}-\left|-1+\left(1+\Upsilon^{IN}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right)^{\frac{\eta\left(\Upsilon\left(\mathcal{M}_{BCFS-\mathfrak{p}}\right)+1\right)}{\sum_{l=1}^{\eta}\left(\Upsilon\left(\mathcal{M}_{BCFS-l}\right)+1\right)}}\right|^{u}\\ \times\left(-\left|-1+\left(1+\Upsilon^{IN}_{\mathcal{M}_{BCFS-\mathfrak{q}}}\right)^{\frac{\eta\left(\Upsilon\left(\mathcal{M}_{BCFS-\mathfrak{q}}\right)+1\right)}{\sum_{l=1}^{\eta}\left(\Upsilon\left(\mathcal{M}_{BCFS-l}\right)+1\right)}}\right|^{v}\right)\end{matrix}\right)\right)\right)\right)^{1/\eta^{2}-\eta}\right|^{1/u+v}\right)\end{pmatrix}$$

$$=BCFPGBM^{u,v}\left(\mathcal{M}_{BCFS-1},\mathcal{M}_{BCFS-2},\ldots,\mathcal{M}_{BCFS-\eta}\right).$$

The BCFPGBM operator holds the following axioms.

1) *Idempotency*: Let $\mathcal{M}_{BCFS-\mathfrak{p}}=\mathcal{M}_{BCFS}=\left(\Upsilon^{P}_{\mathcal{M}_{BCFS}},\ \Upsilon^{N}_{\mathcal{M}_{BCFS}}\right)=\left(\Upsilon^{RP}_{\mathcal{M}_{BCFS}}+\mathfrak{i}\,\Upsilon^{IP}_{\mathcal{M}_{BCFS}},\ \Upsilon^{RN}_{\mathcal{M}_{BCFS}}+\mathfrak{i}\,\Upsilon^{IN}_{\mathcal{M}_{BCFS}}\right)$, $\mathfrak{p}=1,2,..,\eta$. Then,

$$BCFPGBM^{u,v}\left(\mathcal{M}_{BCFS-1},\mathcal{M}_{BCFS-2},\ldots,\mathcal{M}_{BCFS-\eta}\right)=\mathcal{M}_{BCFS}.$$

2) *Commutativity*: Let $\mathcal{M}_{BCFS-\mathfrak{p}}=\left(\Upsilon^{P}_{\mathcal{M}_{BCFS-\mathfrak{p}}},\Upsilon^{N}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right)=\left(\Upsilon^{RP}_{\mathcal{M}_{BCFS-\mathfrak{p}}}+\mathfrak{i}\,\Upsilon^{IP}_{\mathcal{M}_{BCFS-\mathfrak{p}}},\Upsilon^{RN}_{\mathcal{M}_{BCFS-\mathfrak{p}}}+\mathfrak{i}\,\Upsilon^{IN}_{\mathcal{M}_{BCFS-\mathfrak{p}}}\right)$, $\mathfrak{p}=\mathfrak{p}$, 2,

$\ldots$, ŋ have the permutation $\mathcal{M}'_{BCFS-ƥ} = (\Upsilon'^{P}_{\mathcal{M}_{BCFS-ƥ}}, \Upsilon'^{N}_{\mathcal{M}_{BCFS-ƥ}}) = (\Upsilon'^{RP}_{\mathcal{M}_{BCFS-ƥ}} + \mathfrak{i}\ \Upsilon'^{IP}_{\mathcal{M}_{BCFS-ƥ}}, \Upsilon'^{RN}_{\mathcal{M}_{BCFS-ƥ}} + \mathfrak{i}\,\Upsilon'^{IN}_{\mathcal{M}_{BCFS-ƥ}})$, $ƥ = ƥ, 2, \ldots, ŋ$. Then,

$$BCFPGBM^{u,v}(\mathcal{M}_{BCFS-1}, \mathcal{M}_{BCFS-2}, \ldots, \mathcal{M}_{BCFS-ŋ}) = BCFPGBM^{u,v}(\mathcal{M}'_{BCFS-1}, \mathcal{M}'_{BCFS-2}, \ldots, \mathcal{M}'_{BCFS-ŋ}).$$

3) *Boundedness*: Let a group of BCFNs $\mathcal{M}_{BCFS-ƥ} = (\Upsilon^{P}_{\mathcal{M}_{BCFS-ƥ}}, \Upsilon^{N}_{\mathcal{M}_{BCFS-ƥ}}) = (\Upsilon^{RP}_{\mathcal{M}_{BCFS-ƥ}} + \mathfrak{i}\,\Upsilon^{IP}_{\mathcal{M}_{BCFS-ƥ}}, \Upsilon^{RN}_{\mathcal{M}_{BCFS-ƥ}} + \mathfrak{i}\,\Upsilon^{IN}_{\mathcal{M}_{BCFS-ƥ}})$, $ƥ = ƥ, 2, \ldots, ŋ$ and let $\mathcal{M}^{-}_{BCFS} = (\min_{ƥ}\{\Upsilon^{RP}_{\mathcal{M}_{BCFS-ƥ}}\} + \mathfrak{i}\min_{ƥ}\{\Upsilon^{IP}_{\mathcal{M}_{BCFS-ƥ}}\}, \max_{ƥ}\{\Upsilon^{RN}_{\mathcal{M}_{BCFS-ƥ}}\} + \mathfrak{i}\max_{ƥ}\{\Upsilon^{IN}_{\mathcal{M}_{BCFS-ƥ}}\})$ and $\mathcal{M}^{+}_{BCFS} = (\max_{ƥ}\{\Upsilon^{RP}_{\mathcal{M}_{BCFS-ƥ}}\} + \mathfrak{i}\max_{ƥ}\{\Upsilon^{IP}_{\mathcal{M}_{BCFS-ƥ}}\}, \min_{ƥ}\{\Upsilon^{RN}_{\mathcal{M}_{BCFS-ƥ}}\} + \mathfrak{i}\min_{ƥ}\{\Upsilon^{IN}_{\mathcal{M}_{BCFS-ƥ}}\})$. Then,

$$\mathcal{M}^{-}_{BCFS} \le BCFPGBM^{u,v}(\mathcal{M}_{BCFS-1}, \mathcal{M}_{BCFS-2}, \ldots, \mathcal{M}_{BCFS-ŋ}) \le \mathcal{M}^{+}_{BCFS}.$$

Definition 9: Take a group of BCFNs $\mathcal{M}_{BCFS-ƥ} = (\Upsilon^{P}_{\mathcal{M}_{BCFS-ƥ}}, \Upsilon^{N}_{\mathcal{M}_{BCFS-ƥ}}) = (\Upsilon^{RP}_{\mathcal{M}_{BCFS-ƥ}} + \mathfrak{i}\,\Upsilon^{IP}_{\mathcal{M}_{BCFS-ƥ}}, \Upsilon^{RN}_{\mathcal{M}_{BCFS-ƥ}} + \mathfrak{i}\,\Upsilon^{IN}_{\mathcal{M}_{BCFS-ƥ}})$, $ƥ = ƥ, 2, \ldots, ŋ$, then the BCFWPGBM operator is inspected as

$$BCFWPGBM^{u,v}(\mathcal{M}_{BCFS-1}, \mathcal{M}_{BCFS-2}, \ldots, \mathcal{M}_{BCFS-ŋ}) = \frac{1}{u+v}\left(\prod_{\substack{ƥ=1, ɋ=1\\ ƥ\neq ɋ}}^{ŋ}\left(u\left(\mathcal{M}_{BCFS-ƥ}\right)^{\frac{ŋ w_{\mathfrak{v}-ƥ}\left(Ƭ\left(\mathcal{M}_{BCFS-ƥ}\right)+1\right)}{\sum_{l=1}^{ŋ} w_{\mathfrak{v}-l}\left(Ƭ\left(\mathcal{M}_{BCFS-l}\right)+1\right)}} \oplus v\left(\mathcal{M}_{BCFS-ɋ}\right)^{\frac{ŋ w_{\mathfrak{v}-ɋ}\left(Ƭ\left(\mathcal{M}_{BCFS-ɋ}\right)+1\right)}{\sum_{l=1}^{ŋ} w_{\mathfrak{v}-l}\left(Ƭ\left(\mathcal{M}_{BCFS-l}\right)+1\right)}}\right)\right)^{1/ŋ^2-ŋ}, \tag{16}$$

where $u, v > 0$ and $Ƭ(\mathcal{M}_{BCFS-ƥ}) = \sum_{\substack{ɋ=1\\ ɋ\neq ƥ}}^{ŋ} Sup(\mathcal{M}_{BCFS-ƥ}, \mathcal{M}_{BCFS-ɋ})$, and $w_{\mathfrak{v}} = (w_{\mathfrak{v}-1}, w_{\mathfrak{v}-2}, \ldots, w_{\mathfrak{v}-ŋ})$ is the weight vector (WV) with the property that $w_{\mathfrak{v}-ƥ} \in [0, 1]$ and $\sum_{ƥ=1}^{ŋ} w_{\mathfrak{v}-ƥ} = 1$.

Theorem 4: The aggregated outcome from Eq. 16 based on the group of BCFNs $\mathcal{M}_{BCFS-ƥ} = (\Upsilon^{P}_{\mathcal{M}_{BCFS-ƥ}}, \Upsilon^{N}_{\mathcal{M}_{BCFS-ƥ}}) = (\Upsilon^{RP}_{\mathcal{M}_{BCFS-ƥ}} + \mathfrak{i}\,\Upsilon^{IP}_{\mathcal{M}_{BCFS-ƥ}}, \Upsilon^{RN}_{\mathcal{M}_{BCFS-ƥ}} + \mathfrak{i}\,\Upsilon^{IN}_{\mathcal{M}_{BCFS-ƥ}})$, $ƥ = ƥ, 2, \ldots, ŋ$ is a BCFN and is inspected as

$$BCFWPGBM^{u,v}(\mathcal{M}_{BCFS-1}, \mathcal{M}_{BCFS-2}, \ldots, \mathcal{M}_{BCFS-ŋ})$$

$$= \left(\begin{array}{l}
\left(1-\left(1-\left(\prod_{\substack{ƥ=1, ɋ=1\\ ƥ\neq ɋ}}^{ŋ}\left(1-\left(\left(1-\left(\Upsilon^{RP}_{\mathcal{M}_{BCFS-ƥ}}\right)^{\frac{ŋ w_{\mathfrak{v}-ƥ}\left(Ƭ\left(\mathcal{M}_{BCFS-ƥ}\right)+1\right)}{\sum_{l=1}^{ŋ} w_{\mathfrak{v}-l}\left(Ƭ\left(\mathcal{M}_{BCFS-l}\right)+1\right)}}\right)^{u} \times \left(1-\left(\Upsilon^{RP}_{\mathcal{M}_{BCFS-ɋ}}\right)^{\frac{ŋ w_{\mathfrak{v}-ɋ}\left(Ƭ\left(\mathcal{M}_{BCFS-ɋ}\right)+1\right)}{\sum_{l=1}^{ŋ} w_{\mathfrak{v}-l}\left(Ƭ\left(\mathcal{M}_{BCFS-l}\right)+1\right)}}\right)^{v}\right)\right)\right)^{1/ŋ^2-ŋ}\right)^{1/u+v}\right) \\
+\mathfrak{i}\left(1-\left(1-\left(\prod_{\substack{ƥ=1, ɋ=1\\ ƥ\neq ɋ}}^{ŋ}\left(1-\left(\left(1-\left(\Upsilon^{IP}_{\mathcal{M}_{BCFS-ƥ}}\right)^{\frac{ŋ w_{\mathfrak{v}-ƥ}\left(Ƭ\left(\mathcal{M}_{BCFS-ƥ}\right)+1\right)}{\sum_{l=1}^{ŋ} w_{\mathfrak{v}-l}\left(Ƭ\left(\mathcal{M}_{BCFS-l}\right)+1\right)}}\right)^{u} \times \left(1-\left(\Upsilon^{IP}_{\mathcal{M}_{BCFS-ɋ}}\right)^{\frac{ŋ w_{\mathfrak{v}-ɋ}\left(Ƭ\left(\mathcal{M}_{BCFS-ɋ}\right)+1\right)}{\sum_{l=1}^{ŋ} w_{\mathfrak{v}-l}\left(Ƭ\left(\mathcal{M}_{BCFS-l}\right)+1\right)}}\right)^{v}\right)\right)\right)^{1/ŋ^2-ŋ}\right)^{1/u+v}\right), \\
-\left|-1+\left(\prod_{\substack{ƥ=1, ɋ=1\\ ƥ\neq ɋ}}^{ŋ}\left(1-\left(\left(-\left|-1+\left(1+\Upsilon^{RN}_{\mathcal{M}_{BCFS-ƥ}}\right)^{\frac{ŋ w_{\mathfrak{v}-ƥ}\left(Ƭ\left(\mathcal{M}_{BCFS-ƥ}\right)+1\right)}{\sum_{l=1}^{ŋ} w_{\mathfrak{v}-l}\left(Ƭ\left(\mathcal{M}_{BCFS-l}\right)+1\right)}}\right|^{u} \times \left(-\left|-1+\left(1+\Upsilon^{RN}_{\mathcal{M}_{BCFS-ɋ}}\right)^{\frac{ŋ w_{\mathfrak{v}-ɋ}\left(Ƭ\left(\mathcal{M}_{BCFS-ɋ}\right)+1\right)}{\sum_{l=1}^{ŋ} w_{\mathfrak{v}-l}\left(Ƭ\left(\mathcal{M}_{BCFS-l}\right)+1\right)}}\right|^{v}\right)\right)\right)\right)\right)^{1/ŋ^2-ŋ}\right|^{1/u+v} \\
+\mathfrak{i}\left(-\left|-1+\left(\prod_{\substack{ƥ=1, ɋ=1\\ ƥ\neq ɋ}}^{ŋ}\left(1-\left(\left(-\left|-1+\left(1+\Upsilon^{IN}_{\mathcal{M}_{BCFS-ƥ}}\right)^{\frac{ŋ w_{\mathfrak{v}-ƥ}\left(Ƭ\left(\mathcal{M}_{BCFS-ƥ}\right)+1\right)}{\sum_{l=1}^{ŋ} w_{\mathfrak{v}-l}\left(Ƭ\left(\mathcal{M}_{BCFS-l}\right)+1\right)}}\right|^{u} \times \left(-\left|-1+\left(1+\Upsilon^{IN}_{\mathcal{M}_{BCFS-ɋ}}\right)^{\frac{ŋ w_{\mathfrak{v}-ɋ}\left(Ƭ\left(\mathcal{M}_{BCFS-ɋ}\right)+1\right)}{\sum_{l=1}^{ŋ} w_{\mathfrak{v}-l}\left(Ƭ\left(\mathcal{M}_{BCFS-l}\right)+1\right)}}\right|^{v}\right)\right)\right)\right)\right)^{1/ŋ^2-ŋ}\right|^{1/u+v}\right)
\end{array}\right). \tag{17}$$

The BCFWPGBM operator holds these axioms:

1) *Idempotency*: Let $\mathcal{M}_{BCFS-ƥ} = \mathcal{M}_{BCFS} = (\Upsilon^{P}_{\mathcal{M}_{BCFS}}, \Upsilon^{N}_{\mathcal{M}_{BCFS}}) = (\Upsilon^{RP}_{\mathcal{M}_{BCFS}} + \mathfrak{i}\,\Upsilon^{IP}_{\mathcal{M}_{BCFS}}, \Upsilon^{RN}_{\mathcal{M}_{BCFS}} + \mathfrak{i}\,\Upsilon^{IN}_{\mathcal{M}_{BCFS}})$, $ƥ = 1, 2, .., ŋ$. Then,

$$BCFWPGBM^{u,v}(\mathcal{M}_{BCFS-1}, \mathcal{M}_{BCFS-2}, \ldots, \mathcal{M}_{BCFS-ŋ}) = \mathcal{M}_{BCFS}.$$

2) Commutativity: Let $\mathcal{M}_{BCFS-ƥ} = (\Upsilon^{P}_{\mathcal{M}_{BCFS-ƥ}}, \Upsilon^{N}_{\mathcal{M}_{BCFS-ƥ}}) = (\Upsilon^{RP}_{\mathcal{M}_{BCFS-ƥ}} + \mathfrak{i}\,\Upsilon^{IP}_{\mathcal{M}_{BCFS-ƥ}}, \Upsilon^{RN}_{\mathcal{M}_{BCFS-ƥ}} + \mathfrak{i}\,\Upsilon^{IN}_{\mathcal{M}_{BCFS-ƥ}})$, $ƥ = ƥ, 2, \ldots, ŋ$ have the permutation $\mathcal{M}'_{BCFS-ƥ} = (\Upsilon'^{P}_{\mathcal{M}_{BCFS-ƥ}}, \Upsilon'^{N}_{\mathcal{M}_{BCFS-ƥ}}) = (\Upsilon'^{RP}_{\mathcal{M}_{BCFS-ƥ}} + \mathfrak{i}\,\Upsilon'^{IP}_{\mathcal{M}_{BCFS-ƥ}}, \Upsilon'^{RN}_{\mathcal{M}_{BCFS-ƥ}} + \mathfrak{i}\,\Upsilon'^{IN}_{\mathcal{M}_{BCFS-ƥ}})$, $ƥ = ƥ, 2, \ldots, ŋ$. Then,

$$BCFWPGBM^{u,v}(\mathcal{M}_{BCFS-1}, \mathcal{M}_{BCFS-2}, \ldots, \mathcal{M}_{BCFS-ŋ}) = BCFPGBM^{u,v}(\mathcal{M}'_{BCFS-1}, \mathcal{M}'_{BCFS-2}, \ldots, \mathcal{M}'_{BCFS-ŋ}).$$

3) *Boundedness*: Let a group of BCFNs $\mathcal{M}_{BCFS-ƥ} = (\Upsilon^{P}_{\mathcal{M}_{BCFS-ƥ}}, \Upsilon^{N}_{\mathcal{M}_{BCFS-ƥ}}) = (\Upsilon^{RP}_{\mathcal{M}_{BCFS-ƥ}} + \mathfrak{i}\,\Upsilon^{IP}_{\mathcal{M}_{BCFS-ƥ}}, \Upsilon^{RN}_{\mathcal{M}_{BCFS-ƥ}} + \mathfrak{i}\,\Upsilon^{IN}_{\mathcal{M}_{BCFS-ƥ}})$, $ƥ = ƥ, 2, \ldots, ŋ$ and let $\mathcal{M}^{-}_{BCFS} =$

TABLE 4 SVs were determined by the method proposed in this study and from previous studies.

Method	$\mathcal{S}(\mathcal{M}_{BCFS-1})$	$\mathcal{S}(\mathcal{M}_{BCFS-2})$	$\mathcal{S}(\mathcal{M}_{BCFS-3})$	$\mathcal{S}(\mathcal{M}_{BCFS-4})$	Ranking
Hu et al. (2019)	Flopped	Flopped	Flopped	Flopped	Flopped
Riaz et al. (2022)	Flopped	Flopped	Flopped	Flopped	Flopped
Wei et al. Wei et al. (2018)	Flopped	Flopped	Flopped	Flopped	Flopped
Liu and Li (2017)	Flopped	Flopped	Flopped	Flopped	Flopped
BCFBWM (Mahmood et al., 2022b)	0.526	0.514	0.519	0.535	$\mathcal{M}_{BCFS-4} > \mathcal{M}_{BCFS-1} > \mathcal{M}_{BCFS-3} > \mathcal{M}_{BCFS-2}$
BCFPBM	0.863	0.832	0.852	0.85	$\mathcal{M}_{BCFS-1} > \mathcal{M}_{BCFS-3} > \mathcal{M}_{BCFS-4} > \mathcal{M}_{BCFS-2}$
BCFWPBM	0.773	0.749	0.765	0.767	$\mathcal{M}_{BCFS-1} > \mathcal{M}_{BCFS-4} > \mathcal{M}_{BCFS-3} > \mathcal{M}_{BCFS-2}$
BCFPGBM	0.475	0.468	0.49	0.513	$\mathcal{M}_{BCFS-4} > \mathcal{M}_{BCFS-3} > \mathcal{M}_{BCFS-1} > \mathcal{M}_{BCFS-2}$
BCFWPGBM	0.655	0.648	0.672	0.686	$\mathcal{M}_{BCFS-4} > \mathcal{M}_{BCFS-3} > \mathcal{M}_{BCFS-1} > \mathcal{M}_{BCFS-2}$

$(\min_{\beta}\{\Upsilon^{RP}_{\mathcal{M}_{BCFS-\beta}}\} + \mathfrak{i}\min_{\beta}\{\Upsilon^{IP}_{\mathcal{M}_{BCFS-\beta}}\}, \max_{\beta}\{\Upsilon^{RN}_{\mathcal{M}_{BCFS-\beta}}\} + \mathfrak{i}\max_{\beta}\{\Upsilon^{IN}_{\mathcal{M}_{BCFS-\beta}}\})$ and $\mathcal{M}^{+}_{BCFS} = (\max_{\beta}\{\Upsilon^{RP}_{\mathcal{M}_{BCFS-\beta}}\} + \mathfrak{i}\max_{\beta}\{\Upsilon^{IP}_{\mathcal{M}_{BCFS-\beta}}\}, \min_{\beta}\{\Upsilon^{RN}_{\mathcal{M}_{BCFS-\beta}}\} + \mathfrak{i}\min_{\beta}\{\Upsilon^{IN}_{\mathcal{M}_{BCFS-\beta}}\})$. Then, $\mathcal{M}^{-}_{BCFS} \le BCFWPGBM^{u,v}(\mathcal{M}_{BCFS-1}, \mathcal{M}_{BCFS-2}, \ldots, \mathcal{M}_{BCFS-\eta}) \le \mathcal{M}^{+}_{BCFS}$.

Application

In the aforementioned sections, we reviewed pollution and its types and introduced PBM operators based on the BCFS. Thus, here, we would study pollution and its types through introduced operators. As we know that pollution is a worldwide issue, big cities generally have more pollution than small cities or towns. Pollution can easily expand to those areas where no one lives. For instance, some chemicals and pesticides are observed in the Antarctic ice sheet. In the northern Pacific Ocean, a tremendous assortment of tiny plastic particle structures is called the Great Pacific Garbage Patch. Winds can carry radioactive material, inadvertently let out of an atomic reactor, and spread it over the planet. Water and air convey pollutants. Sea flows and migrating fish convey marine toxins all over. Pollution is of four major types, i.e., 1. air pollution, 2. water pollution, 3. soil pollution, and 4. noise pollution, and the causes of every type are different. Every type of pollution has a different effect on the environment. To find out the most harmful type of pollution for the environment is a DM issue. Thus, we would display how we can employ the introduced operators for the BCFS to solve this DM issue.

To employ the introduced operators for the BCFS to solve the DM issue, we define a DM procedure as follows based on the introduced operators in the environment of the BCFS.

Taking η alternatives, i.e., $\mathcal{M}_{BCFS-1}, \mathcal{M}_{BCFS-2}, \ldots, \mathcal{M}_{BCFS-\eta}$ and m attributes $\mathcal{A}_{b-1}, \mathcal{A}_{b-2}, \ldots, \mathcal{A}_{b-m}$ with WV $w_{\mathfrak{v}} = (w_{\mathfrak{v}-1}, w_{\mathfrak{v}-2}, \ldots, w_{\mathfrak{v}-m})$ with a property that $w_{\mathfrak{v}-\mathfrak{q}} \in [0, 1]$ and $\sum_{\mathfrak{q}=1}^{m} w_{\mathfrak{v}-\mathfrak{q}} = 1$, assume that the decision analyst or expert described his/her opinion in the model of BCFS and structured a decision matrix involving BCFNs that is $\mathcal{M}_{BCFS} = (\Upsilon^{P}_{\mathcal{M}_{BCFS}}, \Upsilon^{N}_{\mathcal{M}_{BCFS}}) = (\Upsilon^{RP}_{\mathcal{M}_{BCFS}} + \mathfrak{i}\Upsilon^{IP}_{\mathcal{M}_{BCFS}}, \Upsilon^{RN}_{\mathcal{M}_{BCFS}} + \mathfrak{i}\Upsilon^{IN}_{\mathcal{M}_{BCFS}})$, where $\Upsilon^{RP}_{\mathcal{M}_{BCFS}}, \Upsilon^{IP}_{\mathcal{M}_{BCFS}} \in [0, 1]$ and $\Upsilon^{RN}_{\mathcal{M}_{BCFS}}, \Upsilon^{IN}_{\mathcal{M}_{BCFS}} \in [-1, 0]$. The steps for DM are as follows:

Step 1: In genuine life issues, the information or data can be cost or benefit. Benefits do not require normalization, but costs do:

$$\mathcal{N}_{BCFS} = \begin{cases} (\Upsilon^{P}_{\mathcal{M}_{BCFS}}, \Upsilon^{N}_{\mathcal{M}_{BCFS}}) & for\ benefit, \\ (\Upsilon^{P}_{\mathcal{M}_{BCFS}}, \Upsilon^{N}_{\mathcal{M}_{BCFS}})^{c} & for\ cost, \end{cases} \quad (18)$$

where $(\Upsilon^{P}_{\mathcal{M}_{BCFS}}, \Upsilon^{N}_{\mathcal{M}_{BCFS}})^{c} = (\Upsilon^{RP}_{\mathcal{M}_{BCFS}} + \mathfrak{i}\Upsilon^{IP}_{\mathcal{M}_{BCFS}}, \Upsilon^{RN}_{\mathcal{M}_{BCFS}} + \mathfrak{i}\Upsilon^{IN}_{\mathcal{M}_{BCFS}})^{c} = (1 - \Upsilon^{RP}_{\mathcal{M}_{BCFS}} + \mathfrak{i}(1 - \Upsilon^{IP}_{\mathcal{M}_{BCFS}}), -1 - \Upsilon^{RN}_{\mathcal{M}_{BCFS}} + \mathfrak{i}(-1 - \Upsilon^{IN}_{\mathcal{M}_{BCFS}}))$.

Step 2: After completing step 1, the aggregated values of given information or data are determined by employing any introduced operator (BCFPBM, BCFWPBM, BCFPGBM, and BCFWPGBM).

Step 3: After finding out the aggregated values, the SV of these aggregated values would be determined by employing Eq. 2, and if any two SVs are equal, then accuracy values (AVs) are determined by employing Eq. 3. Furthermore, the ranking would be made based on these SVs and AVs and would reach the best alternative.

In the following section, we solve a numerical example by employing the introduced operators and DM procedure based on the BCFS to address genuine life issues.

Numerical example

Suppose four types of pollution, i.e., $\mathcal{M}_{BCFS-1}$ = Air pollutioŋ, $\mathcal{M}_{BCFS-2}$ = Soil pollutioŋ, $\mathcal{M}_{BCFS-3}$ = Noise pollutioŋ, and $\mathcal{M}_{BCFS-4}$ = Water pollutioŋ, are under consideration and four attributes of the environment, i.e., $\mathcal{A}_{b-1}$ = Lithosphere, $\mathcal{A}_{b-2}$ = hydroshphere, $\mathcal{A}_{b-3}$ = atmosphere, and $\mathcal{A}_{b-4}$ = biosphere, are also considered. The decision analyst or expert provides the values of the effects of these types of pollution on the environment (Table 1). Now, we determine the most harmful type of pollution. For this, we would follow the steps of the DM procedure.

Step 1: In this example, we do not need to perform the first step.

Step 2: The aggregated values of data in Table 1 are determined by employing introduced BCFPBM, BCFWPBM, BCFPGBM, and BCFWPGBM operators, which are explored in Table 2.

Step 3: SVs of these aggregated values determined by employing Eq. 2 are presented in Table 3.

The rankings in Table 3 show that according to the introduced BCFPBM and BCFWPBM, $\mathcal{M}_{BCFS-1}$ = *Air pollutioŋ* has more effect on the environment than the others, and according to the introduced BCFPGBM and BCFWPGBM, $\mathcal{M}_{BCFS-4}$ = *Water pollutioŋ* has more effect on the environment than the others.

Comparison

The introduced PBM operators (BCFPBM, BCFWPBM, BCFPGBM, and BCFWPGBM) and a DM procedure in the environment of the BCFS are more effective than other prevailing operators. We now take prevailing theories, including the CF power and DM procedure defined by Hu et al. (2019), sin trigonometric AOs and the DM procedure in the environment of the BF set defined by Riaz et al. (2022), Hamacher AOs and the DM procedure in the environment of the BF set established by Wei et al. (2018), PBM AOs and the DM procedure in the setting of the interval-valued intuitionistic FS (IVIFS) introduced by Liu and Li (2017), and BM AOs and the DM procedure for the BCFS introduced by Mahmood et al. (2022b). We consider the data provided in Table 1 and aggregate it with the assistance of these prevailing theories and the operators and the DM procedure introduced in this article. The results are shown in Table 4.

The results indicate that existing theories, for example, Hu et al. (2019), Riaz et al. (2022), Wei et al. (2018), and Liu and Li (2017), cannot solve the DM issue because the data form the structure of the BCFS. Hu et al. (2019) can merely solve the data in the model of CFS and cannot tackle the negative aspects, while Riaz et al. (2022) and Wei et al. (2018) merely resolve the information in the setting of the BFS and cannot overcome the second dimension, and Liu and Li (2017) merely resolve the information in the model of IVIFS and cannot address the second dimension and negative aspects. Furthermore, the results show that BM AO (BCF Bonferroni mean AO) (Mahmood et al., 2022b) for the BCFS and the DM approach handle the data and give us the result that $\mathcal{M}_{BCFS-4}$ is the finest choice, but anyone can say that this decision or selection is biased and not fair because of the weight vector that the decision analyst provides to each attribute on his own choice, for example, in this case, we take the weight vector $(0.08, 0.06, 0.06, 0.8)$ and $u = v = 2$. However, in the case of the proposed PBM AOs for BCFS and the DM approach, we can see that the corresponding BCFPBWM gives us that $\mathcal{M}_{BCFS-1}$ is the finest alternative. In the proposed work, the decision analyst cannot provide the weight vector by his/her own choice and has to find it out through the proper formula, and in this example, we used the distance formula presented by Mahmood and Ur Rehman (2022b) in the formula of finding out the weight vector. This shows the investigated AOs get better and fair outcomes than the old ones. The results also show that the introduced operators and the DM procedure are appropriate tools for addressing uncertain and vague information in the environment of the BCFS. Also, the introduced operators can reduce the setting of the FS (by overlooking the NSG and unreal parts in the PSG), BFS (by making the unreal parts in both the PSG and NSG equal to zero), and CFS (by overlooking the NSG). Thus, the introduced operators and the procedure are more robust than other prevailing operators and DM procedures.

Conclusion

In this article, we studied the effect of pollution and its types on the environment by employing the introduced PBM operators and the DM procedure in the setting of the BCFS. The procedure by which the water, land, air, or other components of the natural environment are made unsafe or dirty to use is known as pollution. Pollution occurs by introducing any sort of pollutant into the environment, but the pollutant need not be visible, for example, temperature, light, and sound can be taken as pollutants if introduced into the environment artificially. For this study, we introduced the PBM operator in the setting of the BCFS, which are BCFPBM, BCFWPBM, BCFPGBM, and BCFWPGBM operators. Furthermore, we defined a DM procedure based on these PBM operators in the setting of the BCF set. After that, we described a numerical example in which we took four types of pollution and determined which type of pollution is more harmful and dangerous to the environment. We found that air pollution is more harmful than others by utilizing BCFPBM and BCFWPBM operators, and water pollution is more harmful than others by utilizing BCFPGBM and BCFWPGBM operators. Moreover, we illustrated that the introduced PBM operators and a

DM procedure in the environment of the BCFS are more effective and have a wide model and advantages than certain prevailing works. We also showed that the investigated PBM AOs and the DM procedure for the BCFS gave us better and fair results than the prevailing similar AOs and DM procedures. The investigated operators and the DM procedure for the BCFS have certain limitations as well like they cannot tackle the data in the structure of BCF soft sets, complex bipolar intuitionistic fuzzy sets, BCF linguistic sets, etc.

In the future, our goal would be to review various notions like the bipolar complex fuzzy soft set (SS) (Mahmood et al., 2022c), Pythagorean FS (Li et al., 2022), picture FS (Ullah, 2021), picture fuzzy SSs (Khan et al., 2019), T-spherical FS (Javed et al., 2022), complex bipolar intuitionistic FS (Jan et al., 2022), enhancing digital innovation for the sustainable transformation of manufacturing industry (Yin et al., 2022) and try to employ the introduced work in these notions.

Data availability statement

The original contributions presented in the study are included in the article/Supplementary Material; further inquiries can be directed to the corresponding author.

Author contributions

Conceptualization, XY, TM, and UU; methodology, XY, TM, and UU; software, XY, TM, and UU; formal analysis, XY, TM, and UU; investigation, XY, TM, and UU; writing—original draft, XY, TM, and UU; writing—review and editing, XY, TM, and UU.

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Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

Publisher's note

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Comprehensive evaluation and spatial-temporal differences analysis of China's inter-provincial doing business environment based on Entropy-CoCoSo method

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Introduction: The Doing Business environment affects the operation of enterprises in the region and exerts an important impact on economic development, taxation, and employment.

Methods: According to the characteristics of large differences in the level of economic development between provinces in China, this study builds a set of scientific and reasonable evaluation index systems of inter-provincial Doing Business environment, including five first-level indicators and 11 second-level indicators. Moreover, this study constructs a comprehensive evaluation model based on the Entropy-CoCoSo framework and explores its application in China's inter-provincial Doing Business environment evaluation.

Results: Based on China's provincial panel data from 2007 to 2020, the research found that the Doing Business environment of each province has generally improved in recent years, but the inter-provincial differences are more prominent. The western provinces have obvious late-mover advantages.

Discussion: Finally, we put forward several targeted suggestions, providing a powerful reference for optimizing China's Doing Business environment and promoting high-quality economic development.

KEYWORDS

doing business environment, evaluation index system, CoCoSo, entropy, spatialtemporal differences analysis

1 Introduction

A sound Doing Business (DB) environment indicates an important symbol of the economic soft power of a country or region. In the latest Doing Business 2020 released by the World Bank, China's DB environment ranks 31st in the world, entering the global top 40 for the first time. In 2017, at the 16th meeting of the Central Financial and Economic Leading Group, General Secretary Jinping Xi pointed out that a stable, fair, transparent, and efficient DB environment should be created. In November 2018, Jinping Xi said at the opening ceremony of the first China International Import Expo, "An international first-class DB environment should be created". The quality of the DB environment directly affects the operation of enterprises in the region, related to the stimulation of market vitality, and exerts an important impact on economic development, taxation, and employment. Optimizing the DB environment significantly affects the soft power and comprehensive competitiveness of a region or country, and is effective in promoting economic transformation and upgrading. However, at present, there is a lack of a set of scientific and reasonable evaluation systems to reflect the development of China's existing DB environment at the province level. The existing DB environment evaluation system mostly is overwhelmingly focused on the city level or based on the World Bank (2003) from the micro-enterprise level, while the provincial DB environment index evaluation system is less, and evaluation studies covering all provinces are rare. Although the existing DB environment index evaluation system based on the city level or enterprise level can provide a reference for the leaders or decision-makers to improve local business conditions and business management, it's difficult to take a global perspective to optimize China's business environment from the top-level design and provide a reference for inter-provincial balanced development.

Therefore, departing significantly from the conventional method to the study of DB environment focusing on micro-enterprise or city level, this study aims to construct an evaluation index system in line with the inter-provincial DB business evaluation in China, allowing for better assessment of economic and investment growth at the province level. The changes in China's environment are also analyzed through time and space dimensions by using a novel evaluation framework, and then the development ideas are clarified to provide a reference basis for further optimization of China's DB environment. The proposed system index can provide insights from a more complete framework that approximates the actual development of competitiveness and investment. Therefore, the government can get clues from the system and determine a series of possible provinces according to their unique advantages.

The remainder of this study is structured as follows: In Section 2, we present a brief literature review on DB environment. In Section 3, we present a novel inter-provincial DB environment evaluation index system. We propose the Entropy-CoCoSo evaluation framework for DB environment in Section 4. Section 5 carries out the tpatial-temporal differences analysis of inter-provincial DB environment in China, including the indicators used, data sources, and how the weight of index is calculated. We explore the findings and present several recommendations in Section 6. Finally, we conclude with a brief discussion of research limitations and future directions in Section 7.

2 Literature review

Domestic and foreign scholars have rich research on the evaluation of the DB environment. Among them, the main and widely recognized one is the inter-country DB environment indicators system established by the World Bank from the perspective of enterprises, including 10 first-level indicators and 43 second-level indicators, based on ten aspects of the DB environment (Starting a business, Dealing with construction permits, Getting electricity, Registering property, Getting credit, Protecting minority investors, Paying taxes, Trading across borders, Enforcing contracts) (World Bank, 2003). The World Bank's annual report on the DB environment of various countries is widely adopted by countries because of its authoritativeness and is used as a reference for improving the business environment. Its reports directly or indirectly affect the willingness of investors to invest in the world (Song and He, 2018). Tan et al. (2018) presented a novel index to assess the ease of DB at the sub-national level. Roham et al. (2009) developed a new systematic method employing fuzzy set theory to generate composite indicators of DB environment for assessment and classification problems. Estevo et al. (2022) assessed the relationship between the variables related to the business environment and informality. Khazaei and Azizi (2020) found a positive correlation between business environment indicators and corporate financial performance.

In recent years, with the continuous development of China's economy, more and more scholars have begun to study China's DB environment. Zhang and Long (2022) constructed the evaluation index system of the DB environment in Chinese cities based on the seven dimensions of the government environment and analyzed the realization path of the business environment in the development of the urban tourism industry in China. Miao et al. (2021) presented a business environment evaluation index system consisting of six primary and 33 secondary indicators to optimize the business environment while promoting economic development. Cui et al. (2021) empirically analyzed the impact of the DB environment on economic growth using the DB simplicity scores from the World Bank's Doing Business 2004–2018 project. Wang D. Z. et al. (2022) analyzed the interaction between the business environment, agricultural openness, and high-quality agricultural economy from the perspective of Chinese provinces and find that there is a quantitative interaction between them with significant long-term time effects. By examining the pattern of administrative reform in the Chinese context, Liao (2020) found three main paths to optimize the business environment: balanced development, reform-oriented and resource-driven.

As mentioned earlier, the indicators of the DB environment are vital to the overall understanding and assessment of national business development, and strategy formulation for business regulations and policy. There are now 30 provinces, municipalities, and autonomous regions in China, but their business environment construction level is very different. Judging from the situation of each province and region, the eastern coastal provinces have a good business environment, followed by the central region, and the western provinces are the worst. Thus, the provincial DB environment analysis was indeed useful, and it is necessary to combine China's national conditions to construct an inter-provincial Doing Business environment evaluation index system.

3 Evaluation index system of China's inter-provincial DB environment

The DB environment is an organic and dynamic system, which includes political, economic, social, and cultural factors. Based on the research of the existing DB environment evaluation index system, combined with representation, operability, and practicality, this paper selects five aspects that can best reflect the development of the provincial DB environment, namely, urban and rural development, talent innovation, marketization process, financial environment, and political environment.

(1) Urban and rural development. The imbalance between urban and rural development is a remarkable feature of China's economic and social development, which is directly related to the evaluation of the overall regional DB environment. Coordinating the

TABLE 1 China's inter-provincial DB environment evaluation index system.

Target layer	The first-level indicator	The second-level indicator	References
China's Inter-provincial Doing Business environment evaluation	Urban and rural development (B_1)	National *per capita* disposable income of residents (C_{11})	Zhang and Long (2022); Cui et al. (2021); Khazaei et al. (2020)
		Urbanization rate (C_{12})	Cui et al. (2021); Tan et al. (2018); Zhang and Long (2022)
		The tair index (C_{13})	Roham et al. (2009); Miao et al. (2021); Zhang et al. (2020)
	Talent innovation (B_2)	Labour force mismatch index (C_{21})	Cui et al. (2021); Tan et al. (2018); Cui et al. (2021)
		Innovation utility (C_{22})	Roham et al. (2009); Wang D N et al. (2022); Khazaei et al. (2020)
	Marketization process (B_3)	Marketization index (C_{31})	Cui et al. (2021); Tan et al. (2018); Zhang and Long (2022)
		Financial marketization index (C_{32})	Estevo et al. (2022); Miao et al. (2021); Tan et al. (2018)
	Financial environment (B_4)	Green Finance Index (C_{41})	Estevo et al. (2022); Liao (2020); Khazaei et al. (2020); Zhang et al. (2020)
		Capital mismatch index (C_{42})	Estevo et al. (2022); Tan et al. (2018); Miao et al. (2021)
	Political environment (B_5)	Fiscal transparency (C_{51})	Tan et al. (2018); Cui et al. (2021); Liao (2020)
		Degree of corruption (C_{52})	Cui et al. (2021); Khazaei et al. (2020); Zhang et al. (2020)

integrated development of urban and rural areas, optimizing the DB environment of towns and towns, and introducing supporting policies to encourage enterprises to invest in towns will not only help narrow the gap between urban and rural areas, improve the urbanization rate and promote regional economic development but also form a positive feedback mechanism to make the DB environment develop in a better direction.

(2) Talent innovation. Talent and innovation are the two basic elements for enterprises to realize innovation-driven development. On the one hand, talents are the main body of innovation and entrepreneurship, and the subject of enterprise operation. The creation of a first-class DB environment cannot be achieved without talent. On the other hand, a good innovation environment is conducive to the update and iteration of technology. "Science and technology is the primary productive force". Technological innovation can significantly improve the production efficiency of enterprises, promote the better development of enterprises, and at the same time promote the overall development of the local economy. The DB environment can also be optimized, forming a virtuous cycle.

(3) Marketization process. "Marketization means that we must remove unreasonable institutional barriers and further stimulate the market vitality and social creativity." This is what Chinese Premier Keqiang Li mentioned in a national teleconference. With the rapid economic development and the continuous progress of marketization, various market entities are constantly emerging, especially the vitality of the private economy. Stimulating the vitality of various market entities is an important part of improving the DB environment.

(4) Financial environment. The financial environment is the soil for enterprises to carry out capital management, risk control, investment activities, and financing activities. The optimization of a good financial environment plays a decisive role in the improvement of the DB environment. The daily operation and orderly development of enterprises need the support of capital, ensure the correct allocation of funds, and vigorously develop green finance, which will help to provide strong support for enterprises, and better serve the high-quality development of the economy and society.

(5) Political environment. At the Fifth Plenary Session of the 19th CPC Central Committee, it was proposed to "uphold and improve the basic socialist economic system, give full play to the decisive role of the market in resource allocation, better play the role of the government, and better integrate the effective market with the effective government". Strong government intervention in the market is a distinctive feature of China's economy. The slightest change in the political environment is related to the operation of enterprises, and the registration, approval, production, and sales of enterprises are all completed under the supervision of the government. A relaxed political environment, and a transparent, clean, and promising government, is crucial to the optimization of the DB environment.

According to the above index construction logic, in this study we established China's inter-provincial DB environment index evaluation system, consisting of five first-level indicators of urban and rural development, talent innovation, marketization process, financial environment, and political environment. 11 most representative indicators are the second-level indicators, as shown in Table 1.

4 Entropy-CoCoSo evaluation framework for DB environment

Scholars have applied many methods to measure the level of DB environment, mainly including Analytic Hierarchy Process (AHP) method (Randelovic et al., 2020), Equalization method (Hajduova et al., 2020), Principal Component Analysis method (Borojo and Jiang, 2020), TOPSIS (Hajduova et al., 2020; Wang et al., 2020) and fuzzy linguistic (Roham et al., 2009). However, the weights of indicators are given by experts in advance in most existing research, which has a strong subjectivity according to experts' preferences (Tan et al., 2018). In fact, determining the relative degrees of importance of each indicator objectively is a significant challenging problem. Moreover, the evaluation methods used are relatively simple and traditional among the existing theoretical research, which have some inherent defects. Therefore, we aim to propose a novel multi-criteria decision-making (MCDM) model for the evaluation of the DB environment, namely the Entropy-CoCoSo framework. In the proposed Entropy-CoCoSo evaluation framework, the weight of each index is determined by the entropy method, and the comprehensive evaluation of the Doing Business environment of each province is realized by the combined compromise solution (CoCoSo, Yazdani et al., 2019a) method.

4.1 Index weight based on entropy method

Originally applied to thermodynamics in physics to reflect the degree of system chaos, the entropy method is now widely used in project evaluation and social-economic research. The basic idea of the entropy weight method is to determine the corresponding objective weight referring to the index variability. The greater the uncertainty, the greater the information entropy, which is directly proportional to the information entropy (Ren et al., 2021; Wang X. et al., 2022). The smaller the information entropy of the index, the greater the degree of variability of the indicator, the more information provided, and the greater the role it can play in the comprehensive evaluation, then the greater its weight.

The entropy method can exclude the interference of subjective factors, and objectively and accurately calculate the weight of each index with higher credibility (Ren et al., 2019; Zeng et al., 2020). To reflect the situation of China's inter-provincial DB environment reasonably and accurately, this study uses the entropy method to calculate the weight of each indicator. The main steps are as follows:

(1) Construct the evaluation matrix. Let $A = \{A_1, A_2, \cdots A_m\}$ be a set of alternatives (provinces), $C = \{C_1, C_2, \cdots C_n\}$ be a set of attributes (indicators), $EM = (X_{ij})_{m\times n}$ be the assessment information of alternative A_i with respect to attribute C_j, $1 \le i \le m$, $1 \le j \le n$ and $1 \le k \le K$. The evaluation matrix $EM = (X_{ij})_{m\times n}$ can be described as follows:

$$\mathrm{EM} = \left(X_{ij}\right)_{n\times m} = \begin{pmatrix} X_{11} & X_{12} & \cdots & X_{1m} \\ X_{21} & X_{22} & \cdots & X_{2m} \\ \vdots & \vdots & \cdots & \vdots \\ X_{n1} & X_{n2} & \cdots & X_{nm} \end{pmatrix} \tag{1}$$

(2) Data standardization processing. Since different indicators have different dimensions and units, it is necessary to standardize the indicator assessment. Let the value of each indicator data be X_{ij} before normalization, and the value after normalization be Y_{ij}, and the calculation formula for positive indicators is:

$$Y_{ij} = \frac{X_{ij} - \min(X_i)}{\max(X_i) - \min(X_i)} \tag{2}$$

For negative indicators:

$$Y_{ij} = \frac{\max(X_i) - X_{ij}}{\max(X_i) - \min(X_i)} \tag{3}$$

Then the evaluation matrix $B = (Y_{ij})_{n\times m}$ was obtained after standardizing the raw data matrix according to the above method.

(3) The proportion of each index value, that is, the proportion P_{ij} of the index value X_{ij} of the i-th sample under the j-th index is calculated:

$$P_{ij} = \frac{Y_{ij}}{\sum_{i=1}^{n} Y_{ij}} \tag{4}$$

(4) Find the information entropy of each indicator. According to the definition of information entropy, the information entropy of the j-th index is:

$$E_j = -\frac{1}{\ln n}\sum_{i=1}^{n} P_{ij} \ln P_{ij} \tag{5}$$

where $E_j \ge 0$. If $P_{ij} = 0$, then $E_j = 0$.

(5) Calculate the weight of each index $w_j = (j = 1, 2, 3, \cdots, m)$:

$$w_j = \frac{1 - E_j}{m - \sum_{j=1}^{m} E_j} \tag{6}$$

Apparently, we have $\sum_{j=1}^{m} w_j = 1$.

4.2 Evaluation process based on Entropy-CoCoSo framework

The CoCoSo approach is a comprehensive model that uses arithmetic weighting and geometric weighting to handle MCDM problems (Yazdani et al., 2019b). Its main idea is to use three different metrics (aggregation strategy) to measure the relative importance of each alternative and rank all alternatives according to their relative importance. Combining three evaluation score strategies to make the final decision results more reasonable and superior, which not only avoids the problem of biased results caused by a single decision method but also effectively integrates multiple aggregation ideas to achieve internal balance and thus weakens the variability of decision results. Now the CoCoSo method is widely used in different areas, such as blockchain platform selection (Lai et al., 2022), logistics center site selection (Ulutas et al., 2020; Banihashemi et al., 2021), supplier selection (Yazdani et al., 2019b; Lai et al., 2020; Wang C. N. et al., 2022; Wei et al., 2022) due to its high flexibility and simple calculation.

Considering that no studies have used the technique to handle out the DB environment evaluation issue, this paper presents the Entropy-CoCoSo evaluation framework by integrating the idea of the importance of different indicators, and its main steps are as follows:

(1) Obtain the standardized decision matrix $B = (Y_{ij})_{n\times m}$ using the method described in Section 4.1.
(2) Calculate the arithmetic weighted sum:

$$S_n = \sum_{j=1}^{m} w_j \cdot Y_{ij}, \quad i = 1, 2, \cdots n \tag{7}$$

(3) Calculate the geometric weighted sum:

$$P_n = \sum_{j=1}^{m} \left(Y_{ij}\right)^{w_j}, \quad i = 1, 2, \cdots n \tag{8}$$

(4) The relative importance of the alternative A_i is calculated by using the following three formulas:

$$Ka_i = \frac{P_i + S_i}{\sum_{i=1}^{n} (P_i + S_i)}, \quad i = 1, 2, \cdots n \tag{9}$$

$$Kb_i = \frac{P_i}{\min\limits_i P_i} + \frac{S_i}{\min\limits_i S_i}, \quad i = 1, 2, \cdots, n \tag{10}$$

$$Kc_i = \frac{\lambda P_i + (1-\lambda) S_i}{\lambda \max\limits_i P_i + (1-\lambda) \max\limits_i S_i}, \quad i = 1, 2, \cdots, n \tag{11}$$

where $0 \le \lambda \le 1$.

(5) The comprehensive evaluation index of each alternative A_i is calculated with the following formula:

$$S_i = \sqrt[3]{Ka_i \cdot Kb_i \cdot Kc_i} + \frac{1}{3}(Ka_i + Kb_i + Kc_i), \quad i = 1, 2, \cdots, n \tag{12}$$

(6) Sort the alternatives according to the comprehensive score (S_i). The larger the value of S_i, the better the alternative A_i; the alternative with the largest S_i is the best.

5 Spatial-temporal differences analysis of inter-provincial DB environment in China

To better improve the business environment, the Chinese government has implemented reforms to accelerate enterprise registration and trading, improve the credit mechanism and strengthen investor protection since 2007. The newly revised Company Law has shortened the time required to register an enterprise from 48 days to 35 days, and lowered the minimum capital required, greatly facilitating the establishment of new enterprises. The revision of the Company Law has strengthened the supervision of insider trading and increased the protection of investors. The newly adopted online customs declaration procedure has shortened the customs clearance time of import and export by 2 days, which is conducive to improving the international competitiveness of import and export enterprises. Next, the proposed evaluation index system and Entropy-CoCoSo framework are constructed to measure and evaluate China's inter-provincial DB environment from 2007 to 2020.

5.1 Data source

(1) National *per capita* disposable income of residents and urbanization rate. The relevant data are derived from the "China Statistical Yearbook", the statistical yearbooks of provinces, municipalities, and autonomous regions over the years, and some of the missing data were obtained by the Interpolation method.
(2) Capital mismatch and labor force mismatch. Hsieh and Klenow (2009) and Aoki (2012) put forward the theoretical framework of factor mismatch measurement from the enterprise level, industry level, and national and regional macro level respectively. In this paper, we combine the above theoretical framework and refer to the relative mismatch measurement model of factors proposed by Chen et al. (2011), and present the following model:

$$\mu_{it} = \frac{K_{it} / \sum_{i=1}^{n} K_{it}}{\left(Y_{it} / \sum_{i=1}^{n} Y_{it}\right)\left(\alpha_{it} / \sum_{i=1}^{n} \alpha_{it}\right)} \tag{13}$$

where μ_{it} represents the period t of capital or labor mismatch in region i, Y_{it} represents the output level of the period t in region i, K_{it} represents the period t of the allocation of capital or labor in region i, α_{it} represents the period t of capital or labor output elasticity in region i, $\sum_{i=1}^{n} K_{it}$, $\sum_{i=1}^{n} Y_{it}$, $\sum_{i=1}^{n} \alpha_{it}$ respectively represent the country's total capital or total labor allocation, total output and capital or labor output elasticity.

According to the model, when $\mu_{it} > 1$, it means that the cost of using capital or labor is less than the national average, and the allocation of capital or labor is insufficient; Otherwise, if $\mu_{it} < 1$, the capital or labor is over-allocated, which is a misallocation of capital or labor. On the basis of relative capital mismatch, and referring to the research of Ji et al. (2016), we introduce the absolute mismatch of capital or labor, the model as follows:

$$\gamma_{it} = \left|\mu_{it} - 1\right| \tag{14}$$

γ_{it} can describe the absolute degree of distortion of capital or labor, used in this paper as a proxy variable for the level of capital or labor mismatch.

(3) Innovation utility. The comprehensive score of the provincial-level "Comprehensive Utility Value of China's Regional Innovation Capability" in the "China Regional Innovation Capability Evaluation Report" jointly issued by the China Science and Technology Development Strategy Group and the Chinese Academy of Sciences over the years is used as the proxy variable for the regional innovation level of the explained variable. "World Competitiveness Yearbook", "Global Innovation Index", "Global Competitiveness Report", "Innovative Alliance Index", "National Innovation Index" and other well-known international and domestic reports, and measured according to China's regional characteristics and characteristics, the comprehensive score is the innovation utility, and some missing data were obtained by interpolation.
(4) Green finance index. This paper divides the green financial system into four modules: green credit, green investment, green insurance, and government support, and forms an indicator system based on this, which in turn constitutes a comprehensive measurement system for the level of green financial development. To avoid the subjectivity of the traditional expert weight scoring, this paper adopts the entropy weight method to objectively assign the weight of the index. Then, the green finance index of 30 provinces in mainland China from 2007 to 2020 is measured, and some missing data are obtained by interpolation. The relevant data came from the "China Statistical

TABLE 2 The weight of the indicators.

Target layer	The first-level indicator	Weight	The second-level indicator	Weight
Inter-provincial DB environment evaluation	Urban and rural development	.3163	National *per capita* disposable income of residents	.1508
			Urbanization rate	.0542
			The tair index	.1113
	Talent innovation	.3254	Labour force mismatch index	.1579
			Innovation utility	.1675
	Marketalization process	.2024	Marketization index	.1175
			Financial marketization index	.0849
	financial environment	.0839	Green Finance Index	.0542
			Capital mismatch index	.0296
	political environment	.0720	Fiscal transparency	.0210
			Degree of corruption	.0510

Yearbook", the statistical yearbooks of provinces, municipalities, and autonomous regions over the years, and the "China Insurance Yearbook".

(5) Financial marketization index. As for the measurement of the financial marketization index, this paper refers to the "marketization index of the financial industry" expressed in the "China's Marketization Index - Annual Report on the Relative Progress of Marketization in Various Regions" as the agent variable for financial marketization. Secondly, considering that the current version of the book has adjusted the calculation base period compared with the 2011 version, there are no statistics on the financial marketization index in the central region. Therefore, referring to the method of Zhong et al. (2020), this paper utilizes the deflator index method and the interpolation method to adjust the data, and finally obtains the sample data of the core explanatory variable of this paper, that is, the financial marketization index.

(6) Marketization process. The marketization process refers to the research of Xiao and He (2015), and selects the marketization index to represent the marketization process of the region. Some missing data were obtained by interpolation method, and the data used were all from the "China Statistical Yearbook", "China Marketization Index" and the statistical yearbooks of various provinces and cities.

(7) Fiscal transparency. The comprehensive score of "China's Fiscal Transparency Report - Evaluation of Provincial Fiscal Information Disclosure" issued by the Shanghai University of Finance and Economics, which is currently the most authoritative in China, is used as its proxy variable, and some missing data are obtained by interpolation.

(8) The degree of corruption. The degree of corruption is calculated by the number of cases/the total number of public officials, and some missing data is obtained by the Interpolation method. The relevant data are derived from the "China procurator Yearbook".

5.2 Index weight calculation

Based on the panel data of 30 provinces, municipalities, and autonomous regions in China from 2007 to 2020, the weights of 11 second-lever indicators and five first-lever indicators are determined, the results are shown in Table 2.

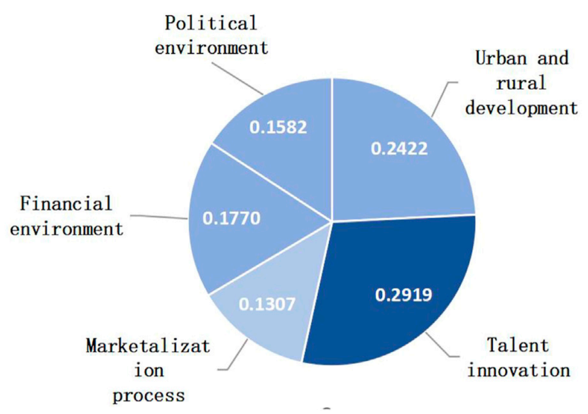

FIGURE 1
Weights of DB environment indicators.

As can be seen from Table 2, talent innovation accounted for 32.54%, urban and rural development status accounted for 31.63%, marketization process accounted for 20.24%, financial environment and political environment accounted for 8.39% and 7.2% respectively, all less than 10%. It shows that in the process of inter-provincial Doing Business environment comparison, there is a great difference between talent innovation data and urban and rural development status data, the data on the marketization process is quite different among provinces, and the data on the financial environment and political environment is relatively small. It basically conforms to the characteristics of unbalanced development among China's regions. The eastern region has a high degree of economic development, and the overall development of urban and rural areas is relatively good. Most of the talents are concentrated in the eastern region with strong innovation capabilities. After years of development, the marketization process is also significantly better than in the central and western regions. The development of the central and western regions is relatively general, the urban and rural development conditions are

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TABLE 3 DB environment scores of each province during 2007–2020.

Region	Province	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	Rate of rise
Eastern region	Beijing	3.21	3.24	3.09	3.19	3.16	3.31	3.26	3.45	3.46	3.51	3.50	3.47	3.32	3.23	.04%
	Tianjin	2.35	2.34	2.23	2.43	2.46	2.47	2.45	2.52	2.54	2.57	2.44	2.47	2.53	2.57	.68%
	Hebei	1.44	1.42	1.31	1.43	1.82	1.63	1.57	1.47	1.47	1.41	1.43	1.39	1.56	1.63	.97%
	Liaoning	2.06	1.98	1.87	1.83	1.93	1.925	1.80	1.97	1.93	1.93	1.80	1.67	1.66	1.64	−1.73%
	Shanghai	3.08	2.98	2.85	3.05	3.06	2.86	2.90	2.99	3.06	3.08	3.05	3.01	2.95	2.85	−.60%
	Jiangsu	2.52	2.48	2.44	2.50	2.65	2.68	2.59	2.55	2.59	2.50	2.78	2.58	2.52	2.44	−.22%
	Zhejiang	2.56	2.45	2.40	2.64	2.59	2.61	2.50	2.62	2.64	2.55	2.51	2.59	2.69	2.72	.48%
	Fujian	2.27	2.31	2.45	2.29	1.98	2.01	2.29	2.23	2.23	2.17	2.06	2.02	2.16	2.24	−.12%
	Shandong	1.98	1.90	1.90	2.12	2.04	2.14	2.03	2.22	2.26	2.14	2.11	2.00	2.00	1.98	.02%
	Guangdong	2.58	2.52	2.34	2.57	2.56	2.53	2.44	2.59	2.76	2.73	2.87	2.98	2.95	2.85	.78%
	Hainan	1.43	1.36	1.33	1.54	1.42	1.81	1.90	1.61	1.73	1.62	1.41	1.50	1.52	1.58	.76%
Central region	Shanxi	1.32	1.26	1.22	1.35	1.30	1.31	1.28	1.51	1.55	1.44	1.25	1.22	1.35	1.44	.68%
	Jilin	1.34	1.33	1.20	1.30	1.40	1.39	1.27	1.45	1.39	1.35	1.12	1.21	1.23	1.29	−.29%
	Heilongjiang	1.34	1.29	1.28	1.47	1.40	1.90	1.70	1.68	1.60	1.60	1.43	1.34	1.36	1.42	.41%
	Anhui	1.51	1.57	1.55	1.45	1.46	1.45	1.42	1.52	1.76	1.70	1.62	1.52	1.55	1.54	.13%
	Jiangxi	1.35	1.36	1.34	1.42	1.55	1.57	1.41	1.34	1.51	1.58	1.45	1.41	1.50	1.60	1.31%
	Henan	1.40	1.39	1.35	1.41	1.46	1.45	1.51	1.38	1.51	1.51	1.37	1.39	1.45	1.47	.43%
	Hubei	1.40	1.42	1.41	1.51	1.53	1.95	1.72	1.68	1.48	1.57	1.45	1.60	1.56	1.47	.37%
	Hunan	1.34	1.37	1.39	1.60	1.63	1.60	1.47	1.61	1.50	1.89	1.77	1.69	1.77	1.84	2.51%
Western region	Inner Mongolia	1.58	1.62	1.57	1.50	1.85	1.96	1.48	1.56	1.56	1.45	1.43	1.29	1.27	1.28	−1.60%
	Chongqing	1.59	1.56	1.47	1.64	1.77	1.78	1.84	1.91	1.87	1.95	1.84	1.83	1.93	1.99	1.73%
	Sichuan	1.46	1.51	1.52	1.58	1.62	1.55	1.41	1.49	1.39	1.43	1.63	1.54	1.65	1.75	1.42%
	Guizhou	.83	.864	.88	.86	.84	.96	.91	1.02	.85	1.05	.97	.89	.98	1.04	1.72%
	Yunnan	.99	.92	.91	.96	.99	1.06	.99	1.12	.87	1.00	1.09	1.11	1.23	1.37	2.53%
	Guangxi	1.08	1.12	1.19	1.36	1.30	1.52	1.44	1.54	1.52	1.41	1.33	1.29	1.38	1.46	2.37%
	Shaanxi	1.22	1.26	1.22	1.30	1.42	1.48	1.35	1.43	1.33	1.43	1.31	1.39	1.41	1.44	1.28%
	Gansu	.90	.86	.82	.94	.91	1.00	.93	1.12	1.12	1.06	1.18	.98	1.03	1.09	1.50%

(Continued on following page)

TABLE 3 (*Continued*) DB environment scores of each province during 2007–2020.

Region	Province	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	Rate of rise
	Qinghai	.99	.95	.91	.98	.92	.91	.91	.99	.88	.92	.94	1.07	1.23	1.47	3.06%
	Ningxia	.98	.97	.89	.93	.91	1.08	1.15	1.20	1.12	1.50	1.28	1.14	1.20	1.22	1.68%
	Xinjiang	.95	.93	.93	1.04	1.47	1.40	1.25	1.40	1.23	1.25	1.09	.99	1.01	1.07	.95%
	nationwide	1.64	1.62	1.58	1.67	1.71	1.78	1.71	1.77	1.76	1.78	1.72	1.69	1.73	1.77	.60%

poor, and the gap is large, and the talents flow from the central and western regions to the eastern region. The lack of talent leads to insufficient innovation ability, and the lack of development efforts and short development time lead to a lag in the marketization process. Although there are certain gaps in the inter-provincial data of financial environment and political environment indicators, they are not as significant as urban and rural development, talent innovation and marketization.

To fully show the different importance among the first-level 5 indicators, it is specially presented in the form of a radar map, as shown in Figure 1.

5.3 Inter-provincial DB environment evaluation

Based on the weights of indicators obtained in Section 5.2, the business environment of 30 provinces, municipalities, and autonomous regions in China from 2007 to 2020 was evaluated by utilizing the CoCoSo method. Two different aggregation processes (arithmetic weighting, and geometric weighting) and three evaluation scoring strategies are integrated to measure the relative importance, that is, the comprehensive evaluation index score of the DB environment of each province in different years S_i, is calculated. The results are shown in Table 3.

According to the results obtained in Table 3, the average scores of each province from 2007 to 2020 were calculated and sorted from high to low. The results are shown in Table 4 and Figure 2.

To fully show the changes in the national DB environment development, a line chart is drawn, shown in Figure 3.

The above results show that, at the national level, the score of China's DB environment has increased from 1.635 in 2007 to 1.767 in 2020, with an average annual growth rate of .6%. The index score shows an upward trend, indicating that China's DB environment is generally optimized, in line with the strategy of "optimizing the DB environment" vigorously advocated by the state in recent years. However, the score of China's DB environment does not increase year by year but shows a fluctuating upward trend. This is because China's business environment is not only affected by the domestic economy, politics, and other aspects, but also by the world economic situation, global politics, and many other external factors. With the further deepening of globalization, the impact of "globalization" on China's DB environment has both advantages and disadvantages. When a global economic crisis occurs, China's DB environment will inevitably be affected by it. Due to the increasingly frequent exchanges of funds, talents, and technologies between China and other countries in the world, it is conducive to China's economic development and the optimization of the business environment. At present, China is gradually changing the way it participates in economic globalization, actively promoting and building a more inclusive economic globalization (Xia et al., 2020).

From the perspective of provinces, the scores of Beijing, Shanghai, Guangdong, Zhejiang, Jiangsu, and Tianjin were always greater than two during the 14 years from 2007 to 2020. In terms of environmental ranking, the above six provinces can be identified as the six provinces with the best DB environment in China, followed by Fujian, Shandong, and Chongqing. Further analysis of the data shows that the top ten provinces in terms of DB environment are concentrated in the eastern region, the provinces ranked ten to twenty are

TABLE 4 Ranking of the DB environment from 2007 to 2020.

Province	14-year average score	Ranking
Beijing	3.312	1
Shanghai	2.984	2
Guangdong	2.661	3
Zhejiang	2.577	4
Jiangsu	2.559	5
Tianjin	2.455	6
Fujian	2.193	7
Shandong	2.059	8
Liaoning	1.856	9
Chongqing	1.784	10
Hunan	1.602	11
Hubei	1.554	12
Hainan	1.553	13
Anhui	1.544	14
Sichuan	1.538	15
Inner Mongolia	1.532	16
Hebei	1.494	17
Heilongjiang	1.484	18
Jiangxi	1.459	19
Henan	1.432	20
Shaanxi	1.357	21
Guangxi	1.352	22
Shanxi	1.341	23
Jilin	1.306	24
Xinjiang	1.146	25
Ningxia	1.113	26
Yunnan	1.045	27
Qinghai	1.005	28
Gansu	.996	29
Guizhou	.924	30

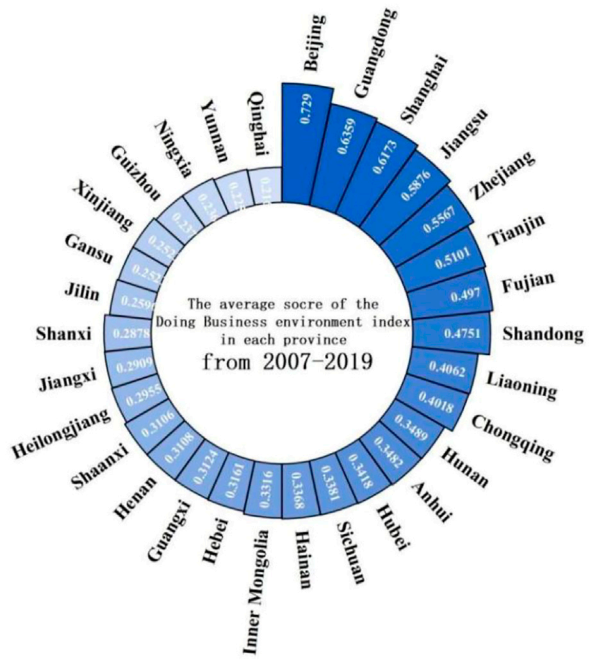

FIGURE 2
The average score of each province from 2007 to 2020.

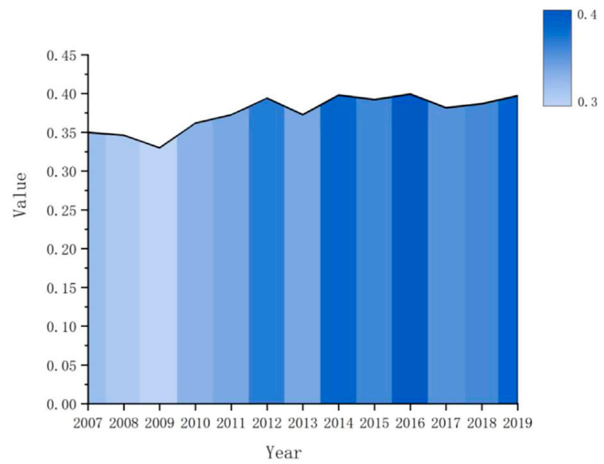

FIGURE 3
The National DB environment score.

concentrated in the central region, and the provinces ranked twenty to thirty basically belong to the western region. From 2007 to 2020, the DB environment of most provinces showed a fluctuating upward trend overall. Some of them experienced large inter-annual changes, and a very small number of provinces experienced negative growth in the DB environment. The major reason for the large inter-annual fluctuations in the province is the violent fluctuations in the talent innovation and political environment of the province. From a deeper level, the behavioral factors, and personnel changes of government officials in each province will have a significant impact on the business environment. The quality of their governance capabilities directly or indirectly affects the province's attraction to outstanding talents and affects innovation capabilities. At the same time, the personnel of provincial officials Changes, and the length of official tenure can affect the disclosure of financial information, which in turn affects the business environment.

To further reveal the spatial and temporal differences in the regional DB environment in China, next we study and calculate the scores of the DB environment of the three geographical regions, shown in Table 5 and Figure 4.

To better illustrate the spatial differences in China's DB environment and the changes in recent years, the spatial and temporal distribution map of China's DB environment is hereby drawn, as shown in Figure 5.

TABLE 5 Score of the operating DB environment from 2007 to 2020.

Region	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
Eastern region	2.32	2.27	2.20	2.33	2.33	2.36	2.34	2.38	2.42	2.38	2.36	2.33	2.35	2.34
Central region	1.38	1.38	1.34	1.44	1.47	1.58	1.47	1.52	1.54	1.58	1.43	1.42	1.47	1.51
Western region	1.16	1.16	1.14	1.21	1.25	1.33	1.24	1.34	1.25	1.32	1.30	1.26	1.33	1.41

According to the above chart, the DB environment in China's three regions shows a significant distribution pattern of "East over West". But without exception, they all show a trend of continuous improvement. The Doing Business environment index score of the eastern region has increased from 2.315 in 2007 to 2.339 in 2020, and the DB environment index score of the central region has increased from 1.374 in 2007 to 1.509 in 2020. The DB environment index score in the western region increased from 1.164 in 2007 to 1.413 in 2020.

In addition, the index score is analyzed, and the index score is generally divided into three intervals, the comprehensive evaluation index $S \geq 1.8$ is a good DB environment, $1.8 > S > 1.4$ is a general environment, and $S \leq 1.4$ is a poor development. Based on this criterion, China's 30 provinces, municipalities, and autonomous regions can be divided into three major regions, the score $S \geq 1.8$ is the developed area, $1.8 > S > 1.4$ is the moderate DB environment area, and $S \leq 1.4$ is the underdeveloped area. It is found that the provinces that are divided into the three major regions according to the division criteria have a high degree of overlap with the provinces that are included in the three major regions in the geographical sense.

All the above characteristics show that the DB environment is completely consistent with the degree of economic development. The eastern region has strong comprehensive economic development strength, a high urbanization rate, early opening to the outside world, and has always been in the vanguard of the domestic DB environment reform wave. Due to its proximity to the eastern region, the central region is easily affected by the radiation effect of the eastern region, and its urban and rural development, talent innovation, marketization process, financial environment, and political environment are better than those in the western region. However, the western region is limited by the natural environment, capital, and other conditions, the industry is backward, and the economy is mainly resource-consuming. Although in terms of index scores and various dimensions of indicators, the eastern region is better than the central and western regions, the average annual growth rate of the index scores in the central and western regions is higher than that of the eastern region, which has obvious late-mover advantages, and the growth potential is not enough. We should accelerate the implementation of measures to optimize the DB environment in the central and western regions.

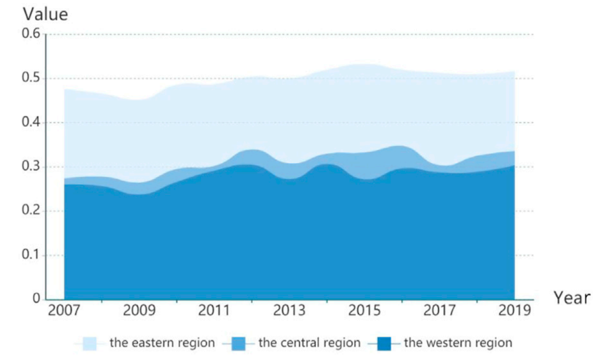

FIGURE 4
The DB environment scores of the three major regions in 2007–2020.

6 Policy recommendations

Based on the above analysis concerning the DB environment of each province in terms of spatial and temporal differences, it is found that the overall environment of various provinces has generally improved in recent years, but the inter-provincial differences are still relatively significant. As a result, this paper makes the following recommendations:

(1) Adhere to the centralized and unified leadership of the central government for steady and effective implementation in all localities and insist on inquiring about politics. Optimizing the business environment is a struggle to correctly handle the relationship between the government and the market, but also a self-revolution on the concept of government, systems and mechanisms, and management techniques. At the same time, the construction of the business environment is an important task that spans all aspects of economic, political, social, cultural, and ecological construction. Therefore, to do a good job in the construction of the business environment, we must give full play to the leading role of the central government in overseeing the overall situation and coordinating all parties to build social consensus. Regional inspection and supervision efforts are made to create a clean and upright political ecology. At the same time, provinces and municipalities across the country should closely follow the pace of the central government, deepen their understanding of policy orientation, and actively and effectively implement the "Regulations on Optimizing the Business Environment" promulgated by the Party Central Committee and the State Council, formulate, and improve the evaluation system, conduct regular evaluations, and fully solve problems for local enterprises. In addition, both the central government and all localities are supposed to insist on inquiring about politics, thinking about problems, handling things, and formulating policies from the perspective of the masses and enterprises, so as to improve the pertinence and effectiveness of the business environment reform policies.

(2) Strengthen inter-regional exchanges and cooperation and promote coordinated and integrated development. The DB environment of provinces has improved in recent years, but

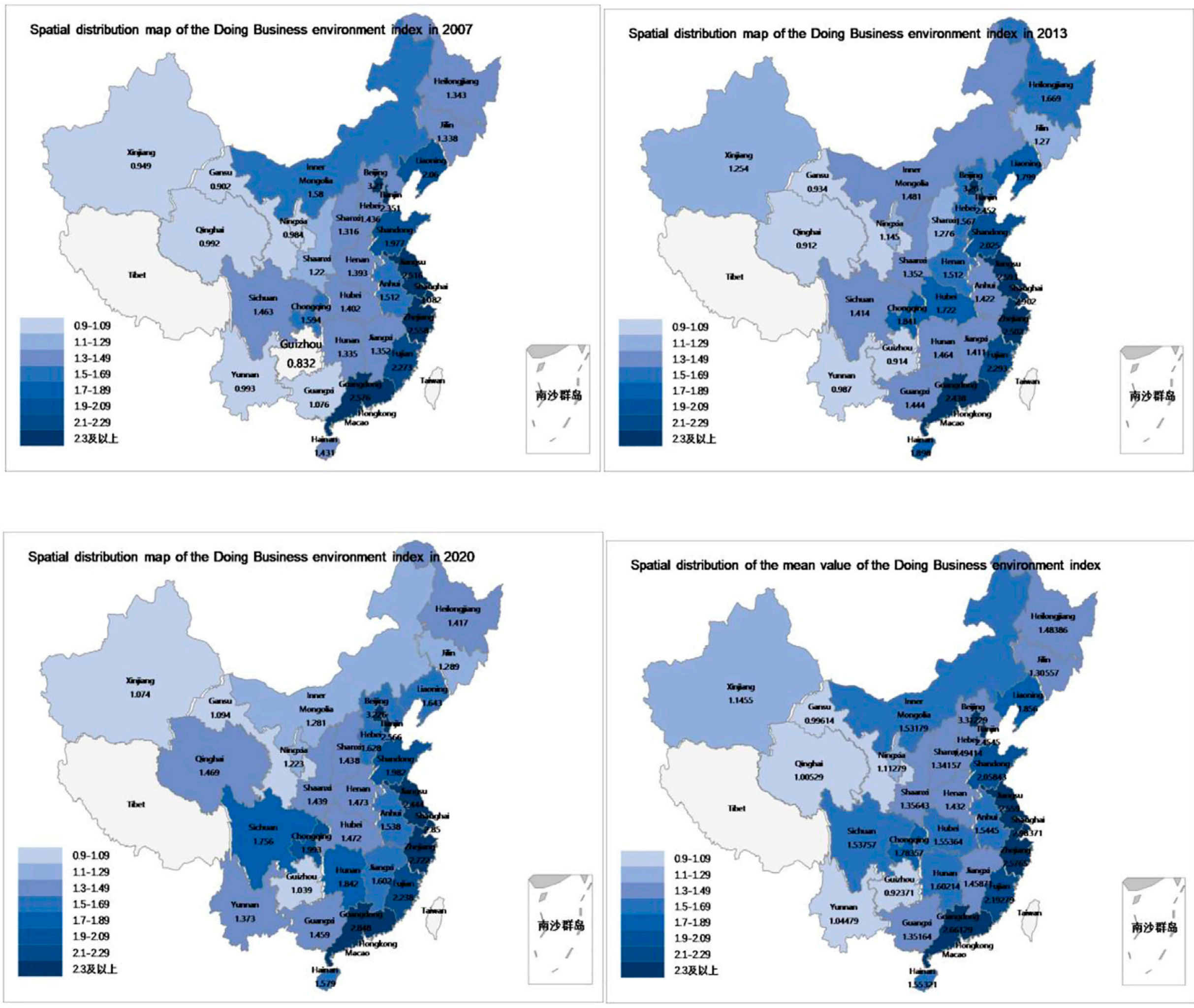

FIGURE 5
The spatial and temporal distribution map of China's DB environment.

the inter-provincial differences are prominent, and the inter-provincial DB environment shows a significant "valuing the east over the west" distribution pattern, it is recommended to strengthen exchanges and cooperation between the eastern and central and western regions and provide targeted assistance. As the eastern region of China has a first-mover advantage in economic development, its economic development has reached a certain extent, and the DB environment has also been significantly improved. Therefore, the eastern region should continue to give full play to its geographical advantages, further optimize the Doing Business environment, and make it the engine of China's economic transformation and development. However, due to the urban and rural development status, talent innovation, marketization process, financial environment, and political environment in the central and western regions all lag behind the eastern region, the central and western regions need to introduce corresponding measures in combination with their own development model to make up for the deficiencies in the above fields. At the same time, all regions should actively respond to national strategies such as "the rise of central China" and "western development", strengthening cooperation and exchanges, and achieving mutual benefit and win-win results in the eastern, central, and western regions. Specifically, the central and western regions should rely on the policy advantages and the human resources, social capital and technological advantages of the eastern regions to seize the development opportunities, vigorously support industrial development, and promote the optimization and transformation of local enterprises and industrial structure, so as to optimize the local DB environment.

(3) Accelerate the coordinated development of urban and rural areas and increase support for talent training and technological innovation. Based on the characteristics of urban and rural development and talent innovation occupying an important position in the inter-provincial DB environment evaluation system, this paper suggests that the coordinated development of urban and rural areas should be accelerated to increase the support for talent training and investment in scientific and technological innovation. Urbanization is the only way to modernization and an important way to achieve common prosperity, which is of great significance for solving the

principal contradiction of Chinese society, promoting economic quality development, and increasing people's income. It is also strong support to narrow the gap between regional development and cannot be ignored for expanding domestic demand and promoting the transformation and upgrading of industrial structure. Therefore, the key to promoting urban and rural coordinated development is to realize urbanization in rural areas. In addition, the competition today is, in the final analysis, the competition of talents, which is the first resource of the country and society, and the most important resource and driving force for technological progress and economic and social development. Which province attracts and gathers more excellent talents means that it occupies the initiative in the development and can form a relatively strong position in the fierce scientific and technological and economic competition.

(4) Accelerate the transformation of the government to a service-oriented government, and vigorously promote the market-oriented process. Based on the research, although the influence of the marketization process in the evaluation of the inter-provincial business environment is not as great as that of urban and rural development and talent innovation, it is equally important. The process of marketization process depends to a large extent on the local government, so it is necessary to straighten out the functions and boundaries of the government in managing the economy, reshape the internal mechanism and process of the government, and regulate and restrain the government to drive the market to be effective, the enterprises to benefit, and the people to benefit, and to promote the transformation of government into a service-oriented government which makes it play its role as a public servant for economic development, while adhering to the market-oriented, in-depth promotion of market-oriented reforms, maintaining an efficient market-oriented operation system, stimulate the vitality and social creativity of various market entities. Give full play to the decisive role of the market in resource allocation, promote the rational flow and distribution of capital and labor, reduce the degree of mismatch, and further improve market efficiency. In addition, it is necessary to deepen the reform of "Streamline administration and delegate power, improve regulation, and upgrade services", and further strengthen innovation supervision. Adhere to the combination of delegating power and regulation, consolidate supervisory responsibilities, and improve the supervision mechanism of the whole chain and the whole process before and after the event. Improving open, transparent, concise, and clear regulatory rules and standards.

7 Conclusion

Considering the actual development situation of China, standing from the macro level, this study first constructed a suitable inter-provincial Doing Business environment index system for China's current economic transition, including five first-level indicators and 11 second-level indicators. This evaluation system departs significantly from conventional methods focusing on DB environment development from the city level or micro-enterprise level, offering an alternative to current popular evaluation systems of DB at the provincial level. Moreover, this study constructs a novel comprehensive evaluation framework based on entropy weight and the classic CoCoSo method, wherein the entropy method is used to determine the objective weight of the selection indicators, and the CoCoSo is applied to comprehensive scores of China's inter-provincial DB environment. Based on China's provincial panel data from 2007 to 2020, the study found that the Doing Business environment of each province has generally improved in recent years, but the inter-provincial differences are more prominent. The western provinces have obvious late-mover advantages. Finally, several corresponding suggestions are then put forward to provide a reference for decision-makers to optimize China's overall DB environment.

There are several methodological issues that need to be highlighted. First, the proposed framework contains 11 indicators, which are relatively simple and easy to manage at the provincial level. We can further expand or add some other relevant indicators in the subsequent use to make them more comprehensive, which may be a potential field for future research. Second, owing to its usefulness and effectiveness, the popular CoCoSo method was applied to measure the development of DB environment in this study. But nevertheless other methods thought needs to go into the consideration regarding the evaluation of DB environment as a typical MCDM, such as the BWM, and EDAS (Zhang et al., 2022; Zeng et al., 2023). Third, a classic issue of establishing an index is to determine how importance should be assigned to the different situations. In this case, we have opted to utilize the entropy method to identify the objective weights of indicators. Our research seems to be objective and empirical manner, but nevertheless, more thought needs to go into the consideration regarding the allocation of weights using a more subjective manner by considering the people's subjective judgment, such as AHP (Saaty, 2006) and social network trust method.

Data availability statement

The original contributions presented in the study are included in the article/supplementary material, further inquiries can be directed to the corresponding author.

Author contributions

Writing—original draft preparation, YZ and ZL; writing—review and editing, SZ; methodology, KU; supervision, SZ. All authors have read and agreed to the published version of the manuscript.

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Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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Environmental pollution liability insurance and green innovation of enterprises: Incentive tools or self-interest means?

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The existing theoretical research on environmental liability insurance mainly focuses on system construction, development status and other aspects, and mainly consists of normative research, with relatively little empirical research. This paper uses empirical research methods to explore the impact of environmental liability insurance on the green innovation of enterprises and examines the driving role of environmental liability insurance in green innovation from the perspective of green governance. This paper, based on the list of enterprises purchasing environmental pollution liability insurance (EPLI) as published by the Ministry of Ecology and Environment, explores the impact of EPLI on the green innovation of enterprises. It is found that purchasing EPLI can significantly improve the green innovation of enterprises. The effect path test shows that EPLI can promote the green innovation of enterprises by relaxing financing constraints and reducing agency costs. The heterogeneity scenario test shows that compared with non-state-owned enterprises, non-heavily polluting enterprises, and enterprises in regions with a lower intensity of environmental regulation, EPLI plays a stronger role in the promotion of green innovation of state-owned enterprises, heavily polluting enterprises and enterprises in regions with a higher intensity of environmental regulation. In addition, it is found that the promotion effect of EPLI on green innovation is mainly reflected in the significant improvement of high-quality green innovation and the optimization of the structure of the green patent output. This paper affirms the positive role of EPLI on the green operations of enterprises and provides empirical evidence that green insurance serves the development of the green innovation of enterprises.

KEYWORDS

environmental pollution liability insurance, green innovation, financing constraints, agency problems, sustainable development

1 Introduction

Extensive economic development has led to the continuous deterioration of the ecological environment and the frequent occurrence of environmental pollution problems in China. Faced with a serious threat to their lives from environmental pollution, the affected people usually seek civil compensation from the enterprises involved by means of litigation. As the main producers of environmental pollution, enterprises are often required to bear huge numbers of litigation claims, pollution compensation, and environmental remediation costs, and their daily production and business activities will also be extremely adversely affected. Taizhou Environmental Protection Federation in Jiangsu Province sued six chemical enterprises for polluting the environment in 2014, and the cost of compensation was up to RMB 160 million.

Such a heavy economic burden may cause the enterprises to go bankrupt and out of business, which undoubtedly slows down the process of high-quality development of China's economy (Yin et al., 2022).

In order to address this problem, the State Council issued "Several Opinions on the Reform and Development of the Insurance Industry" in 2006, which, for the first time, proposed measures "to develop insurance business such as environmental pollution liability insurance through market operation and policy guidance". In 2007, the Ministry of Ecology and Environment and China Banking and Insurance Regulatory Commission jointly issued the "Guidance on Environmental Pollution Liability Insurance", emphasizing that environmental pollution liability insurance, as an important innovation in the environmental management system, is an important means for insured enterprises to use the liability insurance mechanism to protect themselves against the operational risks brought about by pollution accidents and to actively assume social responsibility. In 2013, the Ministry of Ecology and Environment and China Banking and Insurance Regulatory Commission further issued the "Guidance on the Pilot Work of Compulsory Environmental Pollution Liability Insurance", stating that "compulsory environmental pollution liability insurance takes advantage of insurance instruments to deal with environmental pollution damage in a market-oriented and socialized way, which is of positive significance for enterprises to strengthen their business management". In 2015, the State Council issued the "Overall Plan for the Reform of the Ecological Civilization System", emphasizing "the establishment of a compulsory environmental pollution liability insurance system in areas with high environmental risk". In 2016, seven ministries and commissions such as the People's Bank of China and the Ministry of Finance jointly issued the "Guidance on Establishing a Green Financial System", emphasizing the need to vigorously promote the construction of a green financial system that includes compulsory environmental pollution liability insurance, facilitate green innovation and transformation of enterprises in an all-round manner, and promote the green development of China's economy. It is obvious that the Chinese government attaches great importance to the development of this new type of insurance, namely environmental pollution liability insurance.

Environmental pollution liability insurance (hereinafter referred to as "EPLI") usually refers to commercial insurance that takes on the insured party's liability for damages caused to third parties due to the occurrence of environmental pollution accidents as the subject matter (Bie, 2008; Yan, 2009; Li, 2010), which not only has the basic function of insurance but a special function of promoting the green innovation and transformation of the industrial structure (Yin et al., 2022). In theory, EPLI uses market-based means to realize the internalization of the premium expenditure of insured enterprises for operations, directly link insurance premiums to the risk status of enterprises and their risk management level, force enterprises to transform their production processes and upgrade their production processes through economic means, and promote the transformation and upgrade of green innovation (Yin et al., 2022). However, as a special financial system arrangement in China, can environmental liability insurance really promote the green innovation of enterprises? This conclusion is not supported by relevant empirical evidence. Based on this, this paper explores the impact of EPLI on the green innovation of enterprises and its mechanism of action using the method of the multiple linear regression test, in order to understand more comprehensively how green finance supports the high-quality innovative development and the green business of enterprises.

This paper makes the following contributions. Firstly, it enriches the research on the economic consequences of EPLI. The existing research on EPLI mainly consists of normative research, with a focus on system construction, development status, etc. This paper uses an empirical research approach to examine the green innovation-driving role of EPLI from a green governance perspective, providing a theoretical basis for a deeper understanding of the role of EPLI in micro-finance behavior. Secondly, it expands the research on the factors influencing the green innovation of enterprises. There are abundant studies on the factors influencing the green innovation of enterprises, but little research has focused on the relationship between EPLI and the green innovation of enterprises. This paper examines the impact of enterprises purchasing EPLI on green innovation and expands the research on the factors influencing green innovation. Thirdly, it deeply explores the mechanism of action of EPLI in influencing the green innovation of enterprises. The findings of this paper suggest that EPLI acts on the green innovation of enterprises by relaxing financing constraints and reducing agency costs, thus deepening the understanding of how EPLI affects the green innovation of enterprises. Fourthly, it further provides empirical evidence for the specific context in which EPLI affects the green innovation of enterprises. This paper explores whether there are differences in the impact of EPLI on the green innovation of enterprises under different economic scenarios, such as the nature of property rights, the attributes of heavily polluting industries, and the intensity of environmental regulation, clarifies the specific contexts in which EPLI affects green innovation, and provides evidence, support and empirical references for regulators to formulate differentiated green insurance policies and strengthen the green governance effect of EPLI.

2 Literature review

At present, as far as the existing literature on the basic EPLI theory is concerned, scholars at home and abroad have conducted relevant studies mainly from the perspectives of system construction and development status.

2.1 System construction

Wang (2009) pointed out that China should implement strategies to improve environmental protection laws, provide financial support, improve the original insurance and reinsurance markets and form mutual insurance companies; Liu et al. (2019) believed that the design of scientific and reasonable insurance clauses and improvement of the reinsurance mechanism are the keys to the improvement of marine engineering environmental liability insurance. Zheng and Yao (2015) found by comparison that the system of adjusting rates based on the payout ratio has a higher degree of risk differentiation, and therefore recommended that this reward and punishment system be used for EPLI. Fang (2016) pointed out that the innovation of EPLI types and the establishment of a compulsory and voluntary insurance system can promote EPLI development in China. Sun and Gu (2015) argued that the key to the EPLI system lies in the establishment of a mode of "voluntary insurance-based, supported by compulsory insurance".

2.2 Development status

Zhang (2011) pointed out that legislative deficiencies were the main reason for hindering EPLI development; Li (2009)[1] found that China's EPLI faced problems such as low motivation of enterprises to take out insurance, insufficient capacity of insurers, and difficulties in implementation by relevant government departments; Yu et al. (2017) identified narrow coverage areas, unreasonable insurance liabilities, high insurance premium rates, and high insurance payout rates as the main problems in China's EPLI practice; Guo and Zhou (2017) pointed out that the inability to effectively quantify the environmental pollution losses and to set insurance premium rates is the bottleneck in the operation of the EPLI compensation mechanism; Wang et al. (2011); Liu (2015); Chen and Lin (2015),; Li et al. (2015) conducted field research on the EPLI pilot work in different provinces, cities, and regions, and found that there was a general lack of EPLI demand from enterprises and lack of supply from insurers in China (Wang et al., 2011; Chen and Lin, 2015; Li et al., 2015; Liu, 2015). Misheva (2016) analyzed the changing demand among industrial enterprises for EPLI. In combination with the operational mode of highly polluting enterprises, Misheva (2017) analyzed their environmental and risk factors in an integrated manner and pointed out the impact of EPLI on the risk management of industrial enterprises.

In summary, most studies on the EPLI theory are normative studies, with a focus on system construction, development status, etc., while there are relatively few empirical studies. In addition, few of these studies have analyzed the relationship between EPLI and the green innovation of enterprises, which provides an opportunity for innovative study in this paper.

3 Theoretical analysis and research hypothesis

As a high-risk investment, enterprise innovation is often affected by both financing constraints due to imperfect external capital markets and agency costs resulting from poor internal corporate governance (Bai et al., 2018; Yin and Yu, 2022), as is the case with green innovation. EPLI may have a positive impact on the green innovation of enterprises in the following two ways. On the one hand, EPLI can improve the green innovation of enterprises by relaxing financing constraints. As a component of green finance, EPLI has shown two effects in relaxing financing constraints: first, the China Banking Regulatory Commission issued the "Green Credit Guideline" in 2012, so the fulfillment of environmental liability became a necessary access condition for commercial banks to implement green credit (Cai et al., 2019). By taking out EPLI, enterprises transfer their environmental liability to insurance institutions, reducing compensation losses and business risks caused by environmental pollution, reducing the bank lending risk, prompting banks to relax their lending conditions and making it easier to obtain support from green credit, thus relaxing corporate financing constraints. Second, as a manifestation of enterprises' active fulfillment of environmental liability, purchasing EPLI can satisfy investors' preference for social responsibility through a signal mechanism. The fundamental impediment for enterprises to obtain financing in the capital market is information asymmetry (Fazzari et al., 1988), while the fulfillment of social responsibility sends positive financial signals to the outside world, which is conducive to alleviating information asymmetry and reducing the perceived risk of the outside world and cost of capital (Dhaliwal et al., 2011). Through the fulfillment of social responsibility, enterprises can also build a good reputation and credibility, accumulate moral capital for themselves, and improve the level of corporate credit, thus reducing the identified risks of creditors and obtaining recognition in the capital market (Brammer and Pavelin, 2006). It can be seen that purchasing EPLI can help enterprises to alleviate their financing difficulties and improve their green innovation ability.

On the other hand, purchasing EPLI can improve the green innovation of enterprises by reducing agency costs. First, EPLI creates effective incentives for officers. Unlike conventional investment projects, enterprise innovation is characterized by high uncertainty, high possibility of failure, and long investment cycles. Therefore, fostering green innovation requires a stable business environment and green innovation investment (Holmstrom, 1989; Manso, 2011; Ferreira et al., 2014; Dong et al., 2023). Purchasing EPLI can transfer compensation premiums and governance costs to insurers, creating a relatively stable decision-making environment for green innovation and helping to increase officers' tolerance for the risk of failure of green innovation and incentivizing them to boldly pursue green innovation. Second, as an emerging governance mechanism, EPLI can play an external supervising role. Enterprises bring insurance institutions into corporate governance by purchasing EPLI, and the insurance institutions supervise the environmental behavior of enterprises through insurance contracts. According to the insurance underwriting risk theory, insurers, as rational economic actors, will perform an active external governance function through pre-underwriting screening and continuous post-underwriting supervision. In order to reduce their payout risk, insurance institutions will reduce the likelihood of environmental pollution incidents by directing and urging enterprises to implement technological innovations to achieve intensive development. Third, the duty of disclosure and the inquiry mechanism of EPLI can effectively alleviate information asymmetry. The model clauses of EPLI of the Insurance Association of China clearly stipulate that the policyholder should fulfill the duty of disclosure and fill out the insurance policy truthfully. The duty of disclosure and the inquiry mechanism of EPLI strengthen the communication between insurance institutions and insured enterprises, alleviating the information asymmetry within and outside enterprises, thus reducing principal-agent problems such as moral hazard and adverse selection, and promoting green innovation decision-making of enterprises. In summary, EPLI restrains the management's self-interest through incentive, monitoring, and information effects, reduces agency costs, and stimulates management to reduce the probability of environmental incidents through green innovation. Based on the above analysis, this paper proposes the following hypothesis:

Hypothesis 1: Purchasing EPLI can improve the green innovation of enterprises.

4 Research design

4.1 Sample selection and data source

Since the existing regulatory system does not compel listed companies to publicly disclose information related to EPLI, this paper manually collected information on listed companies' purchase of EPLI through the list of enterprises purchasing EPLI as released on the official website of the Ministry of Ecology and Environment. Based on this, this paper used all A-share listed companies on the Shanghai and Shenzhen Stock Exchanges as the initial research samples and screened the initial samples as follows: excluding ST and *ST companies, financial and insurance listed companies, companies with abnormal data and incomplete indicators. The observations of 5222 samples were finally obtained. In this paper, the basic information relating to green patent applications by listed companies was manually collected from the official website of the China National Intellectual Property Administration. The financial data of listed companies and their corporate governance data were obtained from CSMAR, WIND, and RESSET databases. Data related to environmental regulation were taken from the *China Environment Yearbook*. In order to control the effect of anomalous observations, each continuous variable was Winsorized by ±1% in this paper.

4.2 Definition of variables

4.2.1 Green innovation of enterprises

With reference to Qi et al. (2018), this paper used the natural logarithm of the number of green patent applications plus 1 to measure the green innovation of enterprises. The identification of green patents followed the international common identification method, and seven categories of patents (such as energy conservation, nuclear power, alternative energy, transportation, administrative regulations or design, waste management, and agriculture and forestry) were classified as the patents related to environmentally friendly technologies according to the *Green List of the International Patent Classification*.

4.2.2 EPLI

With reference to Hu and Wang (2019), a dummy variable (*Ins*) was set as a proxy variable for EPLI based on whether the enterprise had purchased such insurance. If the listed company or its subsidiary was on the list of enterprises purchasing EPLI as published by the Ministry of Ecology and Environment in 2014 and 2015 and was within the validity of the insurance contract, *Ins* was 1; otherwise, it was 0. That is, if the enterprise was insured with environmental liability insurance, the assigned value was 1; otherwise, it was 0.

4.2.3 Control variables

With reference to Li and Zheng (2016), the following control variables were selected: enterprise size (*Size*), enterprise age (*Age*), capital structure (*Lev*), solvency (*Liquidity*), growth (*Growth*), cash flow (*CF*), return on equity (*Roe*), shareholding ratio of the biggest shareholder (*Big1*), shareholding ratio of institutional investors (*Insti*), board size (*Board*), independent director size (*Indep*), and duality (*Dual*). In order to control the influence of factors that did not change with individuals at the time level and industry level on the regression results, this paper further controlled the time-fixed effect and industry-fixed effect.

4.3 Model design

To examine the impact of EPLI on the green innovation of enterprises, the following Model (1) was developed:

$$\begin{aligned} GI_{i,t} = {} & \beta_0 + \beta_1 \times Ins_{i,t} + \beta_2 \times Size_{i,t} + \beta_3 \times Age_{i,t} + \beta_4 \times Lev_{i,t} \\ & + \beta_5 \times Liquidity_{i,t} + \beta_6 \times Growth_{i,t} + \beta_7 \times CF_{i,t} + \beta_8 \times Roe_{i,t} \\ & + \beta_9 \times Big1_{i,t} + \beta_{10} \times Insti_{i,t} + \beta_{11} \times Board_{i,t} + \beta_{12} \times Indep_{i,t} \\ & + \beta_{13} \times Dual_{i,t} + \sum Ind + \sum Year + \S \end{aligned} \tag{1}$$

where GI is the natural logarithm of the total number of green patent applications by listed companies in the current year plus 1. Ins is set equal to one if a listed company or its subsidiary is included in the list of insured enterprises published by the Ministry of Ecology and Environment, and zero otherwise. Size is the natural logarithm of total assets of the company. Age is the natural logarithm of the number of years since the company was founded. Lev equals total liabilities divided by total assets. Liquidity equals liquid assets divided by liquid liabilities. Growth equals the increase in operating revenue for the period divided by operating revenue for the previous period. CF equals the net cash flow from operating activities divided by total assets. Roe equals the net profit divided by the owner's equity. Big1 equals the number of shares held by the biggest shareholder divided by the total share capital. Insti equals the number of shares held by institutional investors divided by the total share capital. Board equals the natural logarithm of the number of members of the board of directors plus 1. Indep equals the number of independent directors divided by the number of members of the board of directors. Dual is set as equal to one if the posts of chairman and general manager are held by the same person, and zero otherwise. Ind is the dummy variable for an industry. Year is the dummy variable for a year.

5 Empirical results and analysis

5.1 Descriptive statistics

The descriptive statistical results of the main variables are presented in Table 1. The mean value of EPLI (*Ins*) is 0.0779, and the observations of purchasing EPLI only account for 7.79% of all samples, indicating that the coverage of EPLI is generally low for listed companies in China. The mean value of green innovation (*GI*) is 0.2274 and the median value is 0.0000, which indicates that the level of green innovation is generally low among listed companies in China. The minimum value of green innovation (*GI*) is 0.0000, and the maximum value is 3.4012, with a standard deviation of 0.6471, which implies that there is a large individual difference in the green innovation of the samples. In addition, the statistical results of the control variables show that most of the enterprises perform normal production and operation.

5.2 Pearson correlation coefficient

Table 2 shows the Pearson correlation coefficients among the variables. It can be seen from the table that the correlation coefficient

TABLE 1 Descriptive statistics.

Variable	N	Mean value	Standard deviation	Median	Minimum value	Maximum value
Ins	5222	0.0779	0.2681	0.0000	0.0000	1.0000
GI	5222	0.2274	0.6471	0.0000	0.0000	3.4012
Size	5222	22.0633	1.2709	21.9012	19.5522	25.9096
Age	5222	2.8669	0.2818	2.8904	2.0794	3.5264
Lev	5222	0.4315	0.2123	0.4182	0.0530	0.9216
Liquidity	5222	2.3911	2.4586	1.6381	0.2665	16.1452
Growth	5222	0.1449	0.5000	0.0449	-0.5884	3.3236
CF	5222	0.0424	0.0699	0.0409	-0.1631	0.2406
Roe	5222	0.0611	0.1243	0.0683	-0.6113	0.3390
Big1	5222	35.1086	15.0534	33.2150	8.8000	75.5100
Insti	5222	4.3653	4.2327	3.1600	0.0000	19.9000
Board	5222	2.2437	0.1773	2.3026	1.7918	2.7726
Indep	5222	0.3751	0.0536	0.3571	0.3125	0.5714
Dual	5222	0.2652	0.4415	0.0000	0.0000	1.0000

between the explanatory variable Ins and the explained variable GI is significantly positive, indicating that EPLI is helpful to promote the green innovation of enterprises. In addition, the size, age and other factors of the enterprise are also significantly related to GI, so we control these variables in the model. In addition, the correlation coefficient between each variable is less than 0.5, indicating that there is no multicollinearity problem in this model.

5.3 Estimated results of impact of EPLI on green innovation of enterprises

The test results of the main regression are shown in Table 3. It can be seen that the regression coefficient for EPLI is significantly positive with or without controlling the control variables, time effects, and industry effects. After the effect of other factors is considered, the estimated coefficient of *Ins* in Column (3) is 0.0666, which is significantly positive at the level of 5%, indicating that purchasing EPLI can improve the green innovation of enterprises. Hence, Hypothesis 1 is verified. In addition, our research conclusion is also supported by the recent literature on the relationship between environmental regulations, environmental practices and green innovation. Like the research conclusion of Ali et al. (2019), Ali et al. (2022a) and Ali et al. (2022b), our research conclusion demonstrates that governance mechanisms such as environmental laws and regulations and environmental practices can help to improve the environmental performance of enterprises.

5.4 Robustness test

5.4.1 Tobit model

Considering that the explained variable "green patent of enterprise" is censored data with a left-truncated distribution, there is a problem of censoring at zero. Therefore, a Tobit model is built in this paper to re-run the regression with reference to Lin et al. (2011) and Zhang et al. (2017). The regression results are shown in Table 4, consistent with the above.

5.4.2 Propensity score matching

With reference to Ning et al. (2019) and Hao et al. (2016), propensity score matching is carried out, and the enterprise size, enterprise age, capital structure, solvency, growth, cash flow, profitability, shareholding ratio of the biggest shareholder, shareholding ratio of institutional investors, board size, proportion of independent directors, and duality are selected as covariates to establish a Probit model of whether to purchase EPLI, and the enterprises without EPLI are taken as the control group, with the 1:2 and 1:4 closest matching modes for matching. The matched samples are used to perform regression on Model (1) and the results are shown in Table 4, consistent with the main regression.

5.4.3 Instrumental variable method

EPLI may affect the green innovation of enterprises in a significantly positive manner, but green innovation may also affect enterprises' purchasing of EPLI. In other words, the results in this paper may be affected by reciprocal causality. To this end, with reference to Yuan et al. (2018), the mean value of EPLI (Mean_ Ins) purchased by other listed companies in the same industry in the same year is selected as the instrumental variable for EPLI (*Ins*) for a two-stage regression in this paper. The regression results are shown in Table 4, and the regression coefficients remain significantly positive.

6 Further analysis

6.1 Test of the mechanism of action of impact of EPLI on green innovation of enterprises

As mentioned earlier, enterprises purchase EPLI to improve green innovation mainly by relaxing financing constraints and reducing

TABLE 2 Pearson correlation coefficient.

	GI	*Ins*	*Size*	*Age*	*Lev*	*Liquidity*	*Growth*	*CF*	*Roe*	*Big1*	*Insti*	*Board*	*Indep*	*Dual*
GI	1													
Ins	0.0386*	1												
	0.0053													
Size	0.1220*	0.0881*	1											
	0	0												
Age	-0.0613*	0.0102	0.1483*	1										
	0	0.463	0											
Lev	0.0359*	0.0366*	0.4954*	0.1927*	1									
	0.0094	0.0082	0	0										
Liquidity	-0.0250	-0.0715*	-0.3343*	-0.1282*	-0.6327*	1								
	0.0711	0	0	0	0									
Growth	-0.0073	-0.0135	0.0287	-0.0178	0.0035	-0.0157	1							
	0.598	0.329	0.0383	0.198	0.800	0.258								
CF	0.0350	0.0411*	0.0195	-0.0597*	-0.1803*	0.0512*	-0.0217	1						
	0.0114	0.0030	0.159	0	0	0.0002	0.116							
Roe	0.0734*	-0.0121	0.0544*	-0.0777*	-0.2397*	0.1179*	0.1942*	0.2847*	1					
	0	0.381	0.000100	0	0	0	0	0						
Big1	0.0305	0.0103	0.1958*	-0.1399*	0.0708*	-0.0414*	-0.0307	0.1119*	0.1063*	1				
	0.0277	0.455	0	0	0	0.0027	0.0266	0	0					
Insti	0.0149	0.0082	0.0765*	0.0514*	0.0306	-0.0229	0.0768*	-0.0206	0.0680*	-0.1225*	1			
	0.281	0.555	0	0.0002	0.0271	0.0982	0	0.138	0	0				
Board	0.0749*	0.0383*	0.2616*	0.0881*	0.1638*	-0.1436*	-0.0349	0.0303	0.0130	0.0298	-0.0055	1		
	0	0.0057	0	0	0	0	0.0118		0.347	0.0315	0.690			
Indep	0.0083	-0.0271	-0.0592*	-0.0554*	-0.0258	0.0397*	0.0232	0.0283	-0.0227	0.0225	0.0074	-0.5547*	1	
	0.546	0.0506	0	0.0001	0.0619	0.0041	0.0933	0.110	0.101	0.104	0.591	0		
Dual	-0.0126	-0.0112	-0.1773*	-0.1041*	-0.1176*	0.0944*	0.0298	0.0023	0.0614*	-0.0422*	-0.0082	-0.1829*	0.1139*	1
	0.361	0.417	0	0	0	0	0.0315	0.867	0	0.0023	0.556	0	0	

Note: The values in parentheses are t. ***, **, and * indicate that the values are significant at the levels of 1%, 5%, and 10%, respectively.

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TABLE 3 Impact of EPLI on green innovation of enterprises.

Variable	(1)	(2)	(3)
	GI	*GI*	*GI*
Ins	0.0931*** (2.79)	0.0688** (2.07)	0.0666** (2.03)
Control variables	Yes	Yes	Yes
_Cons	0.2201*** (23.62)	-3.2832*** (-7.22)	-4.6980*** (-10.16)
Time fixed effect	No	No	Yes
Industry fixed effect	No	No	Yes
N	5222	5222	5222
adj. R^2	0.001	0.029	0.161

Note: The values in parentheses are t. ***, **, and * indicate that the values are significant at the levels of 1%, 5%, and 10%, respectively, the same below.

TABLE 4 Robustness test.

Variable	Tobit model	PSM1:2	PSM1:4	Ins	GI
Ins	0.3376*** (11.60)	0.0739* (1.72)	0.0793** (2.03)		0.0666** (2.05)
Mean_ Ins				0.9815*** (6.88)	
Control variables	Yes	Yes	Yes	Yes	Yes
_Cons	-45.6941*** (-1072.99)	-4.9068*** (-4.35)	-4.6112*** (-5.14)	-1.1026*** (-5.64)	-4.6980*** (-10.25)
Time fixed effect	Yes	Yes	Yes	Yes	Yes
Industry fixed effect	Yes	Yes	Yes	Yes	Yes
N	5222	1113	1696	5222	5222
adj. R^2	0.1747	0.049	0.034	0.121	0.161

agency costs. In this paper, the mediation model proposed by Wen and Ye (2014) is used to test whether EPLI can promote green innovation by relaxing enterprises' financing constraints and reducing agency costs.

6.1.1 Test of mediating effect based on relaxation of financing constraints

For the measurement of financing constraints, the SA index constructed by Hadlock and Pierce (2010) consists of two completely exogenous variables (enterprise size, and enterprise age), and contains no endogenous variables, which can avoid the subjectivity of other measurement methods and their measurement bias (Hadlock and Pierce, 2010), and has been widely recognized in China. Therefore, the SA index is used to measure the financing constraints in this paper, $SA = -0.737\times Size+0.043\times Size^2-0.04\times Age$. A negative SA index with a higher absolute value indicates a more severe degree of financing constraints on the enterprise. According to the mediation model, the regression results of green innovation of enterprises (*GI*) and EPLI (*Ins*) are shown in Table 3—that is, EPLI promotes the green innovation of enterprises; the regression results of EPLI (*Ins*) to financing constraints (*SA*) are shown in Column (1) of Table 5, and the coefficient of *Ins* is significantly negative at the level of 1%, which indicates that EPLI is conducive to relaxing the financing constraints of enterprises; after the mediating variable of financing constraints (*SA*) is added on the basis of the baseline regression, both *Ins* and *SA* are significant, which indicates that financing constraints have some mediating effects. Thus, EPLI improves the green innovation of enterprises by relaxing financing constraints.

6.1.2 Test of mediating effect based on the reduction in agency costs

Based on the practice of Ang et al. (2000) and Zhou et al. (2019), the administrative expense rate (the proportion of the administrative expenses to the operating revenue) is used in this paper as a measure of agency costs. The steps for testing the mediating effect are similar to those described above and will not be repeated. The regression results in Table 5 show that agency costs are incomplete mediators for the impact of EPLI on the green innovation of enterprises. Thus, EPLI improves the green innovation of enterprises by reducing agency costs.

6.2 Impact of heterogeneous scenarios on the relationship between EPLI and green innovation of enterprises

6.2.1 Test of the impact of EPLI on green innovation of enterprises under different natures of property rights

The nature of property rights is an important institutional context for the study of finance in China, which provides a new perspective for this paper to further study the relationship between EPLI and the

TABLE 5 Test of mediating effect.

Variable	(1)	(2)	(3)	(4)
	SA	*GI*	*Agc*	*GI*
Ins	-0.0211*** (-3.83)	0.0546* (1.68)	-0.0051** (-2.06)	0.0658** (2.01)
SA		-0.7421*** (-9.13)		
Agc				0.7565*** (3.77)
Control variables	Yes	Yes	Yes	Yes
_Cons	0.9999*** (13.28)	-4.1866*** (-9.06)	0.7191*** (21.53)	-5.1999*** (-10.82)
Time fixed effect	Yes	Yes	Yes	Yes
Industry fixed effect	Yes	Yes	Yes	Yes
N	5222	5222	5222	5222
adj. R^2	0.789	0.175	0.286	0.164

TABLE 6 Analysis of scenarios.

Variable	State-owned enterprise	Non-state-owned enterprise	Heavily polluting industry	Non-heavily polluting industry	High-intensity environmental regulation	Low-intensity environmental regulation
Ins	0.0914* (1.70)	0.0314 (0.80)	0.0838** (2.49)	0.0722 (1.40)	0.0905** (2.05)	0.0409 (0.81)
Control variables	Yes	Yes	Yes	Yes	Yes	Yes
_Cons	-5.3463*** (-6.20)	-3.5136*** (-6.49)	-3.4476*** (-5.51)	-5.6629*** (-9.45)	-3.9212*** (-5.96)	-5.5166*** (-8.15)
Time fixed effect	Yes	Yes	Yes	Yes	Yes	Yes
Industry fixed effect	Yes	Yes	Yes	Yes	Yes	Yes
N	1893	3329	1546	3676	2687	2535
adj. R2	0.1345	0.0814	0.096	0.155	0.163	0.150

green innovation of enterprises. In the face of the deteriorating ecological environment, China's performance appraisal system has gradually realized diversification and greenization, and the government has paid more attention to the innovation performance and environmental performance of officers of state-owned enterprises when selecting and promoting officers. For example, the *Business Performance Appraisal Method for Heads of Central Enterprises* incorporates the transformation and upgrade, innovation driving, and social responsibility into the appraisal index system. The officers of state-owned enterprises have an incentive to focus on innovation research and development (R&D) and environmental governance for performance appraisal and position promotion (Liu et al., 2015). In this paper, the samples are divided into two groups according to the nature of the ultimate controller, and the regression of Model (1) is conducted separately. In addition, since the R&D and innovation capability factors of state-owned enterprises and non-state-owned enterprises may affect the regression results of Ins and GI under different property rights, we additionally control the R&D capability of enterprises in Model (1) (where Rd is equal to the R&D investment of enterprises divided by operating income). As shown in Table 6, the results indicate that EPLI plays a more important role in promoting the green innovation of state-owned enterprises.

6.2.2 Test of the impact of EPLI on green innovation of enterprises under different attributes of heavily polluting industries

As the main producers of environmental pollution in China and the industries with high environmental risk, heavily polluting industries are bound to be the key targets of supervision by insurance institutions. In 2013, the Ministry of Ecology and Environment and the China Banking and Insurance Regulatory Commission jointly issued the "Opinions on the Pilot Work of Compulsory Environmental Pollution Liability Insurance", which clearly advises the launch of compulsory environmental pollution liability insurance in heavily polluting industries with high environmental risk, such as the heavy metal and petrochemical industries. Considering that the relationship between EPLI and the green innovation of enterprises may differ in different industries, in this paper, all of the samples are divided into sample groups of heavily polluting industries and non-heavily polluting industries for separate regression. With reference to the study of Shen and Ma (2014), the categories of industries specified as heavily polluting industries in the *List*

TABLE 7 Effect of EPLI on promoting the green innovation of enterprises.

Variable	(1)	(2)
	GInvent	GUtility&GDesign
Ins	0.0604** (2.19)	0.0210 (0.99)
Control variables	Yes	Yes
_Cons	-3.8385*** (-9.86)	-2.4909*** (-8.37)
Time fixed effect	Yes	Yes
Industry fixed effect	Yes	Yes
N	5222	5222
adj. R2	0.142	0.130

of Environmental Verification Sector Classification and Management about Listed Companies published by the Ministry of Ecology and Environment in 2008 are compared in accordance with the *Guidelines on Industry Classification of Listed Companies* revised by the China Securities Regulatory Commission in 2012. As shown in Table 6, the regression results indicate that EPLI plays a more important role in promoting the green innovation of enterprises in heavily polluting industries.

6.2.3 Test of the impact of EPLI on green innovation of enterprises under different intensities of environmental regulation

Environmental regulation is a necessary measure to protect China's ecological environment. The Porter Hypothesis suggests that environmental regulation can force enterprises to carry out technological innovation, and so environmental regulation may have a positive impact on green innovation. In addition, environmental regulation itself can curb the management's opportunism and exert a governance effect on green innovation. With reference to the study of Yang et al. (2008), the "total amount of pollution discharge fees collected/number of pollution discharge fee-paying units" is selected in this paper to measure the intensity of environmental regulation. According to the median value of each industry company each year, the total sample is divided into a group with high environmental regulation and a group with low environmental regulation, and the regression is performed on the model separately. As shown in Table 6, the results indicate that EPLI plays a more important role in promoting the green innovation of enterprises in areas with a high intensity of environmental regulation.

6.3 Further analysis of promotion effect of EPLI on green innovation of enterprises

Compared to utility models and industrial designs, inventions have a higher technical content and newer methods, which can best reflect the quality of innovation (Liu and Zhao, 2019). Therefore, this paper further explores the impact of EPLI on the quality and structure of green innovation outputs. This paper selects green invention patents to measure qualitative green innovation (*GInvent*), the sum of the green utility model and industrial design patents to measure non-qualitative green innovation (*GUtility&GDesign*), and builds Model (3) to test the effect of EPLI on the promotion of different types of green patents:

$$GInvent_{i,t} = \alpha_0 + \alpha_1 \times Ins_{i,t} + \alpha_2 \times Controls + \$ \quad (2)$$

$$GUtility\&GDesign_{i,t} = \alpha_0 + \alpha_1 \, Ins_{i,t} + \alpha_2 \times Controls + \$ \quad (3)$$

The regression results are shown in Table 7. The coefficient of *Ins* in Column (1) has a significantly positive correlation at the level of 5%, and the coefficient of *Ins* in Column (2) does not pass the significance test, which implies that qualitative green invention patents significantly increase after enterprises take out EPLI, while non-qualitative green innovation output does not change significantly. This shows that purchasing EPLI is conducive to promoting high-quality green innovation and optimizing the output structure of green patents.

7 Research conclusions and recommendations

This paper examines the impact of EPLI on the green innovation of listed companies by taking the listed companies in Shanghai and Shenzhen cities as the research samples. It is found that (1) EPLI can significantly improve the green innovation of enterprises; (2) EPLI plays a role in promoting the green innovation of enterprises by relaxing financing constraints and reducing agency costs; (3) compared with non-state-owned enterprises, non-heavily polluting enterprises, and enterprises in regions with a lower intensity of environmental regulation, EPLI plays a stronger role in the promotion of green innovation of state-owned enterprises, heavily polluting enterprises and enterprises in regions with higher intensity of environmental regulation; (4) EPLI plays a role in promoting the green innovation of enterprises, which is mainly reflected in the significant promotion of high-quality green innovation and the optimization of the structure of green patent outputs. This paper demonstrates that EPLI, as a special green financial institutional arrangement, can play a role in the external governance of China's listed companies and exert a positive effect on the green operation and management of enterprises.

Based on the above conclusions, the following recommendations are made in this paper. Firstly, the regulatory authorities should actively improve the system design of environmental liability insurance. Insurance companies should develop a variety of green insurance products to meet the insurance needs of different entities based on the actual needs of insurance companies. Third party assessment agencies should also be introduced to conduct risk assessment to ensure the rationality and scientificity of insurance rates and premiums. At the same time, insurance companies prevent the occurrence of moral hazards and adverse selection by formulating environmental liability insurance contract clauses. Secondly, the research conclusions show that environmental liability insurance has a stronger role in promoting the green innovation of state-owned enterprises, heavily polluting enterprises, and enterprises in regions with higher environmental regulation intensity. Regulators should pay more attention to the environmental liability insurance coverage of non-state-owned enterprises, non-heavily polluting enterprises and enterprises in areas with low environmental regulation intensity. Thirdly, in addition to selecting environmental liability insurance products that are suitable for the enterprise, the enterprise should also improve its environmental risk awareness and risk response ability, and use the governance effect of environmental liability insurance to achieve a win–win situation regarding environmental benefits and economic benefits.

Data availability statement

The raw data supporting the conclusions of this article will be made available by the authors, without undue reservation.

Author contributions

Conceptualization, JN; methodology, SY and JN; software, ZY; validation, FS and JN; formal analysis, SY; investigation, JN; resources, SY and ZY; data curation, FS; writing—original draft preparation, JN; writing—review and editing, SY and JN; supervision, ZY; project administration, FS; funding acquisition, SY. All authors have read and agreed to the published version of the manuscript.

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Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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q-Rung orthopair fuzzy hypersoft ordered aggregation operators and their application towards green supplier

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Green Supply Chain Management (GSCM) is essential to ensure environmental compliance and commercial growth in the current climate. Businesses constantly look for fresh concepts and techniques for ensuring environmental sustainability. To keep up with the new trends in environmental concerns related to company management and procedures, Green Supplier Selection (GSS) criteria are added to the traditional supplier selection processes. This study aims to identify general and environmental supplier selection criteria to provide a framework that can assist decision-makers in choosing and prioritizing appropriate green supplier selection. The development and implementation of decision support systems aimed to solve these difficulties at a rapid rate. In order to manage inaccurate data and simulate decision-making problems. Fuzzy sets introduced by Zadeh, are a useful technique to handle the imperfectness and uncertainty in different problems. Although fuzzy sets can handle incomplete information in different real worlds problems, but its cannot handle all type of uncertainty such as incomplete and indeterminate data. Therefore different extensions of fuzzy sets such as intuitionistic fuzzy, pythagorean fuzzy and q-rung orthopair fuzzy sets introduced to address the problems of uncertainty by considering the membership and non-membership grade. However, these concepts have some shortcomings in the handling uncertainty with sub-attributes. To overcome this difficulties Khan et al. developed the structure of q-rung orthopair fuzzy hypersoft sets by combining q-rung orthopair fuzzy sets with hypersoft sets. A remarkable and beneficial research work is done in the field of q-rung orthopair fuzzy hypersoft sets, and then we think about the application. In this paper, we use the structure of q-rung orthopair fuzzy hypersoft in multi-criteria supplier selection problems. For this, we present aggregation operator to solve multi-criteria decision-making (MCDM) problems with q-rung orthopair fuzzy hypersoft (q-ROFH) information, known as ordered weighted geometric aggregation operator. Since the uncertainty and vagueness is an unavoidable feature of multi-criteria decision-making problems, the proposed structure can be a useful tool for decision making in an uncertain environment. Further, the expert opinions were investigated using the multi-criteria decision-making (MCDM) technique, which helped identify interrelationship and causal preference of green supplier evaluation aspects that used aggregation operators. Finally, a numerical example of the proposed method for the task of Green Supplier Selection is presented.

KEYWORDS

q-rung orthopair fuzzy hypersoft set, decision-making technique, green supplier selection, ordered weighted operator, score function

1 Introduction

Green innovation is an environmental management concept that aims to reduce the deterioration of the environment caused by urban sprawl, industrial waste, and unsustainable use of resources. Nowadays, the government and the public are more concerned about the environment. Several government regulations and GI systems have been implemented to protect the environment. As a result, the study of "green supply chain networks", also known as environmentally friendly supply chains, has gained popularity. It raises specific research questions regarding decisions regarding the product life cycle, inventory, refund procedures, channel conflicts, and coordination information (Hall and Vredenburg, 2003). There are enough products that are well-suited for green innovation. A supply chain network can also enhance green innovation in several ways. Instead of using polyester and polyurethane to make different products such as the textile industry, cold drink companies, and shoe companies can produce them with friendly environmental materials that are easier to recycle or dispose of after use. The objective of the present study is to determine the ideal green innovation level of the product in order to maximize the income of players and minimize the damage caused to consumers. In real life situations, it is impossible to find out the production of environmentally friendly and less harmful green innovative products under uncertain environment and to overcome such glitches. Many researchers used the parameters. Recently, environmental pollution and resource shortages have become global concerns of great importance. In order to meet the urgent needs of resource conservation and environmental friendliness, modern enterprise production management must focus on finding a balance between economic benefits and ecologically sustainable development (Gegovska et al., 2020; Qu et al., 2020). In 2018, Banaeian et al, (2018) discussed green supplier selection by using different techniques of fuzzy group decision. Mabrouk (Mabrouk, 2021) used the determinants of supplier selection process with green consideration. Also analyzed the collection of factors from existing literature of green supplier. In addition there are some studies that use the green supplier's company as the standard for selecting supplier (Badi and Pamucar, 2020; Zhang et al., 2022). Further, some researchers (Erceg and Mularifović, 2019; Pamucar, 2020) used different techniques to select supplier by social, economic and environmental practices.Decision-making technique is used to choose the best option from a range of viable possibilities, each of which contains several criteria. Owing to time restrictions and the lack of understanding of the problem domain, decision makers usually describe their choices using verbal descriptions and ranges rather than specific facts or numbers. Many researchers propose employing fuzzy logic in MCDM scenarios in order to overcome this issue. Zadeh introduced the theme of fuzzy logic to create human logic from incomplete and imprecise information by defining ambiguous statements (Zadeh, 1965; 1973). Moreover, fuzzy logic may also incorporate information from human experience and provide it with an engineering perspective so that it can govern and create ambiguous systems with uncertainty (Siddique and Adeli, 2013). Therefore, the amount of uncertainty in the decision-making process increases when fuzzy logic is added to MCDM tools. As a result, the decision-making outcomes become more accurate and practical (Yang et al., 2017). The earliest use of fuzzy numbers in MCDM was the fuzzy sets theory, which Bellman and Zadeh (Bellman and Zadeh, 1970) proposed. Due to the ambiguity of decision -makers' preferences, this notion was created to solve MCDM challenges. Zadeh's fuzzy sets theory deals only with membership grade.Later, Atanassov, 1986) introduced the notion of the intuitionistic fuzzy set (IFS) in 1986. Since then, real-world MCDM issues and challenges have frequently been addressed using the IFS concept. As IFS deals with both positive and negative membership grade, but only if the sum is less than or equal to 1. In 2002 De et al, (2000) developed some operations on intuitionistic fuzzy sets. In 2011, Wang and Liu, 2011) proposed different operations on IFS and developed some aggregation operators by using the basic operational laws. Also developed multi-attribute decision making (MADM) problem. Many researchers (Li, 2005; Liu and Wang, 2007; Xu, 2011) used intuitionistic fuzzy sets in decision making problems.It is important to keep in mind that the sum of membership and non-membership grades when exceeds 1, then IFS does not fulfill the condition. Alternatively, the Pythagorean fuzzy set (PFS) concept, which is a generalization of IFS, introduced by Yager and Abbasov, (2013). Here, the alternative's membership grade and non-membership grade must fulfill the condition that the square sum of both the membership and non-membership grades are equal to or less than 1. Peng and Yang, (2015) developed some results on pythagorean fuzzy sets. In 2018 Li and Zeng, (2018) proposed a variety of distance measures for pythagorean fuzzy sets and pythagorean fuzzy numbers. Many researchers developed different aggregation operators under pythagorean fuzzy environment, Peng and Yuan, (2016) Pythagorean fuzzy point aggregation operators, Akram et al, (2019) Pythagorean Dombi fuzzy aggregation operators, Wei and Lu, (2018) Pythagorean fuzzy power aggregation operators.Yager, (2016) introduced the theme of generalized orthopair fuzzy sets which is also known as q-rung orthopair fuzzy set.Molodtsov, (1999) suggested the idea of soft set theory, which was completely a new technique for describing uncertainty which traditional mathematical tools cannot handle. Later, Maji et al, (2001) created the fuzzy structure of the soft set by merging the fuzzy set and the soft set. In order to solve decision-making problems, Kong et al, (2008) Kong et al, (2009) used the soft set theoretic technique. Majumdar and Samanta, (2011), have investigated the problem of soft and fuzzy soft sets in terms of similarity measure. Since then, the structures of fuzzy soft sets have been actively used by many researchers, and many studies have also been added to the literature (Feng et al., 2008; Ahmad and Kharal, 2009; Yang et al., 2009; Jiang et al., 2010).Progressively, Smarandache, (2018) developed a novel method for dealing with uncertainties. The soft set was converted into a hypersoft set (HSS) by generalising functionality into a multi-decision function. Subsequently, the HSS theory garnered considerable interest from researchers (Saqlain et al., 2020a; Saqlain et al., 2020b; Gayen et al., 2020; Martin and Smarandache, 2020; Saeed et al., 2020). Furthermore Rahman et al, (2022a) proposed the decision-making methods for possibility intuitionistic fuzzy hypersoft sets (IFHSS) based on the similarity measure. Zulqarnain et al, (2020); (2021a) proposed decision making technique under IFHS environment based on correlation coefficient and aggregation operators. Rahman et al, (2022b) proposed innovative MCDM method employing a well-established method for suppler selection in the construction sector with a rough approximation of the fuzzy hypersoft set (FHS). Zulqarnain et al, (2021b) extended the impression of the IFHSS to Pythagorean fuzzy hypersoft sets

(PFHSSs). The decision support model for the diagnosis of COVID-19 patients was reported by Saeed et al, (2022) using a complex fuzzy hypersoft set (CFHS). The interval-valued complex fuzzy hypersoft set (IVCFHS) was first introduced by Rahman et al, (2021), with a few basic operations. Zulqarnain et al, (2021b) developed the correlation-based technique for order performance by similarity to ideal solution (TOPSIS) strategy for PFHS and used their proven method to choose the best face mask. Siddique et al, (2021) provided aggregation operators (AOs) for PFHSS and developed an MCDM technique with their suggested operators.In the decision-making problem with a complex and vague statement, Yager has provided a q-ROF-set for the decision-making process. It offers a universal platform for handling vagueness and uncertainty. For more appropriate decision analysis, some researchers have used fuzzy hpersoft sets in the decision-making problem. To get a more powerful tool, fuzzy hypersoft set combined with q rung orthopair fuzzy set. It is more effective in decision-making.

Objective: One of the toughest challenge in green supplier selection is handling uncertainty. The detection of green supplier is a challenging problem for decision maker. In this case the decision maker find out the best green supplier selection process by using aggregation operators. Consequently, we observe uncertainty in determining the aggregation operators, and therefore in the green supplier selection. In this study, we take a MADM approach towards green supplier selection by using some attributes to examine their capabilities in encountering uncertainty in green supplier selection.The reminder of this paper is arranged as follows.Section 2, presents literature review about the previous study. In Section 3, we conduct research limitations and motivation. Section 4, presents some basic materials related to our proposed structure. Then we introduce some operational laws and aggregation operators in Section 5, which helpful for the decision making problem. In Section 6, an algorithm for MCDM technique is present to show the uses of anticipated aggregation operators in decision making problems. In addition, in order to show the applicability of the proposed technique, this study present a numerical example to analyze the technique and the final results are similar to q-ROFHSNs. Section 7, presents the comparative study of the existing structures with the proposed structure. Finally, the paper summarizes and present future research direction in Section 8.

2 Literature review

The selection of green suppliers using economic and environmental criteria in the medical industry has shown us the flexibility and importance that are not harmful to the environment (Puška et al., 2022). A critical review of the green supply chain management for a more sustainable manufacturing industry in China (Sheng et al., 2022). The authors found that the existing relevant legal system needs to be improved, especially in terms of tax subsidies, incentive mechanisms, environmental information disclosure, and the range of industries involved. Corresponding suggestions are proposed in response to these identified problems. A new group decision-making model based on the plan-do-check-act (PDCA) to evaluate the green supply chain management performance of manufacturing organizations, and uses an integrated fuzzy multicriteria decision-making approach (MCDM) in which the fuzzy analytical hierarchy process method (FAHP) determines the weights of the criteria and the fuzzy technique for order preference by similarity with the ideal solution method (FTOPSIS) classifies organizations (Ghosh et al., 2022). Finding a selection of green suppliers (ESG) in the best way will help agricultural producers to apply green agricultural production using uncertainty in decision-making. And to avoid the possibility of uncertainty in the decision-making of experts, the Z numbers were used in conjunction with the fuzzy LMAW method (Logarithmic Methodology of Additive Weights) and the Fuzzy CRADIS method (Compromise Classification of Alternatives from the Distance to the Ideal Solution) (Puška et al., 2022). The criteria for choosing suppliers for each company are constant (Sénquiz-Díaz, 2021). In order to determine the supplier that best meets the objective defined by the company, a given number of suppliers that correspond to a specific number of criteria must be taken into account (Stević et al., 2019). A company can save operating expenses and increase its competitiveness by strategically choosing its suppliers (Yazdani et al., 2017). Different criteria are used by companies using the green supplier selecion technique, the most common are related to environmental and economic factors. Multi-criteria decision-making techniques (MCDM) are frequently in the environment of green supplier selection.Practical applications of q-rung orthopair fuzzy hypersoft set include problems for which the complete mathematical description is unavailable, or where the usage of precise model is highly inconvenient. The structure of q-rung orthopair fuzzy hypersoft set have the ability to overcome this difficulties.**Research gap:** Khan et al, (2022b) introduced the theme of q-rung orthopair fuzzy hypersoft sets with certain fundamental operational laws and aggregation operators, which can handle the various interactions between input arguments. Khan et al, (2022a) then suggested aggregation operators based on operational laws, such as weighted average and weighted geometric. To our knowledge, there are no studies on ordered aggregation operators in the literature. However, in spite of the existing works on q-rung orthopair fuzzy and hypersoft sets. Therefore, in this study, we propose the ideas of the ordered weighted average and ordered weighted geometric aggregation operators to fill this gap in the literature. Finally, we develop applications using these aggregation operators in decision-making processes. The ordered aggregation operators are used in a suitable application that is based on a green supplier selection.

Methodology: The methodology used in this study consists of the following steps. Step 1: The first step of research. A) Goal and subject. Step 2: Group of experts. Step 3: Determining weights of criteria. Step 4: Results. Step 5: Analysis of the result and evaluation with the help of experts. Figure 1 show the methodology used in this research work.

3 Research limitations and motivation

The q-ROFHSS is a hybrid of hypersoft and q-rung orthopair fuzzy set, which is a widely used mathematical technique for handling coherence, uncertainty, and incomplete information. Since aggregation operators have been determined to be essential in decision making. To our knowledge, the literature does not use the ordered aggregation operators under q-rung orthopair fuzzy hypersoft environment. However, neither the above techniques nor their deliberate association with membership grade and non-membership grade of many sub-attributes of the studied parameters has a relevant quantitative study of the q-ROFHSN. More precisely, the process is not disturbed by the effects of other membership or non-membership grades on the average geometric operator. Therefore, when certain aggregation operators are taken into account, the results are not favorable and the appropriate preference for alternatives is not found. As a result, it is interesting to consider how to incorporate these q-ROFHSNs into ordered operator. To address these issues, we'll push some ordered aggregation operator under q-ROFHS environment, including the q-ROFHSOWA and q-ROFHSOWG operators.

4 Materials and methods

In this section some basic concepts related to the our proposed structure are collected.

Definition 4.1. *(Zadeh, 1965) In a fuzzy set, A is the Universe of information and M can be defined as a set of ordered pairs and can be represented mathematically as:* $A = \{\langle y, \mu_A(y)\rangle : y \in M\}$. *Here,* $\mu_A(y)$ *represents the degrees of membership of y in A, where* $\mu_A(y) \in [0, 1]$.

Definition 4.2. *(Smarandache, 2018) Let M be a universal set and* $A_1 \times A_2 \times \cdots \times A_n = H \subseteq E$ *(E represents the set of attributes). The pair (T, H) is said to be a hypersoft set over M, where T is a function* $T: A \to P(M)$, *which is defined as*

$$T_{e_{ij}}(m_{ij}) = \{\langle m_{ij}, T_{e_{ij}}(m_{ij})\rangle \in P(M), e_{ij} \in H\}$$

Definition 4.3. *(Zulqarnain et al., 2021b) Let M be a universal set and* $A_1 \times A_2 \times \cdots \times A_n = H \subseteq E$ *(E represents the set of attributes). The pair (T, H) is said to be a Pythagorean fuzzy hypersoft set over M, here T is a function* $T: A \to P(M)$, *which is defined as*

$$T_{e_{ij}}(m_{ij}) = \left\{\langle m_{ij}, \varphi_{e_{ij}}(m_{ij}), \psi_{e_{ij}}(m_{ij})\rangle : m_{ij} \in P(M), e_{ij} \in H\right\}$$

where φ *represents the membership grade and* ψ *represents non-membership grade of an object* $m_{ij} \in M$ *to a set* $T_{e_{ij}}$ *respectively, with the condition that*

$$0 \le \left(\varphi_{e_{ij}}(m_{ij})\right)^2 + \left(\psi_{e_{ij}}(m_{ij})\right)^2 \le 1$$

Definition 4.4. *(Khan et al., 2022b) Let M be a universal set and* $A_1 \times A_2 \times \cdots \times A_n = H \subseteq E$ *(E represent the sets of attributes). The pair (T, H) is said to be a q-ROFHS over M, where T is a function* $T: H \to q\text{–ROFH}(M)$, *which is defined as*

$$T_{e_{ij}}(m_{ij}) = \left\{\langle m_{ij}, \varphi_{e_{ij}}(m_{ij}), \psi_{e_{ij}}(m_{ij})\rangle : m_{ij} \in M, e_{ij} \in H \text{ and } q \ge 1\right\}$$

where q–ROFH(M) denotes the collection of all possible sub sets of q-ROF of M. Here φ_{ij} *and* ψ_{ij} *represent membership and non-membership of an object* $m_{ij} \in M$ *to a set* $T_{e_{ij}}$ *respectively, with the condition that*

$$0 \le \left(\varphi_{e_{ij}}(m_{ij})\right)^q + \left(\psi_{e_{ij}}(m_{ij})\right)^q \le 1, (q \ge 1)$$

For simplicity, $T_{e_{ij}}(m_{ij}) = \langle m_{ij}, \varphi_{e_{ij}}(m_{ij}), \text{and } \psi_{e_{ij}}(m_{ij})\rangle$ *are denoted as* $T_{e_{ij}}(m_{ij}) = \langle \varphi_{e_{ij}}, \psi_{e_{ij}}\rangle$ *is represents a q-ROFHN.*

Definition 4.5. *(Khan et al., 2022a) An average weighted q-rung orthopair fuzzy hypersoft (q-ROFHWA) operator is defined as:*

$$q - ROFHWA(E_{a_{11}}, E_{a_{12}}, \ldots, E_{a_{nm}}) = \oplus_{j=1}^{n} v_j\left(\oplus_{i=1}^{m} w_i E_{a_{ij}}\right) \quad (4.1)$$

where $w_i = \{1, 2, \ldots, n\}$ *and* $v_j = \{1, 2, \ldots, m\}$ *are the weight vectors for the experts and sub-attributes of the selected parameters with the condition that,* $w_i > 0, \sum_{i=1}^m w_i = 1$, *and* $v_j > 0, \sum_{j=1}^n v_j = 1$.*Then the mapping of the q-ROFHWA operator is defined as qROFHWA:* $\Delta^n \to \Delta$ *is the collection of all q-ROFHNs.*

Definition 4.6. *(Khan et al., 2022a) A weighted geometric q-rung orthopair fuzzy hypersoft (q-ROFHWG) operator is defined as:*

$$q - ROFHWG(E_{a_{11}}, E_{a_{12}}, \ldots, E_{a_{nm}}) = \otimes_{j=1}^{n} v_j\left(\otimes_{i=1}^{m} w_i E_{a_{ij}}\right) \quad (4.2)$$

where $w_i = \{1, 2, \ldots, n\}$ *and* $v_j = \{1, 2, \ldots, m\}$ *are the weight vectors for experts and sub-attributes of the selected parameters with the condition that,* $w_i > 0, \sum_{i=1}^m w_i = 1$, *and* $v_j > 0, \sum_{j=1}^n v_j = 1$.*Then the mapping for the q-ROFHWG operator is defined as qROFHWG:* $\Delta^n \to \Delta$ *is the collection of all q-ROFHNs.*

Definition 4.7. *(Khan et al., 2022a) The score function of the q-ROFHNs is defined as* $S_{e_{ij}} = \varphi_{e_{ij}}(m_{ij})^q - \psi_{e_{ij}}(m_{ij})^q$

5 Operational laws and ordered weighted geometric aggregation operator for q-ROFH set

In this section the algebraic operational laws, q-ROFHOWA and q-ROFHOWG aggregation operators with properties are proposed.

Definition 5.1. *An ordered weighted averaging q-rung orthopair fuzzy hypersoft (q-ROFHOWA) operator is defined as:*

$$q - ROFHOWA(E_{a_{11}}, E_{a_{12}}, \ldots, E_{a_{nm}}) = \oplus_{j=1}^{n} v_j\left(\oplus_{i=1}^{m} w_i E_{\sigma a_{ij}}\right) = \left(\sqrt[q]{1 - \prod_{j=1}^{n}\left(\prod_{1=1}^{m}\left(1 - \varphi_{\sigma a_{ij}}^q\right)^{w_i}\right)^{v_j}}, \prod_{j=1}^{n}\left(\prod_{1=1}^{m}\left(\psi_{\sigma a_{ij}}\right)^{w_i}\right)^{v_j}\right) \quad (5.1)$$

where $w_i = \{1, 2, \ldots, n\}$ *and* $v_j = \{1, 2, \ldots, m\}$ *are the weight vectors for experts and sub-attributes of the selected parameters with the condition that,* $w_i > 0, \sum_{i=1}^m w_i = 1$, *and* $v_j > 0, \sum_{j=1}^n v_j = 1$.*Then the mapping for q-ROFHOWA operator is defined as q–ROFHOWA:* $\Delta^n \to \Delta$ *is the collection of all q-ROFHNs.*

Definition 5.2. *Let* $T_{e_{ij}}(m_{ij}) = (\varphi_{e_{ij}}, \psi_{e_{ij}})$, $T_{e_{11}}(m_{11}) = (\varphi_{e_{11}}, \psi_{e_{11}})$ *and* $T_{e_{12}}(m_{12}) = (\varphi_{e_{12}}, \psi_{e_{12}})$ *be two q-ROFHNs. Then algebraic operational laws are define as:*

1. $T_{e_{11}} \oplus T_{e_{12}} = \left(\sqrt[q]{\varphi_{e_{11}}^q + \varphi_{e_{12}}^q - \varphi_{e_{11}}^q \varphi_{e_{12}}^q}, \psi_{e_{11}}\psi_{e_{12}}\right)$
2. $T_{e_{11}} \otimes T_{e_{12}} = \left(\varphi_{e_{11}}\varphi_{e_{12}}, \sqrt[q]{\psi_{e_{11}}^q + \psi_{e_{12}}^q - \psi_{e_{11}}^q \psi_{e_{12}}^q}\right)$
3. $\lambda T_{e_{ij}} = \left(\sqrt[q]{1 - (1 - \varphi_{e_{ij}}^q)^\lambda}, (\psi_{e_{11}})^\lambda\right)$
4. $T_{e_{ij}}^\lambda = \left((\varphi_{e_{ij}})^\lambda . \sqrt[q]{1 - (1 - \psi_{e_{ij}}^q)^\lambda}\right)$

The following aggregation operators for q-ROFHNs are based on algebraic operational laws.

Definition 5.3. *Ordered weighted geometric q-rung orthopair fuzzy hypersoft (q-ROFHOWG) operator is defined as:*

$$q - ROFHOWG(E_{a_{11}}, E_{a_{12}}, \ldots, E_{a_{nm}}) = \otimes_{j=1}^{n} v_j\left(\otimes_{i=1}^{m} w_i E_{\sigma a_{ij}}\right) \quad (5.2)$$

where $w_i = \{1, 2, \ldots, n\}$ *and* $v_j = \{1, 2, \ldots, m\}$ *are weight vectors for the experts and sub-attributes of the selected parameters with the condition that,* $w_i > 0, \sum_{i=1}^m w_i = 1$, *and* $v_j > 0, \sum_{j=1}^n v_j = 1$.*Then the mapping for the q-ROFHOWG operator is defined as q–ROFHOWG:* $\Delta^n \to \Delta$ *is the collection of all q-ROFHNs.*

Theorem 5.4. *Let* $E_{a_{ij}}, (i = 1, 2, \ldots, n)(j = 1, 2, \ldots, m)$ *be the collection of q-ROFHNs, then their aggregated value by using the*

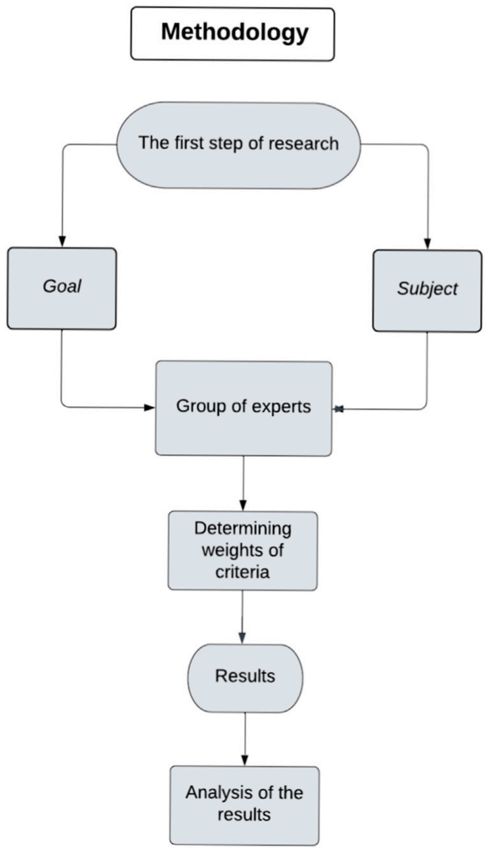

FIGURE 1
Flow chart.

q-ROFHOWG operator is also q-rung orthopair fuzzy hypersoft values, and $q-ROFHOWG(E_{a_{11}}, E_{a_{12}}, \ldots, E_{a_{nm}}) = \left(\prod_{j=1}^{n}(\prod_{1=1}^{m}(\varphi_{\sigma a_{ij}})^{w_i})^{v_j}, \sqrt[q]{1-\prod_{j=1}^{n}(\prod_{1=1}^{m}(1-\psi_{\sigma a_{ij}}^{q})^{w_i})^{v_j}}\right)$ *where* $w_i = \{1,2,\ldots,n\}$ *and* $v_j = \{1,2,\ldots,m\}$ *are the weight vectors for the experts and sub-attributes of the selected parameters respectively, having the condition that,* $w_i > 0, \sum_{i=1}^{m} w_i = 1,$ *and* $v_j > 0, \sum_{j=1}^{n} v_j = 1,$ *and* σ_{ij} *are permutations of (i = 1,2,...m) and (j = 1,2,...,n) such that* $E_{\sigma_{(i-1)j}} \geq E_{\sigma_{(i)j}}$ *and* $E_{\sigma_{i(j-1)}} \geq E_{\sigma_{i(j)}} \forall i, j.$

Proof. The q-ROFHOWG operator can be proven using the principle of mathematical induction as follows:For $m = 1$, we get $w_i = 1$. Then,

$$\begin{aligned} q-ROFHOWG&(E_{a_{11}}, E_{a_{12}}, \ldots, E_{a_{nm}}) \\ &= \otimes_{j=1}^{n} v_j E_{\sigma a_{1j}} \\ &= \left(\prod_{j=1}^{n}\left(\varphi_{\sigma a_{ij}}\right)^{v_j}, \sqrt[q]{1-\prod_{j=1}^{n}\left(1-\psi_{\sigma a_{ij}}^{q}\right)^{v_j}}\right) \\ &= \left(\prod_{j=1}^{n}\left(\prod_{i=1}^{1}\left(\varphi_{\sigma a_{ij}}\right)^{w_i}\right)^{v_j}, \sqrt[q]{1-\prod_{j=1}^{n}\left(\prod_{i=1}^{1}\left(1-\psi_{\sigma a_{ij}}^{q}\right)^{w_i}\right)^{v_j}}\right) \end{aligned} \tag{5.3}$$

For $n = 1$, $v_j = 1$ is obtained.

$$\begin{aligned} q-ROFHOWA&(E_{a_{11}}, E_{a_{12}}, \ldots, E_{a_{nm}}) \\ &= \otimes_{i=1}^{m} w_i E_{\sigma a_{i1}} \\ &= \left(\prod_{i=1}^{m}\left(\varphi_{\sigma a_{ij}}\right)^{w_i}, \sqrt[q]{1-\prod_{i=1}^{m}\left(1-\psi_{\sigma a_{ij}}^{q}\right)^{w_i}},\right) \\ &= \left(\prod_{j=1}^{1}\left(\prod_{i=1}^{m}\left(\varphi_{\sigma a_{ij}}\right)^{w_i}\right)^{v_j}, \sqrt[q]{1-\prod_{j=1}^{1}\left(\prod_{i=1}^{m}\left(1-\psi_{\sigma a_{ij}}^{q}\right)^{w_i}\right)^{v_j}},\right) \end{aligned} \tag{5.4}$$

Therefore, Eq. 3.3 holds for $m = 1$ and $n = 1$. Suppose the equation holds for $m = k_2$, $n = k_1+1$, and $m = k_2+1$, $n = k_1$. Then,

$$= \otimes_{j=1}^{k_1+1} v_j\left(\otimes_{i=1}^{k_2} w_i E_{\sigma a_{ij}}\right) \tag{5.5}$$

$$= \left(\prod_{j=1}^{k_1+1}\left(\prod_{1=1}^{k_2}\left(\varphi_{\sigma a_{ij}}\right)^{w_i}\right)^{v_j} \sqrt[q]{1-\prod_{j=1}^{k_1+1}\left(\prod_{i=1}^{k_2}\left(1-\psi_{\sigma a_{ij}}^{q}\right)^{w_i}\right)^{v_j}}\right)$$

$$= \otimes_{j=1}^{k_1} v_j\left(\otimes_{i=1}^{k_2+1} w_i E_{\sigma a_{ij}}\right) \tag{5.6}$$

$$= \left(\prod_{j=1}^{k_1}\left(\prod_{1=1}^{k_2+1}\left(\varphi_{\sigma a_{ij}}\right)^{w_i}\right)^{v_j}, \sqrt[q]{1-\prod_{j=1}^{k_1}\left(\prod_{i=1}^{k_2+1}\left(1-\psi_{\sigma a_{ij}}^{q}\right)^{w_i}\right)^{v_j}}\right)$$

Now the equation for $n = k_1+1$ and $m = k_2+1$ is proven.

$$\otimes_{j=1}^{k_1+1} v_j\left(\otimes_{i=1}^{k_2+1} w_i E_{\sigma a_{ij}}\right) \tag{5.7}$$

$$\begin{aligned} &= \otimes_{j=1}^{k_1+1} E_{\sigma a_j}\left(\otimes_{i=1}^{k_2} w_i E_{\sigma a_{ij}} \otimes w_{i+1} E_{\sigma a_{(k_2+1)j}}\right) \\ &= \left(\otimes_{j=1}^{k_1+1} \otimes_{i=1}^{k_2} w_i v_j E_{\sigma a_j}\right)\left(\otimes_{j=1}^{k_1+1} v_j w_{i+1} E_{\sigma a_{(k_2+1)(j)}}\right) \end{aligned} \tag{5.8}$$

$$\begin{aligned} &= 2\prod_{j=1}^{k_1+1}\left(\prod_{1=1}^{k_2+1}\left(\psi_{\sigma a_{(i)(j)}}\right)^{w_i}\right)^{v_j} \otimes 2\prod_{j=1}^{k_1+1}\left(\prod_{1=1}^{k_2+1}\left(\psi_{\sigma a_{(k_2+1)(j)}}\right)^{w_{k_2+1}}\right)^{v_j} \\ &= 2\prod_{j=1}^{k_1+1}\left(\prod_{i=1}^{k_2+1}\left(\varphi_{\sigma a_{(i)(j)}}\right)^{w_i}\right)^{v_j}, \sqrt[q]{1-\prod_{j=1}^{k_1+1}\left(\prod_{i=1}^{k_2+1}\left(1-\psi_{\sigma a_{ij}}^{q}\right)^{w_i}\right)^{v_j}} \end{aligned} \tag{5.9}$$

$$= \otimes_{j=1}^{k_1+1} v_j\left(\otimes_{i=1}^{k_2+1} w_i E_{\sigma a_{ij}}\right)$$

Therefore, this is valid for $n = k_1+1$ and $m = k_2+1$.

Example 5.5. *Let* $X = \{X_1, X_2, X_3, X_4\}$ *be set as decision makers with* $w_i = (0.3,0.4,0.1,0.2)^T$*. The group of experts will choose a business location under the set of attributes* $A = \{a_1$ *= skill base in the area,* a_2 *= potential growth for business} with their corresponding sub-attributes* $a_1 = \{a_{11}$ *=communication and negotiation,* a_{12} *= networking},* $a_2 = \{a_{21}$ *=strong sales growth,* a_{22} *= strong leadership}. Let* $A = a_1 \times a_2$ *be a set of sub-attributes where,* $A = \{a_{11}, a_{12}\} \times \{a_{21}, a_{22}\} = \{(a_{11}, a_{21}), (a_{11}, a_{22}), (a_{12}, a_{21}), (a_{12}, a_{22})\} = A = \{\mathring{a}_1, \mathring{a}_2, \mathring{a}_3, \mathring{a}_4\}$ *denotes the set of sub-attributes with: weight* $v_j = (0.3,0.3,0.4,0.1)^T$*. The hypothetical score values for all attributes in q-ROFHSNs with* $q = 5$*, is given as:*

$$(H, A) = \begin{bmatrix} (0.88, 0.99) & (0.77, 0.99) & (0.99, 0.33) & (0.77, 0.66) \\ (0.55, 0.99) & (0.99, 0.44) & (0.77, 0.99) & (0.66, 0.99) \\ (0.77, 0.88) & (0.77, 0.99) & (0.55, 0.66) & (0.99, 0.44) \\ (0.55, 0.88) & (0.88, 0.55) & (0.66, 0.88) & (0.77, 0.88) \end{bmatrix}$$

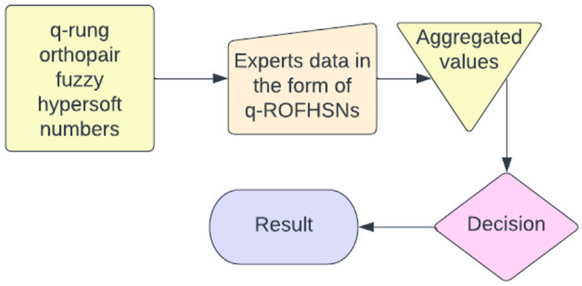

FIGURE 2
Flow chart.

TABLE 1 Comparison of q-ROFHSs operators with some existing operators.

	Fuzzy information	Aggregated parameter information	Aggregated sub parameteric information	Ordered aggregation information
IFSWA (Arora and Garg, 2018)	Yes	Yes	No	No
q-ROFSWA (Hussain et al., 2020)	Yes	Yes	No	No
IFHSWA (Zulqarnain et al., 2021a)	Yes	Yes	Yes	No
PFHSWA (Siddique et al., 2021)	Yes	Yes	Yes	No
PFHSWG (Siddique et al., 2021)	Yes	Yes	Yes	No
Proposed operator	Yes	Yes	Yes	Yes

To obtain the order position of a matrix:

$$(H, A) = \begin{bmatrix} (0.99, 0.33) & (0.88, 0.99) & (0.77, 0.99) & (0.77, 0.66) \\ (0.99, 0.44) & (0.77, 0.99) & (0.66, 0.99) & (0.55, 0.99) \\ (0.99, 0.44) & (0.77, 0.99) & (0.77, 0.88) & (0.55, 0.66) \\ (0.88, 0.55) & (0.77, 0.88) & (0.66, 0.88) & (0.55, 0.88) \end{bmatrix}$$

It is known

$$\begin{aligned} &q - ROFHOWG\left(E_{a_{11}}, E_{a_{12}}, \ldots, E_{a_{nm}}\right) \\ &= \left(\prod_{j=1}^{n}\left(\prod_{i=1}^{m}\left(\varphi_{\sigma a_{ij}}\right)^{w_i}\right)^{v_j}, \sqrt[q]{1 - \prod_{j=1}^{n}\left(\prod_{i=1}^{m}\left(1 - \psi_{\sigma a_{ij}}^{q}\right)^{w_i}\right)^{v_j}}\right) \\ &= \left(\prod_{j=1}^{4}\left(\prod_{i=1}^{4}\left(\varphi_{\sigma a_{ij}}\right)^{w_i}\right)^{v_j}, \sqrt[q]{1 - \prod_{j=1}^{4}\left(\prod_{i=1}^{4}\left(1 - \psi_{\sigma a_{ij}}^{5}\right)^{w_i}\right)^{v_j}}\right) \end{aligned}$$

$$= \left\{ \begin{array}{c} \{(0.99)^{0.3}(0.99)^{0.4}(0.99)^{0.1}(0.88)^{0.2}\}^{0.3} \\ \{(0.88)^{0.3}(0.77)^{0.4}(0.77)^{0.1}(0.77)^{0.2}\}^{0.3} \\ \{(0.77)^{0.3}(0.66)^{0.4}(0.77)^{0.1}(0.66)^{0.2}\}^{0.4} \\ \{(0.77)^{0.3}(0.55)^{0.4}(0.55)^{0.1}(0.55)^{0.2}\}^{0.1}, \\ \sqrt[5]{1 - \left(\left(1 - 0.33^5\right)^{0.3}\left(1 - 0.44^5\right)^{0.4}\left(1 - 0.44^5\right)^{0.1}\left(1 - 0.55^5\right)^{0.2}\right)^{0.3}} \\ \sqrt[5]{1 - \left(\left(10.905 - 0.99^5\right)^{0.3}\left(1 - 0.99^5\right)^{0.4}\left(1 - 0.99^5\right)^{0.1}\left(1 - 0.88^5\right)^{0.2}\right)^{0.3}} \\ \sqrt[5]{1 - \left(\left(1 - 0.99^5\right)^{0.3}\left(1 - 0.99^5\right)^{0.4}\left(1 - 0.88^5\right)^{0.1}\left(1 - 0.88^5\right)^{0.2}\right)^{0.4}} \\ \sqrt[5]{1 - \left(\left(1 - 0.66^5\right)^{0.3}\left(1 - 0.99^5\right)^{0.4}\left(1 - 0.66^5\right)^{0.1}\left(1 - 0.88^5\right)^{0.2}\right)^{0.1}} \end{array} \right\}$$

$$= (0.769, 0.1997)$$

Remark 5.6. *If the value of q is fixed, that is $q = 1$, then the proposed q-ROFHOWG operator reduces to intuitionistic fuzzy hypersoft ordered weighted geometric (IFHOWG) operator.*

Remark 5.7. *If the value of q is fixed, that is $q = 2$, then the proposed q-ROFHOWG operator reduces to Pythagorean fuzzy hypersoft ordered weighted geometric (PFHOWG) operator.*

Remark 5.8. *If there are single parameters, that is $e_1, e_2, \ldots, e_n$, the proposed q-ROFHOWG operator reduces to q-rung orthopair fuzzy soft ordered weighted geometric (q-ROFSOWG) operator.*

Remark 5.9. *From these remarks it is clear that the IFHOWG, PFHOWG and q-ROFSOWG operators are the specials cases of the proposed q-ROFHOWG operator.*

5.1 Properties of q-ROFHOWG operator

The following section proposes the properties of the q-ROFHOWG operator, such as idempotent, bounded, and monotonic. These properties can be proved by the following theorems.

(Idempotency) Let $E_{a_{ij}} = (\varphi_{a_{ij}}, \psi_{a_{ij}})$ be a collection of q-ROFHNs, for all $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, m$. If $E_{\sigma a_{ij}} = E_{11}$ is mathematically identical, then $q - ROFHOWG(E_{a_{11}}, E_{a_{12}}, \ldots, E_{a_{nm}}) = E$.

Proof. As it is known,

$$q-ROFHOWG\left(E_{a_{11}},E_{a_{12}},\ldots,E_{a_{nm}}\right)$$
$$=\left(2\prod_{j=1}^{k_1+1}\left(\prod_{i=1}^{k_2+1}\left(\varphi_{\sigma a_{(i)(j)}}\right)^{w_i}\right)^{v_j},\sqrt[q]{1-\prod_{j=1}^{k_1+1}\left(\prod_{i=1}^{k_2+1}\left(1-\psi_{\sigma a_{ij}}^{q}\right)^{w_i}\right)^{v_j}}\right) \tag{5.10}$$

As $E_{\sigma a_{(i)(j)}}=E_{ij}$, so;

$$=\left(2\left(\left(\varphi_{\sigma a_{(i)(j)}}\right)^{\sum_{i=1}^{m}w_i}\right)^{\sum_{j=1}^{n}v_j},\sqrt[q]{1-\left(\left(1-\psi_{\sigma a_{ij}}^{q}\right)^{\sum_{i=1}^{m}w_i}\right)^{\sum_{j=1}^{n}v_j}}\right)$$
$$=\left(2\left(\varphi_{\sigma a_{(i)(j)}}\right),\sqrt[q]{1-\left(1-\psi_{\sigma a_{ij}}^{q}\right)}\right)$$
$$=\left(\varphi_{a_{ij}},\psi_{a_{ij}}\right)=E \tag{5.11}$$

(Boundedness) Let $E_{a_{ij}}=(\varphi_{a_{ij}},\psi_{a_{ij}})$ be a collection of q-ROFHNs, for all $i=1,2,..m$, and $j=1,2,\ldots,n$. If $E_{\min\min}=\min\min E_{a_{(i)(j)}}$ and $E_{\max\max}=\max\max E_{a_{(i)(j)}}$, then $E_{\min\min}\le q-ROFHOWG(E_{a_{11}},E_{a_{12}},\ldots,E_{a_{nm}})\le E_{\max\max}$, where w_i and v_j are weight vectors for expert's and sub-attributes of the selected parameters, such that $w_i>0,\sum_{i=1}^{m}w_i=1$, and $w_i>0,\sum_{i=1}^{n}v_j=1$.

Proof. Let $f(x)=\sqrt[q]{1-(1-x^q)},\ x\in[0,1]$ then

$$\frac{d}{dx}(f(x))=-qx^{q-1}\left(\sqrt[q]{\left(\frac{1-x^q}{q}\right)-1}\right)<0,$$

So $f(x)$ is decreasing function on [0,1]. As $\psi_{\min\min}\le\psi_{a_{ij}}\le\psi_{\max\max}$, hence $f(\psi_{\min_j\min_i})\le f(\psi_{a_{ij}})\le f(\psi_{\max_j\max_i})$.

$\Leftrightarrow\sqrt[q]{1-\max_j\max_i^q\psi}\le\sqrt[q]{1-\psi_{\sigma a_{ij}}^q}\le\sqrt[q]{1-\min_j\min_i^q\psi}$

$\Leftrightarrow\sqrt[q]{\prod_{j=1}^{n}(\prod_{i=1}^{m}(1-\max_j\max_i^q\psi)^{w_i})}$

${}^{v_j}\le\sqrt[q]{\prod_{j=1}^{n}(\prod_{i=1}^{m}(1-\psi_{\sigma a_{ij}}^q)^{w_i})^{v_j}}\le\sqrt[q]{\prod_{j=1}^{n}(\prod_{i=1}^{m}(1-\min_j\min_i^q\psi)^{w_i})^{v_j}}$

$\Leftrightarrow\sqrt[q]{((1-\max_j\max_i^q\psi)^{\sum_{i=1}^{m}w_i})^{\sum_{j=1}^{n}v_j}}\le\sqrt[q]{\prod_{j=1}^{n}(\prod_{i=1}^{m}(1-\psi_{\sigma a_{ij}}^q)^{w_i})^{v_j}}$

$\le\sqrt[q]{((1-\min_j\min_i^q\psi)^{\sum_{i=1}^{m}w_i})^{\sum_{j=1}^{n}v_j}}\Leftrightarrow\sqrt[q]{-1+\min_j\min_i\varphi^q}\le\sqrt[q]{-\prod_{j=1}^{n}(\prod_{i=1}^{m}(1-}$

$\varphi_{\sigma a_{ij}}^q)^{w_i})^{v_j}}\le\sqrt[q]{-1+\max_j\max_i\varphi^q}$

$\Leftrightarrow\min_j\min_i(\varphi)\le\sqrt[q]{1-\prod_{j=1}^{n}(\prod_{i=1}^{m}(1-\varphi_{\sigma a_{ij}}^q)^{w_i})^{v_j}}\le\max_j\max_i(\varphi)$ (3.12)

As $\psi_{\min\min}\le\psi_{a_{ij}}\le\psi_{\max\max}$, hence $f(\psi_{\min_j\min_i})\le f(\psi_{a_{ij}})\le f(\psi_{\max_j\max_i})$.

$\Leftrightarrow\sqrt[q]{\min_j\min_i\psi}\le\sqrt[q]{\psi_{\sigma a_{ij}}}\le\sqrt[q]{\max_j\max_i\psi}$

$\Leftrightarrow\sqrt[q]{\prod_{j=1}^{n}(\prod_{i=1}^{m}(\min_j\min_i\psi)^{w_i})^{v_j}}\le\sqrt[q]{\prod_{j=1}^{n}(\prod_{i=1}^{m}(\psi_{\sigma a_{ij}})^{w_i})^{v_j}}\le\sqrt[q]{\prod_{j=1}^{n}(\prod_{i=1}^{m}(\max_j\max_i\psi)^{w_i})^{v_j}}$

$\Leftrightarrow\sqrt[q]{((\min_j\min_i\psi)^{\sum_{i=1}^{m}w_i})^{\sum_{j=1}^{n}v_j}}\le\sqrt[q]{\prod_{j=1}^{n}(\prod_{i=1}^{m}(\psi_{\sigma a_{ij}})^{w_i})^{v_j}}\le\sqrt[q]{((\max_j\max_i\psi)^{\sum_{i=1}^{m}w_i})^{\sum_{j=1}^{n}v_j}}\Leftrightarrow\sqrt[q]{\min_j\min_i\psi}\le\sqrt[q]{\prod_{j=1}^{n}(\prod_{i=1}^{m}(\psi_{\sigma a_{ij}})^{w_i})^{v_j}}\le\sqrt[q]{\max_j\max_i\psi}\Leftrightarrow$

$\min_j\min_i(\psi)\le\sqrt[q]{\prod_{j=1}^{n}(\prod_{i=1}^{m}(\psi_{\sigma a_{ij}})^{w_i})^{v_j}}\le\max_j\max_i(\psi)$(3.13)

Let $q-ROFHOWA(E_{a_{11}},E_{a_{12}},\ldots,E_{a_{nm}})=E$. Then equation (3.12)–(3.13) can be written as $\min_j\min_i(\psi^q)\le\psi_{\sigma a_{ij}}^q\le\max_j\max_i(\psi^q)$ and $\min_j\min_i(\varphi)\le\varphi_{\sigma a_{ij}}\le\max_j\max_i(\varphi)$. Thus,

$$S(H)=\psi^q-\varphi^q\le\psi_{\max_j\max_i}^q-\psi_{\min_j\min_i}^q=S(H_{\max})$$
$$S(H)=\psi^q-\varphi^q\ge\psi_{\min_j\min_i}^q-\psi_{\max_j\max_i}^q=S(H_{\min})$$

If $S(H)<S(H_{\max})$ and $S(H)>S(H_{\min})$, then we have

$$H_{\min}<q-ROFHOWA(E_{a_{11}},E_{a_{12}},\ldots,E_{a_{nm}})<H_{\max}$$

If $S(H)=S(H_{\max})$, then we have $\psi^q=\psi_{\max_j\max_i}^q$ and $\varphi^q=\varphi_{\max_j\max_i}^q$. Thus $A(H)=\psi^q+\varphi$. Therefore,

$$q-ROFHOWA(E_{a_{11}},E_{a_{12}},\ldots,E_{a_{nm}})<H_{\max}$$

If $S(H)=S(H_{\min})$, then we have $\psi^q-\varphi=\psi_{\min_j\min_i}^q$ and $\psi^q=\psi_{\min_j\min_i}^q$ and $\varphi=\varphi_{\min_j\min_i}$ Thus $A(H)=\psi^q+\varphi=\varphi_{\min_j\min_i}^q+\psi_{\min_j\min_i}$. Therefore,

$$q-ROFHOWA(E_{a_{11}},E_{a_{12}},\ldots,E_{a_{nm}})<H_{\min}$$

So, we get

$$H_{\min}\le q-ROFHOWA(E_{a_{11}},E_{a_{12}},\ldots,E_{a_{nm}})\le H_{\max}$$

6 Multi criteria decision making technique

In this section, we will develop the methodology to apply the proposed operators in MCDM problems using q-ROFHNs and solve a numerical example. In addition, we also develop the decision-making technique (DM) to reduce MCDM restrictions and to validate the effects of proposed AOs. And also, a quantifiable data are provide to support the practical aspect of the planned approach.Assume that $S=\{S_1,S_2,\ldots S_s\}$ is a set alternative and $O=\{O_1,O_2,\ldots O_r\}$ is a group of experts. Let $w=(w_1,w_2,\ldots w_n)^T$ is a weight vector for experts with $w_i>0$, and $\sum_{i=1}^{n}w_i=1$. Let $T=\{T_1,T_2,\ldots T_m\}$ be a set of parameters with sub-attributes $\acute{Q}=\{q_{1p}\times q_{2p}\times\cdots\times q_{mp}\}$ with weight vector $v_j>0,\sum_{j=1}^{n}v_j=1$. The group of experts examined the alternatives in the form of q-ROFHNs as $(S_{a_{ij}})_{n\times m}=(\varphi_{a_{ij}},\psi_{a_{ij}})_{n\times m}$.Then, we apply he proposed approach to MCDM problem by applying q-ROFHOWA/q-ROFHOWG operators. Figure 2 shows the graphical representation of MCDM method's steps. The following algorithm can be used to make a decision. Step 1: Construct a decision matrix based on the experts' evaluations of each alternative in the form of q-ROFHNs.

$$(O,\acute{Q})=[T_{e_{ij}}^q]_{n\times m}=\begin{pmatrix}T_{e_{11}}^q & T_{e_{12}}^q & \cdots & T_{e_{1m}}^q\\ T_{e_{21}}^q & T_{e_{22}}^q & \cdots & T_{e_{2m}}^q\\ & \cdots\cdots & & \\ & \cdots\cdots & & \\ & \cdots\cdots & & \\ T_{e_{n1}}^q & T_{e_{n2}}^q & \cdots & T_{e_{nm}}^q\end{pmatrix}$$

Step 2: If all criteria are of the same type, there is no need for normalisation; however, there are two types of criteria, which are known as benefit type attribute B_b and cost type attribute B_c in MCDM. In this case, using the normalisation formula, the matrix $S_{a_{ij}}$ is changed into a normalising matrix

$$(D_{a_{ij}})_{m\times n}=\begin{cases}E_{a_{ij}}^c=(\varphi_{a_{ij}},\psi_{a_{ij}})\ \text{Cost type attributes}\\ E_{a_{ij}}=(\psi_{a_{ij}},\varphi_{a_{ij}})\ \text{Benefit type attributes}\end{cases}$$

Step 3: To obtain the order position of the proposed decision matrix.Step 4: Use the developed ordered weighted aggregation operator to accumulate all the alternative in a decision matrix.Step 5: Calculate the score values for each alternatives.Step 6: Select the appropriate alternative as the best option.

6.1 Example

In order to choose appropriate green suppliers, manufacturers invite a group of experts to evaluate the three potential green suppliers using a system of criteria, including product performance (A1), supplier development potential (A2), and pollution control (A3). They find specialists who can confirm links between influential elements of success affecting the performance of green suppliers. The experts are tasked with organizing sets of pairwise comparisons in terms of impacts and orientation between the performance factors of green suppliers, consisting of sub-attributes such as product lines (AS1), increasing efficiency and effectiveness (AS2), lowering supplier developmental potential (AP1), increasing supplier developmental potential (AP2), and reducing or eliminating fireplaces (AC1) which avoid burning leaves (AC2). Let $A = a_1 \times a_2 \times a_3$ be a set of sub-attributes $A = \{a_{11}, a_{12}\} \times \{a_{21}, a_{22}\} \times \{a_{31}, a_{32}\} = \{(a_{11}, a_{21}, a_{31}), (a_{11}, a_{22}, a_{32}), (a_{11}, a_{22}, a_{31}), (a_{11}, a_{21}, a_{32}), (a_{12}, a_{21}, a_{31}), (a_{12}, a_{22}, a_{32}), (a_{12}, a_{22}, a_{31}), (a_{12}, a_{21}, a_{32})\} = A = \{\mathring{a}_1, \mathring{a}_2, \mathring{a}_3, \mathring{a}_4, \mathring{a}_5, \mathring{a}_6, \mathring{a}_7, \mathring{a}_8\}$ denotes the set of sub-attributes with weight.$v_j = (0.15,0.14,0.16,0.12,0.13,0.11,0.10,0.09)^T$. Let $\{O_1,O_2,O_3\}$ be a set of experts with weight $w_i = (0.2,0.3,0.4,0.1)^T$.

Decision matrix for Alternative S_1

$$\begin{bmatrix} (0.66,0.99) & (0.77,0.99) & (0.99,0.33) & (0.77,0.66) & (0.88,0.99) & (0.77,0.99) & (0.99,0.33) & (0.77,0.99) \\ (0.55,0.66) & (0.99,0.44) & (0.55,0.99) & (0.66,0.99) & (0.66,0.99) & (0.66,0.88) & (0.77,0.66) & (0.88,0.66) \\ (0.44,0.88) & (0.77,0.99) & (0.44,0.66) & (0.99,0.44) & (0.77,0.88) & (0.77,0.55) & (0.88,0.33) & (0.77,0.44) \\ (0.55,0.44) & (0.88,0.55) & (0.66,0.88) & (0.77,0.88) & (0.88,0.66) & (0.77,0.66) & (0.99,0.55) & (0.55,0.66) \end{bmatrix}$$

Decision matrix for Alternative S_2

$$\begin{bmatrix} (0.55,0.66) & (0.44,0.99) & (0.88,0.33) & (0.77,0.66) & (0.44,0.99) & (0.77,0.55) & (0.99,0.66) & (0.77,0.99) \\ (0.77,0.88) & (0.77,0.99) & (0.44,0.66) & (0.99,0.44) & (0.77,0.44) & (0.77,0.55) & (0.88,0.33) & (0.88,0.44) \\ (0.88,0.99) & (0.77,0.99) & (0.99,0.44) & (0.77,0.66) & (0.88,0.66) & (0.77,0.99) & (0.99,0.44) & (0.77,0.99) \\ (0.55,0.66) & (0.99,0.44) & (0.33,0.99) & (0.66,0.99) & (0.44,0.99) & (0.33,0.88) & (0.77,0.66) & (0.88,0.66) \end{bmatrix}$$

Decision matrix for Alternative S_3

$$\begin{bmatrix} (0.99,0.66) & (0.44,0.99) & (0.88,0.33) & (0.77,0.66) & (0.44,0.99) & (0.77,0.55) & (0.99,0.66) & (0.77,0.99) \\ (0.44,0.88) & (0.77,0.66) & (0.55,0.88) & (0.99,0.33) & (0.66,0.44) & (0.77,0.66) & (0.55,0.33) & (0.88,0.77) \\ (0.66,0.99) & (0.77,0.99) & (0.99,0.44) & (0.77,0.66) & (0.88,0.66) & (0.77,0.99) & (0.99,0.44) & (0.77,0.99) \\ (0.33,0.88) & (0.77,0.99) & (0.44,0.66) & (0.99,0.44) & (0.77,0.44) & (0.77,0.55) & (0.88,0.33) & (0.88,0.44) \end{bmatrix}$$

Decision matrix for Alternative S_4

$$\begin{bmatrix} (0.99,0.44) & (0.99,0.44) & (0.99,0.33) & (0.77,0.99) & (0.77,0.55) & (0.77,0.44) & (0.44,0.66) & (0.33,0.88) \\ (0.88,0.33) & (0.88,0.77) & (0.77,0.66) & (0.77,0.66) & (0.66,0.44) & (0.55,0.88) & (0.55,0.33) & (0.44,0.88) \\ (0.99,0.44) & (0.99,0.44) & (0.88,0.66) & (0.77,0.99) & (0.77,0.99) & (0.77,0.99) & (0.77,0.66) & (0.66,0.99) \\ (0.99,0.66) & (0.99,0.66) & (0.88,0.33) & (0.77,0.99) & (0.77,0.66) & (0.77,0.55) & (0.44,0.99) & (0.44,0.99) \end{bmatrix}$$

The decision matrices with ordered position as follows:

Decision matrix for Alternative S_1

$$\begin{bmatrix} (0.99,0.33) & (0.99,0.33) & (0.88,0.99) & (0.77,0.99) & (0.77,0.99) & (0.77,0.99) & (0.77,0.66) & (0.66,0.99) \\ (0.99,0.44) & (0.88,0.66) & (0.77,0.66) & (0.66,0.99) & (0.66,0.99) & (0.66,0.88) & (0.55,0.99) & (0.55,0.66) \\ (0.99,0.44) & (0.88,0.33) & (0.77,0.99) & (0.77,0.88) & (0.77,0.55) & (0.77,0.44) & (0.44,0.88) & (0.44,0.66) \\ (0.99,0.55) & (0.88,0.66) & (0.88,0.55) & (0.77,0.88) & (0.77,0.66) & (0.66,0.88) & (0.55,0.66) & (0.55,0.44) \end{bmatrix}$$

Decision matrix for Alternative S_2

$$\begin{bmatrix} (0.99,0.66) & (0.88,0.33) & (0.77,0.99) & (0.77,0.66) & (0.77,0.55) & (0.55,0.66) & (0.44,0.99) & (0.44,0.99) \\ (0.99,0.44) & (0.88,0.44) & (0.88,0.33) & (0.77,0.99) & (0.77,0.88) & (0.77,0.55) & (0.77,0.33) & (0.44,0.66) \\ (0.99,0.44) & (0.99,0.44) & (0.88,0.99) & (0.88,0.66) & (0.77,0.99) & (0.77,0.99) & (0.77,0.99) & (0.77,0.66) \\ (0.99,0.44) & (0.88,0.66) & (0.77,0.66) & (0.66,0.99) & (0.55,0.66) & (0.44,0.99) & (0.33,0.99) & (0.33,0.88) \end{bmatrix}$$

Decision matrix for Alternative S_3

$$\begin{bmatrix} (0.99,0.66) & (0.99,0.66) & (0.88,0.33) & (0.77,0.99) & (0.77,0.66) & (0.77,0.55) & (0.44,0.99) & (0.44,0.99) \\ (0.99,0.33) & (0.88,0.77) & (0.77,0.66) & (0.77,0.66) & (0.66,0.44) & (0.55,0.88) & (0.55,0.33) & (0.44,0.88) \\ (0.99,0.44) & (0.99,0.44) & (0.88,0.66) & (0.77,0.99) & (0.77,0.99) & (0.77,0.99) & (0.77,0.66) & (0.66,0.99) \\ (0.99,0.44) & (0.88,0.44) & (0.88,0.33) & (0.77,0.99) & (0.77,0.55) & (0.77,0.44) & (0.44,0.66) & (0.33,0.88) \end{bmatrix}$$

Decision matrix for Alternative S_4

$$\begin{bmatrix} (0.99,0.44) & (0.99,0.44) & (0.99,0.33) & (0.77,0.99) & (0.77,0.55) & (0.77,0.44) & (0.44,0.66) & (0.33,0.88) \\ (0.88,0.33) & (0.88,0.77) & (0.77,0.66) & (0.77,0.66) & (0.66,0.44) & (0.55,0.88) & (0.55,0.33) & (0.44,0.88) \\ (0.99,0.44) & (0.99,0.44) & (0.88,0.66) & (0.77,0.99) & (0.77,0.99) & (0.77,0.99) & (0.77,0.66) & (0.66,0.99) \\ (0.99,0.66) & (0.99,0.66) & (0.88,0.33) & (0.77,0.99) & (0.77,0.66) & (0.77,0.55) & (0.44,0.99) & (0.44,0.99) \end{bmatrix}$$

The aggregated values using the q-ROFHOWG operator of the alternatives are given as follows.$S_1 = (0.5649, 0.2988)$, $S_2 = (0.7798, 0.1896)$, $S_3 = (0.7654, 0.3877)$, $S_4 = (0.6685, 0.1679)$.The score values are given as follows.$S_1 = 0.05514$, $S_2 = 0.28810$, $S_3 = 0.25392$, $S_1 = 0.13337$, $S_4 = 0.1337$. The order of alternative is $S_2 > S_3 > S_4 > S_1$, hence S_2 is the best alternative.

7 Comparative analysis and discussion

In this section, an evaluation of the proposed model is planned to demonstrate its effectiveness of the proposed method. For this purpose, we compare our proposed method with some existing methods. Zulqarnain et al. (Arora and Garg, 2018) developed intuitionistic fuzzy soft weighted average aggregation operator. If we assign the membership and non-membership i. e 0.88 and 0.99, then their sum exceeds 1. Hussain et al, (2020) developed q-ROFSWA operators, which cannot handle the parameterized values of alternatives. So in this case the method failed to cope the situation. Similarly, if we take IFHSWA aggregation operator, so thus method also fail and did not full the condition that the sum of membership and non-membership may not exceeds 1. On the other side the PFHSWA and PFHSWG aggregation operators also failed to tackle the situation. So the proposed approach full all these situations. The aggregation operators created by Khan et al, (2022a) can expertly compress with the multiple sub-attributes of the alternatives. However, these aggregation operators may sometimes not produce certain undesirable results. We, therefore, opted for the ordered aggregation operators in the q-ROFHS environment, which is achieved by managing several sub-attributes equivalents to existing aggregation operators, in order to manage these complexity in uncertain environment. q-ROFHSS is therefore the greatest interpretation of PFHSS. The advantage of the proposed technique is that they have the ability to solve real worlds problems by using their parameterizations properties with $q > 2$. The planned approach is effective and realistic, using the q-ROFHOWA and q-ROFHOWG operators. A creative MCDM model in the q-ROFH environment is built. The planned model is more capable than common approaches and can generate the most appropriate results in MCDM problems. The proposed approach will therefore be more effective, significant, and improved compared to the other models in different versions of the fuzzy sets. Table 1 show the study of comparative analysis.

8 Conclusion

The circumstances of group decision -making usually involve complex criteria that are influenced by a number of other factors. The selection of appropriate criteria and the choice of green suppliers are essential for the operations of every business. Their prosperity depends directly on how this situation is resolved. An organized approach would divide the examination of the green supplier selection problem into two sections: criteria and methodology. The assessment of decision-makers is expressed in the form of q-ROFHNs. In this study, the ambiguity and incompleteness of the information were resolved using the proposed structure. In addition, q-ROFHOWG aggregation operators were developed. Based on these operators, a novel MCDM strategy for green supplier management was created. Additionally, a real-world example of selecting green suppliers was provided to demonstrate the viability of the suggested operators. The suggested approach could successfully solve various challenges in the selection of green suppliers, including giving decision-makers, a pleasant environment for assessment and encouraging a relatively high degree of consensus among decision-makers, while properly examining the weights of decision-makers. Therefore, this study offers a more useful and effective method for selecting green-suppliers for the actual businesses. In future the proposed structure can be applied in different decision making techniques such as Complex Proportional Assessment (COPRAS) method, TOPSIS method, Analytic Hierarchy Process (AHP) method, VlseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR) method and (an acronym in Portuguese for Interative Multi-criteria Decision Making (TODIM) method. And we would consider the impact of the psychological factors of experts on the results of the decision in the problem of selecting green suppliers. In addition, the new method can also be applied in the selection of investment projects and in various other fields.

Data availability statement

The original contributions presented in the study are included in the article/Supplementary Material, further inquiries can be directed to the corresponding author.

Author contributions

All authors listed have made a substantial, direct, and intellectual contribution to the work and approved it for publication.

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Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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Supplementary material

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Coupling coordination degree and influencing factors of green science and technology innovation efficiency and digital economy level: Evidence from provincial panel data in China

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Green innovation and the digital economy are the new engine and driving force for Chinese high-quality development and will become the mainstream of China's high-quality development. Therefore, it is of great significance to explore the interaction between the two for the formulation of economic development policies. This paper constructed an evaluation system of green science and technology innovation efficiency (GSTIE) and digital economy level (DEL) based on 30 provinces in China. Through the corrected coupling coordination degree (CCD) model, this paper measured the coupling coordination degree of green science and technology innovation efficiency and DEL and analyzed its provincial differences and spatial effects. By employing the fuzzy set Qualitative Comparative Analysis (fsQCA) method, this paper further explored the influencing factors configuration affecting the coupling coordination degree of GSTIE and DEL. The research results are as follows. Compared with the development of green science and technology innovation, the development of the digital economy was relatively backward. The coupling coordination degree between China's provincial GSTIE and DEL showed an overall increasing trend year by year, and there was obvious spatial heterogeneity in which the eastern region was the highest, followed by the western and central regions. A single influencing factor does not constitute a necessary condition for a high coupling coordination degree. There were four paths that improve the coordinated development level between GSTIE and DEL: HC + RD + OP-jointly driven, RD + OP-dual driven, HC + GS-dual driven, and GS-oriented. Finally, based on the research conclusions, this paper proposed corresponding policy suggestions.

KEYWORDS

green science and technology innovation, digital economy, coupling coordination, condition configuration, fsQCA

1 Introduction

Since the reform and opening up, China's economy has developed and grown rapidly and made remarkable achievements. The major strategic evidence that China's economy has entered a stage of high-quality development from a stage of high-speed development was clearly highlighted in the meeting of the Political Bureau of the Central Committee of the Communist Party of China. However, pursuing rapid economic development has caused numerous problems, including resource shortage and environmental degradation, which have restricted the sustainable development of the economy and environment in China (Tang and Li, 2022). To solve these problems and achieve high-quality development, China should take the path of green development and rely on the implementation of an innovation-driven development strategy (Wang Q. et al., 2022). Unlike traditional innovation, green innovation, with innovation-driven and green development two special characteristics, is a combination of technology innovation and environmental protection (Dai et al., 2021). As an important carrier to promoting green development, green science and technology innovation is a kind of technology innovation, which focuses on environmental management and highlights the concept of sustainable development (Zheng et al., 2021). The improvement of green science and technology innovation efficiency (GSTIE) is an important way to accelerate the realization of green and sustainable development (Zhou et al., 2020). With green sustainable development becoming increasingly popular, GSTIE is gradually becoming a hot topic that is studied in academia and practice (Hosan et al., 2022). At the same time, with the rapid development and wide application of digital technology, the digital economy came into being, which is a new type of economy different from the traditional agricultural and industrial economy (Li et al., 2023). The new round of scientific and technological revolution represented by the digital economy has led human society into the era of digital economy (Zhao T. et al., 2020). According to the White Paper on China's Digital Economy (2022) released by the China Institute of Information and Communication, the scale of China's digital economy has reached 45.5 trillion yuan in 2021, with a year-on-year nominal growth of 16.2%, accounting for 39.8% of the proportion of GDP. The rapid rise of the digital economy provides not only opportunities for China to build new national competitive advantages but also an important path for China's green innovation development. Digital technology, represented by big data, artificial intelligence and the Internet, has the advantages of high intelligence, which can provide favorable environmental support and factor supply for green technology innovation (Wang F. et al., 2022). Green science and technology innovation and digital economy will become the mainstream of China's economic development. It can be seen that both green science and technology innovation and the digital economy perform an essential role in promoting the green development in China. At present, improving GSTIE and DEL is the strategic focus in China. According to systems theory, both green science and technology innovation and the digital economy are constantly developing and perfect dynamic systems. What is the coupling relationship between the two subsystems in the same social market environment? What is the degree of coupling development of the two and what factors jointly affect their coordinated development? To answer these questions, this study explores the coordinated development of GSTIE and the digital economy level (DEL) subsystems and its influencing factors, which is conducive to the formulation of a two-way driving strategy and the formation of a virtuous cycle between the two subsystems.

The remaining parts of this paper are arranged as follows: Section 2 reviews relevant literature; Section 3 introduces the construction of indicator system, data sources, and research methods; Section 4 reports the measurement results, temporal and spatial dynamics, and spatial correlation; Section 5 provides information on variable selection and discusses the influencing factors of the coupling coordination degree (CCD); Section 6 summarizes the research results and proposes policy recommendations.

2 Literature review

In academia, many researchers have studied the efficiency of technology or science and technology, whereas there is still a lack of research on GSTIE. Based on the content, the existing literature can be mainly classified into the aspects outlined below. In studies that evaluated GSTIE, most of the researchers built evaluation index systems that contain multiple indicators based on the perspective of input-output and used data envelopment analysis (DEA) (Luo et al., 2019; Wang et al., 2020; Liu B. et al., 2021), stochastic frontier analysis (Faria et al., 2019), and other methodologies to measure or evaluate the innovation efficiency. In addition, some scholars measured the efficiency of green innovation by applying the super-efficiency slack-based measure (SBM) model (Peng et al., 2021). The super-efficient SBM model, which was developed by Tone after making improvements based on the traditional DEA model, could effectively overcome the problem that the undesired output was ignored by the traditional DEA model, as well as achieve the comparison of multiple effective decision units (Zhao P. et al., 2020). It has become a mainstream method widely used by academic researchers to evaluate innovation efficiency.

Other studies focused on the spatial difference of GSTIE. For instance, using the method of the Dagum Gini coefficient and its subgroup decomposition, kernel density estimation, and the spatial Markov chain, Yao et al. (2022) discussed the convergence characteristics and dynamic evolution rule of industrial green technology innovation efficiency in 110 cities of the Yangtze River Economic Belt from 2006 to 2020 (Yao et al., 2022). Utilizing the combination of the global Super-Epsilon-based measure (EBM) model, vector autoregression model, block model, and Moran index, Wang and Zhang (2021) conducted an empirical study on the spatial-temporal features of the green innovation efficiency of China's 30 provinces and analyzed its spatial heterogeneity and spatial correlation network characteristics (Wang and Zhang, 2021). Moreover, some scholars have conducted studies on the spatial differences in green innovation efficiency using different scales. For example, Peng et al. (2019) studied the green innovation efficiency in the Yangtze River Economic Belt, and their results showed that the green innovation efficiency levels of the upper, middle, and lower reaches of the Yangtze River were substantially different (Peng et al., 2019). Besides, Liu et al. (2020), taking China's high-tech industry as the research object, conducted an empirical study on the regional differences and influencing factors of the green technology innovation efficiency of China's high-tech industry clusters, the results revealed that the factors affecting the green innovation efficiency of high-tech industry varied by cluster area (Liu et al., 2020).

In studies on the influencing factors of GSTIE, scholars selected indicators such as economic development level, research and development (R&D) investment in science and technology, education level, government subsidies, and environmental regulation and conducted empirical research and quantitative analysis on the factors influencing green innovation efficiency in combination with econometric models (Yi et al., 2020). Furthermore, other scholars studied the influence of carbon trading policies (Liu B. et al., 2021), upgrading industrial structure (Du et al., 2021), and green finance (Lee and Lee, 2022) on green innovation efficiency.

Moreover, studies on the digital economy paid particular attention to the definition, measurement, and influential mechanism of the digital economy. The digital economy was first formally defined by American scholar Tapscott (1996) as a virtual economic system developed by relying on information infrastructure such as the Internet and e-commerce. At the Hangzhou G20 Summit held in China in 2016, the leadership of all the countries gave a unified definition of the digital economy: an economic activity, in which digital knowledge and information were taken as a key factor of production, the modern information network was regarded as an important carrier, and the effective use of information and communication technology was taken as an important driving force for the improvement of efficiency and optimization of economic structure. Bukht and Heeks (2018) understood the digital economy as a proportion of economic output, which is generated based on digital technology, the digital sector, and platform services (Bukht and Heeks, 2018).

In terms of research methods, increasingly diverse research methodologies have been used for the measurement of DEL in the academic world. Most scholars chose the entropy method (Li and Liu, 2021; Luo and Zhou, 2022), whereas a few scholars chose principal component analysis and the coefficient of variation method based on sub-analysis to conduct relevant studies on the measurement of DEL. Regarding the measurement method, the academic community mainly adopts the comprehensive measurement method of constructing the evaluation system of the digital economy, composed of multiple dimension indicators such as infrastructure construction and internet level (Li and Liu, 2021; Luo and Zhou, 2022; Zhang et al., 2022). At present, there has not yet reached a unified standard on the comprehensive evaluation index system for the development of the digital economy. In the research on the impact mechanism of the digital economy, previous studies paid particular attention to the social and economic impact of the digital economy. Scholars have conducted studies on the impact of the digital economy on green development efficiency (Luo K. et al., 2022), carbon emission performance (Zhang, et al., 2022), resource consumption (Pouri, 2021), regional sustainable development (Yang Q. et al., 2022), and high-quality green development (Ma and Zhu, 2022).

As shown above, the academic community has generated fruitful research achievements related to the construction and measurement of green science and technology innovation index systems; temporal and spatial differentiation or evolution pattern; influencing factors; and the connotation, measurement, and impact mechanism of the digital economy; however, there is still a lack of studies that directly linked green science and technology innovation with the digital economy, and the research on the relationship between the two is still in the stage of further exploration. Although some researchers have studied the relationship between the digital economy and green innovation or green technology innovation (Cao et al., 2021; Lin and Ma, 2022; Ma and Zhu, 2022), most of them have only studied the one-way influence between the two. A few scholars have regarded the two as separately developed but mutually influencing systems and applied the coupling coordination theory to quantitatively study the coupling and coordinated relationship between the two subsystems.

Based on the above, by selecting China's 30 provinces as the research objects, this paper directly connected green science and technology innovation and the digital economy on the foundation of the perspective of coupling and aimed to reveal the coupling and coordinated relationship between the two subsystems and its influencing factors by constructing an index system to evaluate GSTIE and DEL. The innovations of this study mainly lie in the following three aspects. First, this research introduced the "coupling" theory and focused on studying the coupling and interaction between GSTIE and DEL in China's provinces, broadening the research perspective of green science and technology innovation and the digital economy, and further enriching the research results on the relationship between the two subsystems. Second, this study analyzed the evolution characteristics of Chinese provincial GSTIE and DEL from the two dimensions of time and space, which is of benefit to enrich the theoretical system of the research on GSTIE and DEL. Third, in this study, the configuration influencing factors that can affect the CCD between GSTIE and DEL were studied by means of the fuzzy set Qualitative Comparative Analysis (fsQCA) methodology, which could have a more comprehensive relationship with the development of provincial green science and technology innovation, the development of the digital economy, and even the coupling and coordinated relationship between the two subsystems in China.

3 Data and methodology

3.1 Construction of index system

Following systematic, scientific, feasible, and comparable principles, we constructed the evaluation index systems of GSTIE and DEL (Table 1). In this study, to measure the GSTIE of provinces, by referring to the previous research on the construction of a green science and technology innovation evaluation index system (Wang et al., 2020), the GSTIE subsystem was divided into three aspects on the basis of the perspective of input-output: green innovation input, green innovation desired output, and unexpected output. For the green innovation input evaluation, this study chose three

TABLE 1 GSTIE and DEL index system.

Subsystem	Target layer	Index layer	Indicator attributes
GSTIE	Innovation input	R&D personnel full-time equivalent (person-years)	+
		R&D funds internal expenditure (10^4 yuan)	+
		New product development expenses of industrial enterprises above designated size (10,000 yuan)	+
	Expected output of innovation	Technical market turnover (100 million yuan)	+
		Number of domestic patent applications authorized (cases)	+
		Sales revenue of new products of industrial enterprises above designated size (10,000 yuan)	+
	Unexpected output	Total industrial wastewater discharge (10^4 tons)	–
		Industrial SO_2 emission (10^4 tons)	–
		Industrial smoke and dust emission (10^4 tons)	–
DEL	Digital infrastructure construction	Number of cell phone owners (10^4)	+
		Number of fixed telephone owners (10^4)	+
		Length of long-distance optical cable line (10^4/km)	+
	Internet development	Internet port access (10^4)	+
		Internet broadband access users (10,000 households)	+
		Number of web pages (10^4)	+
	Digital industry development	Total telecom business (10^8 yuan)	+
		Total postal business (10^8 yuan)	+
		Employment personnel of urban units in information transmission, software and information technology services industry (10^4 people)	+
	Digital transactions	Proportion of enterprises with e-commerce transactions in total enterprises (%)	+
		Total sum of e-commerce sales (10^8 yuan)	+
		Total sum of e-commerce procurement (10^8 yuan)	+

indicators: the full-time equivalent of R&D personnel, the internal spending on R&D, and the new product development funds of industrial enterprises above the designated size. The indicators of the turnover of the technological market, the number of domestic patent applications granted, and the sales revenue of new products in industrial enterprises above the designated size were chosen as the expected output indicators of GSTIE. The indicators of undesired output mainly included the total emission of industrial wastewater, emission of industrial sulfur dioxide, and emission of industrial smoke and dust.

Scholars and organizations at home and abroad have carried out many explorations and studies on the measurement of DEL and proposed statistical indicators and measurement methods from multiple dimensions; however, no unified evaluation index system has been developed. Therefore, with reference to the existing literature (Chen and Miao, 2021; Wang J. et al., 2022; Xu et al., 2022) and considering the data availability, 12 index factors were selected from the four dimensions of the digital infrastructure, internet development, the development of the digital industry, and the digital transactions, to construct an evaluation index system for measuring DEL of China's provinces. In this study, the evaluation indicators of the number of cell phone users, the number of fixed phone users, and the length of long-distance optical cable lines were selected to measure the level of digital infrastructure development (Li and Liu, 2021). Regarding Internet development, the Internet is the carrier and bedrock of the development of the digital economy (Han and Zhang, 2019). This study selected three evaluation indicators: internet port access, internet broadband access users, and web pages (Ding et al., 2021). Digital industry development is generally used to measure the development of digital production in the digital context, including industrial income and the number of industry practitioners. Therefore, in this study, the total amount of telecom businesses, total postal business, and the number of urban unit employees who are working in information transmission, software, and information technology service industries were selected to measure and reflect the development of the Chinese digital industry (Luo and Zhou, 2022). In this research, we chose three evaluation indicators to reflect the development level of digital transactions: the proportion of enterprises with e-commerce transactions activities, e-commerce sales, and e-commerce purchases (Liu et al., 2020).

3.2 Data sources

In this paper, the objects of study were 30 provinces in mainland China (considering the availability of data, excluding Tibet, Hong Kong, Macao, and Taiwan), and the time span was 2013–2019. All the index data are from the China Statistical Yearbook, China Environmental Statistical Yearbook, China Science and Technology Statistical Yearbook, China Internet Network Information Center, and the National Bureau of Statistics from 2013 to 2019 (http://www.stats.gov.cn/), as well as statistical yearbooks of provinces. In addition, to ensure the availability and scientificity of the research data, the missing data in a few years and provinces are supplemented by linear interpolation during the research process.

3.3 GSTIE evaluation model

Data envelopment analysis (DEA) is a powerful non-parametric technical efficiency evaluation method first proposed by Charnes in 1978 (Charnes et al., 1978). The method is suitable for evaluating the relative efficiency of decision-making units (DMUs) with multiple input and output indicators (Shang et al., 2020). There are different DEA models proposed for different purposes. However, conventional DEA models are radial and fail to consider the slack of inputs and outputs. To solve this problem, Tone (2001) proposed a non-radial SBM-DEA model with slack variables, which perfectly solves the impact of slack variables on the measurement value. The non-radial SBM-DEA model can only calculate the efficiency value of the effective DMUs equal to 1, and cannot further evaluate multiple effective DMUs (Li and Du, 2021), Tone (2002) proposed the super-efficiency SBM-DEA model to remedy this defect. The model can make the efficiency value of the decision unit not limited by the (0, 1) and can more accurately calculate the DMUs with an efficiency value of 1 (Jiang et al., 2021). To significantly reflect the provincial differences in GSTIE, this study constructed a super-efficiency SBM-DEA model based on undesirable output. The specific model is as follows:

$$\min \rho = \frac{1}{m}\sum_{i=1}^{m}\frac{x}{x_{ik}} \Big/ \frac{1}{r+p}\left(\sum_{s=1}^{r_1} y^d y^d_{sk} + \sum_{q=1}^{r_2} y^u y^u_{qk}\right) \quad (1)$$

$$\text{s.t.}\begin{cases} x' \geq \sum_{j=1,\neq k}^{n} x_{ij}\lambda_j;\ y^d \leq \sum_{j=1,\neq k}^{n} y^d_{sj}\lambda_j;\ y^d \geq \sum_{j=1,\neq k}^{n} y^d_{qj}\lambda_j \\ x' \geq xk;\ y^d \leq y^k_d;\ y^u \geq y^u_k \\ \lambda_j \geq 0, i = 1, 2, \cdots, m;\ j = 1, 2, \cdots, n, j \neq 0 \\ s = 1, 2, \cdots, r;\ q = 1, 2, \cdots, p \end{cases} \quad (2)$$

where n represents the number of decision units; m, r, and p denote inputs, expected outputs, and unexpected outputs involved in each DMU, respectively; x, y^d and y^u represent the necessary elements of the input matrix, the desired output matrix, and undesired output matrix, respectively; λ is weight vectors, and ρ $(0 < \rho \leq 1)$ represents the final value GSTIE, with a larger value indicating that there is a higher level of GSTIE.

TABLE 2 Grade division of CCD between GSTIE and DEL.

CCD	Coupling coordination level
0–0.0.20	Low coupling coordination
0.20–0.40	Basic coupling coordination
0.40–0.50	Moderate coupling coordination
0.50–0.80	High coupling coordination
0.80–1.00	Excellent coupling coordination

3.4 Coupling coordination measurement model

3.4.1 Revised CCD model

Coupling refers to the phenomenon by which two or more systems influence each other through interactive mechanisms, and the coupling degree reflects the degree of mutual dependence and mutual restriction among systems (Zhang et al., 2023). However, it cannot reflect the coordinated development level among systems (Sun and Cui, 2018). For example, the coupling degree between GSTIE and DEL is high while the overall level of development is low, which is not the best state of high-quality development. CCD is used to reflect the degree of mutual balance and coordination between different systems (Dong et al., 2021; Yang et al., 2022a). Because traditional CCD has errors in writing, coefficient, weight and model, Wang Shujia and others revised the model, which improves the scientificity and rationality of the research (Wang et al., 2021). Therefore, this paper applied the revised CCD model to assess the coupling coordination development level of the two subsystems of GSTIE and DEL in China's provinces. The specific calculation formula of the revised CCD model is:

$$\begin{cases} C = \sqrt{[1-(U_2-U_1)] \times \frac{U_1}{U_2}} \\ D = \sqrt{C \times T} \\ T = \alpha_1 U_1 + \alpha_2 U_2 \end{cases} \quad (3)$$

where C represents the coupling degree, and $C \in [0,1]$. The greater the C value is, the stronger the interaction between the two systems. U_1 represents the comprehensive evaluation index of the GSTIE system; U_2 represents the comprehensive evaluation index of the DEL subsystem, and $\max U_i$ is assumed to U_2; T indicates the comprehensive coordination index of GSTIE and DEL, α, β is the undetermined coefficient, and $\alpha + \beta = 1$. In addition, it is generally believed that GSTIE is as important as DEL, and therefore $\alpha = \beta = 1/2$; D refers to the CCD, and $D \in [0,1]$. Specifically, the larger it is, the more harmonious and balanced the GSTIE and DEL subsystems are. In this research, after referring to relevant research (Liu T. et al., 2021), we have divided the CCD into five levels by using the median segmentation method, as shown in Table 2.

3.4.2 Spatial autocorrelation model

The spatial autocorrelation model can observe the degree of aggregation in the study area of an attribute value, mainly including global spatial autocorrelation and local spatial autocorrelation (Yang et al., 2022b). This study applied global spatial autocorrelation to reflect the global spatial association pattern of CCD of the GSTIE and DEL subsystems in the whole research area to determine whether there is a spatial correlation in the space. Global Moran's I is generally used for the calculation. The specific calculation formula is:

$$\mathbf{I} = \frac{\mathbf{n} \times \sum_{i=1}^{n} \sum_{j \neq 1}^{n} \mathbf{W_{ij}} (\mathbf{x_i} - \bar{\mathbf{x}}) (\mathbf{x_j} - \bar{\mathbf{x}})}{\left(\sum_{i=1}^{n} \sum_{j=1}^{n} \mathbf{W_{ij}} \right) \times \sum_{i=1}^{n} (\mathbf{x_i} - \bar{\mathbf{x}})^2} \tag{4}$$

where I denotes Moran's I, n represents the number of provinces and cities, x_i and x_j are the CCD of i province and j province, respectively, and $\bar{x}$ is the average value of the CCD; and W_{ij} represents the space weight matrix. Moran's I value ranges from −1 to 1, with $I > 0$ indicating a positive spatial correlation. The larger the value is, the more significant the spatial correlation is, with $I < 0$ indicating a negative spatial correlation. The smaller the value is, the greater the spatial difference is, with $I = 0$ indicating that there is a randomness of the space.

As the global spatial autocorrelation analysis can only reflect the overall situation of spatial correlation and difference of the study area and cannot sufficiently reflect the coupling and coordinated relationship between local elements and adjacent elements (Yang et al., 2022c; Li et al., 2022). The local spatial autocorrelation analysis can effectively identify the spatial dependence and heterogeneity of CCD. The calculation formula is:

$$\mathbf{I_i} = \frac{(\mathbf{x_i} - \bar{\mathbf{x}})}{\mathbf{m_0}} \sum_j \mathbf{w_{ij}} (\mathbf{x_j} - \bar{\mathbf{x}}) \tag{5}$$

where x_i denotes the CCD value in i province, and $\bar{x}$ refers to the average CCD value in all provinces; $I_i > 0$ represents the spatial agglomeration (H-H or L-L) with similar coupling coordination values of a province or city, and $I_i < 0$ represents the spatial agglomeration (L-H or H-L) with different coupling coordination values of a province or city.

3.4.3 Fuzzy set qualitative comparative analysis

On the basis of the perspective of configuration, by utilizing the fsQCA method, this study explored how the seven factors—human capital, R&D investment, openness, industrial structure, economic development, environmental regulation, and government subsidies—interacted to jointly affect the coupling and coordinated development between GSTIE and DEL in each province and identified the condition configuration that can achieve the coordinated development of the two subsystems. Unlike traditional statistical analysis and qualitative analysis methodologies, QCA can analyze the complex causal relationship between different combinations of antecedents and results (Liu et al., 2017). As one of the QCA techniques, fsQCA is a novel research methodology between variable and case orientation and goes beyond qualitative and quantitative research (Guo, and Lin, 2022). This method can be effectively applied to explore the synergy and linkage between various factors and can identify different causal paths leading to the same result by comparing and analyzing multiple causal conditions in each case (Mercado-Caruso et al., 2022).

In this study, fsQCA was applied to identify the factors influencing the provincial CCD, which was mainly based on the following considerations. The coupling coordinated development between GSITE and DEL is comprehensively affected by multiple factors, conforming to the basic assumption of fsQCA. Moreover, traditional research methods are mainly applicable for analyzing the net effect of a single influencing factor, whereas fsQCA can explore the complex non-linear and asymmetric causal relationship between multiple factors and results (Sun et al., 2022). In this paper, 30 provinces are taken as research samples, with small sample size, and fsQCA is the best method to perform a comparative analysis of small and mid-sized samples (Yifan and Bei, 2022). Furthermore, in this study, we aim to investigate the influence of different combinations of seven factors of the CCD between GSITE and DEL, as well as the multiple driving paths behind the coordinated development of the two subsystems, and fsQCA can effectively identify the equivalent paths that have different antecedent condition configurations leading to the same result; therefore, it is more suitable to explore the differentiated driving paths of the coordinated development of GSTIE and DEL in various areas.

4 Analysis of the spatio-temporal evolution of the CCD between GSTIE and DEL

4.1 Analysis of GSTIE and DEL

4.1.1 Spatio-temporal analysis of GSTIE

The measurement results presented in Table 3 indicate that during the research period, the overall level of GSTIE across the country presented an overall upward trend, and the efficiency value rose from 0.684 at the beginning of the period to 0.734 at the end of the period, with an increase of 7.35%. The overall level was high, and the change range was small. This indicates that the development of green science and technology innovation, as an increasingly important topic of national green innovation, has been accelerating in recent years, and its role in promoting energy conservation, emission reduction, and sustainable economic development has become increasingly significant.

To reflect their spatial distribution differences more intuitively, ArcGIS software has been utilized to visualize GSTIE in representative years, as shown in Figure 1. As observed in Figure 1, the spatial change characteristics of GSTIE in 2013, 2015, 2017, and 2019 were relatively similar, showing a spatial distribution pattern of a low level of GSTIE in western provinces and cities and a high level of GSTIE in central and eastern provinces in China. During the investigation period, the efficiency values of GSTIE in Beijing, Jiangsu, Zhejiang, and Qinghai remained unchanged at the level of 1.00, whereas the efficiency values in other provinces and cities fluctuated. Specifically, in 2013, the efficiency values of GSTIE in Hebei, Fujian, Shanxi, Jilin, Henan, Inner Mongolia, Guangxi, Xinjiang, and other regions were below 0.40. By 2015, the efficiency value of Yunnan dropped to below 0.40, and the efficiency values of Liaoning, Anhui, and Heilongjiang rose from below 0.60 to above 0.60. In 2017, the efficiency values of Shanxi, Jilin, and Guangxi broke through the low-efficiency level of below 0.40, and the efficiency values of Jilin and Guangxi reached 1.00, whereas the efficiency values of Hainan and Ningxia decreased from 1.00 to low-efficiency level of below 0.40. By 2019, the efficiency values of Guangxi and Chongqing dropped from

TABLE 3 Measurement results of GSTIE and DEL.

Region	GSTIE value				DEL value			
	2013	2015	2017	2019	2013	2015	2017	2019
National average	0.6835	0.6300	0.6565	0.7338	0.2757	0.2968	0.2885	0.2658
Average value in eastern region	0.7960	0.7201	0.6938	0.8437	0.3891	0.4007	0.3820	0.3651
Average value in central region	0.5041	0.6172	0.7715	0.7848	0.2372	0.2624	0.2576	0.2317
Average in western region	0.7015	0.5493	0.5356	0.5869	0.1902	0.2179	0.2175	0.1913

1.00 to below 0.40, those of Hebei, Hainan, and Xinjiang rose from below 0.40 to above 0.40, and those of Hebei and Xinjiang reached 1.00.

4.1.2 Spatio-temporal analysis of DEL

As shown in Table 3, from 2013 to 2019, the overall DEL of China showed a decreasing trend, decreasing from 0.2757 in 2013 to 0.2658 in 2019, a decrease of 3.59%, and the average annual decline rate was around 1.00%, suggesting that the development of China's digital economy lagged behind the development of green technology innovation during the investigation period. From the regional perspective, there was a regional imbalance in DEL, and during the investigation period, DEL in the eastern region was always higher than DEL in the central and western regions, and even far higher than the national average level. This is mainly due to the fact that areas in the east have a relatively superior economic foundation and technological R&D, which provides a good growth environment for the development of the digital economy, and therefore DEL is the highest in this region.

From the perspective of the spatial evolution characteristic of DEL (see Figure 2), China's DEL showed a similar spatial distribution pattern throughout the study period, and in the four representative years, DEL shows a decreasing trend in the eastern, central, and western regions in succession. During the research period, the DEL of provinces in China was generally at a low level, and there was much room for improvement.

Provinces and cities with high DEL are mainly in Guangdong in the eastern region, while provinces and cities with low DEL are mainly located in Xinjiang, Qinghai, Ningxia, and Gansu in the western region and Jilin in the central region. On the whole, in the four periods, Beijing, Jiangsu, Zhejiang, Shandong, and Guangdong in the eastern region occupy the dominant position in DEL. DEL in these provinces and cities has always been at the forefront of the country. However, for the remaining provinces, DEL is at or below 0.40, and around half is at a low level of 0.20~0.40.

4.2 Analysis of the CCD between GSTIE and DEL

4.2.1 Analysis of the coupled coordination between GSTIE and DEL in provinces

As can be seen from the calculation results, the average CCD of each province has been classified into five grades, as shown in Table 4. The results revealed that there were certain differences in the CCD among provinces, and most provinces and cities were concentrated in the moderate and high coordination stage, with 22 in total, of which there were 15 high coordination provinces and 7 moderate coordination provinces. High-coordination provinces and cities have the potential to develop into high-quality coordinated provinces and cities, and there is much room for improvement. Only Beijing, Jiangsu, Zhejiang, and Guangdong have been in the stage of high-quality coordination. Inner Mongolia, Hainan, Ningxia, and Xinjiang were in the basic coordination stage; however, there were no low-coordination provinces. On the whole, from 2013 to 2019, the average CCD between GSTIE and DEL in each province was above the level of medium coordination, indicating there has been a good development trend, and mainly showed the distribution situation of "more in the middle, less at both ends," that is, the number of provinces in the middle state of moderate coordination and high coordination was the highest and there were fewer provinces at both ends of basic coordination and high-quality coordination. In the future, we need to strengthen the relevant construction of GSTIE and DEL of basic and moderate coordination provinces and cities, improve their coupling and coordination level to improve the overall level of CCD in the whole country, and let more provinces and cities develop towards the stage of high coordination and high-quality coordination. In addition, more attention should be paid to the sustainable development of high-coordination provinces so that these provinces can develop into high-quality coordination provinces as soon as possible.

This study selected the starting and ending years and the middle years in the research period, namely, the values of CCD in 2013, 2015, 2017, and 2019, and utilized ArcGIS software to conduct spatial visualization processing to further explore the inter-provincial spatial distribution characteristics of the CCD of GSTIE and DEL (Figure 3).

As illustrated in Figure 3, overall, the national CCD between GSTIE and DEL showed a steadily increasing trend (Figure 3). The spatial distribution of the coupling and coordinated development between GSTIE and DEL was unbalanced, showing the distribution pattern of east > west > central. During the investigation period, the spatial pattern of the CCD of China's provincial GSTIE and DEL changed significantly, and the CCD in most regions was at or above the level of moderate coordination, which indicated that GSTIE in most provinces and cities of China was well coordinated with DEL. Specifically, in 2013, half of the provinces were in the moderate coordination stage, five provinces were in the stage of high-quality coordination, five provinces were

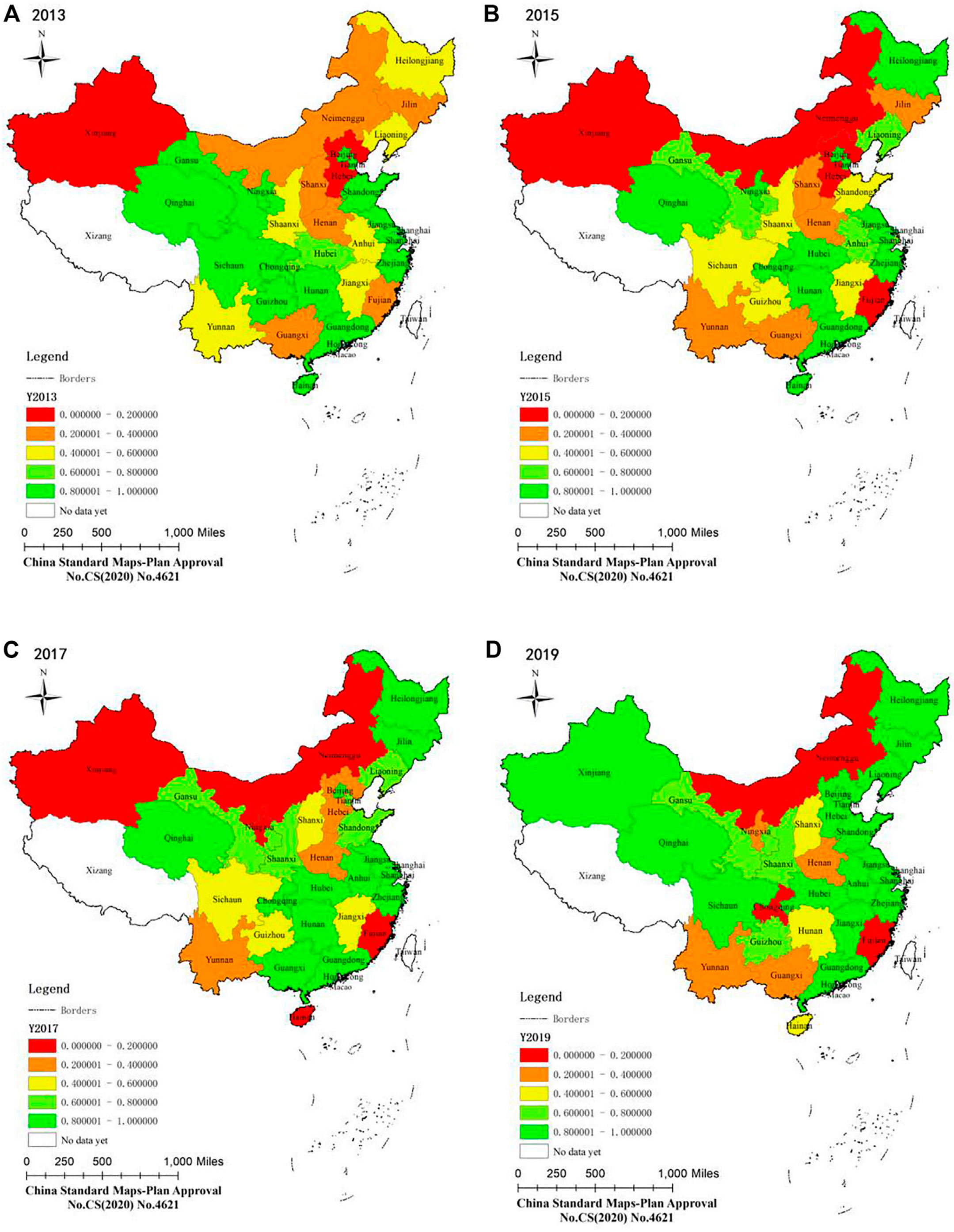

FIGURE 1
Spatial changes in China's provincial GSTIE in representative years.

in the stage of high coordination, four provinces—Jilin, Hebei, Ningxia, and Hainan—were in the basic coordination stage, and only Xinjiang in the west was in the low coordination stage and remained unchanged until 2019 when it moved from the low coordination stage and entered the medium coordination stage. In 2015, the number of high-coordination provinces increased by five: Hebei, Shandong, Hainan, Chongqing, Anhui, and Heilongjiang. At the same time, Guizhou moved from the highly coordinated stage and entered the moderate coordination stage. In 2017, there was relatively little change in the spatial distribution pattern of coupling and coordination. Guangxi had two periods of moderate coordination and then entered the high coordination stage. Jilin left the basic coordination stage and entered the moderate coordination stage. In 2019, the number of excellent coordination provinces increased from four to six—Beijing, Jiangsu, Zhejiang, Guangdong, Sichuan, and Shandong—Hebei

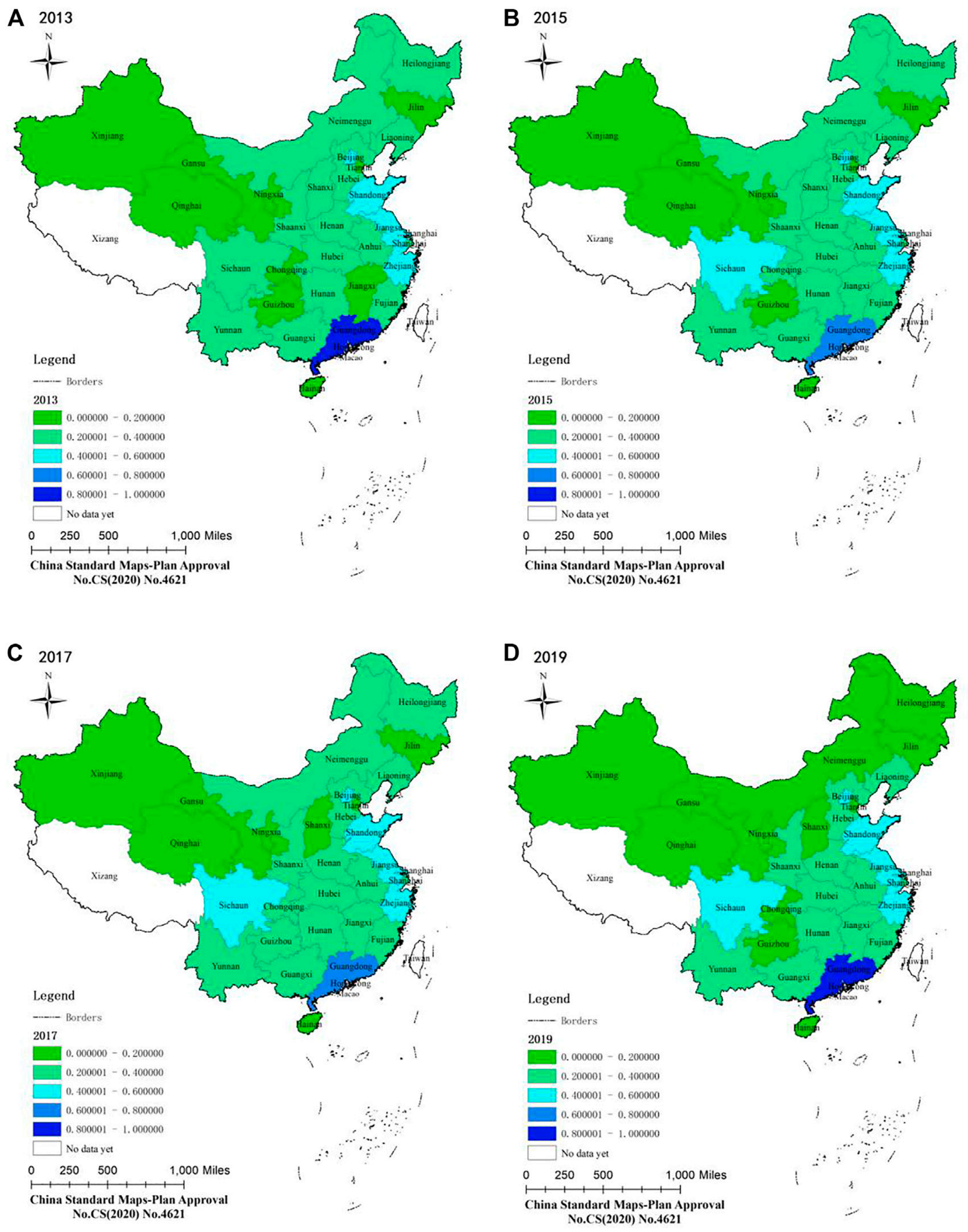

FIGURE 2
Spatial change of China's provincial DEL in representative years.

TABLE 4 Mean level of CCD of China's provinces from 2013 to 2019.

Level	Low coordination	Basic coordination	Moderate coordination	High coordination	High-quality coordination
Provinces	None	Inner Mongolia, Hainan, Ningxia, Xinjiang	Hebei, Shanxi, Tianjin, Jilin, Fujian, Yunnan, Qinghai	Shaanxi, Liaoning, Heilongjiang, Chongqing, Shanghai, Anhui, Jiangxi, Shandong, Hunan, Henan, Hubei, Sichuan, Guangxi, Guizhou, Gansu	Beijing, Jiangsu, Zhejiang, Guangdong

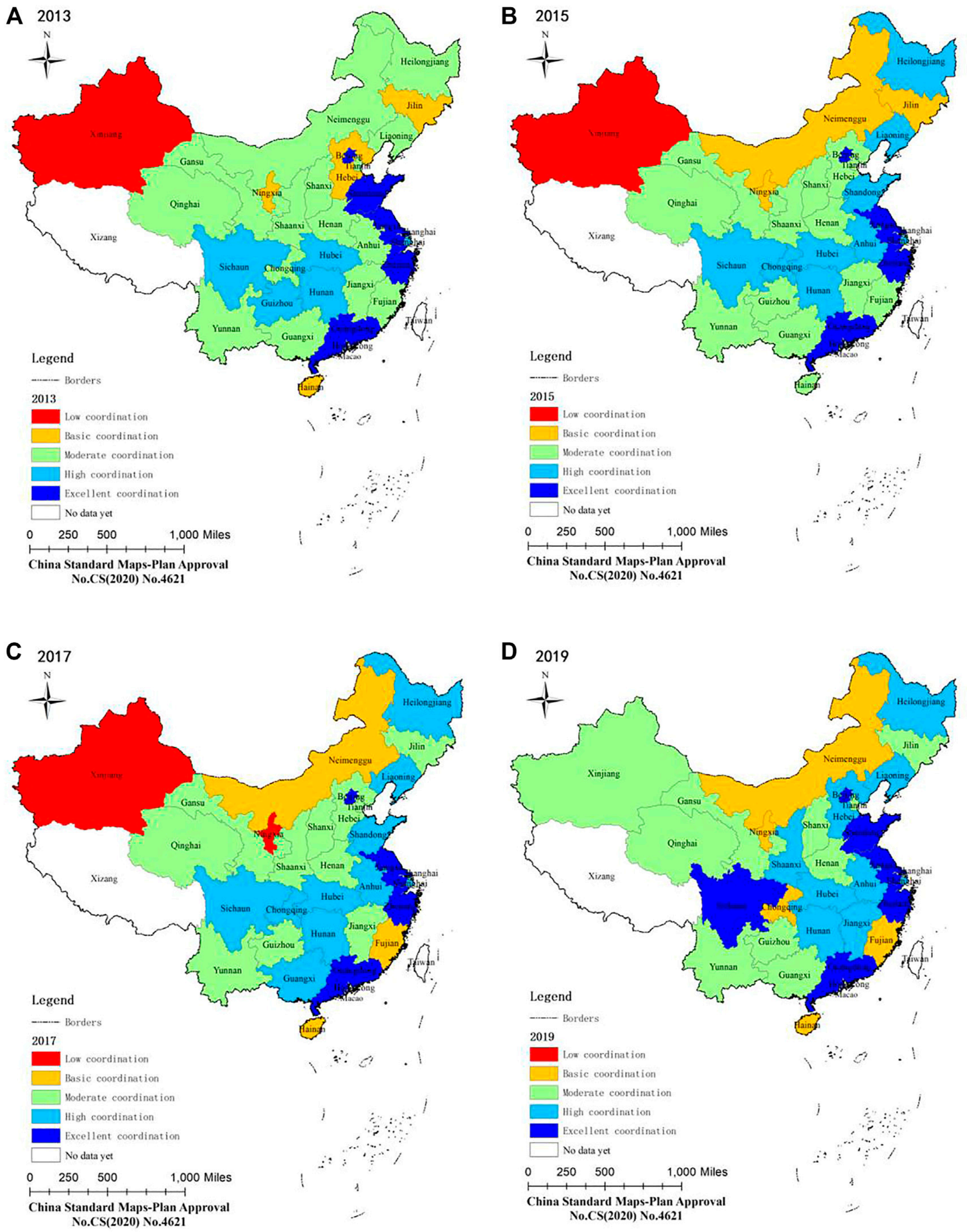

FIGURE 3
Spatial distribution map of the CCD between GSTIE and DEL in China.

and Jiangxi rose from a moderate coordination stage in 2017 to a high coordination stage, and Guangxi and Chongqing dropped to the stage of moderate coordination and basic coordination, respectively. At the same time, Xinjiang and Ningxia left the low coordination stage, and there was no low coordination province, which suggested that the CCD of GSTIE and DEL in the two provinces had been improved to a certain extent, and the relationship between the two had become slightly closer.

4.2.2 Analysis of the regional CCD between GSTIE and DEL

This paper drew the point map and ridge map of CCD of GSTIE and DEL subsystems of provinces in China (see Figure 4) to further explore the regional distribution characteristics of CCD between the two subsystems.

In Figure 4, it can be seen that there was an imbalance in the coupling and coordinated development between GSTIE and DEL

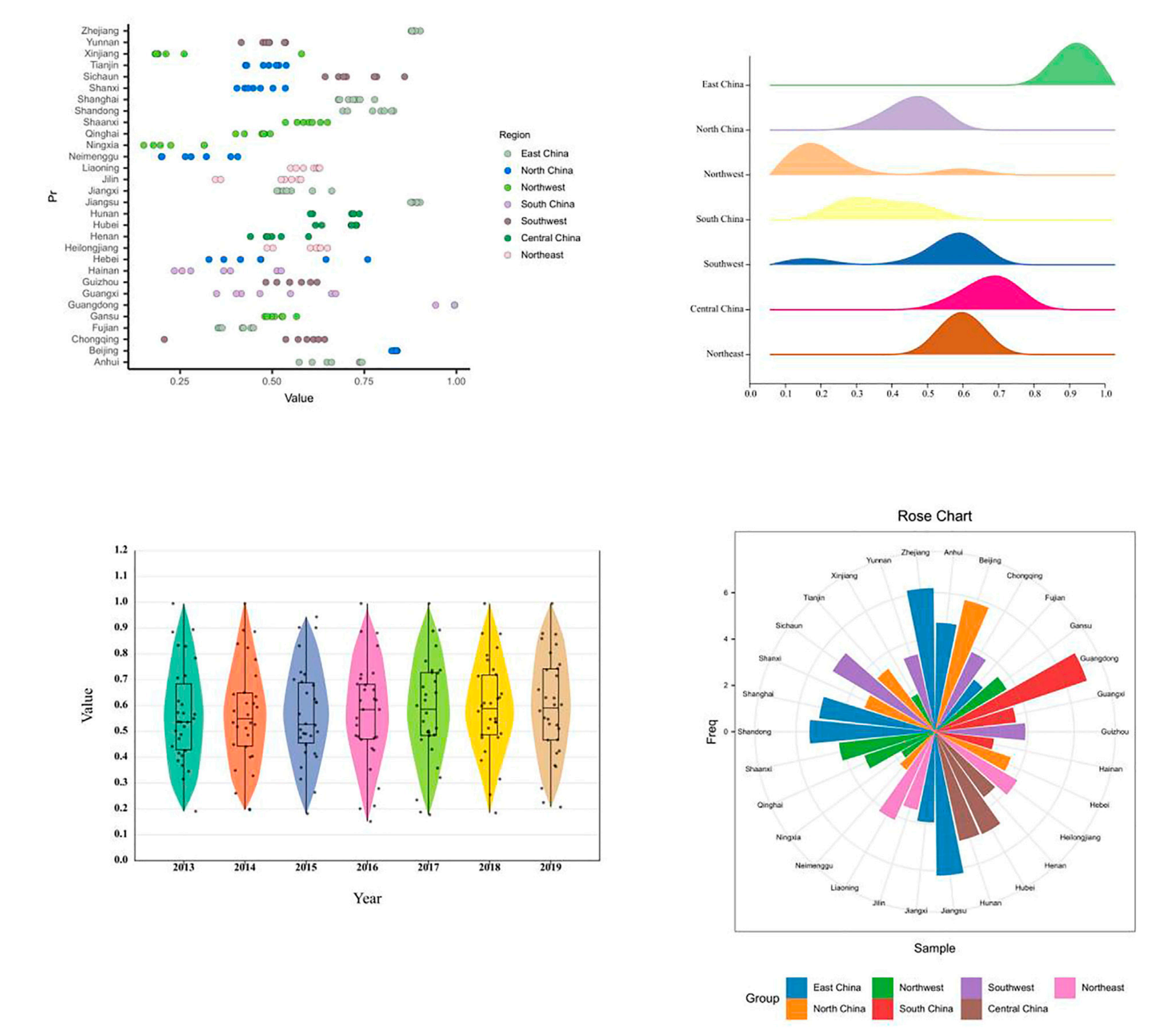

FIGURE 4
Zoning map of CCD between GSTIE and DEL.

among regions. The regional CCD from high to low was: East China, Central China, South China, Southwest China, Northeast China, North China, and Southwest China, and for every region, there is still substantial room for improvement of the coupling coordination relationship between GSTIE and DEL.

According to the distribution of CCD between GSTIE and DEL among provinces in each region, there was a big difference in the CCD of the provinces and cities in East China, of which the CCD in Jiangsu and Zhejiang provinces was the largest, and the level of CCD in the two provinces fluctuated slightly every year, maintaining the same level. Furthermore, there was a big difference in the CCD among the provinces in North China. The CCD in Beijing fluctuated slightly every year and remained at a high level of 0.80–0.90, and the level of the CCD was the highest in this region and the lowest in Inner Mongolia. There was a small difference in the CCD in the four provinces and cities of Southwest China, among which Sichuan had a higher level than that the other three provinces in this region. Conversely, there was a big difference in the CCD among provinces in Northwest China. The CCD in Ningxia fluctuated sharply every year, followed by Xinjiang, and the level of coupling coordination in the other three provinces showed small fluctuations every year. There was a large difference in the CCD between GSTIE and DEL in the three provinces of Southern China, among which the CCD in Guangdong had a small fluctuation annually and was in the highest position and far higher than that in Guangxi and Hainan, with a maximum gap of 0.76. The CCD was relatively different in the three provinces of Central China, and the CCD in Henan was the lowest, but it was rising as a whole. The CCD in Hubei grew slowly during the investigation period and exceeded that of Hunan. There was a small difference in the CCD among provinces in Northeast China, among which Liaoning and Heilongjiang had small fluctuations in the CCD every year.

TABLE 5 Global Moran's I of CCD of GSTIE and DEL in China.

Year	Moran's I	E(I)	Z value	*p*-value
2013	−0.0284	−0.0345	0.0798	0.9364
2015	0.0226	−0.0345	0.0058	0.4551
2017	0.0475	−0.0345	1.0702	0.2845
2019	0.0697	−0.0345	1.3509	0.1767

4.3 Analysis of the spatial autocorrelation of CCD

In this paper, we explored the coupling and coordinated evolution of GSTIE and DEL in different provinces, but cannot fully understand the whole picture of China's provincial GSTIE and DEL. Therefore, the spatial autocorrelation model was applied to further explore the correlation among provinces and deepen the research on the development of China's provinces, so as to provide a reference for the coordinated development of China's provinces (Yang et al., 2022a).

Based on the CCD of GSTIE and DEL during the period 2013–2019, this paper calculated the global Moran's I index by applying ArcGIS 10.2 software (Table 5). The results showed that in all four representative years of the sample selection, the global Moran's *I* values of the CCD were larger than 0 except for 2013 but did not pass the test of significance, indicating that there was positive spatial autocorrelation in CCD, but its global spatial autocorrelation was not significant. In addition, the results also revealed that the global Moran's *I* index showed an upward trend over time, which indicates that the spatial autocorrelation of the CCD is gradually increasing.

The local spatial autocorrelation index can specifically describe the local spatial agglomeration characteristics of the CCD between GSTIE and DEL among provinces. Therefore, this paper calculated the local Moran's *I* index of the CCD between GSTIE and DEL of China's provinces in 2013, 2015, 2017, and 2019, and used ArcGIS 10.2 software to spatially process the results, obtaining the LISA cluster map of CCD between China's provincial GSTIE and DEL, as shown in Figure 5.

It can be seen from the LISA agglomeration map that from 2013 to 2019, there were three types of agglomeration regions, namely, High-High, High-Low, and Low-High, but there was no Low-Low.

High-high concentration areas (H-H). The CCD in the areas and its surrounding areas is high, and the internal difference in space is small. As shown in Figure 5, this type of province existed in all four-time nodes. In general, the number of provinces in the high-high concentration area was at most 4 and at least 2, and all of them were distributed in the east. In 2013, the high-high concentration areas included Shanghai, Jiangsu, and Zhejiang, forming a linkage region that was adjacent to and promoted each other. In this region, both GSTIE and DEL were at a high level, and the region has taken a leading position in the development of green innovation, which has an important position in driving the integration and development of GSTIE and DEL in China and was an important growth factor to drive the development of other provinces and cities. In 2015, due to its limited radiation capacity and the negative impact of other provinces and cities, Shanghai temporarily left the high-high concentration area, leaving only Jiangsu and Zhejiang. In 2017, under the radiation of Jiangsu and Zhejiang, Anhui and Shanghai joined the high-high cluster type, and both left the cluster in 2019. By 2019, Shandong had joined the high-high concentration areas and become a region with its own radiation capacity.

High-low concentration areas (H-L). In this region, the CCD is higher but lower in its surrounding neighborhood, and the spatial internal heterogeneity is large. It can be seen from Figure 5 that there was no province or city in high-low concentration areas in the three-time nodes of 2013, 2015, and 2017, while there was only Sichuan in 2019. Before 2019, the local agglomeration characteristics of the CCD in Chongqing were not significant. From the distribution perspective, this type of province was located in the western region. In the process of coupling and coordinated development between GSTIE and DEL, it did not drive the surrounding provinces and cities to improve the level of the CCD. Therefore, it should strengthen the connection and interaction with cities, play the role of spatial linkage, and improve the overall CCD in the region.

Low-high concentration areas (L-H). This type of region has a low CCD, while the CCD in its surrounding areas is high and the spatial internal heterogeneity is large. As shown in Figure 5, this type of province existed only in 2017 and 2019 among the four-time nodes. In 2017, there were two provinces and cities, Hainan and Fujian. In 2019, Hainan, under the reverse influence of other provinces and cities, moved from the low-high concentration areas, leaving only Fujian. The CCD of such provinces was at a low level, whereas that of their surrounding provinces and cities was high and mainly distributed near high-high agglomeration areas. It is most likely to receive the radiation and driving effect of the surrounding provinces' coupling and coordination development. Therefore, the areas are more likely to develop into high-high concentration areas.

From the perspective of the overall evolution pattern, high-high concentration areas and high-low concentration areas were mainly distributed in eastern China, and low-high agglomeration areas were closely followed by high-high agglomeration areas. In addition, in the study period, the local spatial agglomeration characteristics of the CCD between GSTIE and DEL in most provinces and cities in China were not significant, which indicated that the synergy effect between GSTIE and DEL needed to be strengthened. Therefore, there was a certain spatial structure problem in the overall coupling coordination between China's GSTIE and DEL at this stage. We should build more high-high concentration areas as soon as possible and make the most of the radiation and driving effect of high-high clustering areas to drive the low-high clustering areas closely distributed to develop into high-high clustering areas to improve the level of CCD of China's GSTIE and DEL. To solve the problem of regional imbalance in CCD and reduce differences among regions, it is necessary to further explore the factors affecting the development of the coupling coordination between the two subsystems.

5 Analysis of the influencing factors of the coupling and coordinated development of GSTIE and DEL

5.1 Variable design and calibration

5.1.1 Variable selection and setting

Before using fsQCA for analysis, variables need to be classified into two categories: result variables and conditional variables. With

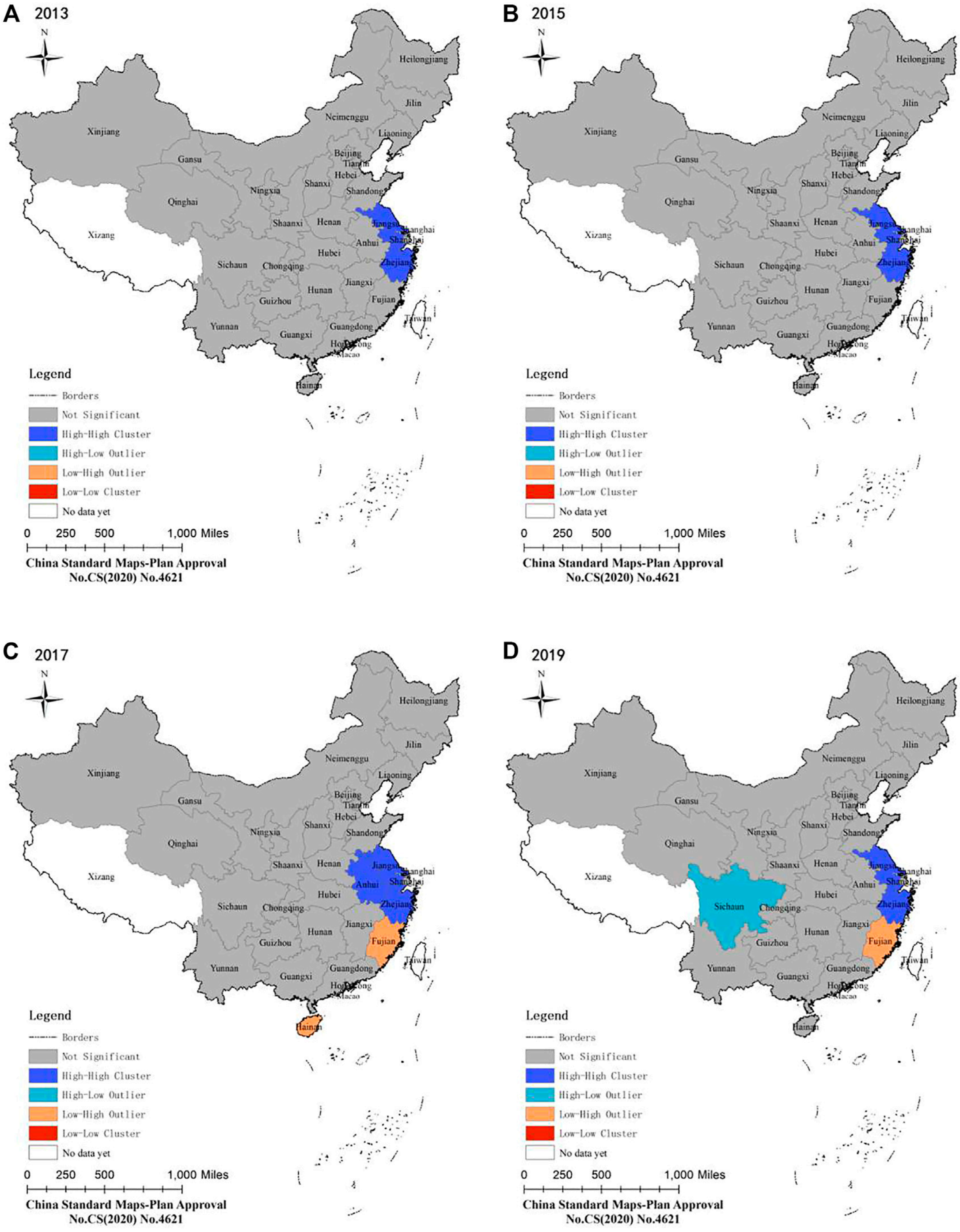

FIGURE 5
LISA cluster diagram of the CCD between GSTIE and DEL of China's 30 provinces.

reference to the research of Luo L. et al. (2022), all the indicator data included in the variables are the average value of relevant statistical data from 2013 to 2019 (Luo L. et al., 2022).

The coupling and coordinated development between GSTIE and DEL can be affected by a variety of factors. Based on previous research and the actual situation, human capital, capital investment, openness, economic development, industrial structure, government subsidies, and environmental regulation were determined as the conditional variables. The full-time equivalent of R&D personnel is a universal indicator that is applied for the measurement of human capital (HC). Therefore, in this study, the full-time equivalent of R&D personnel of each province was selected as the specific evaluation indicator of human capital. R&D investment is commonly used as an indicator reflecting the financial resource investment in technological innovation, and considering the difference in the scale of

TABLE 6 Descriptive statistics of variables.

Variable	Mean value	Standard deviation	Minimum value	Maximum value
HC	133,747	148,976	4,008	803,208
RD	1.75	1.12	0.45	6.30
OP	85.10	78.95	0.04	332.59
ED	58,329	26,931	23,151	164,220
IS	49.92	8.47	34.70	83.70
GS	24.10	13.57	6.87	57.76
ER	0.32	0.31	0.01	2.45
CCD	0.672	0.335	0.058	1.000

economic development in various regions, the proportion of internal R&D expenditure in the gross domestic product (GDP) was selected for the measurement of R&D investment (RD). In this study, the amount of foreign direct investment (FDI) was selected as the proxy variable of the level of openness (OP) (Song et al., 2021). GDP *per capita* was chosen for the measurement of the economic development (ED) level (Yang et al., 2021). The industrial structure (IS) was measured using the proportion of the value of the tertiary industry value to GDP as a specific measure (Hao et al., 2020). The index of "the proportion of government funds in the internal expenditure of R&D funds" was selected to represent the strength of government subsidies (GS). Environmental regulation (ER) represents the intensity of local environmental regulation in a region. To quantitatively evaluate environmental regulation, this study selected the proportion of industrial pollution control investment in the secondary industry as the proxy variable (Li et al., 2020). The descriptive statistics of all the variables are presented in Table 6.

5.1.2 Calibration for variables

Calibration, the first step of fsQCA, refers to the process of assigning set membership to a case (Huang et al., 2022). This study calibrated the relevant variables to a fuzzy set by using the direct calibration method. Referring to existing studies (Romero-Castro et al., 2022), the 75%, 50%, and 25% quantile value of the resulting variable and the conditional variable were selected as the three qualitative anchor points of full subordination, intersection, and non-subordination, respectively. The calibration results are presented in Table 7.

5.2 Single-factor necessity analysis

Before performing configuration analysis, the single-factor necessity test in QCA was used to determine whether a single antecedent variable is a necessary condition for leading to the particular result (Jiao et al., 2020). A single condition with a consistency level greater than 0.90 can be considered a necessary condition (Jiao et al., 2020; Huang et al., 2022). As shown in Table 8, the values of the consistency level of all conditions do not exceed 0.90, and therefore there was no necessary condition producing high CCD or non-high CCD of GSTIE and DEL.

5.3 Configuration analysis

In this study, fsQCA 3.1 software was used to further analyze the condition configurations generating the high CCD and non-high CCD between GSTIE and DEL, respectively, and further explore the combination of various conditions influencing the coupling and coordinated development between GSTIE and DEL. In this study, referring to existing research (Mercado-Caruso et al., 2022; Peiró-Signes et al., 2022), the original consistency threshold was set to 0.80, the Proportional Reduction in Inconsistency threshold was set to 0.70, and the case frequency threshold was set to 1. The core conditions of all solutions were identified by comparing the nesting relationship between the intermediate solution and the parsimonious solution, that is, the conditions that appear in both the intermediate solution and the parsimonious solution are the core conditions, whereas those that only appear in intermediate solutions are auxiliary conditions. The results are presented in Tables 9, 10.

5.3.1 Configuration of high CCD

The results presented in Table 9 reveal that five configurations could generate a high level of CCD between GSTIE and DEL, and the consistency values of these configurations were above the level of 0.90, indicating that all five configurations were sufficient conditions for high CCD between the two subsystems. The consistency level of the solutions was 0.9263, indicating that all the configurations covering most cases were sufficient for high CCD. In addition, the overall coverage of the solutions was 0.6712, which indicated that all the configurations can explain around 67.12% of the cases with high CCD. Configuration H1a had the same core conditions as configuration H1b, and they can form a second-order equivalent configuration. Accordingly, this paper summarized four configuration paths that can produce the high CCD between GSTIE and DEL.

(1) HC + RD + OP-jointly driven. In configuration H1 (H1a and H1b), the core conditions are HC, RD, and OP, so it was named HC + RD + OP-jointly driven. Configuration H1a showed that the conditional configuration with high HC, high RD, high OP, and non-high ER as core conditions and non-high GS as the edge condition can generate high CCD between GSTIE and DEL. H1a indicated that the areas with insufficient GS and weak ER, relying on rich HC, high RD and high OP, and through

TABLE 7 Calibration anchor point for variables.

Variable name		Anchor point		
		Non-subordination	Intersection	Full subordination
Conditional variable	HC	40925.0536	95710.8643	144610.9107
	RD	1.1329	1.4699	2.1429
	OP	19.4310	72.2051	139.4584
	ED	41736.0714	46994.2143	68438.8214
	IS	46.4000	47.8286	50.4750
	GS	13.3661	21.1982	34.6414
	ER	0.1779	0.2589	0.3622
Result variable	CCD	0.4858	0.5488	0.6933

TABLE 8 Single condition necessity analysis results.

Conditional variable	High CCD		Non-high CCD	
	Consistency	Coverage	Consistency	Coverage
HC	0.8178	0.7827	0.2934	0.3188
~HC	0.2883	0.2644	0.8000	0.8329
RD	0.8128	0.8071	0.2420	0.2728
~RD	0.2676	0.2372	0.8288	0.8341
OP	0.8171	0.7757	0.2953	0.3182
~OP	0.2819	0.2605	0.7918	0.8309
ED	0.6740	0.6495	0.3887	0.4252
~ED	0.4036	0.3677	0.6796	0.7030
IS	0.5601	0.5061	0.5467	0.5608
~IS	0.5139	0.4997	0.5185	0.5723
GS	0.4947	0.4810	0.5517	0.6090
~GS	0.5979	0.5402	0.5298	0.5434
ER	0.3587	0.3396	0.7141	0.7675
~ER	0.7544	0.6992	0.3856	0.4057

Note: "~" represents logical operation "No", and all numerical results are calculated by using fsQCA3.1 software.

independent R&D and introduction and absorption, can achieve high CCD between GSTIE and DEL. In this configuration, the values of consistency and unique coverage were 0.8988 and 0.1765, respectively, which indicated that the configuration could explain around 89.88% of the cases, and around 17.65% of the cases should be interpreted only by this configuration path. The provinces and cities represented by this route include Jiangsu, Zhejiang, Anhui, Guangdong, Hunan, and Hubei. Taking Guangdong as an example, the original driving data showed that the full-time equivalent of R&D personnel in this province ranked first in the whole country, with prominent advantages in HC, and RD and OP ranked among the top five in the country. However, Guangdong was ranked lowest in the country in terms of GS and ER. This showed that the provinces and cities represented by Guangdong, under low GS and weak ER, relying on high HC, high RD and high OP, can drive the coupling coordinated development between GSTIE and DEL, which conformed to the typical characteristics of the configuration solution of HC + RD + OP-co-driven. In configuration H1b, the core conditions included high HC, high RD, high OP, and non-high ER, and the marginal conditions were non-high ED and non-high IS, which indicated that if the level of regional ED is not high, IS is not advanced and ER is not high, it could realize the coordinated development between GSTIE and DEL through the combination of high HC, high RD, and high OP. In this

TABLE 9 Configuration of high CCD between GSTIE and DEL.

Conditional variable	High CCD				
	H1a	H1b	H2	H3	H4
HC	●	●	•	●	●
RD	●	●	●	•	●
OP	●	●	●	•	⊗
ED		⊗	•	•	●
IS		⊗	⨂	•	⨂
GS	⊗		⊗	●	●
ER	⨂	⨂			●
Consistency	0.8988	0.9863	0.8355	0.9130	0.9684
Original coverage	0.3858	0.2043	0.1843	0.1943	0.0655
Unique coverage	0.1765	0.0512	0.0406	0.1473	0.0199
Overall consistency	0.9263				
Overall coverage	0.6712				

Note: ● Represents the presence of core condition, ⨂ represents the absence of core condition, • represents the presence of auxiliary condition, ⊗ represents the absence of auxiliary condition, blank represents condition irrelevant to the outcome. This note also applies to the following tables.

TABLE 10 Configuration of non-high CCD between GSTIE and DEL.

Conditional variable	Non-high CCD					
	NH1	NH2a	NH2b	NH 3	NH 4	NH5
HC	⊗	⊗	⊗	•	⊗	•
RD	⨂	⨂	⨂	•	⨂	⨂
OP	⨂	⨂	⨂	⨂	⨂	•
ED		⊗	•	•	⊗	⊗
IS	⨂	⊗	•	⨂		⊗
GS		•	•	⨂	•	⨂
ER	⨂		⊗	⨂	●	●
Consistency	0.8984	0.8517	0.9744	0.8254	0.9012	0.9400
Raw coverage	0.2884	0.2088	0.0715	0.0652	0.3887	0.0884
Unique coverage	0.0884	0.0219	0.0157	0.0376	0.1718	0.0351
Solution consistency	0.9039					
Solution coverage	0.6194					

configuration, the consistency value was around 0.9863, and the unique coverage was around 0.0512, meaning that the configuration path could explain approximately 98.63% of the cases, and around 5.12% of the cases could be interpreted only by this configuration path. Provinces with such typical characteristics included Anhui and Sichuan. Taking Sichuan as an example, the original data showed that ED level, advanced IS and the intensity of ER of Sichuan were in the bottom 50% of the country, but in terms of HC, RD, and OP, it ranked in the top 50% of the country, and the CCD of Sichuan ranked sixth in the country. This showed that the provinces represented by Sichuan mainly relied on the combination of high HC, high RD and high level of OP to produce the high level of coupling coordination development between GSTIE and DEL, which conformed to the typical characteristics of the configuration solution of HC + RD + OP-jointly driven.

(2) RD + OP-dual driven. In configuration H2, high RD, high OP and non-high IS were the core conditions, high HC, high ED, and non-high GS were the marginal conditions, and ER was an independent variable. Since RD and OP had a central role in this configuration, this configuration was named RD + OP-dual driven. H2 showed that regions with low IS and low GS, through high RD and OP, supplemented by higher HC and higher ED, could achieve the coupling and coordinated development between GSTIE and DEL. Under this path, strong or weak ER was not necessary to produce a high CCD between the two subsystems. In this configuration, the consistency and unique coverage were 0.8355 and 0.0406, respectively, that is, it could explain around 83.55% of the cases, and around 4.06% of the cases could be interpreted only by this configuration path. The representative provinces were Hubei and Shandong. In Shandong, for example, the original driving data showed that the level of HC, RD, OP, and ED ranked among the top ten in China, of which RD and OP ranked among the top six, whereas the level of IS and GS ranked among the bottom ten in the country. The provinces represented by Shandong reflected the typical characteristics of high coupling coordination between GSTIE and DEL driven by R&D investment and openness.

(3) HC + GS-dual driven. In configuration H3, high HC and high GS were the core conditions, high RD, high OP, high ED, and high IS were the auxiliary conditions, and ER was an independent variable. As HC and GS played a central role in this configuration, it was named HC + GS-dual driven. H3 showed that regions with high ED, through HC and sufficient GS, and with the help of higher RD and higher IS, could achieve high CCD between GSTIE and DEL. In this configuration, the values of consistency and unique coverage were 0.9130 and 0.1473, respectively, that is, the configuration explained around 91.3% of the cases, and around 14.73% of the cases could only be explained by this configuration path. Beijing and Shanghai were the typical provinces for this type of configuration. In Shanghai, for example, the original driving data showed that Shanghai was at the national leading level in terms of HC, RD, OP, ED, IS, and GS, and the CCD of the province's GSTIE and DEL ranked seventh in the country, with a high CCD. Therefore, in addition to ER, Shanghai has few weaknesses in all aspects. It mainly relied on high HC and high GS as the core driving force and other multiple conditions to achieve high CCD between GSTIE and DEL. This showed that the provinces represented by Shanghai conformed to the typical characteristics of the dual-driven configuration solution of HC and GS.

(4) GS-oriented. In configuration H4, high HC, high RD, high ED, high GS, high ER, and non-high IS were the core conditions, and complementary non-high OP was the marginal condition, which could produce the coordinated development between GSTIE and DEL. H4 showed that areas where the level of OP is not high and the IS is not advanced enough, relying on rich HC, high intensity RD, high ED, high GS, and high-intensity ER, could produce high CCD of GSTIE and DEL. In this configuration, the consistency value was 0.9684 and the unique coverage value was 0.0199, which meant that this configuration path could explain around 96.84% of the cases, and around 1.99% of the cases could be explained only by this configuration path. The representative province was Shaanxi. According to the original driving data, Shaanxi's HC, RD, ED, GS and ER were among the top ten in the country, whereas its OP level was ranked 17th in the country, and the advanced degree of IS was ranked third from the bottom in the country. It was revealed that OP and IS were the development shortcomings in Shaanxi. As GS had obvious development advantages, this configuration was named GS-oriented.

5.3.2 Configuration of non-high CCD

This study also tested the configurations that produced non-high CCD between GSTIE and DEL, and the results are presented in Table 10. The results showed that six configurations generated non-high CCD, the consistency of the solution was 0.9039, and the coverage was 0.6194, indicating that the interpretation of these six configurations was 90.39% and explained around 61.94% of the cases with non-high CCD.

NH1 showed that non-high RD, non-high OP, non-high IS and non-high ER were the core conditions, indicating that in areas with low RD and OP levels, if the IS was not advanced and ER was weak, even if the HC level was low, it would be impossible to achieve high CCD of GSTIE and DEL. NH2a and NH2b had the same core conditions, namely, non-high RD and non-high OP. NH2a showed that irrespective of whether ER was strong, the lack of high HC, high RD, high OP, high ED, and high IS would not produce high CCD even if there was high GS. NH2b showed that the lack of high HC, high RD, high OP and high ER would not achieve high CCD even though DE, IS and GS were high. In NH3, non-high OP, non-high IS, non-high GS, and non-high ER were the core conditions, and high HC, high RD and high ED were the marginal conditions. NH3 showed that areas with a low level of OP and weak ER, where the IS was not advanced enough and GS were insufficient, even if the ED level was high, and even with a high level of HC and a high level of RD, high CCD between GSTIE and DEL would not be achieved. In NH4, non-high RD and non-high OP were the core conditions, and non-high HC, non-high ED, and high GS were the marginal conditions, which showed that in areas with low levels of OP and ED, the lack of adequate RD and HC, even with high GS and high ER, would still lead to non-high CCD between GSTIE and DEL. NH5 showed that the lack of high RD, high ED, high IS and high GS would eventually lead to non-high CCD between GSTIE and DEL even if HC, OP and ER were high.

6 Discussion

Considering environmental pollution and resource constraints, promoting the coordinated development of green scientific and technological innovation and digital economy is the direct embodiment of the core competitiveness of the economy and society, and is also the new focus of high-quality development. This study has the following theoretical and practical values. For theoretical values, on one hand, from the perspective of coupling development, this paper evaluated the coupling and coordinated development of GSTIE and DEL at the provincial level in China, which is a theoretical supplement to the existing research on green innovation and digital economy and expands the idea of studying green innovation and digital economy from the perspective of coupling. On the other hand, considering the causal complexity of the influencing factors, this paper further explored the influencing factors of CCD by using the fsQCA method to reveal the impact of various configurations on the coupling and coordinated development of GSTIE and DEL, which not only complements the studies on the influencing factors of CCD in terms of methods and conclusions but also provides a quantitative basis for the design of the targeted coordinated development policies in various regions. For practical values, first, the evaluation results are conducive to objectively reflecting the current situation and differences in the development of green science and technology innovation and the digital economy in each province, as well as the strength of the coupling coordination relationship between GSTIE and DEL. Second, from the perspective of coupling and configuration, the research results of this study can broaden the direction of regional policymaking and provide a reference for local governments to accurately implement policies to improve the DEL and promote high-quality development.

However, there are still some limitations: First, limited by the availability of public data, the indicator system for measuring GSTIE and DEL may not be comprehensive and accurate enough. Further research can enrich the construction of the indicator system to improve the accuracy of evaluation results (Liu et al., 2022). Secondly, limited by the number of cases, although this study has integrated as many as possible seven important elements that affect the coupling and coordinated development between GSTIE and DEL, it does not include all possible influencing factors. In the future, we can adopt a new analysis perspective or incorporate more influencing factors to study, so as to supplement and enrich the research conclusions of this paper. Third, this paper used static panel data without considering the time effect. It should be the focus of future research to dynamically discuss the configuration relationship by combining QCA and other analysis methods (Luo L. et al., 2022; Sun and Liu, 2022).

7 Conclusion and policy recommendations

7.1.1 Research conclusion

This paper selected 30 provinces and cities as research objects and constructed the CCD index system of GSTIE and DEL. Using the super-efficiency SBM-DEA model to calculate GSTIE, this paper revealed the distribution characteristics of GSTIE and DEL in China's provinces and cities from the perspective of time and space and assessed the coupling coordination level of GSTIE and DEL in various provinces and cities. In addition, the fsQCA method was used to explore the influencing factors of CCD from the perspective of configuration, revealing the differential driving path of the coupling and coordinated development in various provinces and cities. The main conclusions of this study are as follows:

According to the evaluation results, it can be concluded that the level of the digital economy is generally low, and the comprehensive score of China's provincial DEL system has always been lower than that of the GSTIE system, indicating that the provincial DEL system has always been lagging behind the GSTIE system, which is basically consistent with the conclusion of previous studies (Zhao & Meng, 2022). Beijing, Jiangsu, Zhejiang, Shandong, and Guangdong are high-value areas of the DEL, which may be because there is a large scale of digital economy development.

From the CCD evaluation results, it could be found that there were spatial differences in the coupling coordination between GSTIE and DEL systems, showing a spatial characteristic of the eastern region > western region > central region. The coupling coordination of GSTIE and DEL in most provinces has reached a medium or higher level, but the imbalance in some provinces is still obvious. In addition, the results of spatial autocorrelation analysis showed that the local spatial agglomeration characteristics of the CCD between GSTIE and DEL in most provinces and cities were not significant, indicating that it is necessary to strengthen the synergy effect between GSTIE and DEL.

The single-factor necessity test results show that there is no single factor by itself that can lead to the coupling and coordinated development of GSTIE and DEL. However, there are multiple pathways that drives the coordinated development of GSTIE and DEL, each representing a condition configuration composed of different influencing factors. The configuration analysis results showed that five conditional configurations could produce high CCD between GSTIE and DEL, and these five configurations could be categorized into four types: HC + RD + OP-jointly driven, RD + OP-dual driven, HC + GS-dual driven, and GS-oriented.

7.1.2 Policy recommendations

To improve the level of coordinated development of GSTIE and DEL, this research proposes the following policy recommendations on the basis of the above research conclusions.

It is necessary to understand the essence of coordinated development and promote the coordinated development between GSTIE and DEL. First, provinces and cities need to improve their technological innovation capabilities, optimize the innovation environment, and gather innovative talents and innovative capital; increase the R&D funds investment, accelerate the construction of innovation platforms, and provide the essential support for training innovative talents and gathering innovative resources. Second, all provinces and cities should comprehensively optimize the digital economy development environment, improve infrastructure construction of the digital economy, accelerate the deployment of network and information communication facilities, provide policy preferences for the core industries of the digital economy, improve the *status quo* of the low level of regional digital economy development, and reverse its predicament of lagging behind green science and technology innovation. In addition, the government should effectively control the investment in technology resources, attach importance to the transformation and application of effective invention patent achievements, drive the deep integration of green science and technology innovation and industrial development, and empower digital economic development.

Furthermore, it is necessary to overcome regional barriers and strengthen transregional cooperation. All provinces should strengthen cooperation and exchanges on the green science and technology innovation efficiency and the development of the digital economy and jointly build a regional cooperation mechanism for resource sharing to improve China's overall level of coordinated development. We should encourage the eastern region with excellent performance in coordinating the development of GSTIE and DEL to play the role of radiation, guidance, and assistance by improving the incentive mechanism to drive the development of the surrounding areas with a low coupling and coordination level. The central and western regions with a low level of CCD should strengthen cooperation with the eastern developed provinces, draw on excellent experience, conduct appropriate exploration according to their own development status, change the situation where most regions in China have no significant spatial characteristics, and strive for cross-regional linkage cooperation.

Moreover, attempts should be made to give full play to the synergistic role of various elements, and according to local actual conditions, a suitable coordinated development path should be chosen. The linkage and synergy effect of multiple conditions fully reflects the complexity of the coupling and coordinated development of GSTIE and DEL. This means that all regions should, from the overall perspective, establish an effective coordinated, united and unified development mechanism and strengthen the deep integration and optimal allocation of various elements. Additionally, each region should carefully select different coordinated development paths and targeted measures on the basis of its own development status and resource endowment to drive the coupling and coordinated development of green science and technology innovation efficiency and the digital economy.

Data availability statement

The original contributions presented in the study are included in the article/supplementary material, further inquiries can be directed to the corresponding author.

Author contributions

Conceptualization, ZM; Data curation, CX and YG; methodology, ZM; resources, CX; writing—review and editing, ZM and YG; visualization, YG; project administration, YG; funding acquisition, YG. All authors have read and agreed to the published version of the manuscript.

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Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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The impact of the digital economy on carbon emission intensity: Evidence from China

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Evaluating the impact of the digital economy on carbon emission intensity has great significance in promoting sustainable development. Based on the panel data of 30 provinces in China from 2013 to 2019, the level of the digital economy is estimated by using entropy weight and the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) model. The panel vector auto-regressive (PVAR) model is used to analyze the impact of the digital economy on carbon emission intensity. The results show that, first, the comprehensive development level of the digital economy in China is not high, and it tends to rise slowly from 0.208 in 2013 to 0.221 in 2019. Second, the carbon emission intensity of China shows a downward trend from 0.720 in 2013 to 0.607 in 2019. There are significant differences in carbon emission intensity among different regions, and a decreasing trend is seen from the western region to the eastern region. Third, there is a long-term equilibrium relationship between the digital economy and carbon emission intensity. Fourth, the digital economy has a long-term negative effect on carbon emission intensity, but carbon emission intensity has no negative effect on the digital economy. According to the research conclusion, this study puts forward some suggestions.

KEYWORDS

digital economy level, carbon emission intensity, entropy weight method, TOPSIS model, PVAR model

1 Introduction

Since the reform and opening up in 1978, China's economy has developed rapidly. In 2010, China's economy jumped to the second place in the world for the first time (Hou et al., 2021). Although China has made some progress in economic development, the economic development of China mainly relies on fossil fuel consumption. According to the data from the China Energy Statistical Yearbook (2021), the total energy consumption of China was 4.5 billion tons of standard coal in 2020, and the total coal consumption accounted for 62.2%. China's long-term coal-based energy consumption structure has caused serious environmental problems (Zhong et al., 2022). However, with the development of the 5G network, e-commerce, artificial intelligence, and other technologies, the global economy has entered the digital age, and the digital economy has gradually penetrated various fields, such as society and ecology. The digital economy has shown strong carbon emission reduction potential, which has become an important way to alleviate environmental pressure (Yin et al., 2022). In 2021, the scale of the digital economy accounted for 39.8% of China's GDP, ranking second in the world (Yang W et al., 2022). With the huge dividends released by the digital economy, China's economy has gradually changed from extensive development to green and low-carbon development (Zhang et al., 2022). Theoretically, the digital economy is the main engine of high-quality economic development (Li, 2019; Yin and Yu, 2022). The digital economy has a great impact on people's production and life, but there is little research on the relationship between the digital economy and carbon emission intensity. Can digital economy help to curb carbon emissions? Answering this question has practical significance for promoting economic development and reducing carbon intensity. Therefore, based on the provincial panel data of China, this study uses the panel vector auto-regressive (PVAR) model to analyze the relationship between the digital economy and carbon emission. Compared with the existing literature, the contributions of this study are as follows: First, the determination of evaluation indicators is more scientific, and this paper constructs the index system of the digital economy level based on the input and output dimensions. Second, the research perspective of this paper is more comprehensive, and this paper uses the PVAR model to analyze the relationship between the digital economy and carbon emission intensity.

The arrangement of this paper is as follows: The second part is a literature review, which includes the related literature on the digital economy and carbon emission. The third part is the research design, including introducing the research methods and data sources. The fourth part is the research results, which analyze the level of the digital economy in China, the carbon emission intensity, and the relationship between them. The fifth part is the conclusion, which summarizes this research and puts forward suggestions based on the research findings of this paper.

2 Literature review

Research on the digital economy can be traced back to the late 1990s, when economist Don Tapscott first proposed the concept of "digital economy," pointing out that digital technology can produce new economic forms (Ma and Zhu, 2022). Subsequently, many countries accelerated the formulation of development strategies for the digital economy and actively promoted the development of a digital economy based on information networks and data resources (Sorescu and Schreier, 2021). With the emergence of the new generation of digital technologies, the digital economy not only accelerates the upgrading of the traditional industrial structure but also accelerates the process of digital governance (Sama et al., 2022). Compared with the rapid development of the digital economy, the research on how to measure the development level of the digital economy lags behind. Therefore, academia began to research the index construction and measure the development level of the digital economy (Liu, 2022). Li and Liu (2021), based on the cross-sectional data of eight comprehensive economic zones in China in 2018, evaluated the digital economic level from the aspects of digital infrastructure, digital industry development, and digital application, and found that the development level of the digital economy in China was low. Furthermore, some scholars used the analytic hierarchy process (AHP), an entropy method to calculate the level of the digital economy in China, and found that the digital economy in China was on the rise, and the main influencing factors included the construction of digital infrastructure and the application of digital technology (Yang and He, 2022). Considering the regional differences in the development of the digital economy, Shi (2022) used the Theil index to analyze the differences in the digital economy in seven regions of China and found the differences in the digital economy among regions were obvious, and the overall trend was on the rise.

In the field of carbon emissions, the academia mainly focuses on how to measure carbon emission measurement, influencing factors of carbon emission, spatial characteristics, and carbon emission reduction paths, and the research objects are provinces, city clusters, and high energy consumption industries. There are many pieces of research measuring carbon emission, carbon emission intensity, and carbon emission efficiency (Cai et al., 2019; Cary, 2020). Through analysis of the influencing factors of carbon emissions, it is found that the factors that promote carbon emissions include the level of industrialization (Wang and Ma, 2018), economic income (Han et al., 2022), energy consumption (Vujović et al., 2018), etc., while the factors that inhibit carbon emissions include activity intensity (Wang et al., 2018), technological input (Wen et al., 2020), environmental regulations (Zhang et al., 2021), etc. Considering the spatial distribution characteristics of carbon emissions, Su et al. (2018) found the regional differences in urban carbon emissions in China showed a downward trend. Yu et al. (2022a) took the Yangtze River Delta region as the research area, used a spatial correlation network model to analyze the spatial correlations of land-use carbon emissions, and found that land-use carbon emissions in the Yangtze River Delta had a spatial correlation and spillover effects. In addition, it is important to reduce the carbon emissions of the construction industry (Dong et al., 2023). Buildings are considered the last mile toward the carbon-neutral century, and studying the decarbonization potential of commercial building operations has become another mainstream topic (Probst et al., 2021; Xiang et al., 2022).

In terms of the relationship between the digital economy and carbon emissions, there are two different opinions about the relationship between the digital economy and carbon emissions. Some scholars believe the digital economy can curb carbon emissions (Yu et al., 2022b). With the development of the digital economy, more and more cities are implementing the concept of low-carbon development because the digital economy encourages the development of digital inclusive finance and green innovative technologies (Ma et al., 2022a; Zheng and Li, 2022). However, the effect of the digital economy on emission reduction is significantly different in different regions. The effect of the digital economy on emission reduction in eastern China is more prominent than in central China and western China (Yi et al., 2022). In addition, the spatial spillover effect of the digital economy on carbon emissions has also become a hot issue in academia. Improving the development level of the digital economy has a significant impact on the reduction of carbon emissions in neighboring areas (Zhou et al., 2022). Other scholars believe the development of the digital economy contributes to the increase in carbon emissions (Avom et al., 2020). The digital economy accelerated China's import trade, resulting in an increase in consumer demand, which amplified consumption-based carbon emissions (Ma et al., 2022b). Liu et al. (2022) found the increase in information and communication technology can increase carbon emissions. In addition, some scholars found the

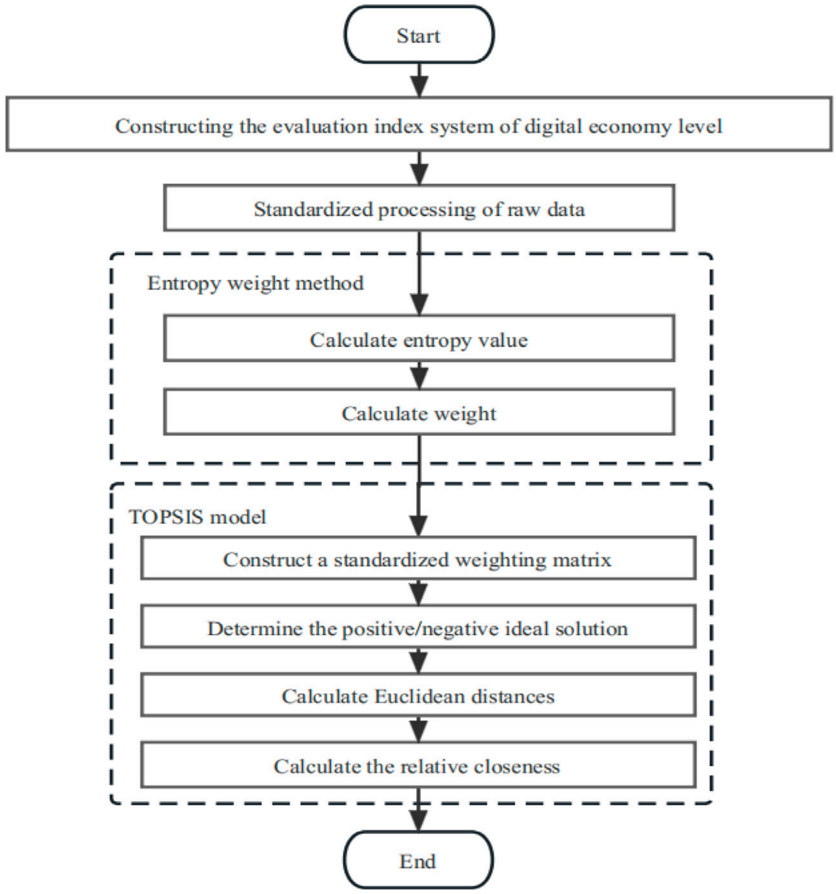

FIGURE 1
Entropy–TOPSIS model.

impact of the digital economy on carbon emissions presents an inverted U-shaped non-linear relationship (Yang Z et al., 2022), which accords with the hypothesis of the environmental Kuznets curve (EKC) (Li et al., 2021).

To sum up, academia has made great achievements in the research of digital economy and carbon emissions, but there are still some research gaps in this field. First, there are many studies on designing digital economic indicators from the perspective of input, but there are few studies on measuring the level of the digital economy from the perspective of input and output. Second, few pieces of literature used the PVAR model to analyze the interactive relationship between the digital economy and carbon emission intensity. The PVAR model not only follows the advantages of the VAR (vector auto-regressive) model but also effectively solves the endogenous and individual heterogeneity problems and accurately reflects the influence of one variable and its lag term on other variables in the model (Dai et al., 2022). Therefore, based on the panel data of 30 provinces in China from 2013 to 2019, this paper first uses the entropy weight–TOPSIS method to evaluate the level of the digital economy in China, then calculates the carbon emission intensity based on the carbon emission coefficient method, and finally, uses the PVAR model to explore the relationship between the digital economy and carbon emission intensity.

3 Research methods and variable description

3.1 Research methods

3.1.1 Entropy–TOPSIS model

In this paper, the entropy weight method and TOPSIS model are combined to evaluate the level of the digital economy in China. The main idea is to standardize the processing of index data, then use the entropy weight method to determine the index weight, and finally use the TOPSIS model to determine the ranking of the digital economy level (Deng et al., 2020; Li S et al., 2022). The process of the entropy–TOPSIS method is as follows (see Figure 1).

Step 1: Standardization of raw data.

$$\text{Positive indicators: } y_{jn} = \frac{x_{jn} - m(x_{jn})}{M(x_{jn}) - m(x_{jn})} \; (1 < j < i, 1 < n < m) \tag{1}$$

$$\text{Negative indicators: } y_{jn} = \frac{m(x_{jn}) - x_{jn}}{M(x_{jn}) - m(x_{jn})} \; (1 < j < i, 1 < n < m) \tag{2}$$

TABLE 1 Digital economic index system.

Target layer	Criteria layer	Primary index	Secondary indicators (units)
Digital economy	Invest	Digital infrastructure	Telephone penetration rate (mobile/hundred people), number of internet broadband access ports (10,000), Number of IPV4 addresses (10,000), number of domain names (10,000)
		Digital innovation elements	Proportion of scientific expenditure to fiscal expenditure (%), number of R&D personnel (person), proportion of R&D funds to GDP (%), number of patent applications (pieces), average number of students in higher education per 100,000 population (person)
	Output	Digital industry	Software business income (10,000 Yuan), telecom business income (100 million Yuan), income from express delivery business (10,000 Yuan)
		Industrial digitalization	Number of websites owned by each hundred enterprises (pieces), E-commerce sales (100 million Yuan), the proportion of enterprises with e-commerce activities (%)

In Formulas 1 and 2, x_{jn} represents the sample value of the n-th index in the j-th object, $M(x_{jn})$ is the maximum value of the sample values, and $m(x_{jn})$ is the minimum value of the sample values.

Step 2: Calculating the index weight.

$$H_n = \left[-\frac{1}{ln(j)}\right]\sum_{j=1}^{i}\left[\frac{y_{jn}}{\sum_{j=1}^{i}y_{jn}} \times ln\left(\frac{y_{jn}}{\sum_{j=1}^{i}y_{jn}}\right)\right] \quad (3)$$

$$w_n = \frac{1-H_n}{\sum_{n=1}^{m}(1-H_n)} \quad (4)$$

In Formulas 3 and 4, H_n represents the entropy value of each index, and w_n represents the weight of each index.

Step 3: Calculating the Euclidean distance and relative closeness.

First, we construct a standardized weighting matrix Q.

$$Q = q_{jn} = w_n \times \frac{y_{jn}}{\sum_{j=1}^{i}y_{jn}} \quad (5)$$

Second, we determine the positive ideal solution $Q+$ and negative ideal solution Q-.

$$Q^+ = [q_1^+, q_2^+, ..., q_m^+] \quad (6)$$

$$Q^- = [q_1^-, q_2^-, ..., q_m^-] \quad (7)$$

Third, we calculate the Euclidean distances $D+$ and D-.

$$D^+ = \sqrt{\sum_{n=1}^{m}(Q-Q^+)^2} \quad (8)$$

$$D^- = \sqrt{\sum_{n=1}^{m}(Q-Q^-)^2} \quad (9)$$

Fourth, we calculate the relative closeness P.

$$P = \frac{D^-}{D^+ + D^-}(0 < P < 1) \quad (10)$$

3.1.2 Panel vector auto-regressive model

This study chose the PVAR and Gaussian mixture model (GMM) methods to analyze the relationship between the digital economy level and carbon emission intensity. The PVAR model not only follows the advantages of the vector auto-regressive (VAR) model but also effectively solves the endogenous and individual heterogeneity problems and accurately reflects the influence of one variable and its lag term on other variables in the model (Dai et al., 2022). The formula is as follows:

$$Z_{it} = \beta_0 + \sum_{j=1}^{n}\beta_j Z_{i,t-j} + f_i + g_t + h_{it}\,(i = 1, 2, ..., 30; t = 1, 2, ..., 7) \quad (11)$$

In Formula (11), Z_{it} is an endogenous variable, including digital economy level (CE) and carbon emission intensity (CO_2). β_0 represents the intercept term, j represents the lag order, f_i represents the individual effect, g_t represents the time effect, and h_{it} represents the random error term.

3.2 Variable description and data sources

3.2.1 Digital economy level

Ding et al. (2021) pointed out that the digital economy is the economic output brought by the digital input. Combined with China's digital economy development report, this study designs two criteria layers of input and output. Furthermore, it is subdivided into four first-level indicators, the digital input is represented by digital infrastructure and digital innovation elements, and the digital output is represented by digital industry and industrial digitalization (see Table 1). Referring to the research of Li and Wang (2022), digital infrastructure includes telephone penetration rate, the number of internet broadband access ports, and the number of domain names. Referring to the research of Zhao et al. (2022a), innovative elements include the number of scientific research and experimental development (*R&D*) personnel, the proportion of R&D funds to gross domestic product (*GDP*), etc. Referring to the research of Wang and Shi (2021), the digital industry refers to software business income and telecom industry income, and industrial digitalization refers to the number of websites owned by every hundred enterprises and e-commerce sales. The data of this paper comes from China Statistical Yearbook (2014–2020), and some missing data are filled in by interpolation.

TABLE 2 Development level of the digital economy in China from 2013 to 2019.

Region	Province	2013	2014	2015	2016	2017	2018	2019
Eastern China	Guangdong	0.869	0.789	0.879	0.879	0.881	0.890	0.896
	Jiangsu	0.580	0.516	0.624	0.619	0.609	0.643	0.660
	Zhejiang	0.480	0.615	0.523	0.513	0.511	0.539	0.552
	Beijing	0.478	0.695	0.474	0.477	0.464	0.452	0.450
	Shanghai	0.454	0.622	0.481	0.470	0.460	0.459	0.468
	Shandong	0.501	0.337	0.441	0.450	0.451	0.428	0.401
	Fujian	0.242	0.435	0.280	0.269	0.281	0.292	0.294
	Liaoning	0.248	0.388	0.275	0.265	0.262	0.269	0.270
	Tianjin	0.205	0.281	0.221	0.206	0.199	0.207	0.198
	Hebei	0.137	0.183	0.159	0.150	0.158	0.165	0.166
	Hainan	0.101	0.256	0.099	0.094	0.092	0.099	0.106
	Average	0.390	0.465	0.405	0.399	0.397	0.404	0.406
Central China	Anhui	0.184	0.136	0.197	0.192	0.192	0.203	0.203
	Hubei	0.166	0.169	0.179	0.171	0.173	0.182	0.181
	Henan	0.157	0.144	0.164	0.159	0.159	0.168	0.169
	Hunan	0.140	0.115	0.156	0.149	0.155	0.165	0.165
	Jiangxi	0.110	0.099	0.139	0.134	0.149	0.153	0.156
	Shanxi	0.096	0.188	0.113	0.106	0.106	0.109	0.106
	Heilongjiang	0.112	0.169	0.105	0.103	0.106	0.101	0.093
	Jilin	0.075	0.213	0.088	0.083	0.082	0.086	0.086
	Average	0.130	0.154	0.143	0.137	0.140	0.146	0.145
Western China	Sichuan	0.184	0.180	0.201	0.195	0.198	0.205	0.206
	Shaanxi	0.118	0.257	0.128	0.119	0.118	0.126	0.126
	Chongqing	0.117	0.183	0.129	0.120	0.121	0.130	0.127
	Inner Mongolia	0.069	0.292	0.086	0.076	0.072	0.082	0.083
	Xinjiang	0.059	0.262	0.076	0.069	0.065	0.071	0.072
	Ningxia	0.072	0.234	0.077	0.070	0.066	0.074	0.076
	Guangxi	0.079	0.053	0.086	0.083	0.084	0.084	0.084
	Qinghai	0.049	0.228	0.059	0.053	0.048	0.056	0.059
	Yunnan	0.067	0.077	0.069	0.068	0.070	0.072	0.073
	Gansu	0.053	0.115	0.055	0.049	0.049	0.054	0.053
	Guizhou	0.049	0.090	0.051	0.049	0.049	0.051	0.051
	Average	0.083	0.179	0.093	0.087	0.086	0.091	0.092
National average		0.208	0.277	0.221	0.215	0.214	0.220	0.221

3.2.2 Carbon emission intensity

Carbon emission intensity is measured by the proportion of carbon emissions to GDP (gross domestic product). The specific formula is as follows:

$$CO_2 = \frac{C_i}{GDP} \tag{12}$$

In Formula(12), referring to the research of Zhao et al. (2022b), this paper selects the eight kinds of carbon sources,

TABLE 3 Carbon emission intensity in China from 2013 to 2019.

Region	Province	2013	2014	2015	2016	2017	2018	2019
Eastern China	Guangdong	0.298	0.276	0.258	0.240	0.227	0.217	0.194
	Jiangsu	0.401	0.367	0.354	0.334	0.299	0.276	0.263
	Zhejiang	0.352	0.328	0.311	0.280	0.270	0.244	0.228
	Beijing	0.188	0.183	0.167	0.142	0.129	0.123	0.105
	Shanghai	0.403	0.338	0.330	0.294	0.276	0.247	0.220
	Shandong	0.630	0.628	0.631	0.629	0.592	0.587	0.649
	Fujian	0.338	0.359	0.322	0.273	0.258	0.254	0.230
	Liaoning	0.801	0.762	0.741	0.964	0.943	0.933	1.043
	Tianjin	0.444	0.393	0.369	0.324	0.313	0.322	0.434
	Hebei	0.948	0.872	0.856	0.797	0.735	0.765	0.793
	Hainan	0.621	0.627	0.655	0.582	0.513	0.495	0.466
	Average	0.493	0.467	0.454	0.442	0.414	0.406	0.420
Central China	Anhui	0.571	0.546	0.517	0.465	0.436	0.408	0.332
	Hubei	0.418	0.384	0.353	0.320	0.303	0.269	0.245
	Henan	0.556	0.517	0.489	0.444	0.383	0.345	0.282
	Hunan	0.369	0.325	0.321	0.296	0.287	0.254	0.232
	Jiangxi	0.415	0.386	0.380	0.350	0.330	0.313	0.284
	Shanxi	1.783	1.813	1.798	1.709	1.684	1.765	1.835
	Heilongjiang	0.745	0.726	0.720	0.722	0.698	0.625	0.788
	Jilin	0.613	0.574	0.527	0.493	0.487	0.457	0.607
	Average	0.684	0.659	0.638	0.600	0.576	0.554	0.576
Western China	Sichuan	0.412	0.399	0.358	0.316	0.273	0.230	0.216
	Shaanxi	0.847	0.817	0.794	0.756	0.683	0.599	0.615
	Chongqing	0.349	0.336	0.308	0.270	0.249	0.222	0.193
	Inner Mongolia	1.298	1.266	1.258	1.251	1.482	1.574	1.754
	Xinjiang	1.543	1.564	1.567	1.598	1.508	1.395	1.328
	Ningxia	2.237	2.129	2.087	1.912	2.160	2.210	2.368
	Guangxi	0.472	0.435	0.387	0.373	0.390	0.367	0.372
	Qinghai	0.904	0.779	0.692	0.751	0.709	0.636	0.609
	Yunnan	0.609	0.505	0.425	0.388	0.372	0.395	0.317
	Gansu	1.011	0.939	0.920	0.836	0.817	0.766	0.733
	Guizhou	1.027	0.865	0.758	0.718	0.615	0.527	0.477
	Average	0.974	0.912	0.868	0.833	0.842	0.811	0.817
National average		0.720	0.681	0.655	0.627	0.614	0.594	0.607

such as coal and natural gas, to calculate the carbon emission (C_i) in combination with the carbon emission coefficient. The formula is as follows:

$$\boldsymbol{C_i} = \sum_{i=1}^{8} \boldsymbol{E_i} \times \boldsymbol{\alpha_i} \times \boldsymbol{\mu_i} \tag{13}$$

In Formula (13), E_i represents the energy consumption, α_i represents the carbon emission coefficient, and μ_i represents the standard coal conversion coefficient. The data comes from China Energy Statistics Yearbook (2014–2020), and some missing data are filled in by interpolation.

4 Results and discussion

4.1 Comprehensive evaluation of the development level of digital economy

The entropy weight–TOPSIS method is used to evaluate the level of the digital economy. The specific results are shown in Table 2.

As can be seen from Table 2, the level of the digital economy in China increased from 0.208 in 2013 to 0.221 in 2019, with an increase of 6.25%, indicating the level of the digital economy is slowly rising. Specifically, the level of the digital economy rose sharply from 2013 to 2014, reached 0.277 in 2014, then showed a downward trend from 2014 to 2017, and slowly rose from 2017 to 2019. In addition, there are significant differences in the level of the digital economy in China, which is consistent with the research of Xu et al. (2021). In 2019, the average value of the digital economy was 0.221 in China, 0.406 in eastern China, 0.145 in central China, and 0.092 in western China. The level of the digital economy in eastern China is higher than the national average, while that in central China and western China is significantly lower than the national average, which shows the development among regions is unbalanced, and the level of the digital economy decreases from the eastern region to western region. It is because the eastern region has a superior geographical location and a good economic and talent base, which creates conditions for technological innovation. However, the economic development level in central China and western China is low, which leads to a weak foundation of digital economy development and a slow growth rate. Second, Guangdong Province in eastern China ranked first in digital economy level, and Hainan Province ranked 19th, which is far behind other provinces in eastern China. The average values of the digital economy in Xinjiang, Ningxia, Guangxi, Qinghai, Yunnan, Gansu, and Guizhou in the western region are below 0.1, and while Sichuan ranks 10th, Sichuan's digital economy is growing faster than other provinces in the western region. As can be seen from Table 2, there are great differences in the development of digital economy in different provinces.

4.2 Measurement of carbon emission intensity

Based on the carbon emission coefficient method, this study evaluates the carbon emission intensity of different provinces in China from 2013 to 2019, and the results are shown in Table 3.

As can be seen from Table 3, China's carbon emission intensity shows a downward trend, with the carbon emission intensity decreasing from 0.720 in 2013 to 0.607 in 2019. This is consistent with the research conclusion by Wang and Zheng (2021). It is worth noting that the carbon intensity levels of Shanxi and Ningxia in 2013 and 2019 were at high levels. The reasons are as follows: First, Shanxi and Ningxia have great responsibilities in safeguarding national energy, so the coal development intensity is high, but the economic level is low, and the growth rate is slow, resulting in the high carbon intensity. Second, it is difficult to transform the energy consumption structure and the industrial structure. The carbon emission intensity of Inner Mongolia increased from 1.298 in 2013 to 1.754 in 2019, an increase of 35.13%. According to the resource curse theory (Wu et al., 2021), Inner Mongolia is constrained by a heavy industrial structure, which has the development dilemma of high energy consumption and low output value. In addition, the preferential electricity price policy in Inner Mongolia has also promoted the transfer of high energy-consuming industries, resulting in the carbon intensity of Inner Mongolia rising instead of falling. In addition, the carbon emission intensity of China has a discrepancy, showing a decreasing trend from western China to eastern China. This is because a series of projects, such as power transmission from western China to eastern China and gas transmission from western China to eastern China, have been started after the western development, and the rapid economic development in western China has caused air pollution. Because the projects have the characteristics of a long cycle and large scale, it is difficult to reduce the carbon emission intensity in western China.

4.3 The relationship between digital economy and carbon emission intensity

4.3.1 Unit root test

To ensure the accuracy of this paper and avoid false regression caused by unstable variables (Duan et al., 2022), this paper uses the unit root test, including the Levin–Lin–Chu test (LLC), different root Im–Pesaran–Shin test (IPS), and short panel Harris–Tzavalis test (HT). The results show that the LLC, IPS, and HT tests of the digital economy level (*lnCE*) all reject the original hypothesis at a 1% significance level. In other words, the original sequence is stable, and the unit root test is passed. The original sequence of carbon emission intensity ($lnCO_2$) failed the IPS and HT tests. This means the original sequence has a unit root. The results in Table 4 show that the digital economy and the carbon emission intensity that the first-order difference data have passed LLC, IPS, and HT tests. It indicates that the two variables are first-order single integration, which can be used for the co-integration test.

4.3.2 Co-integration test

Based on the single integration of the same order, the co-integration test is used to further analyze whether there is a long-term co-integration relationship between variables (Li Z et al., 2022). This paper used the Pedroni and Kao tests to test the co-integration relationship between the digital economy and carbon emission intensity. The original assumption is that there is no co-integration relationship between variables. The results in Table 5 show that the Pedroni and Kao tests reject the original hypothesis at 1% significance level. In other words, there is a long-term equilibrium relationship between digital

TABLE 4 Unit root test.

Variable	LLC	IPS	HT
lnCE	−26.206 (0.000)	−9.589 (0.000)	−0.501 (0.000)
$lnCO_2$	−6.108 (0.000)	2.919 (0.998)	0.176 (0.500)
ΔlnCE	−42.311 (0.000)	−24.029 (0.000)	−0.466 (0.000)
$\Delta lnCO_2$	−8.428 (0.000)	−2.183 (0.015)	−0.077 (0.000)

TABLE 5 Co-integration test.

Test	Statistical indicators	Statistics	*p* value
Pedroni	Modified Phillips–Perron t	4.502	0.000***
	Phillips–Perron t	−29.770	0.000***
	Augmented Dickey–Fuller t	−135.919	0.000***
Kao	Modified Dickey–Fuller t	4.500	0.000***
	Dickey–Fuller t	−6.901	0.000***
	Augmented Dickey–Fuller t	6.624	0.000***
	Unadjusted modified Dickey	−3.341	0.004***
	Unadjusted Dickey–Fuller t	−22.182	0.000***

***, **, and * represent significance levels of 1%, 5%, and 10%, respectively; the robust standard error is in brackets.

economy level and carbon emission intensity, and the PVAR model can be used.

4.3.3 Optimal lag order test

Before using the PVAR model, it is necessary to discriminate the optimal lag order of variables. Among them, the optimal lag order is determined according to the minimum value of AIC (Akaike information criterion), BIC (Bayesian information criterion), and HQIC (Hannan–Quinn information criterion) (Yuan et al., 2022). The results in Table 6 show that the optimal lag order selected by BIC is first order and the optimal lag order selected by AIC and HQIC is second order. According to the majority vote principle, the optimal lag period of the model is determined to be second order.

TABLE 6 Test results of optimal lag order.

Lag	AIC	BIC	HQIC
1	−0.539	−25.671*	−10.369
2	−7.925*	−24.679	−14.479*
3	−6.381	−14.759	−9.658

*The optimal lag order selected by the corresponding criteria.

4.3.4 GMM estimation based on the PVAR model

GMM is used to estimate the parameters of digital economy level (*lnCE*) and carbon emission intensity ($lnCO_2$), and the estimated results are shown in Table 7. Column (1) is the regression result which takes the digital economy level as the explained variable. The result shows that the coefficient between carbon emission intensity lagging by two periods and the digital economy level is negative, but it fails to pass the significance level test, which indicates that the effect of carbon emission intensity on the digital economy level is not strong in the short term. Column (2) is the regression result which takes carbon emission intensity as the explained variable. The result shows that the digital economy level lagging by two periods has a significant inhibitory effect on carbon emission intensity, indicating that the higher the digital economy level the lower the carbon emission intensity. This is because the digital economy is driving the economy from factor-driven to innovation-driven, which not only helps to optimize the industrial structure, but also helps to improve energy efficiency. This transformation stimulates the high-quality development of the economy and reduces carbon emission intensity. It is consistent with the research of Wang et al. (2022a). In other words, the digital economy has a significant negative effect on carbon emissions. The difference between the research of Wang et al. (2022b) and this study is that this paper considers endogenous variables and uses the PVAR model to analyze the two-way interactive relationship between the digital economy and carbon emissions, deepening the relevant research results.

4.3.5 Impulse response analysis

To analyze the impact changes between digital economy level and carbon emission intensity, the impulse response function is used for dynamic analysis. Referring to Wang et al. (2022a), the number of sample periods set by the model is 10. Combined with the Monte Carlo method, the number of repetitions of data simulation is 500, and the impulse response function results are shown in Figure 2. In Figure 2, the horizontal axis indicates the number of lag periods, the vertical axis indicates the degree of the impulse response, the upper and lower dashed lines indicate the upper and lower bounds of the 95% confidence interval, and the solid line in the middle indicates the impulse response value of the response variable after being impacted by a standard deviation unit.

As can be seen from Figure 2A, the level of the digital economy has a positive response to itself, reaching the maximum value in the current period and showing a downward trend as a whole. In the second period, the maximum negative response is reached, and it begins to show an upward trend. In the third period, it rises to zero. It shows that the level of the digital economy is strengthened by itself and has a long-term promoting effect. As can be seen from Figure 2B, the impact of the digital economy on carbon emission intensity is negative and gradually becomes stable. Specifically, the level of the digital economy has a negative impact on carbon emission intensity from the current period, reaching the maximum negative impact in the second period. However, the overall trend rises and is kept stable below zero level. It shows that the improvement of the digital economy can inhibit carbon

TABLE 7 GMM parameter estimation results.

Variable	(1)lnCE	(2)lnCO_2
L2.lnCE	−0.038** [0.019]	−0.699* [0.041]
L2.lnCO_2	−0.022 [0.085]	0.098 [0.124]

***, **, and * represent significance levels of 1%, 5%, and 10%, respectively; the robust standard error is in brackets.

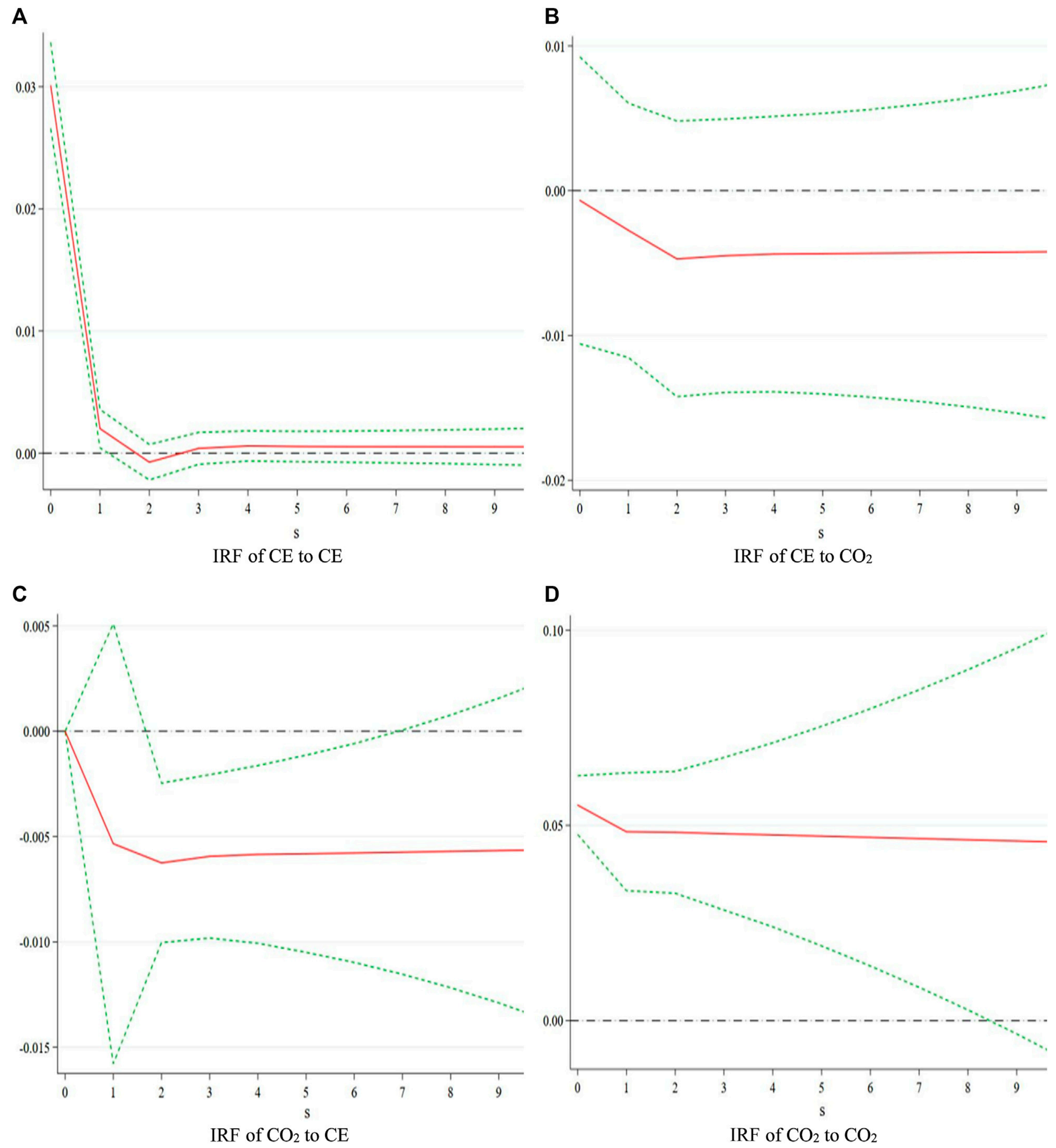

FIGURE 2
Impulse response function. **(A)** IRF of CE to CE, **(B)** IRF of CE to CO_2, **(C)** IRF of CO_2 to CE, **(D)** IRF of CO_2 to CO_2.

TABLE 8 Variance decomposition.

Variable	Variable contribution rate			Variable	Variable contribution rate		
	Stage	lnCE	$lnCO_2$		Stage	lnCE	$lnCO_2$
lnCE	1	1.000	0.000	$lnCO_2$	1	0.000	1.000
	2	0.970	0.030		2	0.001	0.999
	3	0.931	0.069		3	0.004	0.996
	4	0.899	0.101		4	0.005	0.995
	5	0.869	0.131		5	0.006	0.994
	6	0.842	0.158		6	0.006	0.994
	7	0.817	0.183		7	0.006	0.994
	8	0.794	0.206		8	0.007	0.993
	9	0.772	0.228		9	0.007	0.993
	10	0.751	0.249		10	0.007	0.993

emission intensity. On the whole, in the short term, the digital economy plays a prominent role in restraining carbon emission intensity, while in the long term, the impact of the digital economy on carbon emission intensity has a certain time lag, which indicates the impact of the digital economy on carbon emission intensity is more lasting and far-reaching. Existing studies have shown that the digital economy promotes green economic development. For example, Song et al. (2022) found that the development of the digital economy can improve urban air quality. Kong and Li (2022) found that the development of the digital economy is helpful in improving the efficiency of a green economy. This study discusses the relationship between the digital economy and carbon intensity and finds digital economy contributes to the reduction of carbon intensity, expanding related research. Figure 2C shows a downward trend after the impact of carbon emission intensity on the digital economy level and tends to be stable after the third period, indicating carbon emission intensity has a long-term inhibitory effect on the digital economy level. Figure 2D shows that after the impact of one standard deviation of carbon emission intensity, its effect begins to decline and gradually becomes stable after reaching the maximum value in the current period, which indicates carbon emission intensity has a significant promoting effect on itself.

4.3.6 Variance decomposition

In this paper, variance decomposition is utilized to analyze the contribution of various structural impulses to the fluctuation of the digital economy level and carbon emission intensity. The results are shown in Table 8.

As can be seen from Table 8, in terms of *lnCE* (digital economy level), the contribution of *lnCE* to itself reaches 75.1% in period 10, while the variance contribution of $lnCO_2$(carbon emission intensity) is 24.9%. It can be seen that the contribution rate of *lnCE* mainly comes from itself and $lnCO_2$. In terms of $lnCO_2$, the variance contribution of $lnCO_2$ to itself is higher in period 10, which is more than 99% stable, while the contribution rate of *lnCE* to $lnCO_2$ is lower in period 10, which is less than 1% stable. It indicates that the self-strengthening ability of $lnCO_2$ is stronger. From the results of variance decomposition, we can see that the variance contribution rate of the digital economy and carbon emission intensity mainly comes from its own strengthening. In the long run, the impact of carbon emission intensity on the level of the digital economy has been strengthened over time.

5 Conclusion

Based on the data from 30 provinces in China from 2013 to 2019, this study analyzes the impact of digital economy development on carbon emission intensity. The research findings are as follows: 1) The development level of digital economy development in China is not high but has a slow upward trend. The development level of the digital economy in China has significant differences in different provinces. Specifically, the development level of the digital economy is decreasing from eastern China to western China. This indicates that while accelerating the development of the digital economy, China needs to pay attention to the regional differences in the digital economy. 2) China's carbon emission intensity has declined, and the regional differences are significant. Specifically, the carbon emission intensity in eastern China is the lowest, followed by central China, and highest in western China. 3) Through the co-integration test, there is a long-term equilibrium relationship between the digital economy and carbon emission intensity. In addition, the optimal lag period of the model is 2. The GMM estimation results based on the PVAR model show the digital economy lagging behind two periods and significantly inhibiting carbon emission intensity. 4) From impulse response analysis, the digital economy has a long-term inhibitory effect on carbon emission intensity. This shows the digital economy has a certain climate improvement effect, and promoting the development of the digital economy can reduce the carbon emission intensity.

Based on the research findings, the following suggestions are put forward:

1) We should strengthen the construction of digital infrastructure such as internet broadband, 5G base stations, and artificial intelligence and, at the same time, actively apply digital technology to change resource waste. 2) China should strengthen the input of digital innovation elements. On the one hand, the government should rationally arrange financial expenditure and improve the coverage and innovation of information technology to empower the rapid development of the digital economy. On the other hand, high-quality digital talents should be cultivated to enhance the application ability of information technology and to provide a talent guarantee for the improvement of the digital economy. 3) We should steadily promote the development of industrial digitalization and intensify the development of the digital industry. In terms of industrial digitalization, enterprises need to penetrate internet technology into every link of production and operation to create a new format of enterprise development. In terms of the digital industry, the current development level is low, and the speed is slow. Therefore, the government needs to increase the support of innovation policies to create a good business environment for the development of the digital industry. 4) Cooperation should be strengthened and digital economy policies should be issued in line with regional characteristics. The eastern region should increase investment in information technology innovation capability, and the central and western regions should increase investment in digital infrastructure and digital innovation elements, thus helping the high-quality development of the digital economy.

This study analyzes the relationship between the digital economy and carbon emissions of 30 provinces in China, but there are still some shortcomings. First, there are significant differences in the digital economy level and carbon emission intensity in different regions of China, so it is a future research direction to study the relationship between the digital economy level and carbon emission intensity in different regions. Second, the factors related to the two variables of the digital economy level and carbon emission intensity are diverse and complex. In the future, other influencing factors should also be considered to further improve the research conclusions. Third, the study period of this paper is only 7 years, which can be appropriately increased in the future.

Data availability statement

The raw data supporting the conclusion of this article will be made available by the authors, without undue reservation.

Author contributions

XZ and MJ drafted the manuscript. YZ conceptualized and designed the study. XZ contributed to materials and analysis. YZ contributed to the theories and revised the manuscript. All authors read and agreed to the published version of the manuscript.

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Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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How can management ability promote green technology innovation of manufacturing enterprises? Evidence from China

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Management ability improves organizational learning and innovation ability, thereby improving enterprise performance. Higher-ability management has more flexible and adaptive strategic thinking and ability, allowing it to quickly capture market opportunities, formulate long-term digital transformation strategies, gain competitive advantage through product and service innovation, and thus promote enterprise innovation performance. The research sample for this paper is A-share listed companies in the manufacturing industry from 2008 to 2019, and it integrates management capabilities with digital economy and green technology innovation. Our research shows that management ability can significantly boost manufacturing enterprises' green technology innovation. Furthermore, regardless of whether the enterprise is state-owned or non-state-owned, management ability is positively related to green technology innovation performance, and internal control has a significant positive regulating effect on the relationship between them. Given these findings, this study offers important insights for strengthening the integration of the digital economy and green transformation, emphasizing the rationality of management capabilities, and developing policies for various enterprises.

KEYWORDS

management ability, green technological innovation, digital transformation, internal control, nature of the property right

1 Introduction

It is necessary to investigate the digital transformation of the manufacturing industry against the backdrop of the digital economy. In 2016, the CPC Central Committee and the State Council issued the Outline of National Innovation-Driven Development Strategy, stating that innovation-driven development is the core support of national power, the core strategy of international competition, and the new trend of economic development, and that innovation-driven development should be included in the national priority strategy. According to data, China's digital economy will be worth 39.2 trillion yuan in 2020, up

3.3 trillion yuan from last year, accounting for 38.6% of GDP, up 2.4 percentage points year on year. Both the “14th Five-Year Plan” and the long-term goal of 2035 propose that we seize the strategic opportunities of the new round of scientific and green technological revolutions and industrial revolutions, actively accelerate digital development, promote digital transformation, promote deep integration of the Internet, big data, artificial intelligence, and the real economy, and build a digital power with international competitiveness. Green innovation is gradually assuming a dominant position as a result of the rapid advancement of digital technology and the continuous advancement of high-quality development. Green innovation can not only directly generate economic value, but it can also have an impact on the ecological environment, achieve environmental performance, and ultimately realize high-quality development for the entire society and economy (Guo and Ji, 2022). China has a new understanding of the digital economy and green innovation as it enters a new era. This research proposes a novel concept of digital green innovation in manufacturing enterprises. It differs from green innovation, which is based on technological advancement, and digital innovation, which is based on artificial intelligence (Gunasekeran et al., 2021; Stahl, 2021; Yao et al., 2022). Green innovation is the process by which digital technologies such as 5G and the Internet of things are integrated into current green innovation and development (H. H. Zhou et al., 2021). Digital and green innovation are no longer separated from these two dimensions, but instead combine to form a more appropriate innovative form for current economic development. The incorporation of digital technology into the process of green innovation not only improves the efficiency of resource integration and the ability of environmental monitoring, but also serves as a key link between green production, life, and a beautiful environment, thereby realizing high-quality green innovation development (Xu and Zhang, 2020). Green manufacturing aims to save energy and reduce consumption through digital technology, intelligent manufacturing, improving production efficiency, and producing the same or even higher output value with less consumption and emissions (Acquah et al., 2021; Mandal et al., 2021). At the same time, the demand for energy-saving products and technologies in green manufacturing coincides with the demand for new products and technologies in intelligent manufacturing (Zhou et al., 2021; Yin et al., 2022a).

The factors that influence enterprise green technology innovation are broadly classified as government factors and market factors. Most studies concentrate on government factors such as environmental regulations (Berrone et al., 2013; Yu et al., 2017; Qu et al., 2022) and government support (Xie et al., 2015; Wu et al., 2022); however, the positive impact of environmental oversight on enterprise green transformation is gradually fading (Leiter et al., 2011). To some extent, environmental regulation will raise enterprise operating costs (Berrone et al., 2013; Qu et al., 2022), discouraging some businesses from pursuing green transformation. Furthermore, while government assistance partially subsidizes the resources required for green technological innovation, this may lead to a problem of dependence. In reality, businesses must be more dynamic and adaptable in their allocation of innovative resources (Diewert et al., 2018; El-Kassar and Singh, 2019; Feng et al., 2022). Due to the limitations of government assistance, some scholars began to investigate the impact of market factors on enterprise green innovation. Lin et al. (2013) used Vietnam as an example to demonstrate that market demand correlates positively with green product innovation. Cuerva et al. (2014) believe that the advancement of external technology promotes the dynamic capability of businesses, thereby enhancing green innovation. Cao et al. (2021) and Feng et al. (2022) stated that digital finance can solve the green innovation financing problem, avoiding over-reliance on the government and putting enterprises in a better position to promote green technological innovation; however, market factors cannot be ignored (Wei et al., 2015; Cao et al., 2021; Yin et al., 2022b).

The digital economy, as the integration of information technology and economic production mode, offers a new strategic option for China’s industrial transformation (Yang et al., 2021). It opens up a new research avenue for green technology innovation by releasing social productive forces, altering knowledge transfer methods, and lowering transaction costs (He et al., 2021; Li et al., 2022). Dou and Gao. (2022) believe that cloud computing and big data will accelerate the transformation of production modes in areas with a high level of digital economy, requiring enterprises to update their equipment (Wang and Fan, 2020), eliminate the traditional production mode (Cao et al., 2021), and achieve a balance between economic and environmental development. Because green technology innovation is more risky than general innovation, sufficient resources are required to support green technology innovation (Wu et al., 2022). Because green technology innovation in enterprises is expensive, risky, and profitable, enterprise managers will not consider green innovation technology when making business decisions. As a result, green technology innovation will be hampered if manufacturing enterprises cannot rationally allocate limited resources. The digital economy can break time and space constraints with digital technologies, leading to the sharing and transfer of labor and capital resources, thereby contributing to green total factor growth rates (Song et al., 2020; Li et al., 2022). Furthermore, it is worth noting that the digital economy is a driver of economic structural transformation, which will aid in the elimination of “highly polluting, energy-intensive” industries and their transformation into green industrial factories (Lu et al., 2021). Based on the above analysis, we have introduced digital transformation as an intermediary between management capabilities and green technology innovation. Management ability is an important human capital of an enterprise that can create a sustainable competitive advantage for the enterprise, play a critical role in the enterprise's survival and development, and have a direct impact on the enterprise's green technological innovation. Currently, research on the economic consequences of managerial competence focuses primarily on corporate performance, earnings management, tax avoidance activities, and investment efficiency, with few empirical studies on managerial competence and enterprise innovation, and the findings are inconsistent. Based on the above analysis, we introduced management ability as a mediating force between the digital economy and green technology innovation.

The level of digitalization has an increasing impact on the performance of green technological innovation. Currently, research on the impact of digital transformation on enterprise innovation performance has been delayed; previously, empirical research on the impact of digital transformation on enterprise innovation performance focused on the macro-level impact of regional innovation level and factor allocation efficiency, while micro-level research on the relationship between digital transformation and enterprise innovation was limited. There have been numerous studies on innovation investment and patent application, but there have been few studies on innovation performance. To summarize, can management ability improve the performance of innovation? How can traditional manufacturing management effectively improve green technological innovation through digital transformation in the post-epidemic era, when the global economic situation is complicated and severe, and the domestic economic growth rate is slowing? Does internal control promote or inhibit management ability and technological innovation in China, given the late implementation of internal control and the imperfect system? What are the distinctions between the functions and influences mentioned above in China's special property rights? All of these are critical issues that require immediate attention.

This paper's main contributions are as follows: First, this paper identifies the text of sample companies' reports, and then determines the degree of digital transformation by dividing the frequency of indicators in the reports from the application of digital technology, which is useful for expanding and enriching relevant theories of digital transformation measurement. Second, this paper employs the DEA BCC model to improve Demerjian's model for measuring management competence by excluding the influence of production scale and by controlling time, bringing the influence of fiscal and monetary policy into the model, in order to more accurately measure the management competence of Chinese listed companies. Third, from the standpoint of digital transformation, this paper establishes a theoretical framework of management ability to promote green technological innovation of enterprises *via* the action path of "digital transformation," thereby expanding the research boundary of related theories. Fourth, it investigates the moderating effect of internal control on the relationship between managerial ability and green technological innovation performance, providing support for the theory of internal control promotion.

2 Theoretical analysis and research hypothesis

2.1 Management ability and green technological innovation

Green technological innovation, as a driving force for economic development, is an important source of competitive advantage for enterprises, which is critical to China's economic development. Sehmookler (1966) believed that "innovation efficiency" is essentially the input-output ratio of enterprise innovation activities when it comes to measuring green technological innovation performance. Charnes and Cooper (1978) proposed using the CCR model of the data envelopment method to measure innovation efficiency, and Banker (1984) broke the traditional constant return on scale limitation by using the BBC model to accurately reflect innovation efficiency. Many academics use financial indicators to investigate and analyze the innovation efficiency of businesses. Hirshleifer et al. (2013) calculated an enterprise's green technological innovation efficiency by dividing the number of patents granted by the sum of R&D expenditures over the previous 2 years. In terms of research on the influencing factors of enterprises' innovation activities, some scholars believe that preferential tax policies, financial subsidies, and intellectual property protection provided by the government can help stimulate enterprises' innovation; enterprises with higher corporate tax rates have less R&D expenditure; the higher the proportion of institutional investors, the higher the proportion of enterprise innovation investment; and the improvement of CEO competiveness.

Management, as an enterprise's most important human capital, plays a critical role in its daily operations and major decisions. The "Top echelon theory" was proposed by Hambrick and Mason (1984), who argued that top managers have different background characteristics and will inevitably be influenced by these characteristics when making decisions. Bertrand and Schoar (2003) believe that management's background characteristics will influence organizational behavior more than time and company specific characteristics. A large number of behavioral research findings show that competent management can develop better strategies and correctly predict competitor behavior. Some scholars measure management ability using the manager's fixed effect, historical rate of return, stock price fluctuation, media attention, and executive compensation, but these methods have limitations. Demerjian et al. (2012) distinguished management competence from corporate efficiency and proposed a DEA-Tobit two-stage analysis method to assess overall management competence. Scholars in the United States and elsewhere began to investigate the economic consequences of management competence, such as corporate profits, investment activities, tax avoidance activities, financing constraints, corporate strategy, earnings management, on-the-job consumption, and so on.

There are few literatures on managerial competence and green technological innovation at the moment, and the conclusions about their relationship are contradictory. Some academics believe that management ability can promote enterprise green technological innovation. For the first time, Chen et al. (2015) demonstrated that management competence can promote enterprise innovation, and that companies with higher management competence can create more valuable products and have a competitive advantage. According to Yung and Chen (2018), managers with higher ability are generally more willing to take risks, which contributes to an increase in enterprise R&D investment. Based on data from publicly traded companies in the manufacturing industry, Cheng and Wang (2019) discovered that management capabilities are positively related to enterprise green technological innovation. Mishra (2019) discovered a U-shaped relationship between business management capability and enterprise R&D, which means that as business management capability improves, enterprise innovation capability decreases and then increases. According to Yung and Chen (2018) research, management capability is negatively related to enterprise innovation investment, with this relationship being stronger in state-owned enterprises. Furthermore, Zhou and Peng (2019) demonstrated empirically, using data from China's A-share listed companies, that high-capacity management would inhibit enterprise innovation input and output for the purpose of risk prevention.

Management plays a critical role in the process of enterprise innovation, and its ability becomes the key to enterprise innovation success. According to Barker and Mueller (2002), management, as the primary source of strategic decision-making, has a direct impact on enterprise green technological innovation. Based on data from publicly traded companies in the manufacturing industry, Cheng and Wang (2019) discovered that management capabilities are positively related to enterprise green technological innovation. The following four aspects highlight management's role in promoting green technological innovation. To begin with, more capable management has a higher risk tolerance and control ability. In the course of their innovation activities, businesses are bound to face a variety of risks. Because higher-level management has greater risk-taking ability, they are more willing to take risks and pursue high-risk innovation projects. At the same time, more capable management can control risks more calmly and be adept at identifying opportunities in risks, allowing them to make sound decisions and promote the improvement of innovation performance. Second, more capable management can better identify investment opportunities and make sound strategic decisions, thereby improving enterprise innovation performance. According to relevant research, high-capacity management can better understand the current company strategy and the industry's development trend, more accurately predict market demand, look for favorable investment opportunities, evaluate the potential investment value, and choose projects with higher NPV to invest in, making them more likely to invest in high-value innovative projects. Third, improved management skills can effectively alleviate financial constraints and provide financial security for enterprise innovation. According to Andreou et al. (2015), higher management ability can reduce the degree of information asymmetry by effectively communicating enterprise value and thus raising more funds. According to Chemmanur et al. (2010), higher management ability has a stronger signal transmission function, which can effectively avoid friction in information communication between inside and outside the enterprise, lower the risk premium of external financing, and thus alleviate the financing constraint problem. Fourth, competent management has strong integration and allocation abilities, allowing it to fully absorb and utilize idle enterprise resources, fully exploit the innovative advantages of researchers, and better implement innovative projects. According to Barney (1991), management ability is an important human capital of an enterprise that can create a sustainable competitive advantage. According to Lee et al. (2018)'s research, high-capacity managers have more experience in enterprise resource management, which allows them to make better use of enterprise resources and implement innovative projects. Furthermore, Chang et al. (2015) discovered that competent managers can influence innovation outcomes by establishing the best framework for R&D personnel, thereby maximizing their innovation potential. According to Zacher (2015), complementary management capabilities can stimulate employees' innovative ability, thereby promoting the team's innovative performance. As a result, we propose the following assumptions:

H1. There is a positive correlation between managerial competence and green technological innovation performance, that is, higher managerial competence can improve the technological innovation performance of enterprises.

2.2 The intermediary effect of digital transformation between management ability and green technological innovation

The impact of digital level on the performance improvement of green technological innovation is increasing as the digital economy develops, and more and more scholars are beginning to pay attention to digital transformation and green technological innovation of enterprises. Abrell et al. (2016), from the perspective of innovation results, believed that digital application improved the operation status of small and medium-sized enterprises, enhanced product performance and service quality, and thus enhanced the innovation performance of enterprises; The research of Jahanmir and Cavadas (2018) found that the continuous improvement of digitalization level had a profound impact on enterprise organization mode, information transmission efficiency, information utilization efficiency and information decision-making management, thus improving the innovation performance level of enterprises to some extent; The research of Nambisan et al. (2017) pointed out that "internet plus" has a positive impact on innovation efficiency, and economic development and government funding have a regulatory effect; The research of Barrett et al. (2015) proves that the digitalization level of enterprises is positively related to the operational innovation performance and market innovation performance of enterprises, and it is found that the digitalization level of different industries has different influences on innovation performance, and there is an optimal threshold for the improvement of innovation performance, so the digital transformation needs to be further deepened; Aslam et al. (2020) has proved that dynamic capability plays an intermediary role in the relationship between digital transformation and enterprise innovation performance.

Management ability is primarily manifested in learning ability, opportunity identification, networking, strategic positioning, business innovation, organization and management, and so on. Enterprise digital transformation cannot be separated from continuous improvement and advancement of management capabilities, which promotes enterprise green technological innovation. First and foremost, the ability of management to identify opportunities and build networks can lead to the acquisition of more key resources required for digital transformation, thereby improving enterprise green technological innovation performance. Premaratne (2002) discovered that resource acquisition has a positive impact on small business performance; that is, small businesses can only improve their performance by establishing entrepreneur networks to acquire key resources. According to Andreou et al. (2015), strong management ability can effectively improve the efficiency of obtaining key resources, thereby promoting enterprise growth. Second, the strong learning ability of management can assist enterprises in better analyzing, forecasting, and evaluating in the complex digital environment, avoiding the risks of the enterprise's digital transformation period, and avoiding the negative impact brought by the innovative performance of digital applications. Third, the management's strong organizational management ability can promote the mutual transmission and sharing of digital knowledge and ability among organization members, and continuously improve the innovation performance with digital resources as the core, thus improving the innovation performance. Enterprises can improve their innovation performance through organizational collaboration and innovation, according to Subramanian (1996) and Hubert and Xuereb (1997). According to Vial (2019), entrepreneurship improves organizational learning and innovation ability, there by improving enterprise performance. Finally, higher-ability management has more flexible and adaptive strategic thinking and ability, allowing it to quickly capture market opportunities, formulate long-term digital transformation strategies, gain competitive advantage through product and service innovation, and thus promote enterprise innovation performance. Li (2020) discovered that entrepreneurs' strategic and management abilities contributed significantly to the growth of businesses. Simultaneously, increased management ability can significantly improve the company's performance by implementing relevant diversification strategies. As a result, we propose the following assumptions:

H2. The management ability can improve the green technological innovation performance of enterprises through digital transformation, that is, digital transformation plays an intermediary role.

2.3 The moderating role of internal control between management ability and technological innovation

Faced with increasingly fierce market competition and a more complex business environment, higher-level management has a deeper understanding and grasp of the internal control system of enterprise business activities, and can flexibly respond to changes in the internal and external environment of enterprises, effectively design and operate internal controls, dynamically and timely adjust internal controls of enterprises, and improve implementation. Demerjian et al. (2013) discovered through questionnaire research that the more attention management pays, the more effective the enterprise's internal control is. According to Bikkil et al. (2015)'s research, higher management ability can better identify the company's internal control defects, thereby improving the quality of internal control.

In 2008, the Basic Standards of Enterprise Internal Control stated unequivocally: "Internal control is a process carried out by the board of directors, the board of supervisors, the managers, and all enterprise employees with the goal of achieving the control objectives. The manager is in charge of organizing and overseeing the enterprise's internal control system on a daily basis ". As more academics become interested in enterprise internal control, two opposing viewpoints emerge. Some academics believe that internal control is beneficial in promoting enterprise green technological innovation. Cheng and Wang (2019) assessed the internal control quality of enterprises using the internal control index of Chinese listed companies and discovered that internal control can improve enterprise innovation. Internal control and R&D subsidy are significantly positively correlated with enterprise innovation and performance, according to research by Chen et al. (2015). According to Hall's (2002) research, the quality of an enterprise's internal control plays a significant role in promoting its innovation performance. Other scholars believe that overly strict institutionalization frequently leads to rigid management, which is incompatible with the implementation of flexible green technological innovation. Jensen (2010) estimated the productivity of 432 companies' enterprise capital expenditure and R&D expenditure items, revealing that many companies had significant inefficiency, demonstrating the failure of internal control systems. Furthermore, Hu and Jefferson (2004) demonstrates that a strict internal control system increases management's innovation risk, and the innovation environment is not guaranteed, limiting management's innovation efficiency. However, because the implementation of internal control in China has been delayed and an overly strict system has yet to be established, the theory of internal control promotion may be more appropriate for China's national circumstances.

The following three aspects primarily reflect the role of internal control in promoting the relationship between management capabilities and green technological innovation. First, high-quality internal control can effectively control enterprise risks and provide assurance for high-risk innovation activities undertaken by management. Through risk assessment and control activities of internal control, enterprises can analyze the scientificity and feasibility of innovative projects, carry out combined development and innovation, and improve the risk warning system, reducing uncertainty in the innovation process, effectively preventing the company's innovation risks, and thus improving the efficiency of green technological innovation. Second, effective internal control can create a good platform for information and communication, promote the transmission of internal and external information, and improve the reliability of financial reports, thereby increasing investors' trust in financial reports, reducing the degree of internal and external information asymmetry, alleviating enterprise financing constraints, and raising more funds for enterprises' innovative activities. Third, internal control can be an effective supervision mechanism in the innovation process. High-level internal control can effectively supervise enterprise management's behavior, prevent management from taking more private benefits in order to "maximize personal benefits," and increase excessive investment caused by agency conflicts, reducing enterprise innovation efficiency. As a result, the following assumptions are advanced in this paper:

H3. Internal control plays a positive regulatory role in the relationship between management capabilities and green technological innovation.

3 Research design

3.1 Sample selection

From 2008 to 2019, China's A-share manufacturing listed companies in Shanghai and Shenzhen were used as research samples in this paper. The original data are from the CSMAR database, the DIB internal control and risk management database, the RESSET database, and the Osiris database, and are manually collected from reports published by listed companies. The samples are screened based on the research needs: first, financial companies with accounting standards that differ significantly from those of other industries are excluded; second, companies with special treatment, such as ST and *ST, are excluded; and finally, companies with missing data are deleted. Following screening, 2084 unbalanced panel data are obtained. To avoid the influence of outliers, the main continuous variables are truncated at the 1% and 99% levels.

3.2 Variable definition

3.2.1 Explained variables

Green technological innovation (IE). According to Sehmookler (1966), "innovation efficiency" is essentially the input-output ratio of enterprises to innovation activities. According to Mao and Zhang (2018), when a patent is in the application stage, it indicates that the enterprise's innovation activities have yielded results, so the number of patent applications is regarded as the most accurate and direct indicator of innovation output. This paper, based on the research method of Hirshleifer et al. (2013), focuses on the ratio of innovation output to R&D investment as a measure of green technological innovation performance (IE). The precise method is to divide the number of patent applications by the total R&D investment over the previous 3 years. Because the value of green technological innovation performance is small, this paper has done some treatment to facilitate observation, and the detailed calculation method is shown in Table 1.

3.2.2 Explain variables

Management abilities (MA). Demerjian et al. (2012) proposed the DEA-Tobit two-stage analysis method in this paper, which separated management ability from company efficiency and then measured management ability as a whole. The BCC model of Data Envelopment Analysis (DEA) is used in the first stage to estimate the company's efficiency (FE) by industry, as shown in model (2). Operating income (Sales) is

TABLE 1 Definition of main variables.

Variable type	Variable symbol	Variable title	Variable definition
Explained variable	IE	Green technical innovation	Ln (number of patent applications)
Explanatory variable	MA	Management ability	Residual of model (4)
mediator variable	DT	Digital transformation	Ln (the frequency of digital transformation indicators in the report is the second and +1)
regulated variable	IC	Internal control	Ln (Dibo control index)
	Size	Company size	Ln (total assets at the end of the period)
	Age	Established years	Ln (current year -IP0 year +1)
Control variable	Lev	Asset-liability ratio	Ending liabilities/ending total assets
	ROA	return on assets	Net profit/total assets at the end of period
	ATO	turnover of total assets	Operating income/total assets at the end of period
	RD	Research and development intensity	Net R&D expenditure/total assets at the end of the period

the output variable, while the input variables are operating cost (CoGS), sales and management expenses (SG&A), net assets, plant and equipment (PPE), net R&D expenditure (R&D), Goodwill, and net Intangible assets (intangibles). The variables Sales, CoGS, and SG&A are current year data; the variables PPE, R&D, Goodwill, and Intangible are end-of-year data, because management's past decisions on these assets are expected to affect current income.

3.2.3 Intermediary variables

Transformation to digital (DT). At the moment, scholars both at home and abroad use two main measurement methods for digital transformation. The first step is to create a digital transformation questionnaire and assess it using a Likert7-point scale. The second step is to recognize the text of the sample company's report, and finally to determine the degree of digital transformation by calculating the frequency of the breakdown indicators of artificial intelligence technology, blockchain technology, cloud computing technology, big data technology, and digital technology application in the report. Based on the practice of Flammer (2021), this paper employs the Robert frequency of digital transformation indicators in sample company reports and the natural logarithm plus 1 as digital transformation proxy variables. The greater the value of this indicator, the greater the enterprise's degree of digital transformation.

3.2.4 Adjustment variables

Internal control (IC) (IC). Domestic scholars typically assess the quality of internal control based on the internal control defects disclosed by enterprises or the development of comprehensive internal control evaluation indicators. Shenzhen Dibo Company's internal control index (since 2000) comprehensively, objectively, and comprehensively reflects the internal control quality of enterprises across seven dimensions: internal environment, control activities, risk assessment, information communication, supervision and inspection, whether the accounting firm issues an assessment report, and whether independent directors and the board of supervisors express their opinions. As a result, the natural logarithm of "internal control index" is used as a proxy variable for internal control quality in this paper (IC). The higher the value of this index, the more effective the enterprise internal control system and the higher the quality of internal control.

3.2.5 Control variables

In order to control other factors that may affect the green technological innovation performance of enterprises, this paper mainly selects company Size (size), established years (Age), asset-liability ratio (Lev), return on assets (ROA), total asset turnover rate (ATO) and R&D intensity (RD) as control variables, and controls the annual dummy variables. See Table 1 for definitions of variables.

4 Model building

4.1 Benchmark model

Multiple regression analysis is widely used in fields such as econometrics, ecology and finance (Dormann et al., 2013). Many studies often use a set of predictor variables to measure the response to a particular variable. In view of Wei et al. (2015) and He et al. (2021), multiple regression models have been used to measure the relationship between digital technology, management ability, and green innovation. To test the relationship between regional digital economy and green technology innovation, we constructed the following regression models (see model Eq. 1 and model Eq. 2).

The BCC model of Data Envelopment Analysis (DEA) is used in the first stage to estimate the company's efficiency (FE) by industry, as shown in model Eq. 2. Operating income (Sales) is the output variable, while the input variables are operating cost (CoGS), sales and management expenses (SG&A), net assets, plant and equipment (PPE), net R&D expenditure (R&D), Goodwill, and net Intangible assets (intangibles). The variables Sales, CoGS, and SG&A are current year data; the variables PPE, R&D, Goodwill, and Intangible are end-of-year data, because management's past decisions on these assets are expected to affect current income.

$$\min\left[\theta - \varepsilon\left(\sum_{i=1}^{n} s_i^- + \sum_{r=1}^{s} s_r^+\right)\right]$$
$$s.t. \sum_{j=1}^{n} x_{ij}\lambda_j + s_i^- = \theta x_{ij_0}, i \in (1, 2\cdots m)$$
$$\sum_{j=1}^{n} Y_{rj}\lambda_j - s_r^+ = \theta y_{rj_0}, r \in (1, 2, \cdots s)$$
$$\sum_{j=1}^{n} \lambda_j = 1$$
$$\theta, \lambda_j s_i^-, s_r^+ \geq 0, j = 1, 2, \cdots n \quad (1)$$

In the above equation, s_i^-, s_r^+ denotes the slack variable, m and s are the number of input and output indicators, and x_{ij_0}, y_{rj_0} is the i th input term and the j th output term of the j_0 th decision unit. Furthermore, the general value of ε takes the form 10^{-6}, which indicates positive infinity. Where $= 1, s_i^- = 0, s_r^+ = 0$, then the decision unit DMU_j has reached a DEA effective state, i.e. optimal technical efficiency; where $\theta < 1, s_i^- \neq 0, s_r^+ \neq 0$, then DMU_j means that the decision unit is DEA ineffective, with redundant inputs and insufficient outputs.

$$\text{MaxFE} = \frac{\text{Sales}}{\varphi_1\text{COGS} + \varphi_2\text{SG\&}A + \varphi_3\text{PPE} + \varphi_4\text{R\&}D + \varphi_5\text{Goodwill} + \varphi_6\text{Intangible}} \quad (2)$$

In the second stage, this paper employs the Tobit regression model for regression, with the residual error representing the company's management capability (MA), as shown in model Eq. 3. Size denotes the size of the company; Market denotes the market share; F denotes the level of free cash flow; Age denotes the number of years listed; BSC denotes the diversified management level of enterprises, as measured by the Herfindal index of operating income; and FCI denotes foreign business. If there is income from overseas business, the value will be 1; otherwise, it will be 0.

$$\text{FE} = \alpha_0 + \alpha_1\text{Size} + \alpha_2\text{MarketShare} + \alpha_3\text{FCF} + \alpha_4\text{Age} + \alpha_5\text{BSC} + \alpha_6\text{FCI} + \sum\text{Industry} + \sum\text{Year} + \varepsilon \quad (3)$$

4.2 Mediating effect model

Based on the above variable design, we develop model Eq. 4 to investigate the impact of management capabilities on enterprise green technological innovation performance. We use the explained variables that lag by one period because the innovation output has a certain lag. Using the method of Baron and Kenny (1986), this paper develops models Eqs. 5, 6 to test the intermediary effect of digital transformation. We developed a model to investigate the moderating effect of internal control on management ability and green technological innovation performance Eq. 7.

$$\text{IE}_{i,t+1} = \beta_0 + \beta_1\text{MA}_{i,t} + \beta_2\text{Size}_{i,t} + \beta_3\text{Age}_{i,t} + \beta_4\text{Lev}_{i,t} + \beta_5\text{ROA}_{i,t} + \beta_6\text{ATO}_{i,t} + \beta_7\text{RDi}_{i,t} + \sum\text{Year} + \varepsilon \quad (4)$$

$$\text{DT}_{i,t} = \beta_0 + \beta_1\text{MA}_{i,t} + \beta_2\text{Size}_{i,t} + \beta_3\text{Age}_{i,t} + \beta_4\text{Lev}_{i,t} + \beta_5\text{ROA}_{i,t} + \beta_6\text{ATO}_{i,t} + \beta_7\text{RD}_{i,t} + \sum\text{Year} + \varepsilon \quad (5)$$

$$\text{IE}_{i,t+1} = \beta_0 + \beta_1\text{MA}_{i,t} + \beta_2\text{DT}_{i,t} + \beta_3\text{Size}_{i,t} + \beta_4\text{Age}_{i,t} + \beta_5\text{Lev}_{i,t} + \beta_6\text{ROA}_{i,t} + \beta_7\text{ATO}_{i,t} + \beta_8\text{RD}_{i,t} + \sum\text{Year} + \varepsilon \quad (6)$$

$$\text{IE}_{i,t+1} = \beta_0 + \beta_1\text{MA}_{i,t} + \beta_2\text{IC}_{i,t} + \beta_3\text{MA}_{i,t} \times \text{IC}_{i,t} + \beta_4\text{Size}_{i,t} + \beta_5\text{Age}_{i,t} + \beta_6\text{Lev}_{i,t} + \beta_7\text{ROA}_{i,t} + \beta_8\text{ATO}_{i,t} + \beta_9\text{RD}_{i,t} + \sum\text{Year} + \varepsilon \quad (7)$$

where IE is the dependent variable, representing the green technology innovation of enterprises. i represents individual enterprises, and t represents years. Control represents all control variables. In the models, we also control year fixed effects yeart. ε is random error term. β1, β2, β3,β4, β5, β6 and β7 represent the coefficients, indicating the coefficient of digital finance development and control variables. β0 represent the model intercept terms.

TABLE 2 Descriptive statistics of the main variables.

VarName	Obs	Mean	SD	Min	Median	Max
IE	2084	7.9812	3.6174	4.3108	7.7412	27.8173
MA	2084	−0.0713	0.1621	−0.4061	−0.1042	0.3461
DT	2084	1.6102	0.8097	0.6717	1.2963	3.9942
IC	2084	6.4371	0.1191	6.0972	6.4892	7.0134
Size	2084	21.9831	1.1897	19.9137	21.9872	26.2938
Age	2084	2.6914	0.3604	1.5849	2.7462	3.4042
Lev	2084	0.3948	0.1912	0.0664	0.3994	0.8463
ROA	2084	0.0469	0.0464	−0.1124	0.0397	0.1994
ATO	2084	0.6718	0.3472	0.1617	0.6118	2.4316
RD	2084	0.0197	0.0171	0.0001	0.0189	0.0698

4.3 Descriptive statistics

The descriptive statistical results of the main variables are shown in Table 2. The highest value of green technological innovation (IE) among them is 27.8173, the lowest value is 4.3108, the average value is 7.9812, and the standard deviation is 3.6174, demonstrating that there are significant differences in green technological innovation performance among enterprises. The average management competence (MA) is −0.0713, indicating that management competence in

TABLE 3 Regression results.

Variables	(3)	(3)	(3)	(4)	(5)	(6)
	IE	IE	IE	DT	IE	IE
		High-ability	Low-ability			
MA	1.382***	3.643***	−0.261***	0.317***	1.416***	−62.109***
	(2.81)	(3.34)	(−0.18)	(2.19)	(2.71)	(−2.72)
DT					0.186***	
					(2.43)	
IC						1.696**
						(2.24)
MA × IC						8.629***
						(2.82)
Size	−0.098	−0.101	−0.194*	0.029	−0.107	−0.171**
	(−1.39)	(−0.92)	(−1.91)	(1.59)	(−1.48)	(−2.24)
Age	0.142	0.297	−0.074	−0.188***	0.169	0.164
	(0.64)	(0.91)	(−0.33)	(−3.26)	(0.81)	(0.77)
Lev	1.489***	1.737**	1.601***	0.031	1.497***	1.633***
	(2.91)	(2.04)	(2.76)	(0.22)	(2.91)	(3.08)
ROA	−2.534	−1.844	−2.197	0.321	−2.618	−3.304*
	(−1.48)	(−0.79)	(−1.04)	(0.67)	(−1.53)	(−1.79)
ATO	0.486*	0.343	0.387	-0.133**	0.422*	0.331
	(1.77)	(1.22)	(1.02)	(-2.14)	(1.89)	(1.46)
R&D	−39.837***	−43.884***	−33.752***	11.679***	−42.836***	−39.374***
	(−9.09)	(−6.42)	(−6.16)	(9.68)	(−9.37)	(−8.84)
Year	Yes	Yes	Yes	Yes	Yes	Yes
Constant	21.603***	21.081***	22.763***	0.948**	20.836***	10.378**
	(11.97)	(8.03)	(9.61)	(2.02)	(11.94)	(2.03)
Adjusted R^2	0.215	0.218	0.237	0.081	0.223	0.226
N	2084	1061	1023	2084	2084	2084
F	39.924	21.183	24.481	12.689	40.163	36.831

***, ** and * are significant at the level of 1%, 5% and 10% respectively.

China's manufacturing industry is generally low and needs to be improved further. The minimum value is −0.4061, the maximum value is 0.3461, and the standard deviation is 0.1621, indicating that different company managers have varying abilities to use existing assets to generate output, with significant differences. The highest digital transformation degree (DT) is 3.9942, the lowest is 0.6717, and the average is 1.6102, indicating that the digital transformation degree of China's manufacturing industry is low and needs to be popularized and applied further. The highest internal control (IC) is 7.0134, the lowest is 6.0972, the average is 6.4371, and the standard deviation is 0.1191, indicating that there is not a significant difference in internal control quality among listed Chinese manufacturing companies.

5 Empirical results and analysis

5.1 Management ability and innovation performance

According to the regression results in Table 3, there is a 1% significant positive correlation between management ability and enterprise innovation performance (coefficient is 1.382, t value is 2.81), indicating that the stronger the management ability, the higher the enterprise innovation performance, and H1 is supported. To further test the impact of different managers' abilities on enterprise innovation performance, we divided the samples into two groups based on the median of managers' abilities: High-ability and Low-ability, and performed regression using model Eq. 5, as shown in the results. Managers' abilities

(MA) are significantly positively correlated with enterprises' innovation performance (IE) at the 1% level (coefficient is 3.643), indicating that the higher the management ability, the stronger its promotion effect on innovation performance.

5.2 The intermediary role of digital transformation

Models Eqs. 4–6 are constructed in this paper using the intermediary utility test method developed by Baron and Kenny (1986) as a reference to determine whether management capabilities can improve enterprise innovation performance through digital transformation. The results of model Eq. 4 in Table 3 show that managerial competence (MA) is significantly positively correlated with innovation performance (IE) at the 1% level (coefficient is 1.382, t value is 2.81); model Eq. 5 in Table 3 shows that there is a significant positive correlation between management competence (MA) and digital transformation (DT) at the 5% level (coefficient is 0.317, t value is 2.19); (coefficient is 1.416, t value is 2.71). As a result, we believe that digital transformation has a partial intermediary effect and is managed.

5.3 The regulatory role of internal control

The analysis result of model Eq. 7 in Table 3 reflects the influence of internal control quality on the relationship between management capability and innovation performance. The interaction term MA×IC is significantly positive at 1% level (coefficient is 8.629, t value is 2.82). It shows that the degree of internal control has a positive regulating effect on management ability and innovation performance, that is, the higher the quality of internal control, the more obvious the effect of management ability on improving enterprise innovation performance. The hypothesis is supported.

5.4 Heterogeneity of property rights

Table 4 shows that there is a 10% significant positive correlation between management capability and enterprise innovation performance in state-owned enterprises (coefficient is 2.412, t value is 1.89). Management competence (MA) is positively correlated with digital transformation (DT) at the 1% level (coefficient is 0.786, t value is 3.31), at the 5% level (coefficient is 0.397, t value is 2.28), and at the 10% level (coefficient was 11.917, t value was 1.81), indicating There is a significant positive correlation between management capability and enterprise innovation performance in non-state-owned enterprises (coefficient is 1.298, t value is 2.91), and internal control has a significant positive regulating effect (coefficient is 7.013, t value is 1.72), but the intermediary effect of digital transformation is not significant. As a result, when compared to state-owned enterprises, non-state-owned enterprises' management ability does not make good use of digital transformation to improve enterprise innovation performance.

This paper employs the Chow test and the Permutation test to validate the relationship between managerial competence and innovation performance in model Eq. 4 the difference in the moderating effect of internal control between state-owned enterprises and non-state-owned enterprises in model Eq. 7. The findings show that there is no significant difference in the promotion of managerial competence to innovation performance between state-owned enterprises and non-state-owned enterprises. However, in both state-owned and non-state-owned enterprises, internal control has a significant positive moderating effect on the relationship between managerial competence and innovation performance.

5.5 Robustness test

In order to ensure the validity of the research results, this paper further uses two methods to test the robustness. Firstly, this paper uses the method of Goldfarb and Tucker (2019) for reference, and uses the ratio of the sum of R&D investment in the current year and the previous year to the operating income to measure the innovation performance of enterprises, which is recorded as IE1 and regressed again. Secondly, this paper uses the practice of Gurevitch et al. (2018) for reference, and adds some corporate governance factors that may have an impact on the innovation performance of enterprises to make a further robustness test. The added control variables are management Salary, management shareholding ratio (Msharehold), independent director ratio (IndDir), and the concurrent status of chairman and general manager (Dual). It can be seen from the test results in Table 5 that it is completely consistent with the previous empirical results, and the conclusion is robust.

6 Discussion

With the rapid development of the digital economy, some scholars have begun to focus on the relationship between digitalization and green development. Few studies, however, have extended the research to green technology innovation and discussed the impact of management capabilities on green technology innovation. In practice, when implementing green technology innovation, active digitalization and deliberate acceleration of digitalization can help it cross the digital divide and realize green technology innovation more quickly. The conclusion of this paper, in theory, expands on previous research and clarifies the complex linear relationship between management capability and green technology innovation. At the same time, we conducted a heterogeneity test, taking into account

TABLE 4 Heterogeneity of property rights.

Variables	State-owned				Non-state-owned			
	(3)	(4)	(5)	(6)	(3)	(4)	(5)	(6)
	IE	DT	IE	IE	IE	DT	IE	IE
MA	2.412*	0.786***	2.106	−77.103***	1.298***	−0.017	1.289***	−44.167
	(1.89)	(3.31)	(1.72)	(−1.69)	(2.91)	(−0.06)	(2.91)	(−1.72)
DT			0.397**				0.063	
			(2.28)				(0.89)	
IC				1.074				2.083***
				(0.68)				(2.36)
MA × IC				11.917*				7.013*
				(1.81)				(1.72)
Size	−0.281**	0.024	−0.288**	−0.361**	−0.117	0.022	−0.097	−0.129
	(−2.02)	(0.83)	(−2.17)	(−2.29)	(−1.18)	(0.73)	(−1.24)	(−1.61)
Age	−0.568	−0.039	−0.624	−0.557	0.209	−0.197***	0.218	0.224
	(−0.94)	(−0.37)	(−0.91)	(−0.86)	(1.18)	(−3.04)	(1.21)	(1.27)
Lev	1.386	0.117	1.368	1.631	1.291***	0.018	1.357***	1.284***
	(1.27)	(0.46)	(1.17)	(1.39)	(2.74)	(0.12)	(2.73)	(2.68)
ROA	−4.029	1.181	−4.486	−4.903	−1.374	−0.136	−1.396	−2.467
	(−1.12)	(1.47)	(−1.19)	(−1.16)	(−0.88)	(−0.19)	(−0.86)	(−1.39)
ATO	−0.179	−0.172*	−0.097	−0.228	0.607***	−0.161**	0.604***	0.517**
	(−0.34)	(−1.68)	(−0.22)	(−0.47)	(2.79)	(−1.86)	(2.91)	(2.43)
R&D	−56.237***	12.697***	−60.981***	−53.496***	−31.016***	13.714***	−30.908***	−30.971***
	(−5.64)	(6.31)	(−6.18)	(−5.44)	(−7.29)	(8.28)	(−7.34)	(−7.47)
Year	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Constant	27.861***	0.798	27.727***	20.034**	19.017***	0.974	19.018***	4.989
	(7.61)	(1.09)	(7.46)	(2.24)	(9.68)	(1.24)	(9.73)	(0.86)
Adjusted R^2	0.246	0.073	0.261	0.244	0.174	0.079	0.163	0.167
N	813	813	813	813	1271	1271	1271	1271
F	16.742	5.617	17.134	14.906	9.680	8.897	17.849	16.983
Experience p value 1	(3) 0.288				(6) 0.439			
Experience p value 2	(3) 0.184				(6) 0.361			

***, ** and * are significant at the level of 1%, 5% and 10% respectively. "Empirical p-value" is used to test the coefficient difference between groups. The p-values listed in the table are, among which "Empirical p-value 1" is obtained by Chow test, and "Empirical p-value 2" is obtained by 1,000 bootstrap of Fisher combination test.

the unique characteristics of each enterprise. To begin with, the degree of green technology innovation differs greatly between state-owned enterprises and non-state-owned enterprises due to differences in enterprise property rights. The management capabilities of enterprises with varying property rights may have varying effects on their green technology innovation. The reason for this is that, in recent years, the state has paid close attention to green sustainable development, supported green technology behaviors, and provided subsidies and policy incentives for green technology innovation research and development. As a result, in the relationship between internal control and green technology innovation, state-owned enterprises have some advantages over non-state-owned enterprises. External supervision has an impact on the quality of enterprise internal control (Ma et al., 2020). The greater the external supervision, the more self-interested enterprise management will be limited, while the financing environment will be improved and green technology innovation will be encouraged.

Second, the level of green technology innovation varies depending on the enterprise's management capabilities. To begin, this paper discusses the impact of digitalization on enterprise green technology innovation. This research will help enterprises better understand green technology innovation, accelerate the pace of digitalization, reduce the "risk effect" of digitalization, bridge the digital divide, and realize "profit" as soon as possible. Second, the relationship between management capability and green technology

TABLE 5 Robustness test.

Variables	Re-measure innovation performance				Additional control variable			
	(3)	(4)	(5)	(6)	(3)	(4)	(5)	(6)
	IE1	DT	IE1	IE1	IE	DT	IE	IE
MA	0.021**	0.298**	0.017**	−0.612	1.503***	0.334**	1.397***	−61.686***
	(2.24)	(2.19)	(2.13)	(−1.71)	(2.89)	(2.28)	(2.74)	(−2.47)
DT			0.003**				0.204**	
			(1.98)				(2.39)	
IC				0.003				1.697**
				(0.24)				(2.24)
MA × IC				0.086*				9.384***
				(1.62)				(2.73)
Size	−0.001	0.028	−0.002	−0.002	−0.139*	−0.012	−0.138*	−0.194**
	(−0.39)	(1.59)	(−0.49)	(−0.71)	(−1.71)	(−0.63)	(−1.74)	(−2.31)
Age	−0.002	−0.186***	−0.001	−0.003	0.134	−0.189***	0.168	0.163
	(−0.18)	(−3.29)	(−0.07)	(−0.18)	(0.59)	(−3.31)	(0.74)	(0.68)
Lev	−0.048***	0.026	−0.048***	−0.044***	1.427***	0.081	1.396***	1.437***
	(−5.97)	(0.18)	(−5.86)	(−5.91)	(2.73)	(0.57)	(2.71)	(2.86)
ROA	−0.127***	0.319	−0.121***	−0.114***	−3.098*	−0.034	−3.089*	−3.764**
	(−4.48)	(0.67)	(−4.64)	(−4.21)	(−1.79)	(−0.06)	(−1.83)	(−2.02)
ATO	−0.057***	−0.126**	−0.063***	−0.059***	0.371*	−0.118**	0.389*	0.298
	(−17.96)	(−2.09)	(−17.98)	(−18.43)	(1.64)	(−1.97)	(1.74)	(1.33)
R&D	2.749***	12.406***	2.712***	2.664***	−42.017***	11.068***	−44.391***	−40.973***
	(40.28)	(9.97)	(37.94)	(40.07)	(−8.89)	(8.64)	(−9.43)	(−8.67)
Salary					0.148	0.127***	0.124	0.134
					(1.09)	(3.49)	(0.91)	(1.18)
Msharehold					−0.327	0.043	−0.342	−0.436
					(−0.91)	(0.37)	(−0.89)	(−0.91)
IndDir					−1.564	1.112***	−1.813	−1.492
					(−1.21)	(3.08)	(−1.37)	(−1.21)
Dual					−0.297**	−0.024	−0.321**	−0.309**
					(−1.93)	(−0.61)	(−1.84)	(−1.97)
Year	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Constant	0.088***	0.961**	0.089***	0.074	20.334***	−0.371	19.976***	9.647**
	(3.69)	(2.08)	(3.71)	(1.06)	(9.88)	(-0.64)	(9.93)	(2.06)
Adjusted R^2	0.494	0.079	0.498	0.521	0.226	0.079	0.227	0.243
N	2084	2084	2084	2084	2084	2084	2084	2084
F	147.286	13.103	142.127	134.362	31.943	10.882	30.879	31.427

***, ** and * are significant at the level of 1%, 5% and 10% respectively.

innovation is discovered to vary depending on the nature of property rights, digital level, and internal control. It provides an empirical foundation for various types of businesses to implement digital strategies. It enables businesses and governments to examine digital economic performance from a variety of perspectives. Finally, this study contributes to a better understanding of the feasible mode of Chinese enterprises' green technology innovation under China's new normal, as well as the critical role of digitalization in enterprises' green technology innovation. Simultaneously, it provides a new idea for developing countries and countries that pursue economic growth but ignore environmental problems to carry out green technology innovation at the micro level, as well as a foundation for their green policy formulation. As a result, the research conclusion of this paper is beneficial to enterprises in actively and rapidly meeting the challenge of digitalization, shortening the inhibition effect of digitalization, and thus improving the ability and effect of green governance and realizing green transformation.

Thirdly, Both X inefficiency theory and new institutional economics suggest that the internal management of business organizations is the cause of widespread inefficiencies (Bi et al., 2019). Management ability improves organizational learning and innovation ability, thereby improving enterprise performance. Finally, higher-ability management has more flexible and adaptive strategic thinking and ability, allowing it to quickly capture market opportunities, formulate long-term digital transformation strategies, gain competitive advantage through product and service innovation, and thus promote enterprise innovation performance (Vial, 2019). As firms expand in size, the internal pyramid hierarchy increases, and problems such as information friction arise. As a result, information frictions are more serious in large organizations than in SMEs, internal transaction costs are higher, and resources are more wasted. Initially, digitalization may not promote green transformation or even inhibit it.

Fourthly, Internal control plays a positive regulatory role in the relationship between management capabilities and green technological innovation. High-quality internal control can effectively control enterprise risks and provide assurance for high-risk innovation activities undertaken by management. Through risk assessment and control activities of internal control, enterprises can analyze the scientificity and feasibility of innovative projects, carry out combined development and innovation, and improve the risk warning system, reducing uncertainty in the innovation process, effectively preventing the company's innovation risks, and thus improving the efficiency of green technological innovation. Internal control can be an effective supervision mechanism in the innovation process. High-level internal control can effectively supervise enterprise management's behavior, prevent management from taking more private benefits in order to "maximize personal benefits," and increase excessive investment caused by agency conflicts, reducing enterprise innovation efficiency.

7 Conclusion and implications

7.1 Conclusion

In this paper, the research samples are listed companies in the Shanghai and Shenzhen A-share manufacturing industries from 2008 to 2019, and the DEA-Tobit two-stage model proposed by Demerjian et al. (2012) is used as a reference to measure managerial competence, and the influence of managerial competence on enterprise green technological innovation performance is empirically tested. It has been discovered that strong management ability can improve enterprise green technological innovation performance; digital transformation serves as a partial intermediary in the relationship between management ability and enterprise green technological innovation performance; the higher the internal control quality, the more obvious the effect of management ability on improving enterprise innovation performance. Simultaneously, this paper examines the heterogeneity of the property right nature of the aforementioned relationship. We discover that managerial competence is positively correlated with green technological innovation performance in both state-owned and non-state-owned enterprises, with no significant difference. Internal control also has a significant positive moderating effect on the relationship between management ability and innovation performance; however, digital transformation plays a significant intermediary role only in state-owned enterprises. It is clear that the management capability of non-state-owned enterprises has not made good use of digital transformation to improve enterprise innovation performance, and the benefits of digital transformation have not been fully utilized. Finally, to ensure the robustness of the research findings, this paper re-measures green technological innovation performance and incorporates control variables related to corporate governance, and the estimated results are completely consistent with the previous empirical findings.

The following conclusions can be drawn:

(1) Because of its high risk tolerance, opportunity discovery, information transmission, and resource integration, management plays a positive role in promoting green technological innovation and enhancing enterprise competitive advantage. As the digital economy develops, enterprise management can provide more key resources for digital transformation through strong comprehensive ability, reasonably avoid digital transformation risks, continuously strengthen the role of digital resources as the core, formulate long-term digital transformation strategies, and improve enterprise green technological innovation performance by improving product performance and service quality.
(2) Against the backdrop of China's late implementation of internal control and an imperfect system, internal control can effectively control risks, build a good information and communication platform, and exert an effective supervision mechanism, actively promoting the improvement of management's ability to green technological innovation performance and supporting the theory of internal control promotion.
(3) Non-state-owned enterprises' management ability has not made good use of digital transformation to improve enterprise innovation performance, and the role of digital transformation has not been well played. Based on the heterogeneity of ownership, environmental policies should be differentiated for different enterprises. Due to the lack of supervision, green transformation should be added to performance appraisals in SOEs. For non-SOEs,

governments should provide more support to ensure that funding and talents are sufficient for green technology innovation.

7.2 Managerial implication

Green technological innovation is critical as a strategic foundation for resolving the dilemma of green economic development and environmental protection. This paper not only provides a different perspective from the existing literature for the analysis of green technology innovation, but it also provides ideas for how to modify green technology innovation decision-making behaviors among manufacturing enterprises under environmental regulation.

The empirical findings of this paper have significant theoretical and practical implications. The empirical evidence presented above may provide some illumination for promoting green technology innovation. For example, the government should appropriately increase subsidies to manufacturing enterprises. First, the government should provide friendly policies for manufacturing enterprises, such as increasing economic incentives, tax incentives, and institutional guarantees, and strengthen support for green R&D of manufacturing enterprises, to increase the time available and motivation for enterprises to carry out green innovation activities.

First, continue to deepen the digital transformation. Enterprises should increase their capital investment in digital transformation, create a reasonable budget, and keep digital transformation costs under control. Simultaneously, everyone should support R&D, promotion, and application activities in the digital transformation process. Actively construct a digital information network cooperation platform, establish an open digital innovation experimental area, and continue to collaborate with the government and other enterprises *via* the network platform to improve the quality of products and services. Give priority to the introduction of digital talents, as well as the development of digital and intelligent knowledge and skills among employees in this enterprise.

Second, increase government assistance. The government should increase subsidies for green technological innovation activities and provide incentives based on the social benefits of innovation output. At the government level, we should also recruit talents, actively introduce domestic and foreign digital talents, create a digital talent highland, strengthen information infrastructure construction, and provide more accurate services to enterprises by utilizing the national professional concept and technology of digital transformation and upgrading. While developing a digital transformation leader, we should also pay close attention to the digital transformation of non-state-owned enterprises and small and medium-sized businesses. Government agencies should provide targeted subsidies for enterprises of various sizes and economic conditions based on the types of imported technologies, investment amount, and other factors. Furthermore, the government should improve the policy environment for the spread of green technology innovation. First and foremost, the government should improve the system to deal with the risk of green technology innovation spreading. The government should establish a risk response mechanism to reduce the uncertain risk of green technology innovation diffusion in the process of manufacturing enterprises introducing, digesting, absorbing, and applying new technologies, in addition to providing financial and tax policy support. This will assist manufacturing firms in gaining confidence and resolving existing issues related to the development of green innovation. Second, the government should develop appropriate intellectual property policies, such as establishing a green patent and trademark system and managing intellectual property rights confidentially. Its goal is to fully realize the potential of intellectual property rights while also promoting green technology research, application, and popularization.

Then, improve enterprise internal control. Enterprises should strengthen their corporate governance systems, implement a standardized and sound internal control system, and increase the level of internal control. Simultaneously, enterprises should pay attention to the flexibility of their internal control systems in order to avoid impeding the enthusiasm for green technological innovation.

Finally, focus on the development and supervision of management skills. On the one hand, enterprises should fully improve management's skills, learning ability, and networking ability, as well as improve market acumen and formulate a reasonable incentive mechanism for management, in order to better align their own interests with enterprise goals. On the other hand, improve and optimize the appointment, evaluation, elimination, and other related mechanisms of managers, and raise the default cost of managers; develop corresponding rules and regulations to effectively restrict management's behavior, paying special attention to the degree of deviation between control and ownership, as well as the concurrent positions of chairman and general manager, etc., so that the supervisory fun can be fully realized.

7.3 Limitations and future research

Some limitations of this study point to areas for future research. First and foremost, the research sample is from China. Future research should include global samples to broaden the results' applicability. Second, the digital economy is a complex system that is constantly evolving as new digital technology is developed. The indicators of the digital economy must be improved further. Third, the measurement of green technology innovation should be researched further. Although the number of green patent applications is frequently used to measure green technological innovation, the information conveyed by this metric is limited. Additional research could look into the process of green technology innovation, such as the quality of green patents. Finally, the role of management ability in the digital economy and green technology innovation is discussed in this paper. However, this may not be the only path. Further research can investigate the dynamic capabilities and financial constraints of businesses in depth.

Data availability statement

The original contributions presented in the study are included in the article/Supplementary Material; further inquiries can be directed to the corresponding author.

Author contributions

All authors listed have made a substantial, direct, and intellectual contribution to the work and approved it for publication.

Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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Novel decision aid model for green supplier selection based on extended EDAS approach under pythagorean fuzzy Z-numbers

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The main objective of this study is to identify the green suppliers that would most effectively assist manufacturing producers in implementing green manufacturing production while including uncertainty and reliability in their decision-making. For this firstly, we justify and manifest the idea of Pythagorean Fuzzy Z-numbers (PyFZNs). It has significant implications for improving the effectiveness of decision-making processes in several theories of uncertainty. It can more flexibly explain real-world data and human cognition due to its capacity to express imprecise and reliable information. Thus it is a more accurate mathematical tool for addressing accuracy and uncertainty. Secondly, we defined the Pythagorean fuzzy Z-number arithmetic aggregation operators and geometric aggregation operators. Thirdly, based on the proposed operators and EDAS (Evaluation based on distance from average solution) approach, a fast decision model is designed to deal with the issue of multi-criteria decision-making. Finally, using PyFZN data we also provide a numerical example to demonstrate the usability of the created multicriteria decision-making (MDM) approach. Moreover, a case study also proves its efficacy.

KEYWORDS

pythagorean fuzzy z-number, pythagorean fuzzy z-number weighted arithmetic operator, pythagorean fuzzy z-number weighted geometric operator, decision making, EDAS method

1 Introduction

Multi attribute group decision making is a problem in several disciplines, such as management, engineering, and economics. For a very long time, it has been considered that such data which accesses the options in terms of requirement and weight is expressed in actual numbers. The majority of the desired values however are contaminated by uncertainty, which making a decision difficult for the decision-makers to correct judgment as the system becomes more complex every day. To deal with the uncertainties, Zadeh (Zadeh, 1965) introduced the fuzzy set theory notion in 1965, which described the degree of membership. It performed a very significant role in decision making (Ashraf et al., 2022a; Ashraf et al., 2022b; Zhang et al., 2022). In 2011, Zadeh (Zadeh, 2011) further proposed the idea of Z-numbers to better highlight the limitations and reliability of the evaluation by an ordered pair of fuzzy numbers in unpredictable conditions. Compared to the traditional fuzzy number, it is a more extensive concept that is intimately linked to reliability. Given this, the Z-number suggests that a pair of fuzzy numbers in the order of restriction and reliability could be better equipped to explain human knowledge and judgments. Theoretically, the Z-numbers reflect the numeric values of internal machine-mind processes that facilitate adapted real-world understanding (Banerjee and Pal, 2015). The four basic mathematical operations in linear programming addition, subtraction, multiplication, and division as well as algebraic operations like maximum, minimum, square, and square root of continuous Z-numbers were established in 2016 (Aliev et al., 2016). On the basis of the extension principle applied to Z-numbers, a technique for the creation of functions was proposed. This methodology is very helpful in the reduction of uncertainty, while computing the values of Z-valued functions. Z-numbers can be arranged using the proposed total utility of Z-number, which can also be used to make multi criteria judgments under uncertain conditions (Kang et al., 2018b). Ronald proposes a Dempster-Shafer such as belief structure representation of Z-values that incorporates type-2 fuzzy logic (Yager, 2012). Ronald R discusses mixing Z-number and the Dempster-Shafer (D-S) evidence theory, where the Z-number is used to simulate the fuzziness and dependability of sensor data and the D-S evidence theory is used to combine the complex information from Z-numbers (Jiang et al., 2016). Numerous scholars created the TODIM technique based on the Choquet integral for circumstances involving multi-criteria decision-making using linguistic Z-numbers (Wang et al., 2017). Using the Z-number application, the supplier selection problem's idea was illustrated (Jabbarova, 2017). The BWM technique and the Z-number extension were studied in order to handle the informational uncertainty in a multi-criteria decision system (Aboutorab et al., 2018). He created the QUALIFLEX method's linguistic Z-QUALIFLEX extension, which uses linguistic Z-numbers to solve the LGEDM problem (Ding et al., 2020). In (Kang et al., 2018a), reliable methods that can more accurately and flexibly simulate the process of human competition and cooperation are presented for study analysis based on the usefulness of the Z-number in evolutionary games. They present a generalized Z-number that is more in line with human expression tendencies and a multi-criteria decision-making approach based on the Dempster-Shafer (DS) theory and generalized Z-numbers (Ren et al., 2020). To demonstrate the suggested framework and demonstrate its effectiveness in environmental evaluations, the provides an environmental evaluation framework based on the Dempster-Shafer theory and Z-numbers (Kang et al., 2020).

A class of non-standard Pythagorean fuzzy subsets with pairs of membership grades (a, b) meeting the requirement $0 \leq a^2 + b^2 \leq 1$ was introduced using the Pythagorean complement and alternative definitions of complement operations. They provided a number of aggregation techniques and investigated multicriteria decision making in the scenario for these Pythagorean fuzzy set (Yager, 2013). According to (Garg, 2016), some of the aggregator operators discussed include generalized Pythagorean fuzzy Einstein weighted averaging, generalized Pythagorean fuzzy Einstein ordered weighted averaging, and Pythagorean fuzzy Einstein weighted averaging. The Einstein sum, product, and exponentiation as well as geometric aggregation operators and the intuitionistic fuzzy Einstein weighted and ordered weighted geometric operators were only a few of the operations on intuitionistic fuzzy sets that were discussed (Wang and Liu, 2011). The geometric aggregation operators, the intuitionistic fuzzy Einstein weighted and ordered weighted geometric operators, as well as the Einstein sum, product, and exponentiation, were all discussed (Rahman et al., 2017). The Pythagorean Dombi fuzzy aggregation operators introduced by (Akram et al., 2019), ELECTRI-I approach for it by (Akram et al., 2020) and TOPSIS approach by (Akram et al., 2021b). Garg and Sharaf (Garg et al., 2022) presented the spherical fuzzy EDAS approach. Generalized aggregation operators are proposed in (Ashraf and Abdullah, 2019; Chinram et al., 2020; Ashraf et al., 2023). Many scholars worked on its hybrid structures such as cubic Pythagorean linguistic fuzzy numbers (Naeem et al., 2021), rough Pythagorean fuzzy bipolar soft information (Akram and Ali, 2020) and sine trigonometric Pythagorean fuzzy information (Ashraf et al., 2021). It performs better role than fuzzy sets in decision making (Khan et al., 2019; Akram et al., 2021a; Zeng et al., 2023). Saeed et al. (Saeed et al. (2023) presented the refined Pythagorean fuzzy sets and interval-valued complex Pythagorean fuzzy set based methodology is developed in (Yazbek et al., 2023). We refer some decision making techniques (Ashraf et al., 2022c; Garg and Sharaf, 2022) for more details.

We looked at the connection between Z-numbers and linguistic summaries, For instance Dempster-Shafer belief systems and type-2 fuzzy logic were used to describe Z values (Kang et al., 2020). Theoretical components of mathematical operations including addition, subtraction, multiplication, division, and computing the square root of a Z-number over discrete Z-numbers have been proposed (Aliev et al., 2015b). They suggested a Z-number-based computing with words (CWW) algorithm, defined a Z-number-based operator for assessing the degree of requirement satisfaction, described CWW simulation experiments using Z-numbers, examined the benefits and drawbacks of Z-numbers, and offered potential resolution approaches (Pal et al., 2013). The authors developed Z-TOPSIS, a novel variant of the TOPSIS approach that streamlines multi-criteria decision-making problems based on the idea of Z-numbers, in order to support the concept of ranking alternatives using Z-numbers. Additionally, the authors offered a link to some existing information in fuzzy sets (Yaakob and Gegov, 2016). The researchers advise researching the fully Z-number based LP (Z-LP) model and utilizing a method to address Z-LP problems that combines differential evolution optimization and Z-number arithmetic created by the authors in order to better fit real-world problems within the LP framework. Using a benchmark LP problem, the suggested model and solution approach for Z-LP are then shown (Aliev et al., 2015a). For replicating the effects of Pilates exercises on students' motivation, concentration, anxiety, and academic success, they have presented an innovative way. Due to the ambiguity of data relating to cognitive assessment of psychological features and their partial reliability, the use of "Z-if . . . then rules" for modelling the considered relationship has been pushed for the first time (Aliev and Memmedova, 2015). The topic (Kang et al., 2016) is split into two parts: the first explains how to use the fuzzy expectation to convert a Z-number into a concept of fuzzy number, and the second explains how to use the genetic algorithm to determine the best priority weight for supplier selection. This method of calculating the priority weight of the judgment matrix is quick and flexible. The first recommendation in this study is an enhanced ranking approach for generalized fuzzy numbers that take into account the weight of centroid points, fuzziness levels, and fuzzy number spreads. This approach is particularly effective for evaluating symmetric fuzzy numbers and crisp numbers, among other things, and can alleviate some of the drawbacks of existing approaches (Jiang et al., 2017). For the first time, a multi-layer method of grading Z-numbers is proposed in this work. This method has two layers: fuzzy number ranking and Z number conversion (Bakar and Gegov, 2015).

Few articles, to our knowledge, seek to explore the rationality and certainty of information in efficient and effective decision-making in a systematic and comprehensive manner. For handling uncertainty effectively in real-world applications, fuzzy sets, intuitionistic fuzzy sets, Pythagorean fuzzy sets, picture fuzzy sets, hesitant fuzzy sets, etc. all are unable to deal with the reliability of their membership levels. While reliability or certainty is the core component to deal with making decisions efficiently and trustworthy. In order to eliminate this constraint, this research uses the strategic illustration, conceptual evolution map, and major route analysis to explain the development of the novel Pythagorean fuzzy Z-number (PyFZN) field comprehensively. We suggest a PyFZN by considering Pythagorean fuzziness in terms of both membership and non-membership degrees with reliability. We defined some fundamental properties and aggregation operators, which perform a very significant role in making decisions. Furthermore, we suggested a multicriteria decision-making (MDM) approach, which aims to reduce the overall uncertainty of the decision matrix to increase the credibility of the conclusions. To demonstrate the effectiveness of this novel concept we illustrated a real-life example based on it. Moreover, we also presented a comparative analysis with the existing studies to show its supremacy. The primary goals of this study are: 1. To investigate a number of distinct interactive averaging and geometric AOs concerning PyFZNs to circumvent the constraints. 2. Getting some key characteristics and unique cases of the newly defined AOs. 3. To create a novel MDM strategy using the interactive AOs in a fuzzy Pythagorean context that has been suggested. 4. To present the benefits and viability of the developed MDM strategy.

The main goals of this study are to address the aforementioned research topics and close a knowledge gap.

The rest of this article is organized as follows. Preliminary definitions and concepts are presented in Section 2 and are required to support our primary findings. The concept of Pythagorean fuzzy Z-number, their properties, and score function is explained in Section 3. A thorough investigation of this idea is done in Section 4 where we defined a Pythagorean fuzzy Z-number with arithmetic and geometric aggregation operators and also demonstrate some theorems related to these ideas. In Section 5 we defined a Pythagorean fuzzy Z-number ordered weighted arithmetic and geometric aggregation operators and also demonstrate some theorems related to these ideas. The MDM technique based on intended operators clarified in Section 6, which also provided a numerical example for selecting the riskiest companies to invest money as business partners. We provide an EDAS approach for PyFZNs in Section 7 and also illustrate it with the aid of an example. Finally, in Section 8, we presented a comparative analysis and in Section 9, the conclusion and future research directions for this paper are depicted.

2 Preliminaries

Definition 1. (Zadeh, 1965) Let X be a nonempty set, a fuzzy set A in X is characterized by a membership function

where μ_A: X → [0, 1], the function defines the degree of membership of the element, x ∈ X.

That is: A fuzzy set A in X is an object having the form:

$$A = \{\langle x, \mu_A(x)\rangle | x \in X\}.$$

Definition 2. (Yager, 2013) Assume that B is the Pythagorean fuzzy set and here M is a universal set described as

$$B = \{l, \mu_B(l), \nu_B(l) | l \in M\},$$

where the function $\mu_B(l)$: $M \to [0,1]$ and $\nu_B(l)$: $M \to [0,1]$ are the degree of membership and the degree of nonmembership respectively, which satisfies the following requirement:

$$0 \le (\mu_B(l))^2 + (\nu_B(l))^2 \le 1.$$

Definition 3. *The concept of Z-number first introduced by zadeh in 2011 (Zadeh, 2011). The Z-number is discussed as, taking order pair of fuzzy numbers $Z = (S, T)$, where S is a fuzzy ristriction on the values of M and T gives the reliability for S, here M being a universal set.*

3 Pythagorean fuzzy Z-number

Now we define Pythagorean fuzzy Z-number (*PyFZN*) as given below.

Definition 4. *Assume that Gz is a Pythagorean fuzzy Z-number (PyFZN) and M be the universal set:*

$$Gz = \{l, \mu(S,T)(l), \nu(S,T)(l) | l \in M\}$$

where the funtion $\mu(S,T)(l)$: $M \to [0,1]$ and $\nu(S,T)(l)$: $M \to [0,1]$ are constructed as follows:

$$Gz = \{(\mu(S,T)), \nu(S,T)\} = \{(\mu_S, \mu_T), (\nu_S, \nu_T)\}$$

It meets the following requirements:

$$0 \le (\mu(S)(l))^2 + (\nu(S)(l))^2 \le 1$$
$$0 \le (\mu(T)(l))^2 + (\nu(T)(l))^2 \le 1.$$

Now we will discussing the properties of Pythagorean fuzzy Z-numbers which already discuss in Definition 4.

Definition 5. *Let $Gz_1 = \{(\mu_1(S,T)), \nu_1(S,T)\} = \{(\mu_{S_1}, \mu_{T_1}), (\nu_{S_1}, \nu_{T_1})\}$ and $Gz_2 = \{(\mu_2(S,T)), \nu_2 S, T\} = \{(\mu_{S_2}, \mu_{T_2}), (\nu_{S_2}, \nu_{T_2})\}$ be two Pythagorean fuzzy Z-numbers (P_yFZN_S) and ⱡ >0 which satisfies the following characteristics:*

(1) $Gz_1 \supseteq Gz_2$ if and only if $\mu_{S_1} \ge \mu_{S_2}$, $\mu_{T_1} \ge \mu_{T_2}$ and $\nu_{S_1} \le \nu_{S_2}, \nu_{T_1} \le \nu_{T_2}$.

(2) $Gz_1 = Gz_2$ if and only if $Gz_1 \supseteq Gz_2$ and $Gz_1 \subseteq Gz_2$,

(3) $Gz_1 \cup Gz_2 = \{(\mu_{S_1} \vee \mu_{S_2}, \mu_{T_1} \vee \mu_{T_2}), (\nu_{S_1} \wedge \nu_{S_2}, \nu_{T_1} \wedge \nu_{T_2})\}$,

(4) $Gz_1 \cap Gz_2 = \left\{(\mu_{S_1} \wedge \mu_{S_2}, \mu_{T_1} \wedge \mu_{T_2}), (\nu_{S_1} \vee \nu_{S_2}, \nu_{T_1} \vee \nu_{T_2})\right\}$,

(5) $(Gz_1)^c = \left\{(\nu_{S_1}, \nu_{T_1}), (\mu_{S_1}, \mu_{T_1})\right\}$,

(6) $Gz_1 \oplus Gz_2 = \left\{\left(\sqrt{\mu_{S_1}^2 + \mu_{S_2}^2 - \mu_{S_1}^2\mu_{S_2}^2}, \sqrt{\mu_{T_1}^2 + \mu_{T_2}^2 - \mu_{T_1}^2\mu_{T_2}^2}\right) (\nu_{S_1}\nu_{S_2}, \nu_{T_1}\nu_{T_2})\right\}$,

(7) $Gz_1 \otimes Gz_2 = \left\{(\mu_{S_1}\mu_{S_2}, \mu_{T_1}\mu_{T_2}), \left(\sqrt{\nu_{S_1}^2 + \nu_{S_2}^2 - \nu_{S_1}^2\nu_{S_2}^2}, \sqrt{\nu_{T_1}^2 + \nu_{T_2}^2 - \nu_{T_1}^2\nu_{T_2}^2}\right)\right\}$,

(8) $Ⱡ\, Gz_1 = \left\{\left(\sqrt{1-(1-\mu_{S_1}^2)^{Ⱡ}}, \sqrt{1-(1-\mu_{T_1}^2)^{Ⱡ}}\right), (\nu_{S_1}^{Ⱡ}, \nu_{T_1}^{Ⱡ})\right\}$,

(9) $G^{Ⱡ}z_1 = \left\{(\mu_{S_1}^{Ⱡ}\mu_{T_1}^{Ⱡ}), \left(\sqrt{1-(1-\nu_{S_1}^2)^{Ⱡ}}, \sqrt{1-(1-\nu_{T_1}^2)^{Ⱡ}}\right)\right\}$.

Definition 6. *Let $Gz_1 = \left\{(\mu_{S_1}, \mu_{T_1}), (\nu_{S_1}, \nu_{T_1})\right\}$ and $Gz_2 = \left\{(\mu_{S_2}, \mu_{T_2}), (\nu_{S_2}, \nu_{T_2})\right\} \in PyFZN_S$. Then the score funtion is defined as*

$$J(Gz_d) = \frac{1 + \mu_{S_d}\mu_{T_d} - \nu_{S_d}\nu_{T_d}}{2}. \tag{3.1}$$

where $J(Gz_d) \in [0,1]$. If the score of $J(Gz_1) \geq J(Gz_2)$, then $Gz_1 \geq Gz_2$.

Example 1. *Consider two pythagorean fuzzy z-number as $Gz_1 = \{(0.6, 0.8), (0.1, 0.3)\}$ and $Gz_2 = \{(0.5, 0.7), (0.2, 0.4)\}$. Therefore, the base of score funtion the ranking of given pythagorean fuzzy z-number is defined as:Using Eq. 3.1*

$$J(Gz_1) = \left(\frac{1 + (0.6 \times 0.8) - (0.1 \times 0.3)}{2}\right) = 0.725$$
$$J(Gz_2) = \left(\frac{1 + (0.5 \times 0.7) - (0.2 \times 0.4)}{2}\right) = 0.595.$$

Hence, the score of $J(Gz_1) \geq J(Gz_2)$, then $Gz_1 \geq Gz_2$.

4 Pythagorean fuzzy Z-numbers weighted aggregation operators

We may propose the weighted aggregation operators for *PyFZNs* in this part based on actions (6) to (9) in Definition 2.

4.1 PyFZNW operator

We can talk about the *PyFZNW* operator of *PyFZNs* in relation to the basis operations (6) and (8) in Definition 2.

Definition 7. *Let $Gz_d = \left\{(\mu_{S_d}, \mu_{T_d}), (\nu_{S_d}, \nu_{T_d})\right\} (d = 1, 2, \ldots ń)$ be a group of PyFZNsand $PyFZNW: \Omega^{ń} \to \Omega$. Then the PyFZNW operator is defined as*

$$PyFZNW(Gz_1, Gz_2, \ldots Gz_{ń}) = \sum_{d=1}^{ń} Ⱡ_d Gz_d \tag{4.1}$$

where $Ⱡ_d (d = 1, 2, \ldots ń)$ is the weight vector with $0 \leq L = {}_d \leq 1$ and $\sum_{d=1}^{ń} Ⱡ_d = 1$.

Theorem 1. *Let $Gz_d = \left\{(\mu_{S_d}, \mu_{T_d}), (\nu_{S_d}, \nu_{T_d})\right\} (d = 1, 2, \ldots ń)$ be a group of PyFZNs. Then, the collected value of the PyFZNW operator is a PyFZN, which is obtained by the following formula:*

$$PyFZNW(Gz_1, Gz_2, \ldots Gz_{ń}) = \sum_{d=1}^{ń} Ⱡ_d Gz_d = \left\{\begin{array}{c}\left(\begin{array}{c}\sqrt{1-\Pi_{d=1}^{ń}\left(1-\mu_{S_1}^2\right)^{Ⱡ_d}}, \\ \sqrt{1-\Pi_{d=1}^{ń}\left(1-\mu_{T_1}^2\right)^{Ⱡ_d}}\end{array}\right), \\ \left(\Pi_{d=1}^{ń}\nu_{S_1}^{Ⱡ_d}, \Pi_{d=1}^{ń}\nu_{T_1}^{Ⱡ_d}\right)\end{array}\right\} \tag{4.2}$$

where $Ⱡ_d (d = 1, 2, \ldots ń)$ is the weight vector with $0 \leq L = {}_d \leq 1$ and $\sum_{d=1}^{ń} Ⱡ_d = 1$.

Proof. Using mathematical induction to prove the above Theorem 1. In Definition 4 using operation (6) and (8), if $ń = 2$ we obtain the following result:

$$PyFZNW(Gz_1, Gz) = Ⱡ_1 Gz_1 \oplus Ⱡ_2 Gz_2$$
$$= \left\{\begin{array}{c}\left(\begin{array}{c}\sqrt{1-\left(1-\mu_{S_1}^2\right)^{Ⱡ_1}} + \sqrt{1-\left(1-\mu_{S_1}^2\right)^{Ⱡ_2}} - \\ \sqrt{1-\left(1-\mu_{S_1}^2\right)^{Ⱡ_1}}\sqrt{1-\left(1-\mu_{S_1}^2\right)^{Ⱡ_2}}\end{array}\right), \\ \left(\begin{array}{c}\sqrt{1-\left(1-\mu_{T_1}^2\right)^{Ⱡ_1}} + \sqrt{1-\left(1-\mu_{T_2}^2\right)^{Ⱡ_2}} - \\ \sqrt{1-\left(1-\mu_{T_1}^2\right)^{Ⱡ_1}}\sqrt{1-\left(1-\mu_{T_2}^2\right)^{Ⱡ_2}}\end{array}\right), \\ \left(\nu_{S_1}^{Ⱡ_1}\nu_{S_2}^{Ⱡ_2}, \nu_{T_1}^{Ⱡ_1}\nu_{T_2}^{Ⱡ_2}\right).\end{array}\right\}$$
$$= \left\{\begin{array}{c}\left(\sqrt{1-\Pi_{d=1}^{2}\left(1-\mu_{S_d}^2\right)^{Ⱡ_d}}, \sqrt{1-\Pi_{d=1}^{2}\left(1-\mu_{T_d}^2\right)^{Ⱡ_d}}\right), \\ \left(\Pi_{d=1}^{2}\nu_{S_d}^{Ⱡ_d}, \Pi_{d=1}^{2}\nu_{T_d}^{Ⱡ_d}\right)\end{array}\right\}. \tag{4.3}$$

(2). If $ń = m$, using Eq. 5.2 we get the following form:

$$PyFZNW(Gz_1, Gz_2, \ldots Gz_m) = \sum_{d=1}^{m} Ⱡ_d Gz_d$$
$$= \left\{\begin{array}{c}\left(\sqrt{1-\Pi_{d=1}^{m}\left(1-\mu_{S_1}^2\right)^{Ⱡ_d}}, \sqrt{1-\Pi_{d=1}^{m}\left(1-\mu_{T_1}^2\right)^{Ⱡ_d}}\right), \\ \left(\Pi_{d=1}^{m}\nu_{S_1}^{Ⱡ_d}, \Pi_{d=1}^{m}\nu_{T_1}^{Ⱡ_d}\right)\end{array}\right\}. \tag{4.4}$$

(3). If $ń = m + 1$, using the operation (6) and (8) in Definition 2, and also use Eqs 5.3, 5.4 we have the following result:

$$PyFZNW(Gz_1, Gz_2, \ldots Gz_m Gz_{m+1})$$
$$= \sum_{d=1}^{m} Ⱡ_d Gz_d \oplus Ⱡ_{m+1} Gz_{m+1}$$
$$= \left\{\begin{array}{c}\left(\sqrt{1-\Pi_{d=1}^{m}\left(1-\mu_{S_d}^2\right)^{Ⱡ_d}}, \sqrt{1-\Pi_{d=1}^{m}\left(1-\mu_{T_d}^2\right)^{Ⱡ_d}}\right), \\ \left(\Pi_{d=1}^{m}\nu_{S_d}^{Ⱡ_d}, \Pi_{d=1}^{m}\nu_{T_d}^{Ⱡ_d}\right)\end{array}\right\} \oplus Ⱡ_{m+1} Gz_{m+1}.$$
$$= \left\{\begin{array}{c}\left(\sqrt{1-\Pi_{d=1}^{m+1}\left(1-\mu_{S_d}^2\right)^{Ⱡ_d}}, \sqrt{1-\Pi_{d=1}^{m+1}\left(1-\mu_{Td}^2\right)^{Ⱡ_d}}\right), \\ \left(\Pi_{d=1}^{m+1}\nu_{S_d}^{Ⱡ_d}, \Pi_{d=1}^{m+1}\nu_{T_d}^{Ⱡ_d}\right)\end{array}\right\}$$

It is true for all $ń$, then the verification is completed.

Theorem 2. *The PyFZNW operator implies the following properties:*

(1) Idempotency: Let $Gz_d = \left\{(\mu_{S_d}, \mu_{T_d}), (\nu_{S_d}, \nu_{T_d})\right\}$ $(d = 1, 2, \ldots ń)$ PyFZNs. If $Gz_d = Gz (d = 1, 2, \ldots ń)$, there is $PyFZNW(Gz_1, Gz_2, \ldots Gz_{ń}) = Gz$

(2) Boundedness: Let $Gz_d = \left\{(\mu_{S_d}, \mu_{T_d}), (\nu_{S_d}, \nu_{T_d})\right\} (d = 1, 2, \ldots ń)$ as a group of PyFZN and let

$$Gz_{\min} = \{\min(\mu_d(S,T)), \max(\nu_d(S,T))\}$$
$$= \left\{(\min_d(\mu_{sd}), \min_d(\mu_{Td})), (\max(\nu_{sd}), \max \nu(Td))\right\}$$
$$Gz_{\max} = \{\max(\mu_d(S,T)), \min \nu_d(S,T)\}$$
$$= \left\{(\max_d(\mu_{sd}), \max_d(\mu_{Td})), (\min_d(\nu_{sd}), \min_d(\nu_{Td}))\right\}$$

Then,

$$Gz_{\min} \leq PyFZNW(Gz_1, Gz_2, \ldots Gz_{ń}) \leq Gz_{\max}$$

can keep

(3) Monotonoicity: set $Gz_d = \{(\mu_d(S,T)), \nu_d(S,T)\} = \left\{(\mu_{S_d}, \mu_{T_d}), (\nu_{S_d}, \nu_{T_d})\right\} (d = 1, 2, \ldots ń)$ and $G^{\star}z_d = \{(\mu_d^{\star}(S,T)), \nu_d^{\star}(S,T)\} = \left\{(\mu_{S_d}^{\star}, \mu_{T_d}^{\star}), (\nu_{S_d}^{\star}, \nu_{T_d}^{\star})\right\} (d = 1, 2, \ldots ń)$ as

two groups of $PyFZN_S$. When $Gz_d \le G^*z_d$, there is $PyFZNW(Gz_1, Gz_2, \dots Gz_{\acute{n}}) \le PyFZNW(G^*z_1, G^*z_2, \dots G^*z_{\acute{n}})$.

Proof. (1) If $Gz_d = Gz(d = 1, 2, \dots \acute{n})$, then the result of equatuin (3) is given by

$$
\begin{aligned}
PyFZNW(Gz_1, Gz_2, \dots Gz_{\acute{n}}) &= \sum_{d=1}^{\acute{n}} Ⱡ_d Gz_d \\
&= \left\{ \begin{array}{c} \left(\begin{array}{c} \sqrt{1 - \Pi_{d=1}^{\acute{n}} \left(1 - \mu_{S_d}^2\right)^{Ⱡ_d}}, \\ \sqrt{1 - \Pi_{d=1}^{\acute{n}} \left(1 - \mu_{T_d}^2\right)^{Ⱡ_d}} \end{array} \right), \\ \left(\Pi_{d=1}^{\acute{n}} \nu_{S_d}^{Ⱡ_d}, \Pi_{d=1}^{\acute{n}} \nu_{T_d}^{Ⱡ_d}\right) \end{array} \right\}, \\
&= \left\{ \begin{array}{c} \left(\begin{array}{c} \sqrt{1 - \left(1 - \mu_S^2\right)^{\Pi_{d=1}^{\acute{n}} Ⱡ_d}}, \\ \sqrt{1 - \left(1 - \mu_T^2\right)^{\Pi_{d=1}^{\acute{n}} Ⱡ_d}} \end{array} \right), \\ \left((\nu_S)^{\Pi_{d=1}^{\acute{n}} Ⱡ_d}, (\nu_T)^{\Pi_{d=1}^{\acute{n}} Ⱡ_d}\right) \end{array} \right\}, \\
&= \left\{ \left(\sqrt{1 - \left(1 - \mu_S^2\right)}, \sqrt{1 - \left(1 - \mu_T^2\right)} \right), (\nu_S, \nu_T) \right\}, \\
&= \{(\mu_S, \mu_T), (\nu_S, \nu_T)\} = G_Z.
\end{aligned}
$$

(2) Since $Gz_{\min}$ and $Gz_{\max}$ are given by the minimum *PyFZN* and the maximum *PyFZN*, then the inequality $Gz_{\min} \le Gz \le Gz_{\max}$ exists. Thus, there is $\sum_{d=1}^{\acute{n}} Ⱡ_d Gz_{\min} \le \sum_{d=1}^{\acute{n}} Ⱡ_d Gz_d \le \sum_{d=1}^{\acute{n}} Ⱡ_d Gz_{\max}$.

Based on the above property (1) $Gz_{\min} \le \sum_{d=1}^{\acute{n}} Ⱡ_d Gz_d \le Gz_{\max}$ can exists, i,e, and $Gz_{\min} \le PyFZNW(Gz_1, Gz_2, \dots Gz_{\acute{n}}) \le Gz_{\max}$.

(3) $Gz_d \le G^*z_d$, there is $\sum_{d=1}^{\acute{n}} Ⱡ_d Gz_d \le \sum_{d=1}^{\acute{n}} Ⱡ_d G^*z_d$, *d.e.*, $PyFZNW(Gz_1, Gz_2, \dots Gz_{\acute{n}}) \le PyFZNW(G^*z_1, G^*z_2, \dots G^*z_{\acute{n}})$. All the properties of given theorem is complete.

4.2 PyFZNWG operator

Using the operation (7) and (9) in Definition 4, we give the *PyFZNWG* operator of PyFZNs.

Definition 8. *Let $Gz_d = \left\{(\mu_{S_d}, \mu_{T_d}), (\nu_{S_d}, \nu_{T_d})\right\} (d = 1, 2, \dots \acute{n})$ be a group of PyFZNs.Then the PyFZNWG: $\Omega^{\acute{n}} \to \Omega$ operator is defined as:*

$$PyFZNWG(Gz_1, Gz_2, \dots Gz_{\acute{n}}) = \Pi_{d=1}^{\acute{n}} G^{Ⱡ_d} z_d \tag{4.5}$$

where $Ⱡ_d (d = 1, 2, \dots \acute{n})$ with $0 \le \text{L} = {}_d \le 1$ and $\sum_{d=1}^{\acute{n}} Ⱡ_d = 1$.

Theorem 3. *Let $Gz_d = \left\{(\mu_{S_d}, \mu_{T_d}), (\nu_{S_d}, \nu_{T_d})\right\} (d = 1, 2, \dots \acute{n})$ be a group of PyFZNs. Then, the collected value of the PyFZNWG operator is a PyFZN, which is obtained by the following formula:*

$$
\begin{aligned}
&= PyFZNW(Gz_1, Gz_2, \dots Gz_{\acute{n}}) = \Pi_{d=1}^{\acute{n}} G^{Ⱡ_d} z_d \\
&= \left\{ \left(\Pi_{d=1}^{\acute{n}} \mu_{S_d}^{Ⱡ_d}, \Pi_{d=1}^{\acute{n}} \mu_{T_d}^{Ⱡ_d}\right), \left(\begin{array}{c} \sqrt{1 - \Pi_{d=1}^{\acute{n}} \left(1 - \nu_{S_d}^2\right)^{Ⱡ_d}}, \\ \sqrt{1 - \Pi_{d=1}^{\acute{n}} \left(1 - \nu_{T_d}^2\right)^{Ⱡ_d}} \end{array} \right) \right\}
\end{aligned}
\tag{4.6}
$$

where $Ⱡ_d (d = 1, 2, \dots \acute{n})$ with $0 \le \text{L} = {}_d \le 1$ and $\sum_{d=1}^{\acute{n}} Ⱡ_d = 1$. By the similar verification process of Theorem 1

Theorem 4. *The PyFZNWG operator of also implies the following properties*

(1) Idempotency:Set: Let $Gz_d = \{(\mu_d(S, T)), \nu_d(S, T)\} = \left\{(\mu_{S_d}, \mu_{T_d}), (\nu_{S_d}, \nu_{T_d})\right\} (d = 1, 2, \dots \acute{n})$ be a group of PyFZNs. If $Gz_d = Gz(d = 1, 2, \dots \acute{n})$, there is $PyFZNWG(Gz_1, Gz_2, \dots Gz_{\acute{n}}) = Gz$

(2) Boundedness: Set $Gz_d = \{(\mu_d(S, T)), \nu_d(S, T)\} = \left\{(\mu_{S_d}, \mu_{T_d}), (\nu_{S_d}, \nu_{T_d})\right\} (d = 1, 2, \dots \acute{n})$ as a group of P_yFZN and let

$$
\begin{aligned}
Gz_{\min} &= \{\min(\mu_d(S, T)), \max(\nu_d(S, T))\} \\
&= \left\{ \left(\min_d(\mu_{sd}), \min_d(\mu_{Td}) \right), (\max(\nu_{sd}), \max \nu(Td)) \right\} \\
Gz_{\max} &= \{\max(\mu_d(S, T)), \min \nu_d(S, T)\} \\
&= \left\{ \left(\max_d(\mu_{sd}), \max_d(\mu_{Td}) \right), \left(\min_d(\nu_{sd}), \min_d(\nu_{Td}) \right) \right\}
\end{aligned}
$$

Then, $Gz_{\min} \le PyFZNWG(Gz_1, Gz_2, \dots Gz_{\acute{n}}) \le Gz_{\max}$ can keep.

(3) Monotonoicity: set $Gz_d = \{(\mu_d(S, T)), \nu_d(S, T)\} = \left\{(\mu_{S_d}, \mu_{T_d}), (\nu_{S_d}, \nu_{T_d})\right\} (d = 1, 2, \dots \acute{n})$ and $G^*z_d = \{(\mu_d^*(S, T)), \nu_d^*(S, T)\} = \left\{(\mu_{S_d}^*, \mu_{T_d}^*), (\nu_{S_d}^*, \nu_{T_d}^*)\right\} (d = 1, 2, \dots \acute{n})$ as two groups of P_yFZN_S. When $Gz_d \le G^*z_d$, there is

$PyFZNWG(Gz_1, Gz_2, \dots Gz_{\acute{n}}) \le PyFZNWG$

$(G^*z_1, G^*z_2, \dots G^*z_{\acute{n}})$. Can also be confirmed Theorem 2 using the aforementioned properties corresponding to the PyFZNWG operator, which is not repeated here.

5 Pythagorean fuzzy Z-numbers ordered weighted aggregation operators

On the basis of the operations (6)–(9) in Definition 4, we may suggest the weighted aggregation operators of PyFZNs in this section.

5.1 PyFZNOW operator

We are able to provide *PyFZNW* operators for *PyFZNs*.

Definition 9. *Let $\alpha_d = \left\{ (\mu_{S_{\alpha_d}}, \mu_{T_{\alpha_d}})(\nu_{S_{\alpha_d}}, \nu_{T_{\alpha_d}}) \right\} (d = 1, 2, \dots \acute{n})$ be acollection of PFZNs, then the Pythagorean fuzzy Z-number order weighted averaging aggregation operator is defined as:*

$$PyFZNOW(\alpha_1, \alpha_2, \dots \alpha_{\acute{n}}) = Ⱡ_1 \alpha_{\sigma(1)} \oplus Ⱡ_2 \alpha_{\sigma(2)} \oplus \dots \oplus Ⱡ_{\acute{n}} \alpha_{\sigma(\acute{n})}, \tag{5.1}$$

where $Ⱡ_d (d = 1, 2, \dots \acute{n})$ is the weighted vector of $\alpha_d (d = 1, 2, \dots \acute{n})$ with $0 \le Ⱡ_d \le 1$ and $\sum_{d=1}^{\acute{n}} Ⱡ_d = 1$.

Theorem 5. *Let $\alpha_d = \left\{ (\mu_{S_{\alpha_d}}, \mu_{T_{\alpha_d}})(\nu_{S_{\alpha_d}}, \nu_{T_{\alpha_d}}) \right\} (d = 1, 2, \dots \acute{n})$ be acollection of PFZNs, then their aggregated value by using PyFZNOW operators as*

$$PyFZNOW(\alpha_1, \alpha_2, \dots \alpha_{\acute{n}}) = \left\{ \begin{array}{c} \left(\begin{array}{c} \sqrt{1 - \Pi_{d=1}^{\acute{n}} \left(1 - \mu_{S\alpha_{\sigma(d)}}^2\right)}^{Ⱡ_d}, \\ \sqrt{1 - \Pi_{d=1}^{\acute{n}} \left(1 - \mu_{T_{\alpha_{\sigma(d)}}}^2\right)^{Ⱡ_d}} \end{array} \right), \\ \left(\Pi_{d=1}^{\acute{n}} \nu_{S\alpha_{\sigma(d)}}^{Ⱡ_d}, \Pi_{d=1}^{\acute{n}} \nu_{T\alpha_{\sigma(d)}}^{Ⱡ_d}\right) \end{array} \right\} \tag{5.2}$$

where $Ⱡ_d (d = 1, 2, \dots \acute{n})$ is the weighted vector of $\alpha_d (d = 1, 2, \dots \acute{n})$ with $0 \le Ⱡ_d \le 1$ and $\sum_{d=1}^{\acute{n}} Ⱡ_d = 1$.

Proof. Using mathematical induction to to prove the above Theorem 1. In Definition 4 using operation (6) and (8), if $\acute{n} = 2$ we obtain the following result:

$$PyFZNOW\left(\alpha_{\sigma_1},\alpha_{\sigma 2}\right)=Ⱡ_1\alpha_{\sigma_1}\oplus Ⱡ_2\alpha_{\sigma 2}$$
$$=\left\{\begin{array}{c}\left(\dfrac{\sqrt{1-\left(1-\mu^2_{S_{\alpha_{\sigma(1)}}}\right)^{Ⱡ_1}}+\sqrt{1-\left(1-\mu^2_{S_{\alpha_{\sigma(2)}}}\right)^{Ⱡ_2}}-}{\sqrt{1-\left(1-\mu^2_{S_{\alpha_{\sigma(1)}}}\right)^{Ⱡ_1}}\sqrt{1-\left(1-\mu^2_{S_{\alpha_{\sigma(2)}}}\right)^{Ⱡ_2}}}\right),\\ \left(\dfrac{\sqrt{1-\left(1-\mu^2_{T_{\alpha_{\sigma(1)}}}\right)^{Ⱡ_1}}+\sqrt{1-\left(1-\mu^2_{T_{\alpha_{\sigma(2)}}}\right)^{Ⱡ_2}}-}{\sqrt{1-\left(1-\mu^2_{T_{\alpha_{\sigma(1)}}}\right)^{Ⱡ_1}}\sqrt{1-\left(1-\mu^2_{T_{\alpha_{\sigma(2)}}}\right)^{Ⱡ_2}}}\right),\\ \left(\nu^{Ⱡ_1}_{S_{\alpha_{\sigma(1)}}}\nu^{Ⱡ_2}_{S_{\alpha_{\sigma(2)}}},\nu^{Ⱡ_1}_{T_{\alpha_{\sigma(1)}}}\nu^{Ⱡ_2}_{T_{\alpha_{\sigma(2)}}}\right)\end{array}\right\}$$
$$=\left\{\begin{array}{c}\left(\sqrt{1-\Pi^2_{d=1}\left(1-\mu^2_{S_{\alpha_{\sigma(d)}}}\right)^{Ⱡ_d}},\sqrt{1-\Pi^2_{d=1}\left(1-\mu^2_{T_{\alpha_{\sigma(d)}}}\right)^{Ⱡ_d}}\right),\\ \left(\Pi^2_{d=1}\nu^{Ⱡ_d}_{S_{\alpha_{\sigma(d)}}},\Pi^2_{d=1}\nu^{Ⱡ_d}_{T_{\alpha_{\sigma(d)}}}\right)\end{array}\right\}. \quad (5.3)$$

(2). If $\acute{n}=m$, using Eq. 9.6 we get the following form:

$$PyFZNOW\left(\alpha_1,\alpha_2,\ldots\alpha_m\right)=\sum_{d=1}^{m}Ⱡ_d\alpha_{\sigma d}$$
$$=\left\{\begin{array}{c}\left(\sqrt{1-\Pi^m_{d=1}\left(1-\mu^2_{S_{\alpha_{\sigma(d)}}}\right)^{Ⱡ_d}},\sqrt{1-\Pi^m_{d=1}\left(1-\mu^2_{T_{\alpha_{\sigma(d)}}}\right)^{Ⱡ_d}}\right),\\ \left(\Pi^{\acute{n}}_{d=1}\nu^{Ⱡ_d}_{S_{\alpha_{\sigma(d)}}},\Pi^{\acute{n}}_{d=1}\nu^{Ⱡ_d}_{T_{\alpha_{\sigma(d)}}}\right)\end{array}\right\}. \quad (5.4)$$

(3). If $\acute{n}=m+1$, using the operation (6) and (8) in Definition 2, and also use Eqs 9.7, 9.8 we have the following result:

$$PyFZNOW\left(\alpha_1,\alpha_2,\ldots,\alpha_m,\alpha_{m+1}\right)$$
$$=\sum_{d=1}^{m}Ⱡ_d\alpha_{\sigma d}\oplus Ⱡ_{m+1}\alpha_{\sigma d+m}$$
$$=\left\{\begin{array}{c}\left(\sqrt{1-\Pi^m_{d=1}\left(1-\mu^2_{S_{\alpha_{\sigma(d)}}}\right)^{Ⱡ_d}},\sqrt{1-\Pi^m_{d=1}\left(1-\mu^2_{T_{\alpha_{\sigma(d)}}}\right)^{Ⱡ_d}}\right),\\ \left(\Pi^m_{d=1}\nu^{Ⱡ_d}_{S_{\alpha_{\sigma(d)}}},\Pi^m_{d=1}\nu^{Ⱡ_d}_{T_{\alpha_{\sigma(d)}}}\right)\end{array}\right\}\oplus$$
$$Ⱡ_{m+1}\alpha_{\sigma d+m}$$
$$=\left\{\begin{array}{c}\left(\sqrt{1-\Pi^{m+1}_{d=1}\left(1-\mu^2_{S_{\alpha_{\sigma(d)}}}\right)^{Ⱡ_d}},\sqrt{1-\Pi^{m+1}_{d=1}\left(1-\mu^2_{T_{\alpha_{\sigma(d)}}}\right)^{Ⱡ_d}}\right),\\ \left(\Pi^{m+1}_{d=1}\nu^{Ⱡ_d}_{S_{\alpha_{\sigma(d)}}},\Pi^{m+1}_{d=1}\nu^{Ⱡ_d}_{T_{\alpha_{\sigma(d)}}}\right)\end{array}\right\}.$$

It is true for all $\acute{n}$, hence the verification is completed.

Theorem 6. *The PyFZNOW operator implies the following properties:*

(1) Idempotency: Let $\alpha_d=\left\{\left(\mu_{S_{\alpha_d}},\mu_{T_{\alpha_d}}\right)\left(\nu_{S_{\alpha_d}},\nu_{T_{\alpha_d}}\right)\right\}$ $(d=1,2,\ldots\acute{n})$ *PyFZNs. If* $\alpha_d=\alpha\,(d=1,2,\ldots\acute{n})$, *there is* $PyFZNOW\left(\alpha_1,\alpha_2,\ldots\alpha_{\acute{n}}\right)=\alpha$

(2) Boundedness: Let $\alpha_d=\left\{\left(\mu_{S_{\alpha_d}},\mu_{T_{\alpha_d}}\right)\left(\nu_{S_{\alpha_d}},\nu_{T_{\alpha_d}}\right)\right\}$ $(d=1,2,\ldots\acute{n})$ *be a collection of* P_yFZN *and*

$$\alpha_{\min}=\langle\min\left(\mu_{\alpha_d}(S,T)\right),\max\left(\nu_{\alpha_d}(S,T)\right)\rangle$$
$$=\left\{\left(\min_d\left(\mu_{s_{\alpha_d}}\right),\min_d\left(\mu_{T_{\alpha_d}}\right)\right),\left(\max\left(\nu_{s_{\alpha_d}}\right),\max\left(\nu_{T_{\alpha_d}}\right)\right)\right\}$$
$$\alpha_{\max}=\left\{\max\left(\mu_{\alpha_d}(S,T)\right),\min\nu_{\alpha_d}(S,T)\right\}$$
$$=\left\{\max_d\left(\mu_{s_{\alpha_d}}\right),\max_d\left(\mu_{T_{\alpha_d}}\right),\left(\min_d\left(\nu_{s_{\alpha_d}}\right),\min_d\left(\nu_{Td}\right)\right)\right\}$$

Then,

can keep. $\alpha_{\min}\le PyFZNOW\left(\alpha_1,\alpha_2,\ldots,\alpha_{\acute{n}}\right)\le\alpha_{\max}$

(3) Monotonoicity: set $\alpha_d=\left\{\left(\mu_{S_d},\mu_{T_d}\right),\left(\nu_{S_d},\nu_{T_d}\right)\right\}$ $(d=1,2,\ldots\acute{n})$ *and* $\alpha^*_d=\left\{\left(\mu^*_{S_d},\mu^*_{T_d}\right),\left(\nu^*_{S_d},\nu^*_{T_d}\right)\right\}$ $(d=1,2,\ldots\acute{n})$ *be a collection of PyFZNS. When* $\alpha_d\le\alpha^*_d$, *there is* $PyFZNOW\left(\alpha_1,\alpha_2,\ldots\alpha_{\acute{n}}\right)\le PyFZNOW\left(\alpha^*_1,\alpha^*_2,\ldots\alpha^*_{\acute{n}}\right)$.

Proof. (1) If $\alpha_d=\alpha\,(d=1,2,\ldots\acute{n})$, then the result of Eq. 9.2 is given by

$$PyFZNOW\left(\alpha_1,\alpha_2,\ldots,\alpha_{\acute{n}}\right)=\sum_{d=1}^{\acute{n}}Ⱡ_d\alpha_d$$
$$=\left\{\begin{array}{c}\left(\begin{array}{c}\sqrt{1-\Pi^{\acute{n}}_{d=1}\left(1-\mu^2_{S_{\alpha_d}}\right)^{Ⱡ_d}},\\ \sqrt{1-\Pi^{\acute{n}}_{d=1}\left(1-\mu^2_{T_{\alpha_d}}\right)^{Ⱡ_d}}\end{array}\right),\\ \left(\Pi^{\acute{n}}_{d=1}\nu^{Ⱡ_d}_{S_{\alpha_d}},\Pi^{\acute{n}}_{d=1}\nu^{Ⱡ_d}_{T_{\alpha_d}}\right)\end{array}\right\},$$
$$=\left\{\begin{array}{c}\left(\begin{array}{c}\sqrt{1-\left(1-\mu^2_{S_\alpha}\right)^{\Pi^{\acute{n}}_{d=1}Ⱡ_d}},\\ \sqrt{1-\left(1-\mu^2_{T_\alpha}\right)^{\Pi^{\acute{n}}_{d=1}Ⱡ_d}}\end{array}\right),\\ \left(\left(\nu_{S_\alpha}\right)^{\Pi^{\acute{n}}_{d=1}Ⱡ_d},\left(\nu_{T_\alpha}\right)^{\Pi^{\acute{n}}_{d=1}Ⱡ_d}\right)\end{array}\right\},$$
$$=\left\{\left(\sqrt{1-\left(1-\mu^2_{S_\alpha}\right)},\sqrt{1-\left(1-\mu^2_{T_\alpha}\right)}\right),\left(\nu_{S_\alpha},\nu_{T_\alpha}\right)\right\},$$
$$=\left\{\left(\mu_{S_\alpha},\mu_{T_\alpha}\right),\left(\nu_{S_\alpha},\nu_{T_\alpha}\right)\right\}=\alpha.$$

(2) Since $\alpha_{\min}$ and $\alpha_{\max}$ are given by the minimum *PyFZN* and the maximum *PyFV*, then the inequality $\alpha_{\min}\le\alpha\le\alpha_{\max}$ exists. Thus, there is $\sum^{\acute{n}}_{d=1}Ⱡ_d\alpha_{\min}\le\sum^{\acute{n}}_{d=1}Ⱡ_d\alpha_d\le\sum^{\acute{n}}_{d=1}Ⱡ_d\alpha_{\max}$. Based on the above property (1) $\alpha_{\min}\le\sum^{\acute{n}}_{d=1}Ⱡ_d\alpha_d\le\alpha_{\max}$ can exists, i,e., there is $\alpha_{\min}\le PyFZNOW\left(\alpha_1,\alpha_{2,\ldots}\alpha_{\acute{n}}\right)\le\alpha_{\max}$.

(3) $Gz_d\le G^*z_d$, there is $\sum^{\acute{n}}_{d=1}Ⱡ_d\alpha_d\le\sum^{\acute{n}}_{d=1}Ⱡ_d\alpha^*_d$, *d.e.,* $PyFZNOW\left(\alpha_1,\alpha_{2,\ldots}\alpha_{\acute{n}}\right)\le PyFZNOW\left(\alpha^*_1,\alpha^*_2,\ldots,\alpha^*_{\acute{n}}\right)$. All the properties of given theorem is complete.

5.2 PyFZNOWG operator

Using the operation (7) and (9) in Definition 4, we give the *PyFZNOWG* operator of *PyFZNs*.

Definition 10. *Let* $\alpha_d=\left\{\left(\mu_{S_{\alpha_d}},\mu_{T_{\alpha_d}}\right)\left(\nu_{S_{\alpha_d}},\nu_{T_{\alpha_d}}\right)\right\}$ $(d=1,2,\ldots\acute{n})$ *be a collection of PyFZNs and* $PyFZNOWG\colon\Omega^{\acute{n}}\to\Omega$. *Then the PyFZNOWG operator is defined as:*

$$PyFZNOWG\left(\alpha_1,\alpha_{2,\ldots}\alpha_{\acute{n}}\right)=\Pi^{\acute{n}}_{d=1}\alpha^{Ⱡ_d}_{\sigma d}$$

where $Ⱡ_d$ $(d=1,2,\ldots\acute{n})$ *with* $0\le \text{L}={}_d\le 1$ *and* $\sum^{\acute{n}}_{d=1}Ⱡ_d=1$.

Theorem 7. *Let* $\alpha_d=\left\{\left(\mu_{S_{\alpha_d}},\mu_{T_{\alpha_d}}\right)\left(\nu_{S_{\alpha_d}},\nu_{T_{\alpha_d}}\right)\right\}$ $(d=1,2,\ldots\acute{n})$ *be a collection of PyFZNs. Then, the collected value of the PyFZNOWG operator is a PyFZN, which is obtained by the following formula:*

$$PyFZNOWG\left(\alpha_{\sigma(1)},\alpha_{\sigma(2)},\ldots,\alpha_{\sigma(\acute{n})}\right)=\Pi^{\acute{n}}_{d=1}\alpha^{Ⱡ_d}_{\sigma d}$$
$$=\left\{\begin{array}{c}\left(\Pi^{\acute{n}}_{d=1}\mu^{Ⱡ_d}_{S\alpha_{\sigma(d)}},\Pi^{\acute{n}}_{d=1}\mu^{Ⱡ_d}_{T_{\alpha_{\sigma(d)}}}\right),\\ \left(\sqrt{1-\Pi^{\acute{n}}_{d=1}\left(1-\nu^2_{S_{\alpha_{\sigma(d)}}}\right)^{Ⱡ_d}},\sqrt{1-\Pi^{\acute{n}}_{d=1}\left(1-\nu^2_{T\alpha_{\sigma(d)}}\right)^{Ⱡ_d}}\right)\end{array}\right\}$$

where $Ⱡ_d$ $(d=1,2,\ldots\acute{n})$ *with* $0\le Ⱡ_d\le 1$ *and* $\sum^{\acute{n}}_{d=1}Ⱡ_d=1$. *By the similar verification process of Theorem 5.*

Theorem 8. *The PyFZNOWoperator implies the following properties:*

(1) Idempotency: Let $\alpha_d=\left\{\left(\mu_{S_{\alpha_d}},\mu_{T_{\alpha_d}}\right)\left(\nu_{S_{\alpha_d}},\nu_{T_{\alpha_d}}\right)\right\}$ $(d=1,2,\ldots\acute{n})$ *PyFZNs.* If $\alpha_d=\alpha\,(d=1,2,\ldots\acute{n})$, there is $PyFZNOW\left(\alpha_1,\alpha_2,\ldots\alpha_{\acute{n}}\right)=\alpha$

(2) Boundedness: Let $\alpha_d = \left\{(\mu_{S_{\alpha_d}}, \mu_{T_{\alpha_d}})(\nu_{S_{\alpha_d}}, \nu_{T_{\alpha_d}})\right\}(d = 1, 2, \ldots ń)$ be a collection of *PyFZNs* and let

$$\alpha_{\min} = \langle \min\left(\mu_{\alpha_d}(S,T)\right), \max\left(\nu_{\alpha_d}(S,T)\right)\rangle$$
$$= \left\{\left(\min_d(\mu_{s_{\alpha_d}}), \min_d(\mu_{T_{\alpha_d}})\right), \left(\max_d\left(\nu_{s_{\alpha_d}}\right), \max_d\left(\nu_{T_{\alpha_d}}\right)\right)\right\}$$
$$\alpha_{\max} = \left\{\max\left(\mu_{\alpha_d}(S,T)\right), \min \nu_{\alpha_d}(S,T)\right\}$$
$$= \left\{\max_d(\mu_{s_{\alpha_d}}), \max_d(\mu_{T_{\alpha_d}}), \left(\min_d\left(\nu_{s_{\alpha_d}}\right), \min_d(\nu_{Td})\right)\right\}$$

Then,

$$\alpha_{\min} \leq PyFZNOWG(\alpha_1, \alpha_2, \ldots, \alpha_ń) \leq \alpha_{\max}$$

can keep

(3) Monotonoicity: set $\alpha_d = \left\{(\mu_{S_d}, \mu_{T_d}), (\nu_{S_d}, \nu_{T_d})\right\}(d = 1, 2, \ldots ń)$ and $\alpha_d^* = \left\{(\mu_{S_d}^*, \mu_{T_d}^*), (\nu_{S_d}^*, \nu_{T_d}^*)\right\}(d = 1, 2, \ldots ń)$ be a collection of $PyFZńs$. When $\alpha_d \leq \alpha_d^*$, there is

$$PyFZNOW(\alpha_1, \alpha_2, \ldots \alpha_ń) \leq PyFZNOW(\alpha_1^*, \alpha_2^*, \ldots \alpha_ń^*).$$

Can also be confirmed Theorem 2 using the aforementioned properties corresponding to the *PyFZNWG* operator, which is not repeated here.

6 MDM approach using the *PyFZNW* and *PyFZNWG* operator and the score funtion

In order to handle MDM difficulties, this part develops an MDM methodology using assessment data for both Pythagorean values and Pythagorean reliability measures. This approach relates to PyFZNW and PyFZNW Goperators and the score function. Since a set of n criteria $ń = \{l_1, l_2, \ldots l_m,\}$ are used to evaluate a set of m options $Q = \{Q_1, Q_2, \ldots Q_m\}$ in an MDM issue. The weight L = $_d$, are define as the weight vector Ⱡ= (Ⱡ$_1$,Ⱡ$_2$, ...Ⱡ$_ń$), which takes into account the significance of each criterion l_d $(d = 1, 2, \ldots ń)$. Decision-makers are asked to evaluate each criterion's applicability for each alternative $Q_j = \{1, 2, \ldots m\}$ using both membership and non-membership fuzzy values as well as related accuracy measures. $Gz_{jd} = \left\{(\mu_{jd}(S,T)), \nu_{jd}(S,T)\right\} = \left\{(\mu_{S_{jd}}, \mu_{T_{jd}}), (\nu_{S_{jd}}, \nu_{T_{jd}})\right\}$, where $\mu_{S_{jd}}, \mu_{T_{jd}} \in [0,1]$ and $\nu_{S_{jd}}, \nu_{T_{jd}} \in [0,1]$. Now the decision matrix of PyZN is determined as $Gz = (Gz_{jd})_{m\times ń}$. The decision process is defined in MDM problem as step1: Using Eqs 5.2, 6.2, the *PyZN* is defined by:

$$Gz_j = PyFZNWGz_{j1}, Gz_{j2,\ldots}Gz_{jń} = \sum_{d=1}^{ń} Ⱡ_d Gz_{jd}$$
$$= \left\{\begin{array}{c}\left(\sqrt{1 - \Pi_{d=1}^{ń}\left(1 - \mu_{S_{jd}}^2\right)^{Ⱡ_d}}, \sqrt{1 - \Pi_{d=1}^{ń}\left(1 - \mu_{T_{jd}}^2\right)^{Ⱡ_d}}\right), \\ \left(\Pi_{d=1}^{ń}\nu_{Sjd}^{Ⱡ_d}, \Pi_{d=1}^{ń}\nu_{Tjd}^{Ⱡ_d}\right)\end{array}\right\} \tag{6.1}$$

and

$$Gz_j = PyFZNWG\left(Gz_{jd}, Gz_{jd,\ldots}Gz_{jń}\right) = \Pi_{d=1}^{ń} G^{Ⱡ_d} z_{jd}$$
$$= \left\{\begin{array}{c}\left(\Pi_{d=1}^{ń}\mu_{Sjd}^{Ⱡ_d}, \Pi_{d=1}^{ń}\mu_{Tjd}^{Ⱡ_d}\right), \\ \left(\sqrt{1 - \Pi_{d=1}^{ń}\left(1 - \nu_{Sjd}^2\right)^{Ⱡ_d}}, \sqrt{1 - \Pi_{d=1}^{ń}\left(1 - \nu_{T_{jd}}^2\right)^{Ⱡ_d}}\right)\end{array}\right\} \tag{6.2}$$

Step 2: Using Eq. 3.1, the score values of $J(G_{Zj})(j = 1, 2, \ldots m)$ are calculated.

Step 3: The best option among the rated options is chosen based on the score values.

Step 4: End.

6.1 An illustrative example and relative comparative analysis

To illustrate the relevance and efficacy of the developed MDM technique with PyFZN information, this section gives an example concerning the challenge of choosing business partners or suppliers. An example of a complex piece of machinery is an aircraft, which has intricate manufacturing processes and strict supplier criteria. Due to the complexity of airplanes, it is challenging for the major manufacturers to complete production on their own time. Therefore, other suppliers work with the primary manufacturers to finish the production of airplanes with a high degree of personalization in the aircraft's features. For instance, the structure, a component of an aircraft, is crucial to the manufacturing process. An essential component of the skeleton and aerodynamic form of the aircraft body, an aircraft structural part is available in a wide range of complex designs and a number of materials. The weight and strength requirements must be fully taken into account throughout the design and production of aircraft structural elements. The fabrication of structural components for airplanes is a challenging process with stringent standards (Tong and Zhu, 2020a).

Suppose a manufacturer needs to select a reliable supplier from among potential suppliers. Expert panel presents a set of five suppliers or alternatives $Q = \{Q_1, Q_2, \ldots Q_5\}$, which must meet the assessment standards of the criteria: l_1 is the product cost; l_2 is the product quality and l_3 is the delivery lead time.The three criteria's weight vector is written as (0.2, 0.5, 0.3) to denote their relative importance. Then, the PyZNs that are made up of their membership and non-membership fuzzy values and the measurements of associated reliabilities encourage the experts/decision-makers to evaluate the four suppliers/alternatives over the three criteria. As a result, the following PyZN decision matrix can be used to create all PyZNs:

$Gz = (Gz_{jd})_{5\times 3}$

{(0.6, 0.4), (0.5, 0.3)}	{(0.7, 0.1), (0.3, 0.5)}	{(0.4, 0.1), (0.8, 0.2)}
{(0.3, 0.1), (0.4, 0.5)}	{(0.4, 0.3), (0.6, 0.1)}	{(0.2, 0.7), (0.1, 0.3)}
{(0.2, 0.8), (0.4, 0.1)}	{(0.6, 0.1), (0.6, 0.2)}	{(0.3, 0.5), (0.6, 0.2)}
{(0.5, 0.6), (0.7, 0.2)}	{(0.3, 0.6), (0.4, 0.1)}	{(0.6, 0.3), (0.1, 0.4)}
{(0.3, 0.4), (0.1, 0.5)}	{(0.6, 0.1), (0.7, 0.2)}	{(0.5, 0.4), (0.6, 0.1)}

On the other hand we can apply PyFZNW

Step 1: To find PyZNs G_{Zj} $(j = 1, 2, 3, 4, 5)$ using (7.1) equation is defined as:

$$Gz_1 = \{(0.266926, 0.09527), (0.445945, 0.34294)\},$$
$$Gz_2 = \{(0.104892, 0.037337), (0.323212, 0.191836)\},$$
$$Gz_3 = \{(0.078034, 0.271836), (0.553265, 0.17411)\},$$
$$Gz_4 = \{(0.194665, 0.252877), (0.295155, 0.17411)\},$$
$$Gz_5 = \{(0.125337, 0.114306), (0.452892, 0.195123)\}.$$

Step 2: The score values $J(Gz_j)$ of PyFZNW for the alternatives $Q_j = \{1, 2, 3, 4, 5\}$ are given below:

$$J(Gz_1) = 0.436249, J(Gz_2) = 0.470956,$$
$$J(Gz_3) = 0.462442, J(Gz_4) = 0.498918,$$
$$J(Gz_5) = 0.462979.$$

Step 3: According to the score values $J(Gz_4) \geq J(Gz_2) \geq J(Gz_5) \geq J(Gz_3) \geq J(Gz_1)$, the five alternatives are ranked as $Q_4 \geq Q_2 \geq Q_5 \geq Q_3 \geq Q_1$.Hence the best supplier is Q_4

Now we can apply PyFZNWGA, in MDM problem can be solved using the invented MDM approach using the PyFZNWG operator, which is illustrated by the following decision-making process:

Step 1: The overall collected PyFZN Gz_j ($j = 1, 2, 3, 4, 5$) are obtained as follow:

$$Gz_1 = \{(0.573849, 0.131951), (0.198604, 0.109134)\},$$
$$Gz_2 = \{(0.306735, 0.31052), (0.138146, 0.141117)\},$$
$$Gz_3 = \{(0.391217, 0.245646), (0.17265, 0.029779)\},$$
$$Gz_4 = \{(0409072., 0.487351), (0.247477, 0.055679)\}.$$
$$Gz_5 = \{(0.494528, 0.2), (0.042637, 0.142772)\}.$$

Step 2: The score values $J(Gz_j)$ of PyFZNWG for the alternatives $Q_j = \{1, 2, 3, 4, 5\}$ are given below:

$$J(Gz_1) = 0.527023, J(Gz_2) = 0.537876,$$
$$J(Gz_3) = 0.54548, J(Gz_4) = 0.592791,$$
$$J(Gz_5) = 0.546409.$$

Step 3: According to the score values $J(Gz_4) \geq J(Gz_5) \geq J(Gz_3) \geq J(Gz_2) \geq J(Gz_1)$, the five alternatives are ranked as $Q_4 \geq Q_5 \geq Q_3 \geq Q_2 \geq Q_1$.Hence the best supplier is Q_4. Here we first order the given matrix with the help of score funtion then the origional matrix becomes: $Gz = \quad (Gz_{jd})_{5\times3}$

$\{(0.6, 0.4), (0.5, 0.3)\}$	$\{(0.7, 0.1), (0.3, 0.5)\}$	$\{(0.4, 0.1), (0.8, 0.2)\}$
$\{(0.2, 0.7), (0.1, 0.3)\}$	$\{(0.4, 0.3), (0.6, 0.1)\}$	$\{(0.3, 0.1), (0.4, 0.5)\}$
$\{(0.2, 0.8), (0.4, 0.1)\}$	$\{(0.3, 0.5), (0.6, 0.2)\}$	$\{(0.6, 0.1), (0.6, 0.2)\}$
$\{(0.5, 0.6), (0.7, 0.2)\}$	$\{(0.3, 0.6), (0.4, 0.1)\}$	$\{(0.6, 0.3), (0.1, 0.4)\}$
$\{(0.5, 0.4), (0.6, 0.1)\}$	$\{(0.3, 0.4), (0.1, 0.5)\}$	$\{(0.6, 0.1), (0.7, 0.2)\}$

On the other hand we can apply PyFZNOW

Step 1: To find PyZNs G_{Zj} ($j = 1, 2, 3, 4, 5$) using Eq. 7.1 equation is defined as:

$$Gz_1 = \{(0.266926, 0.09527), (0.445945, 0.34294)\},$$
$$Gz_2 = \{(0.072905, 0.23382), (0.371273, 0.20189)\},$$
$$Gz_3 = \{(0.074235, 0.311844), (0.553265, 0.17411)\},$$
$$Gz_4 = \{(0.194665, 0.252877), (0.295155, 0.17411)\},$$
$$Gz_5 = \{(0.194665, 0.129066), (0.256543, 0.275292)\}.$$

Step 2: The score values $J(Gz_j)$ of *PyFZNOW* for the alternatives $Q_j = \{1, 2, 3, 4, 5\}$ are given below:

$$J(Gz_1) = 0.436249, J(Gz_2) = 0.471045,$$
$$J(Gz_3) = 0.46341, J(Gz_4) = 0.498918,$$
$$J(Gz_5) = 0.47725.$$

Step 3: According to the score values $J(Gz_4) \geq J(Gz_5) \geq J(Gz_2) \geq J(Gz_3) \geq J(Gz_1)$, the five alternatives are ranked as $Q_4 \geq Q_5 \geq Q_2 \geq Q_3 \geq Q_1$.Hence the best supplier is Q_4

Now we can apply PyFZNOWG, in MDM problem can be solved using the invented MDM approach using the PyFZNOWG operator, which is illustrated by the following decision-making process:

Step 1: The overall collected PyFZN Gz_j ($j = 1, 2, 3, 4, 5$) are obtained as follow:

$$Gz_1 = \{(0.573849, 0.131951), (0.198604, 0.109134)\},$$
$$Gz_2 = \{(0.319428, 0.255611), (0.040122, 0.086437)\},$$
$$Gz_3 = \{(0.340574, 0.338925), (0.17265, 0.029779)\},$$
$$Gz_4 = \{(0409072., 0.487351), (0.247477, 0.055679)\}.$$
$$Gz_5 = \{(0.409072, 0.263902), (0.190216, 0.035776)\}.$$

Step 2: The score values $J(Gz_j)$ of PyFZNOWG for the alternatives $Q_j = \{1, 2, 3, 4, 5\}$are given below:

$$J(Gz_1) = 0.527023, J(Gz_2) = 0.539091,$$
$$J(Gz_3) = 0.555144, J(Gz_4) = 0.592791,$$
$$J(Gz_5) = 0.550575.$$

Step 3: According to the score values $J(Gz_4) \geq J(Gz_3) \geq J(Gz_5) \geq J(Gz_2) \geq J(Gz_1)$, the five alternatives are ranked as $Q_4 \geq Q_3 \geq Q_5 \geq Q_2 \geq Q_1$. Hence the best supplier is Q_4

According to the created *MDM* approach that makes use of the *PyFZNW*, *PyFZNOW*, *PyFZNWG* and *PyFZNOWG* operators as well as the score function, we can observe that the four types of ranking orders mentioned above for the five options and the best option are the same. As a result, the developed MDM strategy works.

7 The extended EDAS method based on novel pythagorean fuzzy Z-number

Evaluation Based on Distance from Average Solution (EDAS), a brand-new and powerful MCDM technique, was created. This method estimates the desirableness of an option based on how far from the average answer they are. In order to verify the efficacy of the Pythagorean fuzzy z-number weighted geometric AOs, a novel extended EDAS approach is developed here to manage the complex uncertain data in real-life DS situations. Assume there are a number of "alternatives" $\{\varnothing_1, \varnothing_2, \ldots, \varnothing_l\}$, and a satisfactory rating $\{R_1, R_2, \ldots, R_m\}$ for each. Then, $Ⱡ_d = (Ⱡ_1, Ⱡ_2, \ldots Ⱡ_m)^T$ specifies the usefulness of various characteristics R_d ($d = 1, 2, \ldots, m$), such that $L =_d > 0$ and $\Sigma_{d=1}^{m} Ⱡ_d = 1$. Let $Gz_{jd} = \{U_{Sjd}U_{Tjd} \; V_{Sjd}V_{Tjd}\}$ where $0 \leq (U_{Sjd})^2 + (U_{Tjd})^2 \leq 1$ be the permissible rating for each attribute for each option.

Step 1: Choose a series of attributes that can be applied to assess the issue: Through a review of the literature, prospective assessment characteristics are gathered, and an expert DM committee is formed to screen the characteristics in order to create a respectable set of evaluation R_d ($d = 1, 2, \ldots, m$).

$$Gz_{jd} = \begin{pmatrix} (U_{S11}U_{T11} & V_{S11}V_{T11}) & (U_{S12}U_{T12} & V_{S12}V_{T12}) & \ldots & (U_{S1m}U_{T1m} & V_{S1m}V_{T1m}) \\ (U_{S21}U_{T21} & V_{S21}V_{T21}) & (U_{S22}U_{T22} & V_{S22}V_{T22}) & \ldots & (U_{S2m}U_{T2m} & V_{S2m}V_{T2m}) \\ \ldots & & \ldots & & \ldots & \ldots & \\ (U_{Sr1}U_{Tr1} & V_{Sr1}V_{Tr1}) & (U_{Sr2}U_{Tr2} & V_{Sr2}V_{Tr2}) & \ldots & (U_{Srm}U_{Trm} & V_{Srm}V_{Trm}) \end{pmatrix}$$

Step 2: The normalized decision matrix is created using normalization as follows:

$$Gz_{jd} = \begin{matrix} \left(U_{Sjd}U_{Tjd} \quad V_{Sjd}V_{Tjd}\right), \text{ if C1} \\ \left(V_{Sjd}V_{Tjd} \quad U_{Sjd}U_{Tjd}\right), \text{ if c11} \end{matrix}$$

if R_d ($d = 1, 2, \ldots, m$) is a benefit criterion, the statement use CI, if R_d ($d = 1, 2, \ldots, m$) is a cost criterion, the statement CII use.

Step 3: Aggregated Data: The skilled uncertain data of required situations are aggregated using established *PyFZNWG* operators.

$$PyFZNW(Gz_1, Gz_2, ... Gz_{\acute{n}}) = \Pi_{d=1}^{\acute{n}} G^{ł_d} z_d = \left\{ \begin{array}{c} \left(\Pi_{d=1}^{\acute{n}} \mu_{S_d}^{ł_d}, \Pi_{d=1}^{\acute{n}} \mu_{T_d}^{ł_d}\right), \\ \left(\sqrt{1 - \Pi_{d=1}^{\acute{n}}\left(1 - \nu_{S_d}^2\right)^{ł_d}}, \sqrt{1 - \Pi_{d=1}^{\acute{n}}\left(1 - \nu_{T_d}^2\right)^{ł_d}}\right) \end{array} \right\}$$

Step 4: Verify the average solution ($A_V S$), which is based on all the criteria given. $A_V S = [A_V S_d]_{1\times m} = \left\{\frac{\Sigma_{j=1}^{\acute{n}} Gz_{jd}}{\acute{n}}\right\}_{1\times m}$ Using Definition 3, we obtain $A_V S = [A_V S_d]_{1\times m} = \left\{\frac{\Sigma_{j=1}^{\acute{n}} Gz_{jd}}{\acute{n}}\right\}_{1\times m}$

$$= \left\{ \begin{array}{c} \left(\Sigma_{j=1}^{\acute{n}} \sqrt{1 - \left(1 - \mu_{S_j}^2\right)^{ł}}, \sqrt{1 - \left(1 - \mu_{T_j}^2\right)^{ł}}\right), \\ \left(\Sigma_{j=1}^{\acute{n}} \nu_{S_j}^{ł}, \nu_{T_j}^{ł}\right) \end{array} \right\}$$

Step 5: Using the $A_V S$ values, the positive distance from average (PDA_v) and the negative distance from average (NDA_v) can be calculated:

$$PDA_v = \frac{\max 0, \left(Gz_{jd} - A_V S\right)}{A_V S}$$

$$NDA_v = \frac{\max 0, \left(A_V S - Gz_{jd}\right)}{A_V S}$$

To compute the PDA and NDA, we can use the score function of $PyFZNs$ mentioned in Definition 3 as follows:

$$PDA_v = \frac{\max 0, \left(J\left(Gz_{jd}\right) - J(A_V S)\right)}{J(A_V S)}$$

$$NDA_v = \frac{\max 0, \left(J(A_V S) - J\left(Gz_{jd}\right)\right)}{J(A_V S)}$$

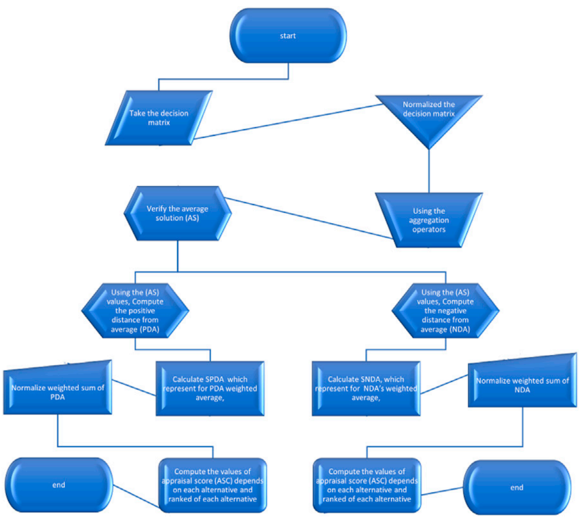
where W shows the score value.

Step 6: Calculate $SPDA$ and $SNDA$, which represent for PDA and $NDA's$ weighted average, respectively: $SPDA = \sum_{d=1}^{m} ł_d PDA_d$, $SNDA = \sum_{d=1}^{m} ł_d NDA_d$ $ł_d \in [0, 1]$ and $\Sigma_{d=1}^{m} ł_d = 1$.

Step 7: Normalize weighted sum of PDA and NDA is defined as repectively:

$$NSPDA = \frac{SPDA}{max(SPDA)}$$

$$NSPDA = \frac{SNDA}{max(SNDA)}$$



Step 8: Compute the values of appraisal score (ASC) depends on each alternative's as

$$ASC = \frac{1}{2}((NSPDA + 1 - NSNDA)$$

Step 9: Depending on the ASC calculations, alternatives are sorted in decreasing order, and the higher the ASC number, the better options will be chosen.

7.1 An illustrative example

Step 1: Consider the decision matrix as discussed in previous example.

$Gz = \quad (Gz_{jd})_{5\times 3}$

{(0.6, 0.4), (0.5, 0.3)}	{(0.7, 0.1), (0.3, 0.5)}	{(0.4, 0.1), (0.8, 0.2)}
{(0.3, 0.1), (0.4, 0.5)}	{(0.4, 0.3), (0.6, 0.1)}	{(0.2, 0.7), (0.1, 0.3)}
{(0.2, 0.8), (0.4, 0.1)}	{(0.6, 0.1), (0.6, 0.2)}	{(0.3, 0.5), (0.6, 0.2)}
{(0.5, 0.6), (0.7, 0.2)}	{(0.3, 0.6), (0.4, 0.1)}	{(0.6, 0.3), (0.1, 0.4)}
{(0.3, 0.4), (0.1, 0.5)}	{(0.6, 0.1), (0.7, 0.2)}	{(0.5, 0.4), (0.6, 0.1)}

Step 2: The normalized decision matrix is created using normalization as follows:

$$Gz_{jd} = \begin{array}{l} \left(U_{Sjd} U_{Tjd} \quad V_{Sjd} V_{Tjd}\right), \text{ if C1} \\ \left(V_{Sjd} V_{Tjd} \quad U_{Sjd} U_{Tjd}\right), \text{ if c11} \end{array}$$

if R_d $(d = 1, 2, \ldots, m)$ is a benefit criterion, the statement use CI, if R_d $(d = 1, 2, \ldots, m)$ is a cost criterion, the statement CII use.Here the given system is already normalized

Step 3: Now we can apply PyFZNWGA, in MDM problem can be solved using the invented MDM approach using the PyFZNWG operator, which is illustrated by the following decision-making process.The overall collected PyFZN Gz_j $(j = 1, 2, 3, 4, 5)$ are obtained as follow:

$$\begin{array}{l} Gz_1 = \{(0.552113, 0.159033), (0.268361, 0.141392)\}, \\ Gz_2 = \{(0.288809, 0.276248), (0.174972, 0.20492)\}, \\ Gz_3 = \{(0.330559, 0.342362), (0.224104, 0.041134)\}, \\ Gz_4 = \{(0.4485, 0.476574), (0.315567, 0.193374)\}. \\ Gz_5 = (0.4485, 0.252332), 0.055274, 0.132527' \end{array} \quad (7.1)$$

Step 4: The score values J (Gz_j) of $PyFZNWG$ for the corresponding $Q_j = \{1, 2, 3, 4, 5\}$ are given below:

$$\begin{array}{l} J(Gz_1) = 0.52493, J(Gz_2) = 0.521964, \\ J(Gz_3) = 0.551976, J(Gz_4) = 0.593963, \\ J(Gz_5) = 0.551241. \end{array}$$

And verify the average solution ($A_V S$) as:

$$\begin{array}{l} A_V(Gz_1) = 0.413696, A_V(Gz_2) = 0.30131, \\ A_V(Gz_3) = 0.207656, A_V(Gz_4) = 0.132527. \end{array}$$

Score funtion of average solution ($A_V S$) we have:

$$J[A_V(Gz)] = 0.548565$$

Step 5: Using the $A_V S$ values, the positive distance from average (PDA_v) and the negative distance from average (NDA_v) can be be calculated:

Positive distance from average

0.334586	0	0.292333	0.066889
0	0	0	0.546247
0	0.136246	0.079207	0
0.08413	0.581673	0.519664	0
0.08413	0	0	0.459126

Negative distance from average

0	0.472196	0	0
0.301882	0.083177	0.157394	0
0.200962	0	0	0.68962
0	0	0	0.382649
0	0.162551	0.733819	0

To compute the *PDA* and *NDA*, we can use the score function of *PyFZNs* mentioned in Definition 3 as follows:

Find *PDA* using average of *PyFZNWG*

0	0	0.00621865	0.082756939	0.004878476

Find *NDA* using average of *PyFZNWG*

0.043085	0.48492	0	0	0

Step 6: Calculate *SPDA* and *SNDA*, which represent for *PDA* and *NDA′s* weighted average, and attributes weighting vector L = = (0.333, 0.333, 0.333, 0.333, 0.333), we can obtain the results as: respectively:

Find *SPDA* using weight vector

0	0	0.00621865	0.082756939	0.004878476

Find *NPDA* using weight vector

0	0	0.00621865	0.082756939	0.004878476

Step 7: Normalize weighted sum of *PDA* and *NDA* is defined as repectively:

NSPDA

0	0	0.07514371	1.000002199	0.05894958

NSNDA

0.888491579	0.999996804	0	0	0

Step 8: Compute the values of appraisal score (*ASC*) depends on each alternative's as:

appraisal score (ASC)

0.05575421	0.0000015	0.537571856	1.000001099	0.529474792

Step 9: Ranking of *EDAS* method based on Pythagorean fuzzy Z-number weighted geometric aggregation operator

Ranking of EDAS method based on PyFZNWG

0.05575421	Q_1
0.0000015	Q_2
0.537571856	Q_3
1.000001099	Q_4
0.529474792	Q_5

$Q_4 \geq Q_3 \geq Q_5 \geq Q_1 \geq Q_2$.Hence the best supplier is Q_4.

An analysis has been performed using the suggested operators, and the results are summarised in the table below in order to study the trend of variation in score and ranking of all options with the change in the aggregation procedure. From this tabular number, we see that the best choice stays the same, indicating that the outcomes are unbiased and cannot be affected by decision-makers preferences on aggregating processes. So the ranking results are trustworthy. While the comparison graph of all methods which are used in this article is represented in Figure 1. Therefore, our suggested technique is more flexible since the decision-maker(s) may pick operators based on their preferences and actual scenarios.

Comparison of all method

PyFZNW	$Q_4 \geq Q_2 \geq Q_5 \geq Q_3 \geq Q_1$	best Q_4
PyFZNWG	$Q_4 \geq Q_5 \geq Q_3 \geq Q_2 \geq Q_1$	Q_4
PyFZNOW	$Q_4 \geq Q_5 \geq Q_2 \geq Q_3 \geq Q_1$	Q_4
PyFZNOWG	$Q_4 \geq Q_3 \geq Q_5 \geq Q_2 \geq Q_1$	Q_4
EDAS	$Q_4 \geq Q_3 \geq Q_5 \geq Q_1 \geq Q_2$	Q_4

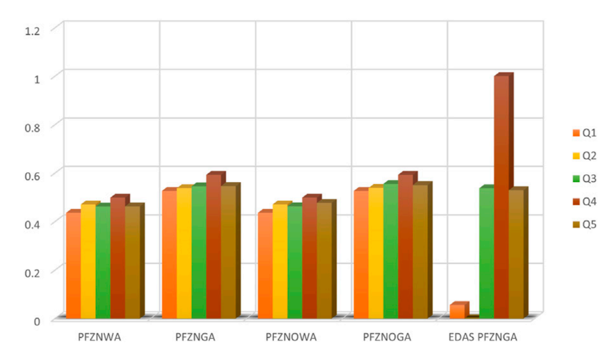


FIGURE 1
Ranking results.

TABLE 1 Comparison with existing studies.

Methods	$\mu(S)$	$\nu(S)$	Reliability	Range
Zadeh (1965)	yes	no	No	$0 \leq \mu(S) \leq 1$
Zadeh (2011)	yes	no	yes	$0 \leq \mu(S, T) \leq 1$
Atanassov and Atanassov (1999)	yes	yes	no	$0 \leq \mu(S) + \nu(S) \leq 1$
Yager (2013)	yes	yes	no	$0 \leq \mu(S)^2 + \nu(S)^2 \leq 1$
Proposed approach	yes	yes	yes	$0 \leq \mu(S,T)^2 + \nu(S,T)^2 \leq 1$

8 Comparasion analysis

Using the idea of constraints in combination with multiattribute and multiobjective decision making approaches, we devised a way to address challenging real-world problems. To demonstrate the usefulness of the proposed approach in contrast to the existing ones for multi-criteria decision-making, a comparative study was conducted using various structures developed by different researchers. Table 1 provides an analytical comparison of the CPHFNS technique to the existing approaches. When such data as CPHFNSS is supplied to a decision maker, none of the existing works can appropriately address it. While the suggested technique is capable of handling existing approaches data. Thus, our proposed methods are superior and more reliable than those now used.

9 Conclusion

In this study, we analyzed the limitation of the current Pythagorean sets and presented a new set, PyFZN, that may handle the issue of hybrid information representation that occurs when Pythagorean values and their related reliability measures are stated simultaneously. The novel score function, basic operations, and the PyFZNW and PyFZNWG operators of PyFZNs also introduced to aggregate information and MDM modeling in the PyFZN context. We also defined their properties and theorems with proofs. To deal with multicriteria decision making issues we offered an algorithm. We view the decision matrix Gz assessment measures of corresponding reliabilities as a special case of the definite. In this paper an example of supplier selection problem at large scale showed how well the created MDM technique worked in the PyFZN environment more effectively than the existing approaches. However, the created MDM strategy offers a fresh approach to resolving MDM issues with PyFZNs. We also presented the EDAS technique on the perposed concept and a comparative analysis with the existing studies to check the efficacy and supermecy of this study.

In the future to enhance the quality of the information provided we plan to apply various aggregation operators such as Einstein, Dombi, average hybrid, etc., with TOPSIS and VIKOR technique and justify their application with the help of medical diagnostics, network signaling, and artificial intelligence.

Data availability statement

The raw data supporting the conclusion of this article will be made available by the authors, without undue reservation.

Author contributions

All authors listed have made a substantial, direct, and intellectual contribution to the work and approved it for publication.

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Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

Publisher's note

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Assessing the environmental impact of industrial pollution using the complex intuitionistic fuzzy ELECTREE method: a case study of pollution control measures

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Environmental pollution has become a major issue in today's world, and controlling it is crucial for the sustainable development of our planet. Industries play a significant role in environmental pollution, and their impact must be controlled and minimized. In this paper, we present a novel approach, the complex intuitionistic fuzzy ELECTREE method (CIF−ELECTREE), for environmental impact assessment of industries. The method combines the principles of CIF sets and the ELECTREE method to provide a comprehensive and reliable assessment of the environmental impact of industries. The proposed method has been applied to real-world data, and the results obtained demonstrate the effectiveness of the CIF−ELECTREE method in controlling and reducing the environmental impact of industries. The results show that the CIF−ELECTREE method can provide a more accurate assessment of the environmental impact of industries compared to traditional methods. This study contributes to the existing literature by introducing a novel approach for environmental impact assessment and highlights the importance of considering both the uncertainty and vagueness in real-world data for reliable decision making.

KEYWORDS

complex intuitionistic fuzzy set, ELECTREE method, multi-attribute decision making, decision making, CIF−ELECTREE method

1 Introduction

Multi-attribute and multi-criteria decision making plays a crucial role in everyday life as well as in many complex decision-making scenarios. These techniques are used to make informed decisions based on multiple objectives, criteria, and attributes, which helps in taking into account the trade-off between conflicting goals. For example, in personal life, multi-attribute decision making is often used when making choices such as buying a car, choosing a school, or purchasing a home. These decisions require weighing multiple factors such as cost, location, quality, and personal preferences, and therefore, multi-attribute decision making provides a useful framework for making such choices. In business and other organizations, multi-criteria decision making is frequently used in decision-making scenarios such as selecting suppliers, choosing a marketing strategy, or making investment decisions. These decisions often involve multiple objectives that must be considered, such as financial performance, customer satisfaction, and environmental impact. Despite the importance of multi-attribute and multi-criteria decision making, the process of making decisions based on multiple factors is often difficult due to the complexity of the data and the uncertainty surrounding the information. In many cases, the information used in decision making is incomplete, uncertain, and ambiguous, making it difficult to reach a reliable conclusion. Additionally, conflicting criteria and objectives can lead to trade-offs that are difficult to reconcile. To overcome these difficulties, the use of multi-attribute and multi-criteria decision making in the context of fuzzy set theory provides a useful framework for representing and handling uncertainty and ambiguity in decision-making problems. This allows for a more robust and informed decision-making process, taking into account the trade-off between conflicting objectives and criteria.

The process of making decisions is becoming more difficult as a consequence of the ambiguity that exists in the information, particularly when the data are accessible in a crisp form. In order to accomplish this goal, Zadeh invented the fuzzy set (Zadeh et al., 1996), and the philosophy behind fuzzy sets is an important consideration when it comes to making decisions in the face of ambiguity. He devised the membership grade as a defense mechanism against uncertainty. The development of mathematics has reached an all-time high thanks to fuzzy theory, since it is so useful in decision making. Many researchers work on fuzzy theory. For example, fuzzy theory has been used to improve digital mammography (Hassanien and Badr, 2003), play a big role in pharmacology (Sproule et al., 2002), create a system to help make decisions about nitrogen fertilizer (Yu et al., 2018), develop a system for risk assessment (Ebadi and Shahraki, 2010) and audit detection (Chang et al., 2008), and look at the social factors of migrant workers with HIV/AIDS (Kandasamy and Smarandache, 2004). The FSDEs (Jafari et al., 2021) are used in real-world systems such as those found in economics and finance, where the phenomena are tied to randomness and fuzziness as two separate forms of uncertainty. However, there are certain limitations to Zadeh's fuzzy sets. For example, when there are more than two alternative values to address uncertainty, the method does not function well.

It has been found that intuitionistic fuzzy sets (IFSs), which were first introduced by Atanassov (1986), Atanassov (1989), and Atanassov (1999), are among the best higher-order fuzzy sets for dealing with ambiguity. In situations in which the information at hand is insufficient for the precise definition of an imprecise concept using a traditional fuzzy set, the idea of an IFS can be viewed as an alternative method for defining a fuzzy set. This can be performed by considering the concept of an IFS as an alternative approach to defining a fuzzy set. IFS theory may be thought of as a broader version of the original fuzzy set theory. Consequently, it is anticipated that IFSs might be used to imitate human decision-making processes and other tasks involving human experience and knowledge (Li, 1999; Li, 2003) which are inherently imprecise or unreliable. The idea of ambiguous sets was first described in the work of Gau and Buehrer (1993). Burillo and Bustince (Dengfeng and Chuntian, 2002; Xiao and Pedrycz, 2022) showed that the concepts of IFSs and ambiguous sets are synonymous with one another. Some operations on IFSs were defined by De et al. (2000). Szmidt and Kacprzyk (2000) studied the distances between IFSs. Szmidt and Kacprzyk (1996) and Szmidt and Kacprzyk (1997) considered the use of IFSs for building soft decision-making models with imprecise information. They proposed two solution concepts: the intuitionistic fuzzy core and the consensus winner for group decision making utilizing IFSs.

In terms of membership functions, making crisp sets into fuzzy sets is the same as making a set of integers into a set of real numbers. Changing the range of the membership function from {0, 1} to [0,1] is the same as changing I to R. Obviously, the development of numbers did not stop with the introduction of the real number system. In the past, we started with real numbers and then added complex numbers to their range. As a result, this modification might be the starting point for future work in fuzzy set theory. In the context of fuzzy set theory, a complex fuzzy set is the result of this kind of extension; it is a fuzzy set that differs from other fuzzy sets by having a membership function that accepts values in the complex number range. The theories that have been proposed by Ramot et al. have examined the decision-making problems that arise when using the fuzzy set and its generalizations, which can only deal with the ambiguity and uncertainty present in the data. These cannot account for the differences in data at a given moment. For this reason, Ramot et al. (2002) looked into the CFS, whereby the degree of membership is a complex integer that belongs to a unit disc on a complex plane. A complex fuzzy set (CFS) processes the data in a single set, which is just two dimensions deep. Despite the fact that Nguyen et al. (2000) were the ones to first propose a CFS, Ramot et al.'s conceptual space is more accommodating to decision makers. Over the last several years, the CFS has received a lot of research and media coverage. CFS operations (Zhang et al., 2009), power aggregation operators (Hu et al., 2019), and continuity of complex fuzzy operations (Hu et al., 2018) all had different operation features and distance measures. (Li and Chiang (2012) proposed a method through the use of CFSs for conducting the dual-output forecasting experiments (Xiao et al., 2022) with real-world financial time series, such as the Dow Jones Industrial Average, the Taiwan Stock Exchange Capitalization Weighted Stock Index, and the National Association of Securities Dealers Automated Quotation System.

In light of the significance of similarity measures (SMs), many researchers have recently opted for SMs based on the fuzzy set (FS) (Beg and Ashraf, 2009), CFS (Bi et al., 2019; Xiao, 2020a), and hesitant fuzzy set (HFS) (Xu and Xia, 2011). However, when a decision maker assigns membership grades in the form of groups, the complex values render the existing measures inadequate. In this work, the authors establish a complex intuitionistic fuzzy set (CIFS), a hybrid of the IFS and CFS , to deal with such problems while retaining the benefits of the SMs. Complex-valued membership degrees are presented in a polar form in CIFS theory. When a decision maker is presented with a set of data that only has two dimensions, all of the available theories, including the fuzzy set and CFS, perform admirably.

Benayoun et al. (1966) were the pioneers of the ELECTREE technique. The original inspiration for its core notions of concordance, discordance, and outranking was drawn from practical use cases. In addition, it uses concordance and discordance indices to examine the ordering of options. Energy (Beccali et al., 1998; Xiao, 2022), environment or water management (Ahrens and Kantelhardt, 2009), finance (Doumpos and Zopounidis, 2001), and decision analysis (Almeida, 2005; Xiao, 2020b) are just some of the domains where ELECTREE methodologies have been put to use. We have already shown that the ELECTREE technique, one of the most well-known outranking models, may be used to address the multi-criteria decision making (MCDM) issue. This approach is excellent because of its obvious logic, but it has been used in very little CIFS research. The ELECTREE technique involves contrasting potential solutions head-to-head in light of the decision maker's own evaluations of the available options. Relationships of concordance, discordance, and superiority are of interest to this methodology. In this study, we provide a novel approach to using the ELECTREE technique in CIF systems, making it possible to solve MCDM issues there. People may use the CIFS to express uncertain scenarios in a decision-making dilemma, and its features are concerned with the degree of membership, the degree of complex membership, the degree of non-membership, the degree of complex non-membership grades, and the intuitionistic index all at once. In the ELECTREE assessment process, decision makers use CFS data with several values rather than a single value, and they are provided CFS data with multiple criteria to choose from.

The rest of the paper is organized as follows: In Section 2 we define some fundamental preliminaries, basic operations, and properties, and the construction of the CIF decision matrix is described. In Section 3, the proposed complex intuitionistic fuzzy ELECTREE method is described briefly along with the algorithm. In Section 4, the case study of the paper is discussed. Last, in Section 6, we discuss the paper's final findings and conclusions.

2 Preliminaries

Definition 2.1. Suppose we have a finite universal set M such that $M = \{m_1, m_2, m_3, \ldots m_n\}$; then, an IFS B in M is defined in the following form:

$$B = \{\langle m_k, \hat{\mu}_B(m_k), \hat{\nu}_B(m_k)\rangle | m_k \in M\}, \tag{1}$$

where the functions $m_k \in M \to \hat{\nu}_B(m_k) \in [0,1]$ and $m_k \in M \to \hat{\mu}_B(m_k) \in [0,1]$ describe the degree of non-membership and degree of membership, respectively, of the given element $m_k \in M$ to the set $B \subseteq M$, and for each $m_k \in M$,

$$0 \le \hat{\nu}_B + \hat{\mu}_B \le 1.$$

Definition 2.2. A set C of the form given below defined upon M universal set is called as the CFS.

$$C = \{(h, \hat{\mu}_C(h)) | h \in H\},$$

where

$$\hat{\mu}_C(h) = \left\{\psi C(h) \cdot \hat{e}^{2\pi\iota\left(\omega_{\psi C}(h)\right)}\right\}$$

are the representation of the complex-valued certainty grade generally, which is subset of the unit disc in a complex plane having condition $\psi C(h), \omega_{\psi C}(h) \in [0,1]$.

Definition 2.3. A CIFS W defined on the universal set M is given as follows:

$$W = \{(z, \hat{\mu}_W(z), \hat{\nu}_W(z)) : z \in M\}, \tag{2}$$

where $\hat{\nu}_w$ and $\hat{\mu}_w$ are complex-valued non-membership and membership functions, respectively, and $\hat{\mu}_W(z) = \hat{Y}_W(z)\hat{e}^{2\pi\iota\hat{W}_{\hat{Y}W}(z)}$, and $\hat{\nu}_W(z) = \hat{S}_W(z)\hat{e}^{2\pi\iota\hat{W}_{\hat{S}W}(z)}$ such that $\hat{Y}_W(z) \ge 0$, and $\hat{S}_W(z) \le 1; 0 \le \hat{Y}_W(z) + \hat{S}_W(z) \le 1$ and $0 \le \hat{W}_{\hat{Y}W}(z)$, $\hat{W}_{\hat{S}W}(z) \le 2\hat{\pi}; 0 \le \hat{W}_{\hat{Y}W}(z) + \hat{W}_{\hat{S}W}(z) \le 2\hat{\pi}\ \forall z \in M$. We can also denote the CIFS W as

$$\left\{\left(z, \left(\hat{Y}_W(z), \hat{W}_{\hat{Y}W}(z)\right), \left(\hat{S}_W(z), \hat{W}_{\hat{S}W}(z)\right)\right) : z \in M\right\}.$$

2.1 Some basic operations upon the intuitionistic fuzzy set and complex intuitionistic fuzzy set We considered

$$\hat{\pi}_B(m_k) = 1 - \hat{\nu}_B - \hat{\mu}_B \tag{3}$$

as the intuitionistic index of the member m_k in the set M. This is the degree of indeterminacy membership of the element m_k to the set M. It is sure that for each $m_k \in M, 0 \le \hat{\pi}_B(m_k) \le 1$.

The operations of the IFS (Atanassov, 1986; Atanassov, 1989; Atanassov, 1999) are given as follows. For every $B, C \in$ A-IFS (M),

(1) $B \subset C$ if $\forall\ m \in M$, $(\hat{\mu}_B(m) \le \hat{\mu}_C(m)$ and $\hat{\nu}_B(m) \ge \hat{\nu}_C(m))$.
(2) $B = C$ if $B \subset C$ and $C \subset B$.
(3) $\bar{B} = \{(m, \hat{\nu}_B(m), \hat{\mu}_B(m))\}$.
(4) $\hat{d}(B, C) =$

$$\sqrt{\frac{1}{2n}\sum_{k=1}^{n} \begin{array}{c}(\hat{\mu}_B(m_k) - \hat{\mu}_C(m_k))^2 + (\hat{\nu}_B(m_k) - \hat{\nu}_C(m_k))^2 \\ + (\hat{\pi}_B(m_k) - \hat{\pi}_C(m_k))^2\end{array}}, \tag{4}$$

where $\hat{d}(B, C)$ is the normalized Euclidian distance between A and B.

Definition 2.4. Let $W = \{(z, (\hat{Y}_W(z), \hat{W}_{\hat{Y}W}(z)), (\hat{S}_W(z), \hat{W}_{\hat{S}W}(z))) : z \in M\}$ and $H = \{(z, (\hat{Y}_H(z), \hat{W}_{\hat{Y}H}(z)), (\hat{S}_H(z), \hat{W}_{\hat{S}H}(z))) : z \in M\}$ be the two CIFSs defined upon H. Then, we can say that

(1) $W^c = \{(z, (\hat{S}_W(z), \hat{W}_{\hat{S}W}(z)), (\hat{Y}_W(z), \hat{W}_{\hat{Y}W}(z))) : z \in M\}$.
(2) $H \subseteq W$ if $\hat{Y}_H(z) \le \hat{Y}_W(z), \hat{S}_H(z) \ge \hat{S}_W(z)$ and $\hat{W}_{\hat{Y}H}(z) \le \hat{W}_{\hat{Y}W}(z), \hat{W}_{\hat{S}H}(z) \ge \hat{W}_{\hat{S}W}(z)$.
(3) $H = W$ if $H \subseteq W$ and $W \subseteq H$.
(4) $H \cup W = \left\{\begin{array}{c}\left(z, (\max\{\hat{Y}_H(z), \hat{Y}_W(z)\}, \max\{\hat{W}_{\hat{Y}H}(z), \hat{W}_{\hat{Y}W}(z)\}),\right. \\ \left.(\min\{\hat{S}_H(z), \hat{S}_W(z)\}, \min\{\hat{W}_{\hat{S}H}(z), \hat{W}_{\hat{S}W}(z)\})\right) : z \in M\end{array}\right\}$.

(5) $d(H, W) =$

$$\sqrt{\frac{1}{2n}\sum_{k=1}^{n}\begin{pmatrix}\left(\hat{Y}_H(z)-\hat{Y}_W(z)\right)^2+\left(\hat{W}_{\hat{Y}H}(z)-\hat{W}_{\hat{Y}W}(z)\right)^2\\ +\left(\hat{S}_H(z)-\hat{S}_W(z)\right)^2+\left(\hat{W}_{\hat{S}H}(z)-\hat{W}_{\hat{S}W}(z)\right)^2\end{pmatrix}}. \tag{5}$$

2.2 Construction of the complex intuitionistic fuzzy decision matrix

A decision matrix may be used to represent an MCDM problem in which each element represents the evaluation or value of the *kth* alternative $\hat{A}_k$ in relation to the *nth* criteria $\hat{Z}_l$. We extend the canonical matrix format to a CIF decision matrix P in this study. In other words, decision makers are expected to assign the non-membership and membership degree based on their perspectives and encapsulate the extent to which the alternative $\hat{A}_k$ fulfils the criterion $\hat{Z}_l$. They can offer evaluation data for each criterion's options. Let M be the MCDM issue setting's discussion universe, including the decision criteria. The set of all criteria is shown as $C = \{c_1, c_2, c_3, \ldots, c_n\}$. A CIFS G_k of the *kth* alternative on M is given as $G_k = \{\langle m_l, M_{kl}\rangle | m_l \in M\}$, where $M_{kl} = (\hat{\mu}_{kl}, \hat{\nu}_{kl})$, where $\hat{\nu}_{kl}$ and $\hat{\mu}_{kl}$ are complex-valued non-membership and membership functions, respectively, and $\hat{\mu}_{kl} = \hat{Y}_k(l)\hat{e}^{2\pi\iota\hat{W}_{\hat{Y}k}(l)}$, and $\hat{\nu}_{kl} = \hat{S}_k(l)\hat{e}^{2\pi\iota\hat{W}_{\hat{S}k}(l)}$ such that $\hat{Y}_k(l) \geq 0$, and $\hat{S}_k(l) \leq 1; 0 \leq \hat{Y}_k(l) + \hat{S}_k(l) \leq 1$ and $0 \leq \hat{W}_{\hat{Y}k}(l), \hat{W}_{\hat{S}k}(l) \leq 2\hat{\pi}; 0 \leq \hat{W}_{\hat{Y}k}(l) + \hat{W}_{\hat{S}k}(l) \leq 2\hat{\pi}\ \forall l \in M$, and also denotes the degree of membership and non-membership of the *kth* alternative $\hat{A}_k$ in relation to the *nth* criteria $\hat{Z}_l$ such that $\hat{\pi}_{kl} = 1 - \hat{\mu}_{kl} - \hat{\nu}_{kl}$ and $k = 1, 2, 3, \ldots m$ *and* $l = 1, 2, 3, \ldots, n$. The CIF decision matrix, $\bar{Z}$, is described as follows:

$$\bar{Z} = \begin{bmatrix}(\hat{\mu}_{11}, \hat{\nu}_{11}) & . & . & . & (\hat{\mu}_{1n}, \hat{\nu}_{1n})\\ . & . & & & .\\ . & & . & & .\\ . & & & . & .\\ (\hat{\mu}_{m1}, \hat{\nu}_{m1}) & . & . & . & (\hat{\mu}_{mn}, \hat{\nu}_{mn})\end{bmatrix}. \tag{6}$$

The decision makers assign a set of grades of importance, $\hat{W}$, because it is impossible to assume that all factors are equally important. An IFS $\hat{W}$ in M is defined as follows:

$$\hat{W} = \{\langle m_l, \hat{\omega}_l\rangle | m_l \in M\} \tag{7}$$

such that $0 \leq \hat{\omega}_l \leq 1$ and $\sum_{l=1}^{n}\hat{\omega}_l = 1$, and $\hat{\omega}_l$ is the grade of reliability that is given to each criteria, which we can also be said as the weight of each criteria. In order to build the CIF decision matrix, the decision maker must pay four times to acquire the assessed data: twice for the degrees of membership and twice for the non-membership data. In terms of mathematics, the interval-valued fuzzy set (IVFS) theory and the IFS theory are interchangeable (Deschrijver and Kerre, 2003; Dubois et al., 2005; Montero et al., 2007). Due to the restriction of the total of membership and non-membership degrees, the decision maker's assessment using IVFS data is simpler than that with CIF data. The collection of all closed subintervals of $[0, 1]$ is represented by interval $([0, 1])$. A complex interval value intuitionistic fuzzy set (CIVFS) G_m of the *kth* alternative on M is given as $G_k = \{\langle m_l, M_{kl}\rangle | m_l \in M\}$, where M_{kl}: $M \to \text{Int}([0, 1])$ such that $m_l \to M_{kl} = [M_{kl}^-, M_{kl}^+]$. The probable level at which the alternative G_k meets the requirement m_l is indicated by M_{kl}. M_{kl}^+ and M_{kl}^- are the upper and the lower bounds, sequentially, of the aforementioned interval M_{kl}.

Starting at the beginning of the closed interval $[M_{kl}^-, M_{kl}^+]$, the decision maker considers each option. Suppose $M_{kl}^- = \hat{\mu}_{kl}$, and $M_{kl}^+ = 1 - \hat{\nu}_{kl}$ such that $[M_{kl}^-, M_{kl}^+] = (\hat{\mu}_{kl}, 1 - \hat{\nu}_{kl})$. A complex interval can be mapped into the CIFS, i.e., $(\hat{\mu}_{kl}, 1 - \hat{\nu}_{kl})$. The idea that the CIFS and CIVFS are mathematical equivalents may be used to convert CIVF data into CIFS data. Additionally, a decision maker must assess a lot of information using CIVFS data, making it difficult to compile all available options based on their expertise and experience. Decision makers may provide ranking and incomplete or missing data and convert it into CIF data. The approach determines the number of alternatives that are categorically better and worse than a given option. Given that not all options may be rated in accordance with a criteria, it permits partial ordinal data. We create two functions, $\hat{A}_{kl}$ and $\hat{E}_{kl}$, for each G_m with regard to m_l in order to account for missing data or non-comparable results. Let $\hat{A}_{kl}$ represent the number of alternatives that are unquestionably worse than G_m, such as $G_1, G_2, \ldots, G_{m-1}, G_{m+1}, G_{m+2}, \ldots$, and G_m, while $\hat{E}_{kl}$ indicates the number of alternatives that are unquestionably superior than G_m, such as $G_1, G_2, \ldots, G_{m-1}, G_{m+1}, G_{m+2}, \ldots$, and G_m. The following are the levels of non-membership and membership, respectively:

$$\hat{\nu}_{kl} = \frac{\hat{E}_{kl}}{m-1}, \tag{8}$$

$$\hat{\mu}_{kl} = \frac{\hat{A}_{kl}}{m-1}. \tag{9}$$

3 ELECTREE methods based on complex intuitionistic fuzzy data

The CIF–ELECTREE technique (with the algorithm) and concordance and discordance sets are introduced in this section. For numerical examples, we will utilize the CIF–ELECTREE method algorithm. Binary outranking relations are used to simulate ELECTREE procedures; the connection is constructed by the decision maker and need not be transitive. Non-dominant alternative partial ordering is enabled by the connection. For each pair of alternatives x and y $(x, y = 1, 2, \ldots, m$, and $x \neq y)$, each criterion can be divided into two separate subsets. The concordance set A_{xy} of G_x and G_y includes all of the criteria for which G_x is preferable to G_y. We can also say that $A_{xy} = \{l | m_{xl} \geq m_{yl}\}$, where $L = \{l | l = 1, 2, 3, \ldots, n\}$. The discordance set, which is a complementary subset, is $B_{xy} = \{l | m_{xl} < m_{xy}\}$. The concepts of scoring function, accuracy function, and complex intuitionistic index are used in the proposed CIF-ELECTREE technique to categorize various concordance and discordance sets, and concordance and discordance sets are used to generate concordance and discordance matrices, respectively. Using the ideas of optimum and non-optimum points, decision makers may select the optimum option.

3.1 Discordance and concordance sets

The ideas of scoring function, accuracy function, and hesitant degree of the CIF value allow us to evaluate many alternatives to their CIF values. When two alternatives have the same score degree, the better option has a higher score degree or a higher accuracy degree. A greater accuracy degree denotes a lower hesitation degree, and a higher score degree denotes a bigger membership degree or a smaller non-membership degree. With the principles of the score function and accuracy function, we categorize various concordance sets as "concordance sets," medium concordance sets," and "weak concordance sets." The discordance set, intermediate discordance set, and weak discordance set are further terms for the many sorts of discordance sets. The score function was developed by Chen and Tan (1994) to measure how well an option meets a decision maker's needs. Suppose $M_{kl} = (\hat{\mu}_{kl}, \hat{\nu}_{kl})$ is an CIF value such that $\hat{\mu}_{kl} = \hat{Y}_k(l)\hat{e}^{2\pi\iota\hat{W}_{\hat{Y}k}(l)}$ and $\hat{\nu}_{kl} = \hat{S}_k(l)\hat{e}^{2\pi\iota\hat{W}_{\hat{S}k}(l)}$ such that $\hat{Y}_k(l) \geq 0$, and $\hat{S}_k(l) \leq 1; 0 \leq \hat{Y}_k(l) + \hat{S}_k(l) \leq 1$ and $0 \leq \hat{W}_{\hat{Y}k}(l), \hat{W}_{\hat{S}k}(l) \leq 2\hat{\pi}$; $0 \leq \hat{W}_{\hat{Y}k}(l) + \hat{W}_{\hat{S}k}(l) \leq 2\hat{\pi}$. We can evaluate the score function $\check{S}$ as $\check{S}(M_{kl}) = \hat{\mu}_{kl} - \hat{\nu}_{kl}$. Although a higher $\check{S}(M_{kl})$ score correlates with a higher CIF value M_{kl}, we are unable to compare alternatives with equal scores. In order to assess the level of accuracy of ambiguous values, Hong and Choi (2000) introduced the accuracy function. The accuracy function $\check{D}$ can be used to assess how accurate M_{kl} is. The degree function $\check{D}(M_{kl})$ is evaluated as $\check{D}(M_{kl}) = \hat{\mu}_{kl} + \hat{\nu}_{kl}$. The correctness of the CIF value membership grade increases with an increase in the value of $\check{D}(M_{kl})$. We may infer from (2) and the accuracy function that a lower hesitation degree $\hat{\pi}_{kl}$ correlates with a greater accuracy degree $\check{D}(M_{kl})$. As said previously, the concordance set A_{xy} of G_x and G_y includes all of the criteria for which G_x is preferable to G_y. We calculate it with the principles of the score function and accuracy function and a hesitant degree for the classification of the concordance sets of CIF. It is possible to formulate the concordance set A_{xy} as follows:

$$A_{xy} = \left\{l | \hat{\mu}_{xl} \geq \hat{\mu}_{yl}, \hat{\nu}_{xl} < \hat{\nu}_{yl} \text{ and } \hat{\pi}_{xl} < \hat{\pi}_{yl}\right\}, \tag{10}$$

where $L = \{l | l = 1, 2, 3, \ldots, n\}$, a higher degree of accuracy translates to a lower degree of hesitation, a higher score translates to a higher CIF value, and Eq. 10 is more concordant than Eq. 11 or Eq. 12.The midrange concordance set A°_{xy} is defined as follows:

$$A^{\circ}_{xy} = \left\{l | \hat{\mu}_{xl} \geq \hat{\mu}_{yl}, \hat{\nu}_{xl} < \hat{\nu}_{yl} \text{ and } \hat{\pi}_{xl} \geq \hat{\pi}_{yl}\right\}. \tag{11}$$

The hesitancy degree is the main distinction between Eq. 10 and Eq. 11; in the midrange concordance set, the hesitancy degree at the *xth* alternative with regard to the *lth* criterion is higher than at the *yth* alternative with respect to the *lth* criterion. Thus, Eq. 10 is more quadrant than Eq. 11.The least concordant set is given as follows:

$$A^{\circ\circ}_{xy} = \left\{l | \hat{\mu}_{xl} \geq \hat{\mu}_{yl}, \hat{\nu}_{xl} \geq \hat{\nu}_{yl}\right\}. \tag{12}$$

Eq. 11 is more concordant than Eq. 12 because the degree of non-membership at the *xth* alternative with respect to the *lth* criteria in the weak concordance set Eq. 12 is greater than the degree of non-membership at the *yth* alternative with respect to the *lth* criteria.

The discordance set consists of all criteria where G_x is inferior to G_y. Using the aforementioned principles, the discordance set B_{xy} can be expressed as follows:

$$B_{xy} = \left\{l | \hat{\mu}_{xl} < \hat{\mu}_{yl}, \hat{\nu}_{xl} \geq \hat{\nu}_{yl} \text{ and } \hat{\pi}_{xl} \geq \hat{\pi}_{yl}\right\}. \tag{13}$$

The middle discordant set is defined as follows:

$$B^{\circ}_{xy} = \left\{l | \hat{\mu}_{xl} < \hat{\mu}_{yl}, \hat{\nu}_{xl} \geq \hat{\nu}_{yl} \text{ and } \hat{\pi}_{xl} < \hat{\pi}_{yl}\right\}. \tag{14}$$

Eq. 13 is more discordant than Eq. 14. The least discordant set is given as follows:

$$B^{\circ\circ}_{xy} = \left\{l | \hat{\mu}_{xl} < \hat{\mu}_{yl}, \hat{\nu}_{xl} < \hat{\nu}_{yl}\right\}. \tag{15}$$

Eq. 14 is more discordant than Eq. 15.

We use the idea of discordant and concordant sets to compute matrices of concordant and discordant and the CIF–ELECTREE technique to calculate the aggregate dominance matrix. We then select the optimal option.

3.2 The complex intuitionistic fuzzy ELECTREE method

The CIF–ELECTREE technique is a combination of the CIFS and ELECTREE methods using evaluation data. The concordant index is a way to figure out the worth of the concordance set of the CIF–ELECTREE method. The concordance index is equal to the sum of the weights for the criteria and relationships in the concordance sets. So, for this paper, the concordance index $\hat{A}_{xy}$ between G_x and G_y is as follows:

$$\hat{g}_{xy} = \hat{\omega}_A \times \sum_{l \in A_{xy}} \hat{\omega}_l + \hat{\omega}_{A^{\circ}} \times \sum_{l \in A^{\circ}_{xy}} \hat{\omega}_l + \hat{\omega}_{A^{\circ\circ}} \times \sum_{l \in A^{\circ\circ}_{xy}} \hat{\omega}_l, \tag{16}$$

where $\hat{\omega}_{A^{\circ\circ}}$, $\hat{\omega}_{A^{\circ}}$, and $\hat{\omega}_A$ are the weights of the least, middle, and the concordance sets, respectively. The $\hat{\omega}_l$ is the weight of the defined criteria already defined in (7). Based on how much weight is given to each successive decision criterion, the concordance index shows how much more important one alternative choice is than another. Here is how we characterize the $\hat{G}$ concordance matrix:

$$\hat{G} = \begin{bmatrix} - & \hat{g}_{12} & \cdots & \cdots & \cdots & \hat{g}_{1m} \\ \hat{g}_{21} & - & \hat{g}_{23} & \cdots & \cdots & \hat{g}_{2m} \\ \cdots & \cdots & - & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & - & \cdots & \cdots \\ \hat{g}_{(m-1)1} & \cdots & \cdots & \cdots & - & \hat{g}_{(m-1)1} \\ \hat{g}_{m1} & \hat{g}_{m2} & \cdots & \cdots & \hat{g}_{m(m-1)} & - \end{bmatrix}, \tag{17}$$

where the highest possible value of $\hat{g}_{xy}$ is represented by the symbol $\hat{g}^{*}$, which stands for the positive ideal point, and a larger value of $\hat{g}_{xy}$ implies that G_x is preferred over G_y.

A certain G_x has lower ratings than an alternative G_y. Here, we provide a formal definition of the discordance index:

$$\hat{h}_{xy} = \frac{\max_{l \in B_{xy}} \hat{W}^{**}_B \times \hat{d}(M_{xl}, M_{yl})}{\max_{l \in L} \hat{d}(M_{xl}, M_{yl})}, \tag{18}$$

where $\hat{d}(M_{xl}, M_{yl})$ is defined in Eq. 5 and $\hat{W}^{**}_B$ is equal to $\hat{W}_B$, $\hat{W}'_B$, or $\hat{W}''_B$ depending on different classification of discordance sets. These sets have the settings for light discordance, medium discordance, and heavy discordance, in that order.

Here is how we characterize $\hat{H}$, the matrix of discordance:

$$\hat{H} = \begin{bmatrix} - & \hat{h}_{12} & \cdots & \cdots & \cdots & \hat{h}_{1m} \\ \hat{h}_{21} & - & \hat{h}_{23} & \cdots & \cdots & \hat{h}_{2m} \\ \cdots & \cdots & - & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & - & \cdots & \cdots \\ \hat{h}_{(m-1)1} & \cdots & \cdots & \cdots & - & \hat{h}_{(m-1)1} \\ \hat{h}_{m1} & \hat{h}_{m2} & \cdots & \cdots & \hat{h}_{m(m-1)} & - \end{bmatrix}, \tag{19}$$

where $\hat{h}^{\star}$, the negative ideal point, is the highest value of $\hat{h}_{xy}$, and a bigger $\hat{h}_{xy}$ value means that G_x is less desirable than G_y.

The way to figure out the concordance dominance matrix is based on the idea that the best choice is the one that is closest to the positive ideal solution. This is how the concordance dominance matrix $\hat{K}$ is defined:

$$\hat{K} = \begin{bmatrix} - & \hat{k}_{12} & \cdots & \cdots & \cdots & \hat{k}_{1m} \\ \hat{k}_{21} & - & \hat{k}_{23} & \cdots & \cdots & \hat{k}_{2m} \\ \cdots & \cdots & - & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & - & \cdots & \cdots \\ \hat{k}_{(m-1)1} & \cdots & \cdots & \cdots & - & \hat{k}_{(m-1)1} \\ \hat{k}_{m1} & \hat{k}_{m2} & \cdots & \cdots & \hat{k}_{m(m-1)} & - \end{bmatrix}, \quad (20)$$

where

$$\hat{k}_{xy} = \hat{g}^{\star} - \hat{g}_{xy}. \quad (21)$$

It means how far away each potential option is from the optimum answer. If $\hat{k}_{xy}$ is larger than zero, then G_x is less preferable than G_y. The discordance dominance matrix $\hat{L}$ is used to find the best option based on the idea that it should be the most different from the negative ideal solution. For anyone interested, this is how we characterize $\hat{L}$ in the discordance dominance matrix:

$$\hat{L} = \begin{bmatrix} - & \hat{l}_{12} & \cdots & \cdots & \cdots & \hat{l}_{1m} \\ \hat{l}_{21} & - & \hat{l}_{23} & \cdots & \cdots & \hat{l}_{2m} \\ \cdots & \cdots & - & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & - & \cdots & \cdots \\ \hat{l}_{(m-1)1} & \cdots & \cdots & \cdots & - & \hat{l}_{(m-1)1} \\ \hat{l}_{m1} & \hat{l}_{m2} & \cdots & \cdots & \hat{l}_{m(m-1)} & - \end{bmatrix}, \quad (22)$$

where

$$\hat{l}_{xy} = \hat{h}^{\star} - \hat{h}_{xy}. \quad (23)$$

This describes how to distinguish each alternative from the negative optimum point. For G_x to be preferred over G_y, $\hat{l}_{xy}$ must increase.

The distance from both negative and positive optimal points can be used in the aggregate dominance matrix decision procedure to rank the many possibilities. The definition of the total dominance matrix $\hat{R}$ is as follows:

$$\hat{R} = \begin{bmatrix} - & \hat{r}_{12} & \cdots & \cdots & \cdots & \hat{r}_{1m} \\ \hat{r}_{21} & - & \hat{r}_{23} & \cdots & \cdots & \hat{r}_{2m} \\ \cdots & \cdots & - & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & - & \cdots & \cdots \\ \hat{r}_{(m-1)1} & \cdots & \cdots & \cdots & - & \hat{r}_{(m-1)1} \\ \hat{r}_{m1} & \hat{r}_{m2} & \cdots & \cdots & \hat{r}_{m(m-1)} & - \end{bmatrix}, \quad (24)$$

where

$$\hat{r}_{xy} = \frac{\hat{l}_{xy}}{\hat{l}_{xy} + \hat{k}_{xy}} \quad (25)$$

such that $\hat{l}_{xy}$ and $\hat{k}_{xy}$ are as defined in Eq. 23 and Eq. 21, respectively, and the degree to which a solution is near is optimal, denoted by the symbol $\hat{r}_{xy}$. When comparing two options, if $\hat{r}_{xy}$ for option G_x is larger than that for option G_y, it means G_x is more optimal since it is closer to the positive ideal point and further from the negative ideal point. Choosing the most appropriate alternative procedure is defined as follows:

$$\hat{T}_x = \frac{1}{m-1} \sum_{x=1, x\neq y}^{m} \hat{r}_{xy}, \quad y = 1, 2, 3, \ldots, m, \quad (26)$$

and $\hat{T}_x$ is the final evaluation. $\hat{T}_x$ allows for the ranking of all alternatives under consideration. The optimal solution $\hat{A}^{**}$ may be constructed and defined as follows: $\hat{A}^{**}$ is the distance from the positive ideal point and the maximum distance from the negative ideal point, which is defined as follows:

$$\hat{A}^{**} = \max\left[\hat{T}_x\right], \quad (27)$$

where the best alternative is $\hat{A}^{**}$.

The CIF–ELECTREE method is described here as a new MCDM method for making decisions. It combines the CIFS and ELECTREE techniques with assessment data. The algorithm for the suggested method will be built in three separate steps: evaluation, aggregation, and selection. In the assessment phase, decision makers use the CIFS to choose relevant criteria, recognize alternatives (with varying weights assigned to various criteria), and establish a decision matrix based on the assessed data. During the aggregation phase, we use the suggested method to build concordance and discordance dominance matrices by comparing each option to the others to confirm the dominance connection. After that, we calculate a matrix that represents the overall dominant structure. The CIF–ELECTREE technique is used during the selection phase to determine which option is the best and to rank the options in the order of preference.

Algorithm: The CIF–ELECTREE method's algorithm and decision-making process may be summed up in the following eight phases:

Step (1). We can use evaluation data to build the choice matrix and include input from decision makers in the form of CIF values or comparisons between options. There are three substeps inside this larger phase.

(1) We can select the problem-specific criteria and non-inferior alternatives; various MCDM issues need different criteria. Most criteria may be classified as either subjective or objective. The decision makers find and take into account potential options.

(2) We can obtain a scorecard with relative weights for various factors to consider. With the help of Eq. 7, $\sum_{l=1}^{n} \hat{\omega}_l = 1$, we can figure out how important each criterion is.

(3) Decision makers gather cardinal information and use it to build an CIF decision matrix $\bar{Z}$, Eq. 6. If the people making decisions give us some baseline information, we can use transformations Eq. 8 and Eq. 9 to come to some good conclusions.

Step (2). We can use the scoring function, the accuracy function, and the degree of hesitation of the CIF value to tell the difference between the different concordance and discordance sets. Using Eqs 10–15, we can obtain the chi-squared values A_{xy}, A°_{xy}, $A^{\circ\circ}_{xy}$, B_{xy}, B°_{xy}, and $B^{\circ\circ}_{xy}$ for paired comparisons of alternatives to be chosen.

Step (3). We can determine the matrix $\hat{G}$ of concordance by combining the definitions of the various types of concordance sets and their weights given in Eq. 16 and Eq. 17.

Step (4). Here is how to figure out matrix $\hat{H}$ of discordance: the matrix of discordance index is the result of the different types of discordance sets and their weights, as shown in Eq. 18 and Eq. 19.
Step (5). We can assemble the dominance matrix $\hat{K}$ of concordance, whereby its index is calculated as the difference between the concordance matrix's maximum index and its own index, as described by Eqs 20, 21.
Step (6). We can construct the dominant matrix $\hat{L}$ discordance, whose index is the difference between the discordance matrix's maximum index and its own index, as stated by Eqs 22, 23.
Step (7). The indices of the dominant matrices of concordance and discordance, defined in Eq. 24 and Eq. 25, respectively, are used to form the aggregate dominant matrix $\hat{R}$.
Step (8). We can pick the optimal solution: the sum of the evaluation's final values using Eq. 26 and Eq. 27. All possible options are ranked, and the one with the highest score is deemed the best.

4 Case study: the environmental impact of three industries

Introduction: This case study aims to compare the environmental impact of three industries: oil and gas, coal mining, and paper manufacturing. These industries have been selected as they are known to have a significant impact on the environment, and the study will focus on four common criteria: air pollution, water pollution, land degradation, and waste generation.

Oil and gas industry: The oil and gas industry is known to have a significant impact on air pollution, primarily due to the emissions from drilling and transportation. The burning of fossil fuels also contributes to the release of greenhouse gases, which contribute to climate change. In addition, oil spills and leaks can cause water pollution and damage to marine life. The industry also requires large amounts of water for extraction, which can put a strain on local water resources. However, the industry also creates jobs, income, and energy resources that are important for the human economy.

Coal mining industry: The coal mining industry is known to cause land degradation, as the process of extracting coal requires the removal of large areas of land, including forests and wildlife habitats. This can also lead to soil erosion and landslides. The burning of coal also contributes to air pollution and the release of greenhouse gases. The industry also generates large amounts of waste, including coal ash and slurry, which can contaminate water resources.

Paper manufacturing industry: The paper manufacturing industry generates large amounts of waste, including wood waste and chemical by-products. The industry also requires large amounts of water for production, which can put a strain on local water resources. The production of paper also contributes to air pollution, primarily due to the emissions from the burning of fossil fuels used in the production process. The industry also causes deforestation which is one of the major causes of land degradation and loss of biodiversity.

Conclusion: All three industries have a significant impact on the environment, with the oil and gas industry primarily affecting air pollution, the coal mining industry primarily affecting land degradation, and the paper manufacturing industry primarily affecting waste generation. It is important to note that the study is based on the common criteria only, and there are many other factors that should be considered when evaluating the environmental impact of an industry. It is important to find a balance between economic development and environmental protection and to implement sustainable practices in these industries to minimize their impact on the environment.

4.1 Numerical example

Phase **(1):**

(1) Taking expert advice into account, we consider the four criteria listed as follows: c_1(air pollution), c_2(water pollution), c_3(land degradation), and c_4(waste generation) in the problem. As an alternative, we consider the following industries: I_1, I_2, and I_3.
(2) In the end, it is up to the specialists making the decisions to give each factor a subjective value $(\hat{W})$ as $\hat{W} = [\hat{w}_1, \hat{w}_2, \hat{w}_3, \hat{w}_4] = [0.3, 0.35, 0.25, 0.1]$.
(3) The specialists making the decisions to give each factor a relative value $(\hat{W}^{\circ})$ as; $\hat{W}^{\circ} = [\hat{w}_A, \hat{w}_{A^{\circ}}, \hat{w}_{A^{\circ\circ}}, \hat{w}_B, \hat{w}_{B^{\circ}}, \hat{w}_{B^{\circ\circ}}] = [1, \frac{2}{3}, \frac{1}{3}, 1, \frac{2}{3}, \frac{1}{3}]$.

Given the CIVF decision-making matrix $\bar{Z}$, the CIF matrix decision with cardinal information is changed.

$$\bar{Z} = \begin{bmatrix} & c_1 & c_2 \\ I_1 & [0.12e^{2\pi\iota(0.3)}, 0.29e^{2\pi\iota(0.4)}] & [0.14e^{2\pi\iota(0.2)}, 0.81e^{2\pi\iota(0.4)}] \\ I_2 & [0.33e^{2\pi\iota(0.5)}, 0.42e^{2\pi\iota(0.9)}] & [0.33e^{2\pi\iota(0.3)}, 0.52e^{2\pi\iota(0.6)}] \\ I_3 & [0.12e^{2\pi\iota(0.7)}, 0.84e^{2\pi\iota(0.8)}] & [0.62e^{2\pi\iota(0.1)}, 0.33e^{2\pi\iota(0.3)}] \\ & c_3 & c_4 \\ I_1 & [0.42e^{2\pi\iota(0.2)}, 0.54e^{2\pi\iota(0.5)}] & [0.13e^{2\pi\iota(0.1)}, 0.31e^{2\pi\iota(0.3)}] \\ I_2 & [0.32e^{2\pi\iota(0.1)}, 0.64e^{2\pi\iota(0.7)}] & [0.42e^{2\pi\iota(0.3)}, 0.53e^{2\pi\iota(0.6)}] \\ I_3 & [0.81e^{2\pi\iota(0.5)}, 0.11e^{2\pi\iota(0.7)}] & [0.51e^{2\pi\iota(0.7)}, 0.39e^{2\pi\iota(0.9)}] \end{bmatrix},$$

$$\begin{bmatrix} & c_1 & c_2 \\ I_1 & [0.12e^{2\pi\iota(0.3)}, 0.71e^{2\pi\iota(0.6)}, 0.17e^{2\pi\iota(0.1)}] & [0.14e^{2\pi\iota(0.2)}, 0.19e^{2\pi\iota(0.6)}, 0.67e^{2\pi\iota(0.2)}] \\ I_2 & [0.33e^{2\pi\iota(0.5)}, 0.58e^{2\pi\iota(0.1)}, 0.09e^{2\pi\iota(0.4)}] & [0.33e^{2\pi\iota(0.3)}, 0.48e^{2\pi\iota(0.4)}, 0.19e^{2\pi\iota(0.3)}] \\ I_3 & [0.12e^{2\pi\iota(0.7)}, 0.16e^{2\pi\iota(0.2)}, 0.72e^{2\pi\iota(0.1)}] & [0.62e^{2\pi\iota(0.1)}, 0.27e^{2\pi\iota(0.7)}, 0.11e^{2\pi\iota(0.2)}] \\ & c_3 & c_4 \\ I_1 & [0.42e^{2\pi\iota(0.2)}, 0.46e^{2\pi\iota(0.5)}, 0.12e^{2\pi\iota(0.3)}] & [0.13e^{2\pi\iota(0.1)}, 0.69e^{2\pi\iota(0.7)}, 0.18e^{2\pi\iota(0.2)}] \\ I_2 & [0.32e^{2\pi\iota(0.1)}, 0.36e^{2\pi\iota(0.3)}, 0.32e^{2\pi\iota(0.6)}] & [0.42e^{2\pi\iota(0.3)}, 0.37e^{2\pi\iota(0.4)}, 0.21e^{2\pi\iota(0.3)}] \\ I_3 & [0.81e^{2\pi\iota(0.5)}, 0.09e^{2\pi\iota(0.3)}, 0.1e^{2\pi\iota(0.2)}] & [0.51e^{2\pi\iota(0.7)}, 0.26e^{2\pi\iota(0.1)}, 0.23e^{2\pi\iota(0.2)}] \end{bmatrix}.$$

Phase (2). Using the outcomes of Steps 1–3, we determine which sets exhibit concordance and which exhibit discordance.

The concordance set compiled using Eq. 10 is given as follows:

$$A_{xy} = \begin{bmatrix} \{-,-\} & \{-,-\} & \{-,-\} \\ \{1,-\} & \{-,-\} & \{-,-\} \\ \{3,3\} & \{-,1\} & \{-,-\} \end{bmatrix},$$

where in matrix A_{xy}, the first element of $A_{31} = (3, 3)$ present in the third (horizontal) row and first (vertical) column represents the simple membership concordant, and the second element represents the complex membership concordant. $A_{11} = (-, -)$ is empty.

The midrange concordance set compiled using Eq. 11 is given as follows:

$$A^{\circ}_{xy} = \begin{bmatrix} \{-,-\} & \{-,-\} & \{-,2\} \\ \{4,(1,2,4)\} & \{-,-\} & \{4,2\} \\ \{(1,4),(1,4)\} & \{-,-\} & \{-,-\} \end{bmatrix}.$$

The weak concordance set compiled using Eq. 12 is given as follows:

$$A^{\circ\circ}_{xy} = \begin{bmatrix} \{-,-\} & \{3,3\} & \{1,-\} \\ \{2,-\} & \{-,-\} & \{-,-\} \\ \{2,-\} & \{1,(1,3)\} & \{-,-\} \end{bmatrix}.$$

The discordance set compiled using Eq. 13 is given as follows:

$$B_{xy} = \begin{bmatrix} \{-,-\} & \{1,-\} & \{3,(3,4)\} \\ \{-,-\} & \{-,-\} & \{-,(3,4)\} \\ \{-,2\} & \{(2,3),-\} & \{-,-\} \end{bmatrix}.$$

The midrange discordance set compiled using Eq. 14 is given as follows:

$$B^{\circ}_{xy} = \begin{bmatrix} \{-,-\} & \{4,(1,2,4)\} & \{4,-\} \\ \{-,-\} & \{-,-\} & \{-,-\} \\ \{-,-\} & \{4,2\} & \{-,-\} \end{bmatrix}.$$

The least discordance set compiled using Eq. 15 is given as follows:

$$B^{\circ\circ}_{xy} = \begin{bmatrix} \{-,-\} & \{2,-\} & \{2,1\} \\ \{3,3\} & \{-,-\} & \{1,1\} \\ \{-,-\} & \{-,-\} & \{-,-\} \end{bmatrix}.$$

Phase (3)**.** The concordance matrix $\hat{G}$ compiled using Step (3) is given as follows:

$$\hat{G} = \begin{bmatrix} - & 0.083e^{2\pi\iota(0.083)} & 0.3e^{2\pi\iota(0.23)} \\ 0.476e^{2\pi\iota(0.5)} & - & 0.067e^{2\pi\iota(0.234)} \\ 0.633e^{2\pi\iota(0.516)} & 0.1e^{2\pi\iota(0.85)} & - \end{bmatrix}.$$

For example, $\hat{g}_{31} = \hat{w}_A \times \hat{w}_3 + \hat{w}_{A^\circ} \times (\hat{w}_1 + \hat{w}_4) + \hat{w}_{A^{\circ\circ}} \times \hat{w}_2 = [1 \times 0.25e^{2\pi\iota(0.25)}] + [\frac{2}{3} \times 0.4e^{2\pi\iota(0.4)}] + [\frac{1}{3} \times 0.35e^{2\pi\iota(-)}] = 0.633e^{2\pi\iota(0.516)}.$

.

Phase (4)**.** The disconcordance matrix $\hat{H}$ compiled using Step (4) is given as follows:

$$\hat{H} = \begin{bmatrix} - & 0.4874e^{2\pi\iota(0.6667)} & 0.6916e^{2\pi\iota(1.0)} \\ 0.1379e^{2\pi\iota(0.2023)} & - & 0.333e^{2\pi\iota(1.0)} \\ 0.0e^{2\pi\iota(0.25)} & 0.7651e^{2\pi\iota(0.4410)} & - \end{bmatrix}.$$

For example, $\hat{h}_{13} = \frac{\max_{l\in B_{13}} \hat{W}_B^{**} \times \hat{d}(M_{1l},M_{3l})}{\max_{l\in L} \hat{d}(M_{1l},M_{3l})} = \frac{0.3804e^{2\pi\iota(0.6)}}{0.5500e^{2\pi\iota(0.6)}} = 0.6916e^{2\pi\iota(1.0)}$, where $\hat{d}(M_{11},M_{31}) = [\frac{1}{2}\{(0.12e^{2\pi\iota(0.3)} - 0.12e^{2\pi\iota(0.7)})^2 + (0.71e^{2\pi\iota(0.6)} - 0.16e^{2\pi\iota(0.2)})^2 + (0.17e^{2\pi\iota(0.1)} - 0.72e^{2\pi\iota(0.1)})^2\}]^{\frac{1}{2}} = 0.55e^{2\pi\iota(0.4)}$, $\hat{d}(M_{12},M_{32}) = [\frac{1}{2}\{(0.14e^{2\pi\iota(0.2)} - 0.62e^{2\pi\iota(0.1)})^2 + (0.19e^{2\pi\iota(0.6)} - 0.27e^{2\pi\iota(0.7)})^2 + (0.67e^{2\pi\iota(0.2)} - 0.11e^{2\pi\iota(0.2)})^2\}]^{\frac{1}{2}} = 0.525e^{2\pi\iota(0.1)}$, $\hat{d}(M_{13},M_{33}) = [\frac{1}{2}\{(0.42e^{2\pi\iota(0.2)} - 0.81e^{2\pi\iota(0.5)})^2 + (0.46e^{2\pi\iota(0.5)} - 0.09e^{2\pi\iota(0.3)})^2 + (0.12e^{2\pi\iota(0.3)} - 0.1e^{2\pi\iota(0.2)})^2\}]^{\frac{1}{2}} = 0.380e^{2\pi\iota(0.264)}$, and $\hat{d}(M_{14},M_{34}) = [\frac{1}{2}\{(0.13e^{2\pi\iota(0.1)} - 0.51e^{2\pi\iota(0.7)})^2 + (0.69e^{2\pi\iota(0.7)} - 0.26e^{2\pi\iota(0.1)})^2 + (0.18e^{2\pi\iota(0.2)} - 0.23e^{2\pi\iota(0.2)})^2\}]^{\frac{1}{2}} = 0.407e^{2\pi\iota(0.6)}$ and $\hat{W}_B \times \hat{d}(M_{13}, M_{33}) = (1 \times 0.380)e^{2\pi\iota(1\times 0.264)} = (0.380)e^{2\pi\iota(0.264)}$, $\hat{W}_B' \times \hat{d}(M_{14},M_{34}) = (\frac{2}{3} \times 0.407)e^{2\pi\iota(1\times 0.6)} = (0.271)e^{2\pi\iota(0.6)}$, $\hat{W}_B'' \times \hat{d}(M_{12},M_{32}) = (\frac{1}{3} \times 0.525)e^{2\pi\iota(1\times 0.1)} = (0.175)e^{2\pi\iota(0.1)}$, and $\hat{W}_B'' \times \hat{d}(M_{11},M_{31}) = (0.0 \times 0.55)e^{2\pi\iota(\frac{1}{3}\times 0.4)} = (0.0)e^{2\pi\iota(0.133)}$.

Phase (5)**.** The dominant concordance matrix $\hat{K}$ compiled using Step (5) is given as follows:

$$\hat{K} = \begin{bmatrix} - & 0.55e^{2\pi\iota(0.767)} & 0.333e^{2\pi\iota(0.62)} \\ 0.157e^{2\pi\iota(0.35)} & - & 0.566e^{2\pi\iota(0.616)} \\ 0.0e^{2\pi\iota(0.334)} & 0.533e^{2\pi\iota(0.0)} & - \end{bmatrix}.$$

Phase (6)**.** The dominant discordance matrix $\hat{L}$ compiled using Step (6) is given as follows:

$$\hat{L} = \begin{bmatrix} - & 0.2777e^{2\pi\iota(0.0)} & 0.0735e^{2\pi\iota(-0.333)} \\ 0.6272e^{2\pi\iota(0.464)} & - & 0.4321e^{2\pi\iota(-0.333)} \\ 0.7651e^{2\pi\iota(0.416)} & 0.0e^{2\pi\iota(0.225)} & - \end{bmatrix}.$$

Phase (7)**.** The aggregate dominant matrix $\hat{R}$ compiled using Step (7) is given as follows:

$$\hat{R} = \begin{bmatrix} - & 0.336e^{2\pi\iota(0.0)} & 0.181e^{2\pi\iota(-1.163)} \\ 0.8e^{2\pi\iota(0.570)} & - & 0.433e^{2\pi\iota(-1.179)} \\ 1.0e^{2\pi\iota(0.555)} & 0.0e^{2\pi\iota(1.0)} & - \end{bmatrix}.$$

Phase (8)**.** The best alternative using Step (8) is chosen as follows:

$$\begin{aligned} \hat{T}_1 &= 0.2582e^{2\pi\iota(-0.5813)} \\ \hat{T}_2 &= 0.6164e^{2\pi\iota(-0.3044)} \\ \hat{T}_3 &= 0.5e^{2\pi\iota(0.7775)} \end{aligned}.$$

The optimized order of alternatives is given as $\hat{A}_2^{**} > \hat{A}_3^{**} > \hat{A}_1^{**}$.

5 Comparison analysis

So far, we have compared the suggested technique to the intuitionistic ELECTREE method (Benayoun et al., 1966). We look at the five options $\{\hat{A}_1, \hat{A}_2, \hat{A}_3, \hat{A}_4, \hat{A}_5\}$ and decide which one is best by giving each one a weight $\hat{w}_i = [0.449, 0.293, 0.177, 0.081]$ based on four standard criteria $\{\hat{c}_1, \hat{c}_2, \hat{c}_3, \hat{c}_4\}$.

The relative weights of the criteria are given as $\hat{W}^{\circ} = [\hat{w}_A, \hat{w}_{A^\circ}, \hat{w}_{A^{\circ\circ}}, \hat{w}_B, \hat{w}_{B^\circ}, \hat{w}_{B^{\circ\circ}}] = [1, \frac{2}{3}, \frac{1}{3}, 1, \frac{2}{3}, \frac{1}{3}]$.

The intuitionistic fuzzy decision matrix $\hat{M}$ is given as follows:

$$\hat{M} = \begin{bmatrix} \hat{c}_1 & \hat{c}_1 & \hat{c}_1 & \hat{c}_1 \\ (0.805, 0.084, 0.111) & (0.713, 0.176, 0.111) & (0.759, 0.100, 0.141) & (0.805, 0.084, 0.111) \\ (0.699, 0.194, 0.107) & (0.727, 0.162, 0.111) & (0.699, 0.194, 0.107) & (0.727, 1.162, 0.111) \\ (0.784, 0.129, 0.087) & (0.784, 0.129, 0.087) & (0.794, 0.094, 0.112) & (0.851, 0.05, 0.099) \\ (0.762, 0.085, 0.153) & (0.698, 0.194, 0.108) & (1.000, 0.000, 0.000) & (1.000, 0.000, 0.000) \\ (1.000, 0.000, 0.000) & (0.804, 0.085, 0.111) & (0.828, 0.065, 0.107) & (1.000, 0.000, 0.000) \end{bmatrix}.$$

The concordance set compiled using Eq. 10 is given as follows:

$$A_{xy} = \begin{bmatrix} - & - & - & [1] & - \\ - & - & - & [1] & - \\ [2,3,4] & [1,2,4] & - & [2] & - \\ [3,4] & [3,4] & [3,4] & - & [3] \\ [1,3,4] & [1,4] & [1,3,4] & [1] & - \end{bmatrix}.$$

The midrange concordance set compiled using Eq. 11 is given as follows:

$$A^{\circ}_{xy} = \begin{bmatrix} - & [1,3,4] & [1] & [2] & - \\ [2] & - & [1] & [2] & - \\ - & [3] & - & - & - \\ - & [1] & - & - & - \\ [2] & [2,3] & [2] & [2] & - \end{bmatrix}.$$

The weak concordance set compiled using Eq. 12 is given as follows:

$$A^{\circ\circ}_{xy} = \begin{bmatrix} - & - & - & - & - \\ - & - & - & - & - \\ - & - & - & [1] & - \\ - & - & - & - & [4] \\ - & - & - & [4] & - \end{bmatrix}.$$

The discordance set compiled using Eq. 13 is given as follows:

$$B_{xy} = \begin{bmatrix} - & [2] & [2,3,4] & [3,4] & [1,2,3,4] \\ [4] & - & [1,2,4] & [3,4] & [1,2,3,4] \\ - & - & - & [3,4] & [1,3,4] \\ [1] & - & [2] & - & [1] \\ - & - & - & [3] & - \end{bmatrix}.$$

The midrange discordance set compiled using Eq. 14 is given as follows:

$$B^{\circ}_{xy} = \begin{bmatrix} - & - & - & - & - \\ [1,3] & - & [3] & [1] & - \\ [1] & - & - & - & [2] \\ [2] & [2] & - & - & [2] \\ - & - & - & - & - \end{bmatrix}.$$

The least discordance set compiled using Eq. 15 is given as follows:

$$B^{\circ\circ}_{xy} = \begin{bmatrix} - & - & - & - & - \\ - & - & [1,2,3,4] & - & - \\ - & - & - & - & - \\ - & - & [1] & - & - \\ - & - & - & - & - \end{bmatrix}.$$

The concordance matrix $\hat{G}$ compiled using Step (3) is given as follows:

$$\hat{G} = \begin{bmatrix} - & 0.471 & 0.299 & 0.644 & 0.0 \\ 0.195 & - & 0.299 & 0.644 & 0.0 \\ 0.551 & 0.941 & - & 0.443 & 0.0 \\ 0.258 & 0.557 & 0.258 & - & 0.204 \\ 0.902 & 0.843 & 0.902 & 0.671 & - \end{bmatrix}.$$

The disconcordance matrix $\hat{H}$ compiled using Step (4) is given as follows:

$$\hat{H} = \begin{bmatrix} - & 0.132 & 1.0 & 1.0 & 1.0 \\ 0.722 & - & 0.55 & 1.0 & 1.0 \\ 0.414 & 0.0 & - & 1.0 & 1.0 \\ 0.205 & 0.078 & 0.436 & - & 1.0 \\ 0.0 & 0.0 & 0.0 & 0.718 & - \end{bmatrix}.$$

The dominant concordance matrix $\hat{K}$ compiled using Step (5) is given as follows:

$$\hat{K} = \begin{bmatrix} - & 0.470 & 0.642 & 0.297 & 0.0 \\ 0.746 & - & 0.642 & 0.297 & 0.0 \\ 0.390 & 0.0 & - & 0.498 & 0.0 \\ 0.683 & 0.384 & 0.683 & - & 0.737 \\ 0.039 & 0.098 & 0.039 & 0.270 & - \end{bmatrix}.$$

The dominant discordance matrix $\hat{L}$ compiled using Step (6) is given as follows:

$$\hat{L} = \begin{bmatrix} - & 0.868 & 0.0 & 0.0 & 0.0 \\ 0.278 & - & 0.450 & 0.0 & 0.0 \\ 0.586 & 1.0 & - & 0.0 & 0.0 \\ 0.795 & 0.922 & 0.564 & - & 0.0 \\ 1.0 & 1.0 & 1.0 & 0.282 & - \end{bmatrix}.$$

TABLE 1 Comparison table.

Author	Ranking
Rouyendegh (2018)	$\hat{A}_5 > \hat{A}_4 > \hat{A}_3 > \hat{A}_2 > \hat{A}_1$
Proposed approach	$\hat{A}_5 > \hat{A}_4 > \hat{A}_3 > \hat{A}_2 > \hat{A}_1$

The aggregate dominant matrix $\hat{R}$ compiled using Step (7) is given as follows:

$$\hat{R} = \begin{bmatrix} - & 0.649 & 0.0 & 0.0 & 0.0 \\ 0.271 & - & 0.412 & 0.0 & 0.0 \\ 0.6 & 1.0 & - & 0.0 & 0.0 \\ 0.538 & 0.706 & 0.452 & - & 0.0 \\ 0.962 & 0.911 & 0.962 & 0.511 & - \end{bmatrix}.$$

The best alternative using Step (8) is chosen as follows:

$$FX1.$$

The optimized order of alternatives is given as $\hat{A}_5 > \hat{A}_4 > \hat{A}_3 > \hat{A}_2 > \hat{A}_1$.

Table 1 provides a comparison of the final assessments of the suggested approach to those of another technique.

6 Conclusion

In this study, we proposed a CIF–ELECTREE and TODIM method to evaluate the ranking of different alternatives discussed under the available criteria. In the proposed ELECTREE method, for the selection criteria of the optimum alternative, we use the relative weights of the criteria given by the decision makers to construct the discordant and discordant matrices. The complex intuitionistic concept of "fuzzy distance" is used to determine the distance of each point from the positive and negative optimum points. While the CIFS, which is made up of membership and non-membership functions represented by a set of possible values, is a new way to express human uncertainty in everyday life, in this paper, we also propose a new method, called the CIF–TODIM method, for solving MCDM problems with uncertain fuzzy information. The best thing about the CIF–TODIM method is that it can deal with decision-making problems in which CIFEs show how alternatives rank on each criterion while also taking into account how the decision makers (DMs) act psychologically. The proposed complex intuitionistic fuzzy ELECTREE and TODIM methods are effective in dealing with the issues of decision making in ambiguous and complex situations. The proposed approaches are effective as they allow for multi-attribute decision making with multiple criteria and also allow us to visualize the membership grades in dual dimensions (2D). In this study, CIF–ELECTREE and TODIM methods are used along with four criteria to figure out how different sectors affect the environment. In conclusion, the CIF–ELECTREE and TODIM methods have different strengths and can be used in different decision-making situations depending on the issue and the preferences of the person making the decision. However, in the future, researchers may use complex IFSs in fields as different as weather forecasting, fuzzy time series forecasting, and data analysis, in addition to studying how they might be used in MADM.

Data availability statement

The original contributions presented in the study are included in the article/Supplementary Material, further inquiries can be directed to the corresponding authors.

Author contributions

All authors listed have made a substantial, direct, and intellectual contribution to the work and approved it for publication.

Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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Green innovation risk index screening under the global value chain based on the group decision characteristic root method

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As there are a large number of risk factors affecting green innovation under the global value chain, the screening of risk indicators is a key link in developing green innovation activities. Through the analysis of the influencing factors of current green innovation risk, the primary index of green innovation risk is established. The group decision characteristic root method is used to identify the key importance of primary indicators of green innovation risk and construct the global value chain's green innovation risk index system, which provides a scientific method and new ideas for measuring and evaluating green innovation risk in the global value chain. The empirical results of this paper are as follows: 1) green R&D risk measurement indicators under the global value chain include the proportion of global green R&D personnel investment, the proportion of global green R&D capital investment, the stability of global green R&D in manufacturing, the ease of international transfer of green technology, and the international protection of green technology patents; 2) Green manufacturing risk measurement indicators under the global value chain have seven risk factors: the proportion of global green manufacturing personnel investment, the proportion of global green manufacturing capital investment, the global green manufacturing product production scale, the global green manufacturing product quality performance level, the global outsourcing scale of green products, the global green manufacturing technology transformation degree of manufacturing industry, and the global green manufacturing cost increase; 3) green marketing risk measurement indicators under the global value chain have seven risk factors: the proportion of global green marketing personnel investment, the proportion of global green marketing capital investment, the international demand level for green products, the international market competition intensity of green products, the possession of green products to existing international marketing channels, the commercialization and internationalization level of green technology, and the intensity of green technical barriers to trade in the international community; and 4) green service risk measurement indicators under the global value chain have five risk factors: the proportion of global green service personnel investment, the proportion of global green service capital investment, the [1]coverage of global green service outlets, the globalization level of global green product supply chain, and the global after-sales technical service capability of green products.

KEYWORDS

global value chain, green innovation, risk identification, group decision characteristic root, index screening

1 Introduction

Since the 1980s, the global economy has witnessed rapid advancements in production and the globalization of trade. This phenomenon of globalization has emerged as a significant characteristic of the modern economy, facilitating the accelerated international movement of production factors such as capital, goods, and technology. As a result, it effectively stimulates the optimal allocation of resources on a global scale., but also changes original production, consumption, and management methods and creates new market opportunities. The realization of globalization is governed and organized through global value chains (Sun Y. Y. et al, 2020). Within the global value chain's division of labor system, China's manufacturing industry currently finds itself at nearly the bottom of the global value chain, facing the unfavorable situation of being "low-end locked" (Sun Y. Y. et al, 2020). In order for China's manufacturing industry to break out of the low-end lock of the global value chain, the country needs to actively engage in the international division of labor by integrating itself into the global value chain, improve its own innovation ability and green market entry ability through green innovation, transition to the high value-added link in the global value chain, and complete the value promotion in the global value chain to enhance its competitiveness in the international market. The word "innovation" itself is related to risk. While green innovation in manufacturing has the potential to create economic, social, and ecological value (Dong et al, 2023a), it is important to recognize that it is also a high-risk endeavor. All technological innovation activities accompanied by risks (PIETROBELLI and RABELLOTTI, 2011). A series of current uncertain risk factors have become the biggest obstacles to improving green innovation in the manufacturing industry. The value of green innovation must be created and protected through risk management (Luo and Jia, 2023). At the same time, green innovation ability also directly determines the international market competitiveness of the company itself. Only with strong green R&D ability can a company develop and produce green products that meet the needs of the international market, to continuously improve the international competitiveness of the company and remain invincible in the face of international competition (Wang et al, 2022). Therefore, measuring and identifying the level of green innovation risk is essential for improving companies' ability to develop green innovation, enhance market competitiveness, and achieve sustainable development. Correctly identifying and analyzing green innovation risk is key to the success of innovation activities (Xiao et al, 2022).

2 Green innovation risk index design under the global value chain

2.1 Analysis of constituent factors

Global value chain is made up of four important links: R&D, production, marketing, and service (Zhang et al, 2020). However, not all links create the same amount of value in the global value chain's value creation process. As the famous "smile curve" shows, the value created by R&D, marketing and service links is higher, while the value created by manufacturing links is lower (Wu and Fan, 2021). From the perspective of current development of the world economy, the global value chain not only brings opportunities for products to enter the international market (Ye et al, 2021), it also plays a crucial role in integrating green resources throughout various stages of the global value chain. This integration enhances the green innovation capabilities of the manufacturing industry and contributes to the advancement of green industries (Sui et al, 2015). Therefore, taking into account the influence of the global value chain on the green innovation process, this study divides the green innovation process of the manufacturing industry within the global value chain into four main stages: global green innovation R&D, global green innovation manufacturing, global green innovation marketing, and global green innovation service. The risks faced by green innovation in the global value chain of the manufacturing industry are the products of the combined effects of global value chain, green economic development needs, and innovation risks (Ren et al, 2021). Based on the four important stages of R&D, the linear process of R&D, manufacturing, The process of green innovation in the manufacturing industry within the global value chain encompasses various stages such as manufacturing, marketing and service of green innovation in the manufacturing industry includes the risk factors of green innovation in manufacturing industry under global value chain include global green innovation R&D risk, global green innovation manufacturing risk, global green innovation marketing risk, and global green innovation service risk (Ye, 2021), as shown in Figure 1.

2.2 Construction of risk indicators

The evaluation index design of intellectual property rights of high-tech companies should follow the principles of being comprehensive, scientific, operable, and quantifiable. In this study, we adopted the grounded theory research method to identify the risks associated with green innovation in the manufacturing industry within the global value chain. Furthermore, we explored the factors that influence these risks, using them as the theoretical foundation for our analysis. Based on the existing research results, from the four dimensions of global green R&D risk, global green manufacturing risk, global green marketing risk, and global green service risk, the global value chain green innovation risk index system was initially constructed, including four secondary indicators and 31 tertiary indicators. See Table 1 for details.

3 Green innovation index screening under the global value chain

Academic research on green innovation risk indicators under the global value chain is in its exploratory stage. The existing research mostly uses fuzzy analytic hierarchy process (AHP) and matter-element methods to comprehensively analyze relevant indicators and obtain evaluation results. Due to the large number of green innovation risk indicators, secondary indicators cannot be accurately discarded to retain effective information unless the weight of each indicator under the criterion layer can be accurately identified. In 1996, Professor Yuanhua Qiu proposed the group decision characteristic root method, which is a new characteristic root method that combines the experience and wisdom of scholars

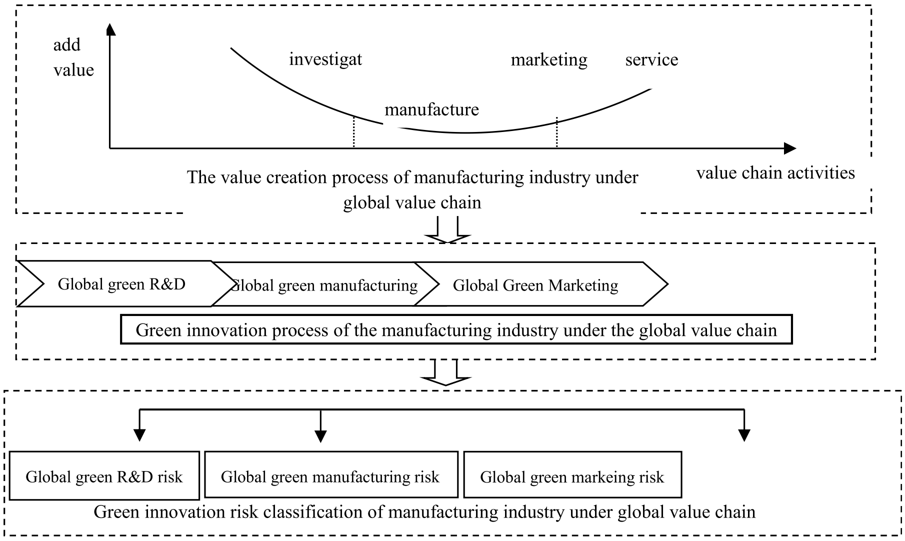

FIGURE 1
Green innovation risk classification of the manufacturing industry under the global value chain.

in related fields to judge and make decisions on multiple evaluated objects. It is a mathematical decision-making method, which is a derivative branch of the analytic hierarchy process proposed by T. L. Saaty, but the group decision eigenvalue method is superior to the analytic hierarchy process. It avoids the inconsistency of the objectives of the analytic hierarchy process judgment matrix of the (Meltzer, 2014). Therefore, this paper adopts the group decision feature root method (GEM) to solve this problem. Group Decision Eigenroot Method (GEM) is a new feature root method for expert group decision-making system (G) to judge and make decisions on multiple evaluation targets. Using GEM only requires experts to score each index, before transposing the score matrix into matrix F. In elaborating the shortcomings of the expert weighting method, the group decision feature root method can compensate for these shortcomings. In group decision-making, the most commonly used method is the comprehensive weighting method. The weight selected by this method has many shortcomings, however. These include complex operation, overly strong subjectivity, and insensitivity to results. The group decision eigenvalue method does a good job of solving these shortcomings. By using an ideal expert, that the issue of the expert's evaluation result of the evaluation object being untrue because of too strong subjectivity or insensitivity to the result is overcome (Quan, 2017). Compared with AHP, not only can GEM overcome the inconsistency of the judgment matrix, but it also needs no consideration of the weight of experts. Thus, the object of evaluation can be properly evaluated using a simple calculation. (Xiao et al, 2009).

3.1 Theoretical model of group eigenvalue method

3.1.1 Ideal expert definition

The definition of ideal expert S*: The expert with the smallest angle between the score vector and the score vector of each expert in the group is called the ideal expert of the group. Ideal expert scoring vector X* is the vector satisfying the maximum value of the function $f = \sum_{i=1}^{m} (b^T x_i)^2$, $\forall b = (b_1, b_2, \ldots, b_n)^T \in E^n$, with no loss of generality, and can be set $\|b\|_2 = 1$ (Jiang, 2007; Wang and Yin, 2013; Tan, 2015; Wang, 2017). That is, $\max_{b \in E^n} \sum_{i=1}^{m} (b^T x_i)^2 = \sum_{i=1}^{m} (x^{*T} x_i)^2, \|b\|_2 = 1$

Finding the ideal expert S * is the first step in this method. In the decision-making system G (S1, S2, . . ., Sm), m experts and n evaluated objects are selected, that is (A1,A2,. . .,An). The score of the i'th expert Si to the j'th evaluated target Aj is recorded as $x.x \in [I, J]$ $(i = 1, 2, .., m; j = 1, 2, ..n)$, The higher the score, the more important the evaluated object Aj is, and the better the goal is. M experts evaluate n evaluation objects to form an n-dimensional column vector xi; the score of the expert group G constitutes an m × n scoring matrix x (Qiu, 1997).

$$x_i = (x_{i1}, x_{i2}, \ldots, x_{in})^T \in E^n \tag{1}$$

$$x = \left(x_{ij}\right)_{m\times n} = \begin{pmatrix} x_{11} & . & . & . & x_{1n} \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ x_{m1} & . & . & . & x_{mn} \end{pmatrix} \tag{2}$$

The decision-making level of scoring experts is affected by many factors, such as personal professional aspects and personal emotions. Professional influence includes knowledge, experience, and personal comprehensive ability. Aspects of personal emotions include personal preferences, physical status, mental status, and emotions at the time. Based on the above factors, there is no ideal expert decision in reality (Wang and Tan, 1999). However, in the study, we assume that the ideal expert exists and is S*, and thus score each index. The result is the result of the expert's score on the index, which is the estimated value of the evaluated object. The score vector of the ideal expert is x*=(x1*,x2*,. . .xn*)T∈En, The ideal expert has the smallest angle between the scoring vector of the evaluated object and the scoring vector of other experts. When making group decisions, people

TABLE 1 Primary selection of green innovation risk indicators in the global value chain.

Risk of green innovation in manufacturing under the global value chain *U*)	Global green innovation R&D risk (C1)	Proportion of global green R&D personnel input in the manufacturing industry (C_{11})
		Proportion of global green R&D investment in the manufacturing industry (C_{12})
		Stability of global green R&D in the manufacturing industry (C_{13})
		Manufacturing global green R&D life cycle uncertainty (C_{14})
		Stability of global green technology application in the manufacturing industry (C_{15})
		Ease of international transfer of green technology in the manufacturing industry (C_{16})
		International protection of green technology patents in the manufacturing industry (C_{17})
	Global green innovation manufacturing risk (C_2)	Proportion of global green manufacturing personnel input in the manufacturing industry (C_{21})
		Proportion of global green manufacturing capital investment in the manufacturing industry (C_{22})
		Manufacturing global green manufacturing production scale (C_{23})
		Manufacturing global green manufacturing products on the technical performance of raw material requirements (C_{24})
		Manufacturing global green manufacturing product quality performance level (C_{25})
		Global outsourcing scale of manufacturing green products (C_{26})
		International OEM scale of green manufacturing products in the manufacturing industry (C_{27})
		Manufacturing global green manufacturing technology transformation degree (C_{28})
		Compatibility of existing international production system and green innovation in the manufacturing industry (C_{29})
		Manufacturing global green manufacturing cost increase (C_{30})
	Global green innovation marketing risk (C_3)	Proportion of global green marketing personnel input in the manufacturing industry (C_{31})
		Manufacturing global green marketing capital investment proportion (C_{32})
		International demand level of green products in the manufacturing industry (C_{33})
		The international market's recognition of manufacturing green products (C_{34})
		International market share of manufacturing green products (C_{35})
		International market competition intensity of manufacturing green products (C_{36})
		Proportion of green products in existing international marketing channels (C_{37})
		Commercialization and internationalization level of green technology in the manufacturing industry (C_{38})
		International community green technical barriers to trade intensity (C_{39})
	Global green innovation service risk (C_4)	Proportion of global green service personnel input in the manufacturing industry (C_{41})
		Proportion of global green service capital investment in the manufacturing industry (C_{42})
		Global green service network coverage of the manufacturing industry (C_{43})
		Manufacturing global green product supply chain globalization level (C_{44})
		Manufacturing green products global after-sales technical service capabilities (C_{45})

habitually find more authoritative experts to participate each time, so they think that ' ideal experts ' are experts with high consistency with the expert group's understanding of the evaluated object. The conclusion of the ideal expert on the evaluated object is completely consistent with the conclusion of the other experts of the expert group on the evaluated object, and the difference between the expert conclusions is the smallest. Following the above definition, x^* is an n-ary column vector. It is obtained by formula $f = \sum_{i=1}^{m} (b^T x_i)^2, \forall b = (b_1, b_2, \ldots, b_n)^T \in E^n$, And without loss of generality it can be set as $\|b\|_2 = 1$. That is:

$$\max_{b \in E^n} \sum_{i=1}^{m} \left(b^T x_i\right)^2 = \sum_{i=1}^{m} \left(x^{*T} x_i\right)^2 \tag{3}$$

Where x^* is the total score of the decision system G for the selected index.

Index solving theorem.

Theorem 1. $\forall b \in E^n, \max_{b \in E^n} \sum_{i=t}^{m} (b^T x_i)^2 = \rho_{max}$.

In the formula, ρ_{max} is the largest eigenvalue of the matrix; x^* is the positive eigenvector of ρ_{max} corresponding to $X^T X$, and $\|\hat{x}^*\| = 1$.

Theorem 2. Suppose A is an m×n matrix and B is an n×m matrix. Then AB and BA have the same (including multiplicity) non-zero eigenvalues.

Theorem 3. a_0 is the eigenvector corresponding to the maximum eigenvalue of $X^T X$, and $\|a_0\| = 1$, then $x^T a_0 = kx^*$, that is, a_0 is the weight vector corresponding to m experts.

This method only needs all the experts involved in the decision-making to directly score the green innovation risk indicators of each evaluated manufacturing industry, and the scoring matrix obtained is then transposed and multiplied by matrix F. Therefore, the optimal decision conclusion is the eigenvector corresponding to the maximum eigenvalue of F. The scoring result of the ideal expert to the evaluated object obtained by the group decision characteristic root is the ranking of multiple evaluated objects.

3.1.2 Treatment of single and multiple roots

When the maximum eigenvalue of the judgment matrix is a single root, the eigenvector corresponding to the maximum eigenvalue is the optimal solution. If the maximum characteristic root of the judgment matrix is a multiple root, it is necessary to find the two maximum characteristic roots at the same time, calculate the corresponding feature vector and determine the corresponding manufacturing green innovation risk index, and then rank the two risk indicators. Other risk indicators are based on the feature vector of the manufacturing green innovation risk index corresponding to the second largest characteristic root or deleting the ranking score in the existing judgment matrix. The above steps should be repeated until the single root of the largest characteristic root is found.

3.2 Index identification

Under the guidance of the principle of index selection, GEM is used to screen the importance of manufacturing green innovation risk measurement indicators under the global value chain. The index portion combines the overall risk of manufacturing green innovation and invites 15 experts to form an expert group. The 15 experts come from universities, government low-carbon management functional departments and Chinese manufacturing enterprise management departments, green product R&D departments, green manufacturing departments, green marketing departments, and green service departments. Qualitative evaluation is made by scoring the questionnaire (see Appendix A questionnaire). The design of the questionnaire is based on the Likert five-point scale, and the importance of the green innovation risk measurement index is divided into five levels. From 1 to 5, they represent "very unimportant," "unimportant," "important," "very important," and "most important" (see Table 2).

The scores of experts on green innovation risk indicators are obtained after collecting the questionnaires. The expert scores of green innovation risk indicators after statistics are shown in Table 3.

The expert scoring results of the above green innovation risk indicators are then processed as follows:

calculation $F = X\hat{T}$

$$F = \begin{pmatrix} 286 & 197 & 289 & 233 & 255 & . & . & . & 246 & 238 & 227 & 215 \\ 197 & 152 & 207 & 164 & 183 & . & . & . & 179 & 169 & 159 & 160 \\ 289 & 207 & 305 & 242 & 267 & . & . & . & 260 & 254 & 239 & 232 \\ 233 & 164 & 242 & 208 & 209 & . & . & . & 215 & 197 & 198 & 185 \\ 255 & 183 & 267 & 209 & 256 & . & . & . & 230 & 227 & 205 & 212 \\ . & . & . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . & . & . \\ 246 & 179 & 260 & 215 & 230 & . & . & . & 238 & 213 & 206 & 199 \\ 238 & 169 & 254 & 197 & 227 & . & . & . & 213 & 228 & 196 & 195 \\ 227 & 159 & 239 & 198 & 205 & . & . & . & 206 & 196 & 209 & 180 \\ 215 & 160 & 232 & 185 & 212 & . & . & . & 199 & 195 & 180 & 202 \end{pmatrix}$$

Using MATLAB software to calculate the maximum eigenvalue, the results show that the maximum eigenvalue is a single root. The eigenvalue of maximum $P_{Max} = 5848.9$, the unstandardized and standardized results of the corresponding feature vector B^T are shown in Table 4:

On the basis of summarizing previous studies, this study selected 0.027 as the standard value of index screening, that is, the index with the feature vector less than 0.027 after eliminating the standardized results, and the index with the feature vector greater than 0.027 after retaining the standardized results. According to the results in Table 4, the standardized results of the six risk factors of C17, C24, C28, C31, C39, and C310 are respectively 0.0269439, 0.0242547, 0.0260108, 0.0203634, 0.0226152, and 0.025095. All are less than the standard value of 0.027 for index screening so these indicators are removed. The values of the standardized results of the remaining indicators are greater than the standard value of 0.027, so these indicators are retained. After the above screening indicators, the retained indicators are C11, C12, C13, C14, C15, C16, C21, C22, C23, C25, C26, C27, C32, C33, C34, C35, C36, C37, C38, C41, C42, C43, C44, and C45.

These relate to the proportion of global green R&D personnel investment in the manufacturing industry, the proportion of global green R&D investment in the manufacturing industry, the stability of global green R&D in the manufacturing industry, the ease of international transfer of green technology in the manufacturing industry, the international protection of green technology patents in

TABLE 2 Expert opinion rating scale.

Score values	1	2	3	4	5
Importance	Very unimportant	Unimportant	Important	Very important	Most important

TABLE 3 Green innovation risk index expert scoring table.

	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}
C_{11}	5	4	5	5	5	3	4	4	2	5	3	4	5	5	5
C_{12}	2	3	3	4	2	3	4	5	3	3	2	2	3	4	3
C_{13}	5	4	4	5	4	5	5	4	4	4	3	5	5	5	5
C_{14}	4	3	3	5	3	3	3	4	2	3	5	5	5	3	3
C_{15}	5	4	5	2	5	3	5	5	5	3	3	4	3	3	5
C_{16}	4	4	5	5	5	4	4	4	3	5	5	5	3	4	4
C_{17}	3	3	2	3	2	3	3	4	3	3	3	4	1	4	3
C_{21}	5	3	2	5	2	4	3	3	5	2	2	3	5	3	5
C_{22}	4	4	3	5	5	4	3	4	3	4	4	4	3	4	4
C_{23}	3	4	4	2	5	5	3	5	5	5	5	5	3	2	4
C_{24}	4	3	3	5	4	1	3	1	1	3	3	3	2	3	3
C_{25}	3	5	4	4	5	5	4	3	2	4	5	4	5	4	2
C_{26}	5	3	3	4	5	2	2	4	4	3	2	2	5	4	4
C_{27}	5	3	4	3	5	2	2	2	4	5	3	2	5	5	5
C_{28}	1	3	4	3	4	4	1	3	3	3	3	2	2	1	2
C_{29}	3	3	3	1	4	3	3	4	2	3	3	3	3	2	2
C_{210}	2	3	3	1	4	2	2	1	1	1	3	3	3	2	2
C_{31}	4	3	3	3	4	4	3	5	5	4	3	2	2	2	2
C_{32}	4	3	5	3	5	4	2	4	3	5	3	2	2	2	1
C_{33}	3	4	4	4	4	5	3	5	5	4	4	3	4	4	3
C_{34}	4	4	4	1	5	5	4	5	5	3	4	4	1	2	4
C_{35}	4	5	4	1	5	5	4	5	4	5	5	3	2	2	3
C_{36}	4	5	4	3	5	5	5	5	5	4	5	3	1	2	3
C_{37}	4	5	4	1	5	3	4	5	4	4	5	4	2	2	5
C_{38}	1	3	4	2	1	2	2	4	4	1	3	3	2	3	2
C_{39}	3	4	3	2	2	3	2	2	2	4	3	4	3	3	1
C_{41}	4	3	5	2	5	3	3	2	5	3	5	5	2	3	3
C_{42}	4	4	5	5	2	4	4	4	3	2	5	5	3	4	4
C_{43}	3	5	3	4	5	5	5	2	4	3	2	4	4	2	5
C_{44}	5	2	2	5	2	3	3	2	4	4	5	3	5	3	5
C_{45}	5	3	2	4	5	5	3	5	5	2	3	3	2	3	2

the manufacturing industry, the proportion of global green manufacturing personnel investment in the manufacturing industry, the proportion of global green manufacturing personnel investment in the manufacturing industry, the proportion of global green manufacturing capital investment in the manufacturing industry, the production scale of global green manufacturing

TABLE 4 Feature vectors are standardized and unstandardized results.

Index	Unstandardized (feature vectors)	Standardized results (feature vectors)
C_{11}	0.214896	0.039078
C_{12}	0.153715	0.027953
C_{13}	0.224574	0.040838
C_{14}	0.181224	0.032955
C_{15}	0.203876	0.037074
C_{16}	0.216339	0.039341
C_{17}	0.148168	0.026944
C_{21}	0.173769	0.031599
C_{22}	0.196098	0.03566
C_{23}	0.204639	0.037213
C_{24}	0.141418	0.025717
C_{25}	0.198809	0.036153
C_{26}	0.175294	0.031877
C_{27}	0.184808	0.033607
C_{28}	0.13338	0.024255
C_{29}	0.143036	0.026011
C_{31}	0.111981	0.020363
C_{32}	0.167281	0.03042
C_{33}	0.164554	0.029924
C_{34}	0.199186	0.036221
C_{35}	0.188788	0.034331
C_{36}	0.195336	0.035521
C_{37}	0.201935	0.036721
C_{38}	0.194906	0.035443
C_{39}	0.124364	0.022615
C_{310}	0.138004	0.025096
C_{41}	0.180545	0.032832
C_{42}	0.194747	0.035414
C_{43}	0.188998	0.034369
C_{44}	0.176911	0.032171
C_{45}	0.177539	0.032285

products in the manufacturing industry, the quality and performance level of global green manufacturing products in the manufacturing industry, the global outsourcing scale of green products in the manufacturing industry, the technological transformation degree of global green manufacturing in the manufacturing industry, the increase of global green manufacturing costs in the manufacturing industry, the proportion of global green marketing personnel investment in the manufacturing industry, the proportion of global green marketing capital investment in the manufacturing industry, and the international demand level of green products.

3.3 Analysis of key indicators

3.3.1 Global green R&D risk

Global green innovation R&D risk refers to the various risks that may occur during the green innovation research and development stage on a global scale. At this stage, these risks can include personnel-related risks, financial, technical, and policy risks (Wang and Wang, 2020). As a highly professional activity, the R&D process of green innovation requires personnel with extensive knowledge and experience, as well as sufficient funding to support the innovation process. Insufficient R&D personnel and capital investment can have a negative impact on the success rate of green innovation. Furthermore, R&D risk stems from the immaturity of green innovation technology. The uncertainty and substitutability of the innovation R&D life cycle will lead to irreparable loss of innovation benefits (Hou et al, 2019). The international transfer of green innovation introduces uncertainties that can hinder the progress of green innovation. The uncertainty arises from factors such as differences in technological capabilities, regulatory frameworks, and market conditions between countries. Additionally, the effectiveness of international intellectual property rights' protection plays a pivotal role. If intellectual property rights are not adequately protected, innovations can be easily imitated by competitors, jeopardizing the benefits derived from the innovation and thereby increasing the risk associated with green innovation.

3.3.2 Global green manufacturing risk

Global green innovation manufacturing risk primarily refers to the potential failure of innovation caused by uncertainties and changes within the system during the manufacturing process (Deng, 2017). To achieve mass production of green products, it is crucial to have an adequate number of skilled manufacturing personnel and sufficient funding (Lu, 2015). Green innovation in the manufacturing industry introduces new requirements for processes, equipment, and raw materials. Global outsourcing is an effective approach to address challenges such as limited production capabilities and low value added in the manufacturing sector. However, changes in the scale of global outsourcing can also introduce risks. Additionally, the competition for orders among enterprises in the international market can be influenced by the scale of subcontracting. At the same time, the government or international organization's production constraints on companies will increase the cost burden in the production process.

3.3.3 Global green marketing risk

In the global green innovation marketing stage, if there is no sufficient guarantee of marketing personnel and funds, the risk of the entire marketing activity will increase. Uncertainty in the international market also has a greater impact at this stage. If the international market changes the demand for manufacturing green products (processes, services), the degree of recognition is not high, the international market share of manufacturing green products (processes, services) is too small, the international market competition is fierce, and it is difficult to use existing international marketing channels (Dong et al, 2023b). Therefore, a high degree of possession of existing marketing channels is an important way to achieve success in new product marketing. In addition, the globalization of the green supply chain in manufacturing and the presence of green technical barriers to trade in developed countries also influence green marketing activities. (Mao and Huo, 2002).

3.3.4 Global green service risks

Human capital and capital investment are essential resources for service activities in manufacturing enterprises, and they have a direct impact on the efficiency and effectiveness of innovative product services (Xiao et al, 2014). The coverage of a service network is a critical factor in balancing the service capabilities of the manufacturing industry. Establishing a comprehensive service network can enhance service efficiency, reduce service costs, and support the successful promotion of product innovation activities (Jie and Zhu, 2021). Simultaneously, the adoption of "green supply chain management" by large multinational enterprises can pose challenges for Chinese enterprises and become a new threshold for their development. After-sales technical service, also known as "after-sales technical support," plays a crucial role in enhancing customer satisfaction and loyalty. It involves providing assistance, guidance, and troubleshooting for customers using the manufactured products. Overall, the availability of skilled human resources, sufficient capital investment, a well-established service network, and effective after-sales technical service are all significant factors in ensuring the success of innovative product services in the manufacturing sector. involves installations and configuarations, use of instructions, and troubleshooting for products (processes, services) sold, as well as serving as a platform for information queries and customer information acquisition, consulting and technical training (Jiang, 2013). Helping to win high customer satisfaction plays a huge role in improving the market share of products (processes, services).

4 Screening results of green innovation indicators under the global value chain

On the basis of referring to the existing relevant research at home and abroad, considering the particularity of the green innovation process in the manufacturing industry, following the scientific, effective, comparable, and operable principles of index selection, through the previous global value chain manufacturing green innovation risk identification, risk factor analysis, and 15 experts ' scoring of risk factor indicators, the group decision eigenvalue method is used to screen out the indicators with a score lower than 0.027. A total of 25 risk factors that have a significant impact on green innovation activities of the global value chain manufacturing industry and that can lead to risk consequences were identified. The risk indicators after screening are shown in Table 5.

4.1 Green R&D risk measurement index under the global value chain

The risk measurement index of green R&D under the global value chain encompasses six key risk factors. The first is proportion of global green R&D personnel investment. This factor refers to the allocation of human resources towards green research and development activities on a global scale. The higher the proportion of investment in skilled personnel focused on green R&D initiatives, the lower the associated risks. The second, proportion of global green R&D capital investment, pertains to

TABLE 5 Global value chain green innovation risk indicators.

Goal layer	First indexes	Second index	Shorthand
Risk of green innovation in manufacturing under the global value chain	Global green R&D risk	Proportion of global green R&D personnel input in the manufacturing industry	C11
		Proportion of global green R&D investment in the manufacturing industry	C12
		Stability of global green R&D technology in the manufacturing industry	C13
		Stability of global green technology application in the manufacturing industry	C14
		Ease of international transfer of green technology in the manufacturing industry	C15
		International protection of green technology patents in the manufacturing industry	C16
	Global green manufacturing risk	Proportion of global green manufacturing personnel input in the manufacturing industry	C21
		Proportion of global green manufacturing capital investment in the manufacturing industry	C22
		Manufacturing global green manufacturing production scale	C23
		Manufacturing global green manufacturing product quality performance level	C24
		Global outsourcing scale of manufacturing green products	C25
		Manufacturing global green manufacturing technology transformation degree	C26
		Manufacturing global green manufacturing cost increase	C27
	Global green marketing risk	The proportion of global green marketing personnel input in manufacturing industry	C31
		Manufacturing global green marketing capital investment proportion	C32
		International demand level of green products in manufacturing industry	C33
		International competition intensity of green products in the manufacturing industry	C34
		Manufacturing green products international marketing channel share	C35
		The commercialization and internationalization level of green technology in the manufacturing industry	C36
		International community green technical barriers to trade intensity	C37
	Global green service risks	Proportion of global green service personnel input in the manufacturing industry	C41
		Proportion of global green service capital investment in the manufacturing industry	C42
		Global green service network coverage in the manufacturing industry	C43
		The green product supply chain globalization level	C44
		Manufacturing green products global after-sales technical service capabilities	C45

the allocation of financial resources towards global green R&D efforts. A higher proportion of capital investment in green R&D indicates a lower level of risk. Stability of global green R&D in the manufacturing industry, the third factor, evaluates the stability and consistency of green R&D efforts within the manufacturing industry on a global scale. A stable and consistent approach to green R&D lowers the associated risks. Fourth is ease of international transfer of green technology, which considers the ease with which green technology can be transferred internationally. Effortless international transfer of green technology reduces the risks involved in green R&D activities. Next, international protection of green technology patents focuses on the level of international protection available for green technology patents. Strong protection measures minimize the risks associated with intellectual property theft and infringement. These risk factors collectively contribute to measuring the potential risks involved in green R&D activities under the global value chain. By assessing and managing these risks effectively, companies can enhance their green innovation capabilities and navigate the challenges associated with global green R&D initiatives.

4.2 Green manufacturing risk measurement index under global value chain

The risk measurement index of green manufacturing under the global value chain includes seven risk factors: the proportion of global green manufacturing personnel input, the proportion of global green manufacturing capital investment, the scale of global green manufacturing products, the quality and performance level of global green manufacturing products, the scale of global outsourcing of green products, the degree of technological transformation of global green manufacturing in manufacturing industry, and the increase in global green manufacturing costs.

4.3 Green marketing risk measurement index under global value chain

The green marketing risk measurement index under the global value chain comprises seven significant risk factors. Proportion of global green marketing personnel investment refers to the allocation of human resources dedicated to green marketing activities on a global scale. A higher proportion of investment in skilled personnel for green marketing signifies a lower level of risk. The second factor, proportion of global green marketing capital investment, pertains to the allocation of financial resources towards global green marketing efforts. A higher proportion of capital investment in green marketing indicates a lower level of risk. Third, international demand level of green products, assesses the level of demand for green products in international markets. Higher demand for green products signifies lower marketing risks associated with their promotion and acceptance. The fourth factor, international market competition intensity of green products, evaluates the level of competition in international markets specifically for green products. Higher competition intensity may indicate higher marketing risks due to the need to differentiate and stand out in the market. The next factor, degree of green products' integration into existing international marketing channels, considers the extent to which green products can be integrated into existing international marketing channels. Greater integration implies lower marketing risks, as established channels can facilitate product distribution and customer reach. The sixth factor, level of green technology commercialization and internationalization, focuses on the extent to which green technologies can be successfully commercialized and internationalized. Higher levels of commercialization and internationalization reduce marketing risks associated with new technology adoption and market penetration. Finally, intensity of green technical trade barriers in the international community assesses the presence and intensity of trade barriers specific to green products and technologies in the global market. Higher barriers pose greater marketing risks, such as restricted market access or added compliance requirements. By considering and managing these risk factors effectively, companies can better evaluate the risks associated with green marketing activities in the global value chain. This enables them to develop appropriate strategies to overcome challenges and leverage opportunities in promoting green products internationally.

4.4 Green service risk measurement index under global value chain

The green service risk measurement index under the global value chain consists of five key risk factors. The first risk factor is proportion of global green service personnel investment and refers to the allocation of human resources towards green service activities on a global scale. A higher proportion of investment in skilled personnel dedicated to green services indicates a lower level of risk. The second factor, proportion of global green service capital investment, pertains to the allocation of financial resources towards global green service initiatives. A higher proportion of capital investment in green services suggests a lower level of risk. Next, coverage of global green service outlets assesses the geographic coverage and distribution of green service outlets worldwide. A wider coverage of outlets decreases the risk associated with fulfilling customer demands and providing timely services. The fourth factor, globalization level of the global green product supply chain, evaluates the extent to which the global green product supply chain is integrated and interconnected internationally. A higher level of globalization reduces risks related to supply chain disruptions, ensuring smooth delivery of green services. The final factor, global after-sales technical service capability of green products, focuses on the ability to provide after-sales technical services for green products on a global scale. A strong global after-sales technical service capability minimizes risks associated with product malfunctioning, customer dissatisfaction, and warranty claims. Stability of global green technology application in the manufacturing industry. These risk factors collectively form the basis of assessing the potential risks involved in green service activities under the global value chain. By understanding and managing these risks effectively, companies can enhance their green service capabilities, ensure customer satisfaction, and establish a strong presence in the global market.

5 Conclusion

Due to the particularity of green innovation, the problem of innovation risk is particularly important (Wang and Liu, 2020). The design of risk indicators and the identification of important indicators needs to be more rigorous. The use of group decision characteristic root method can solve the problem of screening multiple indicators and improve the scientific methodology, independence, and effectiveness of risk indicators. The results show that the selected indicators meet the research requirements and provide a scientific basis for the measurement and evaluation of green innovation risk under the global value chain. Four major findings emerged. First, the risk measurement index of green R&D under the global value chain was found to include six risk factors: the proportion of global green R&D personnel investment, the proportion of global green R&D capital investment, the stability of global green R&D in manufacturing industry, the ease of international transfer of green technology, and the international protection of green technology patents. Second, the green manufacturing risk measurement index under the global value chain was found to include seven risk factors: the proportion of global green manufacturing personnel investment, the proportion of global green manufacturing capital investment, the global green manufacturing product production scale, the global green manufacturing product quality performance level, the global outsourcing scale of green products, the global green manufacturing technology transformation degree of the manufacturing industry, and the global green manufacturing cost increase. Third, the green marketing risk measurement index under the global value chain was found to include seven risk factors: the proportion of global green marketing personnel investment, the proportion of global green marketing capital investment, the international demand level for green products, the international market competition intensity of green products, the degree of green products occupying the existing international marketing channels, the level of green technology commercialization and internationalization, and the intensity of green technical barriers to trade in the international community. Finally, the green service risk measurement index under the global value chain was found to include six risk factors: the proportion of global green service personnel investment, stability of global green technology application in the manufacturing industry, the proportion of global green service capital investment, the coverage of the global green service network, the globalization level of global green product supply chain, and the global after-sales technical service capability of green products.

This research has important theoretical value and practical significance. In the early stage, the grounded theory research method was used to identify the green innovation risk of manufacturing enterprises and screened out the extension of the risk influencing factors, as well as risk measurement indicators that could accurately, objectively, and truly reflect the actual situation of green innovation risk in the manufacturing industry under the global value chain, with as few indicators as possible. This research, which contains the most comprehensive content to date, provides a theoretical basis for the next step in green innovation risk measurement modeling in the manufacturing industry under the global value chain. The limitation of this study is that there is no empirical study of specific manufacturing enterprises, so the author's next research content is to conduct empirical analysis of specific manufacturing enterprises to test the applicability of risk indicators.

Data availability statement

The raw data supporting the conclusion of this article will be made available by the authors, without undue reservation.

Ethics statement

Ethical review and approval was not required for the study on human participants in accordance with the local legislation and institutional requirements. Written informed consent for participation was not required for this study in accordance with the national legislation and the institutional requirements. Written informed consent was obtained from the individual(s) for the publication of any potentially identifiable images or data included in this article.

Author contributions

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Conflict of interest

The author declares that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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Identification and classification of digital green innovation based on interaction Maclaurin symmetric mean operators by using T-spherical fuzzy information

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The digital green concept refers to the devotion to digital technology, i.e., techniques of procedures in the area of ecological or sustainable conservation. It contains leveraging digital techniques, procedures, and new tools to evaluate environmental problems and promote sustainable development. The major influence of this article is to evaluate the selection of the best digital green technology. For this, we aim to propose the idea of Maclaurin symmetric mean (MSM) operators based on interaction operational laws for T-spherical fuzzy (TSF) information, such as TSF interaction weighted averaging (TSFIWA), generalized TSF interaction weighted averaging (GTSFIWA), TSF interaction weighted geometric averaging (TSFIWGA), TSF interaction MSM (TSFIMSM), TSF interaction Bonferroni mean (TSFIBM), and TSF interaction weighted Maclaurin symmetric mean (TSFIWMSM) operators. Some dominant and reliable properties are also invented for evaluation. Moreover, to address the best digital green innovation (DGI) among the top five DGIs, we illustrate the procedure of the multi-attribute decision-making (MADM) technique under the presence of the derived operators. Finally, we demonstrate a numerical example for evaluating the comparative study between the proposed and existing or prevailing operators to enhance the worth of the derived theory.

KEYWORDS

T-spherical fuzzy set, Maclaurin symmetric mean, interactive Maclaurin symmetric mean operators, digital green innovation, decision-making

1 Introduction

The term "digital green sustainable" describes how digital technology and procedures are used to support environmental sustainability and lower carbon emissions. It includes many facets of technology with an emphasis on reducing their environmental effect, including data centers, cloud computing, software applications, and gadgets. The goal of digital green sustainable practices is to maximize technology's benefits while reducing its environmental harm. Organizations and people may help create a digital ecosystem that is more environmentally friendly and sustainable by using these practices. Moreover, to discover the best or finest decision from the collection of preferences, the theory of the multi-attribute decision-making (MADM) technique is very famous and reliable for evaluating the best pick from the array of choices, where the idea of the MADM approach is the sub-division of the decision-making procedure. However, in the presence of classical information, we faced a lot of complications because we have only two possibilities, such as 0 or 1, but not between unit intervals. Therefore, the theory of fuzzy set (FS) was proposed by Zadeh (1965), which described the membership whose values are contained in the unit interval. Furthermore, many scholars have raised the question of what would happen if we involved the falsity grade in the FS because the negative or falsity information is an initial and central part of every real-life problem. Therefore, Atanassov (1986) discovered the idea of intuitionistic FS (IFS). The IFS deals with yes or no types of theory, with the role that the sum of both degrees in the duplet should be contained in the unit interval. Furthermore, Yager (2013) improved or modified the theory of Pythagorean FS (PyFS). Moreover, Yager (2016) initiated the idea of q-rung orthopair FS (qROFS) with a valuable and robust condition such as $0 \leq \nu^{ƫ} + d^{ƫ} \leq 1, ƫ \in \mathbb{Z}^{+}$. When there are more than three possibilities, such as when there is voting between two parties, some individuals choose party A, some choose party B, some damage their vote by stamping both, and others choose not to vote, structures like IFS, PyFS, and qROPFS fail. Cuong and Kreinovich (2014) proposed the picture fuzzy set (PFS) to handle this information. The PFS requires that the sum of the MD, abstinence degree (AD), and NMD must lie within $[0, 1]$. Decision-makers feel hesitant to resolve their issues because the sum of MD, AD, and NMD does not contain $[0, 1]$, and PFS fails. Mahmood et al. (2019) proposed a generalization of PFSs termed spherical fuzzy sets (SFSs) which cover those types of information that the picture fuzzy set fails to explain. For example, we consider the information $(0.6, 0.5, 0.4)$ where $0 \leq 0.6 + 0.5 + 0.4 = 1.5 \geq 1$. However, $0 \leq 0.6^2 + 0.5^2 + 0.4^2 = 0.67 \leq 1$, which shows that the SFS is more easily applicable than the PFS. However, at some point, even the SFS is not enough to deal with uncertain information and $\nu^2 + u^2 + d^2$ exceeds to 1, e.g., we consider $(0.8, 0.6, 0.4)$, for which $0.8^2 + 0.6^2 + 0.4^2 = 1.16 \geq 1$, so to overcome this flaw without any restriction, Mahmood et al. (2019) introduced T-spherical fuzzy sets (TSFS) allowing $0 \leq \nu^{ƫ} + u^{ƫ} + d^{ƫ} \leq 1$, where $ƫ \in \mathbb{Z}^{+}$ and because of such a framework, any information of the type $(0.8, 0.6, 0.4)$ can be of use as for $ƫ = 3$ we have $0.8^3 + 0.6^3 + 0.4^3 = 0.792 \leq 1$. The impressive literature is discovered in Mahmood (2020); Mahmood and Ali (2020); Akram et al. (2022); Hussain et al. (2022); Khan et al. (2022).

The most accurate way for an information alliance is an aggregate operator. Numerous writers have presented multiple aggregation operators over the past 10 years. The average mean (AM) operator is the most commonly used AO since it makes it simple to aggregate all the different data into a comprehensive form. Several practical AOs have also been developed that are useful for collecting data in ambiguous and complex fuzzy decision-making contexts, such as geometric mean (GM), Bonferroni mean (BM), and Heronian mean (HM). Other AM AOs for MADM have been created in past years. Any of the operators mentioned previously do not consider the relationships between the values being used. To resolve this issue, Yager developed the concept of power AO (Yager, 2001). Power AOs significantly impact the relationship of the data being aggregated. Maclaurin symmetric mean (MSM) operators in the aggregation theory are one of the research topics that have gained the most significant interest. The MSM was first proposed by Maclaurin (Jstor, 2022) and later made famous by Detemple and Robertson (Jstor, 2022). The relationship of numerous input arguments can be overcome by MSM, which defines its characteristics. The significant difference between MSM and BM is that MSM may represent interactions among more than input data arguments, but BM cannot. For the given statements, MSM monotonically decreases concerning the parameter value. MSM accepts the fact that opinions are transformed into crisp numbers.

However, Liu et al. (2020) investigated partitioned MSM operators for MADM applications within IFSs. Qin and Liu. (2014) introduced IF MSM (IFMSM) operators. In the technology domain, Wei and Lu. (2018) investigated the Pythagorean fuzzy MSM (PyFMSM) operators. By creating transactional PyFMSM operators for the applications of MADM, Yang and Pang. (2018) also enhanced the efficiency of PyFMSM operators. For the MADM, Wei et al. (2019) proposed the q-Rung orthopair fuzzy MSM (qROFMSM) operators. At the same time, Wang et al. (2019) enhanced the concept of the qROFMSM operator. MSM operations for linguistic variables were examined by Liu and Qin. (2017) using an IF layout, while Qin et al. (2015) introduced the MSM operators in hesitant fuzzy contexts. Further information and applications of MSM operators are shown in Wang et al. (2018); Yu et al. (2018); Ju et al. (2016); Ullah (2021); Ashraf et al. (2022).

In particular circumstances, the aggregating outcomes from IFS's traditional operations are illogical, especially when there is zero NMD of the TSFS. Because operations of IFSs cannot consider interactions between truth and non-membership, He et al. (2014b) suggested the operational interaction laws of IFNs to solve this issue. When the NMD of IFNs is 0, the issues may be resolved, and these rules appropriately consider interactions between the membership and non-membership functions. Our proposed work is more appropriate than the that of existing operators because if any of the NMDs is 0, utilizing the existing operators affects all the other NMD values and gives zero aggregated value, whereas utilizing the proposed work zero does not affect the other ones. In our proposed work, we are using the TSFS as it is more useful than the other extensions of the FS because it gives independency to choose NMD by our own choice between $[0, 1]$ and also to take $ƫ$ of our choice. There are many extensions of the TSFS to get some more information regarding TSFS from more literature that can be seen here. Karaaslan and Al-Husseinawi. (2022) presented the concept of hesitant T-spherical fuzzy (HT-SF) set (HT-SFS) by combining concepts of the HF set and T-SFS, and present some set theoretical operations of HT-SFSs. Özlü and Karaaslan. (2022) introduced the concept of type-2 hesitant fuzzy set (HFS), which is a generalization of the HFS. The concept of complex T-spherical fuzzy Dombi aggregation operators was developed by Karaaslan and Dawood (2021).

Green innovation" is the technological innovation that involves energy conservation, pollution prevention, waste recycling, green product design, or corporate environmental management. Digital technology is crucial for green innovation, and FS theory is typically the best option when it comes to assessing and modeling human knowledge during the process of digital green innovation (DGI). The purpose of this article is to discuss new developments in FS models and their use in DGI in engineering processes and practical accomplishments. Green innovation indicates all forms of invention that minimize green destruction and guarantee that biological supplies are utilized most effectively and efficiently as possible. A study on DGI can be seen in Yin et al. (2022); Yin and Yu (2022); Dong et al. (2023). Furthermore, fuzzy set theory is a valuable and dominant technique for modeling and evaluating human knowledge regarding DGI. Inspired by the aforementioned information, the major influence of this analysis is stated as follows:

1. To derive the theory of GTSFIWA, GTSFIA, TSFIWA, TSFIA, and TSFIWGA operators.
2. To discover the idea of TSFIMSM, TSFIBM, and TSFIWMSM operators.
3. To expose some valuable properties for the evaluated work.
4. To discuss the MADM technique under the presence of the derived operators.
5. To illustrate, a numerical example for evaluating the comparative study between proposed and existing or prevailing operators is applied to enhance the worth of the derived theory.

The rest of our work is divided into different sections. We primarily introduce the basic ideas of T-SFSs and interaction MSM operators in Section 2. We introduce the GTSFIWA, GTSFIA, TSFIWA, TSFIA, TSFIWGA, TSFIGA, and GTSFIWGA operators in Section 3. We defined the TSFIMSM and TSFIWMSM operators in Section 4 and described their properties. Using the proposed operators, we provide a new MADM technique in Section 5 and step-by-step instructions. In Section 6, we illustrate the effectiveness of the proposed MADM method and compare it with that of the recent techniques using a real-world application. The conclusions of this paper are presented in Section 7.

2 Preliminaries

Here, we aim to revise the theory of TSF information and its operational laws, such as algebraic and interaction laws, for TSF information. Throughout the paper, $\tilde{\imath}$ will denote the indexing term, and $\mathcal{R}$ will represent a non-empty set. Furthermore, we have used the $\nu(r), u(r), d(r)$ and $\mathcal{R}$ for truth, abstinence, falsity, and universal set, respectively, where $\oplus$ and $\otimes$ are used for addition and multiplications, respectively.

Definition 1: (Ullah et al., 2018) A TSFS $\breve{t}$ is expressed by

$$\breve{t} = \{\langle r, \nu(r), u(r), d(r)\rangle | r \in \mathcal{R}\}. \tag{1}$$

With a valuable characteristic:

$$0 \leq \nu^{ŀ} + u^{ŀ} + d^{ŀ} \leq 1, ŀ \in \mathbb{Z}^{+}.$$

Under the presence of the aforementioned information, we derive the theory of neutral information $\mathfrak{R}$, such as

$$\mathfrak{R} = \sqrt[ŀ]{1 - (\nu^{ŀ} + u^{ŀ} + d^{ŀ})}. \tag{2}$$

From now onward, the information in triplet $\breve{t} = (\nu, u, d)$ is used as the TSF value (TSFV). Various exceptional cases are stated and derived; for instance, to put the value of $ŀ = 2$ in the TSFS, we obtain the theory of the SFS to set the value of $ŀ = 1$ in the TSFS. We obtain the theory of the PFS to set the value of $u = 0$ in the TSFS, then we obtain the view of q-ROFS, and finally, to set the value of $u = 0$ and $ŀ = 2$ in the TSFS. Then we get the idea of PyFS.

Definition 2: (Garg et al., 2018) Assume any two TSFVs be $\breve{t}_i = (\nu_{\tilde{\imath}}, u_{\tilde{\imath}}, d_{\tilde{\imath}})$. Then, some operational laws of TSFVs are as follows:

$$\breve{t}_1 \oplus \breve{t}_2 = \left(\sqrt[ŀ]{(\nu_1)^{ŀ} + (\nu_2)^{ŀ} - (\nu_1)^{ŀ}(\nu_2)^{ŀ}}, u_1 u_2, d_1 d_2\right), \tag{3}$$

$$\breve{t}_1 \otimes \breve{t}_2 = \left(\nu_1 \nu_2, \sqrt[ŀ]{(u_1)^{ŀ} + (u_2)^{ŀ} - (u_1)^{ŀ}(u_2)^{ŀ}}, \sqrt[ŀ]{(d_1)^{ŀ} + (d_2)^{ŀ} - (d_1)^{ŀ}(d_2)^{ŀ}}\right), \tag{4}$$

$$\zeta\breve{t} = \left(\sqrt[ŀ]{1 - (1 - \nu^{ŀ})^{\zeta}}, u^{ŀ}, d^{ŀ}\right), \tag{5}$$

$$(\breve{t})^{\zeta} = \left(\nu^{ŀ}, \sqrt[ŀ]{1 - (1 - u^{ŀ})^{\zeta}}, \sqrt[ŀ]{1 - (1 - d^{ŀ})^{\zeta}}\right), \tag{6}$$

$$\left(\breve{t}^{c}\right) = (d, u, \nu). \tag{7}$$

Definition 3: (Naseem et al., 2022) Assume any two TSFVs be $\breve{t}_i = (\nu_{\tilde{\imath}}, u_{\tilde{\imath}}, d_{\tilde{\imath}})$. The score and accuracy values are stated by

$$\mathfrak{s}\left(\breve{t}\right) = \nu^{ŀ} - u^{ŀ}.\mathfrak{R}^{ŀ} \in [-1, 1], \tag{8}$$

$$\text{Å}\left(\breve{t}\right) = \left(\nu^{ŀ} + u^{ŀ} + d^{ŀ}\right) \in [0, 1]. \tag{9}$$

Example 1: Let three TSFVs be $\breve{t}_1 = (0.6, 0.5, 0.4)$, $\breve{t}_2 = (0.5, 0.7, 0.6)$, and $\breve{t}_3 = (0.7, 0.4, 0.0)$, and $\omega = (\omega_1, \omega_2, \omega_3) = (0.25, 0.4, 0.35)$ be the weight vector where $ŀ = 3$.

Solution: As we know $\breve{t} = \omega_1\breve{t}_1 \oplus \omega_2\breve{t}_2 \oplus \omega_3\breve{t}_3 = (0.61263, 0.52906, 0)$. Thus, d_{i} of $\breve{t}_3$ is "0", that affects all the other d_{i} by using the previously required operations, and d of $\breve{t}$ becomes 0, which is illogical. To overcome the drawback of these operational laws, several updated operational laws for the TSFS are given as follows:

Definition 4: Let $\breve{t}_1 = (\nu_1, u_1, d_1)$ and $\breve{t}_2 = (\nu_2, u_2, d_2)$ be two TSFVs, $\lambda > 0$. Then, the operational laws can be expressed as follows:

$$\breve{t}_1 \oplus \breve{t}_2 = \left(\begin{array}{c} \sqrt[k]{\nu_1^k + \nu_2^k - \nu_1^k\nu_2^k}, \sqrt[k]{u_1^k + u_2^k - u_1^k u_2^k - \nu_1^k u_2^k - u_1^k \nu_2^k}, \\ \sqrt[k]{d_1^k + d_2^k - d_1^k d_2^k - \nu_1^k d_2^k - d_1^k \nu_2^k} \end{array} \right), \tag{10}$$

$$\breve{t}_1 \otimes \breve{t}_2 = \left(\sqrt[k]{\nu_1^k + \nu_2^k - \nu_1^k\nu_2^k - u_1^k\nu_2^k - \nu_1^k u_2^k}, \sqrt[k]{u_1^k + u_2^k - u_1^k u_2^k}, \sqrt[k]{d_1^k + d_2^k - d_1^k d_2^k} \right), \tag{11}$$

$$\lambda\breve{t} = \left(\sqrt[k]{1 - (1-\nu^k)^\lambda}, \sqrt[k]{(1-\nu^k)^\lambda - (1-(\nu^k + u^k))^\lambda}, \sqrt[k]{(1-\nu^k)^\lambda - (1-(\nu^k + d^k))^\lambda} \right), \tag{12}$$

$$(\breve{t})^\lambda = \left(\sqrt[k]{(1-d^k)^\lambda - (1-(d^k + \nu^k))^\lambda}, \sqrt[k]{1-(1-u^k)^\lambda}, \sqrt[k]{1-(1-d^k)^\lambda} \right). \tag{13}$$

The first two equations can be written in the respective ways shown in the following equations:

$$\begin{aligned} \breve{t}_1 \oplus \breve{t}_2 &= \left(\begin{array}{c} \sqrt[k]{1-(1-\nu_1^k)(1-\nu_2^k)}, \sqrt[k]{(1-\nu_1^k)(1-\nu_2^k) - (1-(\nu_1^k+u_1^k))(1-(\nu_2^k+u_2^k))}, \\ \sqrt[k]{(1-\nu_1^k)(1-\nu_2^k) - (1-(\nu_1^k+d_1^k))(1-(\nu_2^k+d_2^k))} \end{array} \right) \\ &= \left(\begin{array}{c} \sqrt[k]{1-\prod_{j=1}^{2}(1-\nu_j^k)}, \sqrt[k]{\prod_{j=1}^{2}(1-\nu_j^k) - \prod_{j=1}^{2}(1-(\nu_j^k+u_j^k))}, \\ \sqrt[k]{\prod_{j=1}^{2}(1-\nu_j^k) - \prod_{j=1}^{2}(1-(\nu_j^k+d_j^k))} \end{array} \right), \end{aligned} \tag{14}$$

$$\begin{aligned} \breve{t}_1 \otimes \breve{t}_2 &= \left(\begin{array}{c} \sqrt[k]{(1-d_1^k)(1-d_2^k) - (1-(\nu_1^k+d_1^k))(1-(\nu_2^k+d_2^k))}, \\ \sqrt[k]{1-(1-u_1^k)(1-u_2^k)}, \sqrt[k]{1-(1-d_1^k)(1-d_2^k)} \end{array} \right) \\ &= \left(\sqrt[k]{\prod_{j=1}^{2}(1-d_j^k) - \prod_{j=1}^{2}(1-(\nu_j^k+u_j^k+d_j^k))}, \sqrt[k]{1-\prod_{j=1}^{2}(1-u_j^k)}, \sqrt[k]{1-\prod_{j=1}^{2}(1-d_j^k)} \right). \end{aligned} \tag{15}$$

Example 2: Let three TSFVs be $\breve{t}_1 = (0.6, 0.5, 0.4)$, $\breve{t}_2 = (0.5, 0.7, 0.6)$, and $\breve{t}_3 = (0.7, 0.4, 0.0)$, and $\omega = (\omega_1, \omega_2, \omega_3) = (0.25, 0.4, 0.35)$ be the weight vector where $k = 3$.

Solution: Solving Example 1 by using Definition 3, we overcome the drawbacks and obtain that $\breve{t} = \omega_1\breve{t}_1 \oplus \omega_2\breve{t}_2 \oplus \omega_3\breve{t}_3 = (0.61263, 0.58198, 0.46232)$. As $min(d_1, d_2, d_3) = 0 \leq 0.46232 \leq 0.6 = max(d_1, d_2, d_3)$. From the perspective of averages, the outcomes obtained by Definition 3 are more authentic than those of Definition 2.

3 Generalized TSF interaction aggregation operators

In the section on the development of green manufacturing engineering, we concentrate on utilizing the theory IMSM in the environment of TSF set theory, such as GTSFIWA, GTSFIA, TSFIWA, TSFIA, and TSFIWGA operators. Some dominant and reliable properties are also invented for evaluated work.

Definition 5: The GTSFIWA operator is particularized by

$$GTSFIWA_\lambda(\breve{t}_1, \breve{t}_2, \ldots, \breve{t}_q) = \left(\oplus_{i=1}^{q} \omega_i \breve{t}_i^\lambda \right)^{1/\lambda} = \left(\omega_1\breve{t}_1^\lambda \oplus \omega_2\breve{t}_2^\lambda \oplus \ldots \oplus \omega_q\breve{t}_q^\lambda \right)^{1/\lambda}. \tag{16}$$

Theorem 1: Using the information in Eq. 16, we derive the following theory, such as

$$GTSFIWA_{\lambda}(\breve{t}_1, \breve{t}_2, \ldots, \breve{t}_q) = \left(\begin{array}{c} \sqrt[\text{k}]{\left(\begin{array}{c} \left(1 - \prod_{j=1}^{q}\left(1 - \left(1 - d_j^{k}\right)^{\lambda} + \left(1 - \left(\nu_j^{k} + d_j^{k}\right)\right)^{\lambda}\right)^{\omega_j} + \prod_{j=1}^{q}\left(1 - \left(\nu_j^{k} + d_j^{k}\right)\right)^{\lambda\omega_j}\right)^{1/\lambda} \\ -\prod_{j=1}^{q}\left(1 - \left(\nu_j^{k} + d_j^{k}\right)\right)^{\omega_j} \end{array} \right)}, \\ \sqrt[\text{k}]{1 - \left(1 - \prod_{j=1}^{q}\left(1 - \left(1 - u_j^{k}\right)^{\lambda} + \left(1 - \left(\nu_j^{k} + u_j^{k}\right)\right)^{\lambda}\right)^{\omega_j} + \prod_{j=1}^{q}\left(1 - \left(\nu_j^{k} + u_j^{k}\right)\right)^{\lambda\omega_j}\right)^{1/\lambda}}, \\ \sqrt[\text{k}]{1 - \left(1 - \prod_{j=1}^{q}\left(1 - \left(1 - d_j^{k}\right)^{\lambda} + \left(1 - \left(\nu_j^{k} + d_j^{k}\right)\right)^{\lambda}\right)^{\omega_j} + \prod_{j=1}^{q}\left(1 - \left(\nu_j^{k} + d_j^{k}\right)\right)^{\lambda\omega_j}\right)^{1/\lambda}} \end{array} \right). \quad (17)$$

Proof: By using mathematical induction, we concentrate to derive the theory in Eq. 17. For this, we have the following procedure. When $q = 2$, then

$$\omega_1 \breve{t}_1^{\lambda} = \left(\begin{array}{c} \sqrt[\text{k}]{\left(1 - \left(1 - \left((1 - d_1^{k})^{\lambda} - (1 - (\nu_1^{k} + d_1^{k}))^{\lambda}\right)\right)^{\omega_1}\right)}, \\ \sqrt[\text{k}]{\left(1 - \left((1 - u_1^{k})^{\lambda} - (1 - (\nu_1^{k} + u_1^{k}))^{\lambda}\right)\right)^{\omega_1} - (1 - (\nu_1^{k} + u_1^{k}))^{\omega_1\lambda}}, \\ \sqrt[\text{k}]{\left(1 - \left((1 - d_1^{k})^{\lambda} - (1 - (\nu_1^{k} + d_1^{k}))^{\lambda}\right)\right)^{\omega_1} - (1 - (\nu_1^{k} + d_1^{k}))^{\omega_1\lambda}} \end{array} \right),$$

$$\omega_2 \breve{t}_2^{\lambda} = \left(\begin{array}{c} \sqrt[\text{k}]{\left(1 - \left(1 - \left((1 - \nu_2^{k})^{\lambda} - (1 - (\nu_2^{k} + d_2^{k}))^{\lambda}\right)\right)^{\omega_2}\right)}, \\ \sqrt[\text{k}]{\left(1 - \left((1 - u_2^{k})^{\lambda} - (1 - (\nu_2^{k} + u_2^{k}))^{\lambda}\right)\right)^{\omega_2} - (1 - (\nu_2^{k} + u_2^{k}))^{\omega_2\lambda}}, \\ \sqrt[\text{k}]{\left(1 - \left((1 - d_2^{k})^{\lambda} - (1 - (\nu_2^{k} + d_2^{k}))^{\lambda}\right)\right)^{\omega_2} - (1 - (\nu_2^{k} + d_2^{k}))^{\omega_2\lambda}} \end{array} \right).$$

Thus,

$$\omega_1 \breve{t}_1^{\lambda} \oplus \omega_2 \breve{t}_2^{\lambda} = \left(\begin{array}{c} \sqrt[\text{k}]{\left(1 - \left(1 - (1 - d_1^{k})^{\lambda} + (1 - (\nu_1^{k} + d_1^{k}))^{\lambda}\right)^{\omega_1} * \left(1 - (1 - d_2^{k})^{\lambda} + (1 - (\nu_2^{k} + d_2^{k}))^{\lambda}\right)^{\omega_2}\right)}, \\ \sqrt[\text{k}]{\left(\left(1 - (1 - u_1^{k})^{\lambda} + (1 - (\nu_1^{k} + u_1^{k}))^{\lambda}\right)^{\omega_1} * \left(1 - (1 - u_2^{k})^{\lambda} + (1 - (\nu_2^{k} + u_2^{k}))^{\lambda}\right)^{\omega_2} - (1 - (\nu_1^{k} + u_1^{k}))^{\omega_1\lambda}(1 - (\nu_2^{k} + u_2^{k}))^{\omega_2\lambda}\right)}, \\ \sqrt[\text{k}]{\left(\left(1 - (1 - d_1^{k})^{\lambda} + (1 - (\nu_1^{k} + d_1^{k}))^{\lambda}\right)^{\omega_1} * \left(1 - (1 - d_2^{k})^{\lambda} + (1 - (\nu_2^{k} + d_2^{k}))^{\lambda}\right)^{\omega_2} - (1 - (\nu_1^{k} + d_1^{k}))^{\omega_2\lambda}(1 - (\nu_2^{k} + d_2^{k}))^{\omega_2\lambda}\right)} \end{array} \right),$$

$$= \left(\begin{array}{c} \sqrt[\text{k}]{1 - \prod_{i=1}^{2}\left(1 - \left(1 - d_i^{k}\right)^{\lambda} + \left(1 - \left(\nu_i^{k} + d_i^{k}\right)\right)^{\lambda}\right)^{\omega_i}}, \\ \sqrt[\text{k}]{\prod_{i=1}^{2}\left(1 - \left(1 - u_i^{k}\right)^{\lambda} + \left(1 - \left(\nu_i^{k} + u_i^{k}\right)\right)^{\lambda}\right)^{\omega_i} - \prod_{i=1}^{2}\left(1 - \left(\nu_i^{k} + u_i^{k}\right)\right)^{\lambda\omega_i}}, \\ \sqrt[\text{k}]{\prod_{i=1}^{2}\left(1 - \left(1 - d_i^{k}\right)^{\lambda} + \left(1 - \left(\nu_i^{k} + d_i^{k}\right)\right)^{\lambda}\right)^{\omega_i} - \prod_{i=1}^{2}\left(1 - \left(\nu_i^{k} + d_i^{k}\right)\right)^{\lambda\omega_i}} \end{array} \right),$$

$$\left(\omega_1 \breve{t}_1^{\lambda} \oplus \omega_2 \breve{t}_2^{\lambda}\right)^{1/\lambda} = \left(\begin{array}{c} \sqrt[\text{k}]{\left(\begin{array}{c} \left(1 - \left(\prod_{i=1}^{2}\left(1 - \left(1 - d_i^{k}\right)^{\lambda} + \left(1 - \left(\nu_i^{k} + d_i^{k}\right)\right)^{\lambda}\right)^{\omega_i} - \prod_{i=1}^{2}\left(1 - \left(\nu_i^{k} + d_i^{k}\right)\right)^{\lambda\omega_i}\right)\right)^{1/\lambda} \\ -\prod_{i=1}^{2}\left(1 - \left(\nu_i^{k} + d_i^{k}\right)\right)^{\lambda\omega_i} \end{array} \right)}, \\ \sqrt[\text{k}]{1 - \left(1 - \left(\prod_{i=1}^{2}\left(1 - \left(1 - u_i^{k}\right)^{\lambda} + \left(1 - \left(\nu_i^{k} + u_i^{k}\right)\right)^{\lambda}\right)^{\omega_i} - \prod_{i=1}^{2}\left(1 - \left(\nu_i^{k} + u_i^{k}\right)\right)^{\lambda\omega_i}\right)\right)^{1/\lambda}}, \\ \sqrt[\text{k}]{1 - \left(1 - \left(\prod_{i=1}^{2}\left(1 - \left(1 - d_i^{k}\right)^{\lambda} + \left(1 - \left(\nu_i^{k} + d_i^{k}\right)\right)^{\lambda}\right)^{\omega_i} - \prod_{i=1}^{2}\left(1 - \left(\nu_i^{k} + d_i^{k}\right)\right)^{\lambda\omega_i}\right)\right)^{1/\lambda}} \end{array} \right).$$

For $q = 2$, the information in Eq. 17 is valid. Therefore, we assume that the information in Eq. 17 is also accurate for $q = \rho$, such as

$$\left(\oplus_{\tilde{\imath}=1}^{\rho}\omega_{\tilde{\imath}}\breve{t}_{\tilde{\imath}}^{\lambda}\right)^{1/\lambda}=\begin{pmatrix}\sqrt[\mathfrak{k}]{\begin{pmatrix}\left(1-\left(\prod_{\tilde{\imath}=1}^{\rho}\left(1-\left(1-d_{\tilde{\imath}}^{\mathfrak{k}}\right)^{\lambda}+\left(1-\left(\nu_{\tilde{\imath}}^{\mathfrak{k}}+d_{\tilde{\imath}}^{\mathfrak{k}}\right)\right)^{\lambda}\right)^{\omega_{\tilde{\imath}}}-\prod_{\tilde{\imath}=1}^{\rho}\left(1-\left(\nu_{\tilde{\imath}}^{\mathfrak{k}}+d_{\tilde{\imath}}^{\mathfrak{k}}\right)\right)^{\lambda\omega_{\tilde{\imath}}}\right)\right)^{1/\lambda}\\ -\prod_{\tilde{\imath}=1}^{\rho}\left(1-\left(\nu_{\tilde{\imath}}^{\mathfrak{k}}+d_{\tilde{\imath}}^{\mathfrak{k}}\right)\right)^{\lambda\omega_{\tilde{\imath}}}\end{pmatrix}},\\ \sqrt[\mathfrak{k}]{1-\left(1-\left(\prod_{\tilde{\imath}=1}^{\rho}\left(1-\left(1-u_{\tilde{\imath}}^{\mathfrak{k}}\right)^{\lambda}+\left(1-\left(\nu_{\tilde{\imath}}^{\mathfrak{k}}+u_{\tilde{\imath}}^{\mathfrak{k}}\right)\right)^{\lambda}\right)^{\omega_{\tilde{\imath}}}-\prod_{\tilde{\imath}=1}^{\rho}\left(1-\left(\nu_{\tilde{\imath}}^{\mathfrak{k}}+u_{\tilde{\imath}}^{\mathfrak{k}}\right)\right)^{\lambda\omega_{\tilde{\imath}}}\right)\right)^{1/\lambda}},\\ \sqrt[\mathfrak{k}]{1-\left(1-\left(\prod_{\tilde{\imath}=1}^{\rho}\left(1-\left(1-d_{\tilde{\imath}}^{\mathfrak{k}}\right)^{\lambda}+\left(1-\left(\nu_{\tilde{\imath}}^{\mathfrak{k}}+d_{\tilde{\imath}}^{\mathfrak{k}}\right)\right)^{\lambda}\right)^{\omega_{\tilde{\imath}}}-\prod_{\tilde{\imath}=1}^{\rho}\left(1-\left(\nu_{\tilde{\imath}}^{\mathfrak{k}}+d_{\tilde{\imath}}^{\mathfrak{k}}\right)\right)^{\lambda\omega_{\tilde{\imath}}}\right)\right)^{1/\lambda}}\end{pmatrix}.$$

Then, we aim to derive it for $\mathfrak{q}=\rho+1$, such as

$$\left(\oplus_{\tilde{\imath}=1}^{\rho+1}\omega_{\tilde{\imath}}\breve{t}_{\tilde{\imath}}^{\lambda}\right)^{1/\lambda}=\left(\left(\oplus_{\tilde{\imath}=1}^{\rho}\omega_{\tilde{\imath}}\breve{t}_{\tilde{\imath}}^{\lambda}\right)\oplus\left(\omega_{\rho+1}\breve{t}_{\rho+1}^{\lambda}\right)\right)^{1/\lambda},$$

$$=\begin{pmatrix}\begin{pmatrix}\sqrt[\mathfrak{k}]{1-\prod_{\tilde{\imath}=1}^{\rho}\left(1-\left(1-d_{\tilde{\imath}}^{\mathfrak{k}}\right)^{\lambda}+\left(1-\left(\nu_{\tilde{\imath}}^{\mathfrak{k}}+d_{\tilde{\imath}}^{\mathfrak{k}}\right)\right)^{\lambda}\right)^{\omega_{\tilde{\imath}}}},\\ \sqrt[\mathfrak{k}]{\prod_{\tilde{\imath}=1}^{\rho}\left(1-\left(1-u_{\tilde{\imath}}^{\mathfrak{k}}\right)^{\lambda}+\left(1-\left(\nu_{\tilde{\imath}}^{\mathfrak{k}}+u_{\tilde{\imath}}^{\mathfrak{k}}\right)\right)^{\lambda}\right)^{\omega_{\tilde{\imath}}}-\prod_{\tilde{\imath}=1}^{\rho}\left(1-\left(\nu_{\tilde{\imath}}^{\mathfrak{k}}+u_{\tilde{\imath}}^{\mathfrak{k}}\right)\right)^{\lambda\omega_{\tilde{\imath}}}},\\ \sqrt[\mathfrak{k}]{\prod_{\tilde{\imath}=1}^{\rho}\left(1-\left(1-d_{\tilde{\imath}}^{\mathfrak{k}}\right)^{\lambda}+\left(1-\left(\nu_{\tilde{\imath}}^{\mathfrak{k}}+d_{\tilde{\imath}}^{\mathfrak{k}}\right)\right)^{\lambda}\right)^{\omega_{\tilde{\imath}}}-\prod_{\tilde{\imath}=1}^{\rho}\left(1-\left(\nu_{\tilde{\imath}}^{\mathfrak{k}}+d_{\tilde{\imath}}^{\mathfrak{k}}\right)\right)^{\lambda\omega_{\tilde{\imath}}}}\end{pmatrix}\oplus\\ \begin{pmatrix}\sqrt[\mathfrak{k}]{1-\left(1-\left(1-d_{\rho+1}^{\mathfrak{k}}\right)^{\lambda}+\left(1-\left(\nu_{\rho+1}^{\mathfrak{k}}+d_{\rho+1}^{\mathfrak{k}}\right)\right)^{\lambda}\right)^{\omega_{\rho+1}}},\\ \sqrt[\mathfrak{k}]{\left(1-\left(1-u_{\rho+1}^{\mathfrak{k}}\right)^{\lambda}+\left(1-\left(\nu_{\rho+1}^{\mathfrak{k}}+u_{\rho+1}^{\mathfrak{k}}\right)\right)^{\lambda}\right)^{\omega_{\rho+1}}-\left(1-\left(\nu_{\rho+1}^{\mathfrak{k}}+u_{\rho+1}^{\mathfrak{k}}\right)\right)^{\lambda\omega_{\rho+1}}},\\ \sqrt[\mathfrak{k}]{\left(1-\left(1-d_{\rho+1}^{\mathfrak{k}}\right)^{\lambda}+\left(1-\left(\nu_{\rho+1}^{\mathfrak{k}}+d_{\rho+1}^{\mathfrak{k}}\right)\right)^{\lambda}\right)^{\omega_{\rho+1}}-\left(1-\left(\nu_{\rho+1}^{\mathfrak{k}}+d_{\rho+1}^{\mathfrak{k}}\right)\right)^{\lambda\omega_{\rho+1}}}\end{pmatrix}\end{pmatrix}^{1/\lambda},$$

$$=\begin{pmatrix}\sqrt[\mathfrak{k}]{\begin{pmatrix}\left(1-\left(\prod_{\tilde{\imath}=1}^{\rho+1}\left(1-\left(1-u_{\tilde{\imath}}^{\mathfrak{k}}\right)^{\lambda}+\left(1-\left(\nu_{\tilde{\imath}}^{\mathfrak{k}}+u_{\tilde{\imath}}^{\mathfrak{k}}\right)\right)^{\lambda}\right)\right)^{\omega_{\tilde{\imath}}}-\prod_{\tilde{\imath}=1}^{\rho+1}\left(1-\left(\nu_{\tilde{\imath}}^{\mathfrak{k}}+d_{\tilde{\imath}}^{\mathfrak{k}}\right)\right)^{\lambda\omega_{\tilde{\imath}}}\right)^{1/\lambda}\\ -\prod_{\tilde{\imath}=1}^{\rho+1}\left(1-\left(\nu_{\tilde{\imath}}^{\mathfrak{k}}+d_{\tilde{\imath}}^{\mathfrak{k}}\right)\right)^{\lambda\omega_{\tilde{\imath}}}\end{pmatrix}},\\ \sqrt[\mathfrak{k}]{\left(1-\left(1-\prod_{\tilde{\imath}=1}^{\rho+1}\left(1-\left(1-u_{\tilde{\imath}}^{\mathfrak{k}}\right)^{\lambda}+\left(1-\left(\nu_{\tilde{\imath}}^{\mathfrak{k}}+u_{\tilde{\imath}}^{\mathfrak{k}}\right)\right)^{\lambda}\right)^{\omega_{\tilde{\imath}}}-\prod_{\tilde{\imath}=1}^{\rho+1}\left(1-\left(\nu_{\tilde{\imath}}^{\mathfrak{k}}+u_{\tilde{\imath}}^{\mathfrak{k}}\right)\right)^{\lambda\omega_{\tilde{\imath}}}\right)\right)^{1/\lambda}},\\ \sqrt[\mathfrak{k}]{\left(1-\left(1-\prod_{\tilde{\imath}=1}^{\rho+1}\left(1-\left(1-d_{\tilde{\imath}}^{\mathfrak{k}}\right)^{\lambda}+\left(1-\left(\nu_{\tilde{\imath}}^{\mathfrak{k}}+d_{\tilde{\imath}}^{\mathfrak{k}}\right)\right)^{\lambda}\right)^{\omega_{\tilde{\imath}}}-\prod_{\tilde{\imath}=1}^{\rho+1}\left(1-\left(\nu_{\tilde{\imath}}^{\mathfrak{k}}+d_{\tilde{\imath}}^{\mathfrak{k}}\right)\right)^{\lambda\omega_{\tilde{\imath}}}\right)\right)^{1/\lambda}}\end{pmatrix}.$$

Hence, the theory in Eq. 17 is also valid for $\mathfrak{q}=\rho+1$.
Furthermore, by setting the value of the weight vector $\left(\frac{1}{\mathfrak{q}},\frac{1}{\mathfrak{q}},\ldots,\frac{1}{\mathfrak{q}}\right)$ in the information in Eq. 17, then we obtain the GTSFIA operator, such as

$$GTSFIA\left(\breve{t}_{1},\breve{t}_{2},\ldots,\breve{t}_{\mathfrak{q}}\right)=\left(\oplus_{\tilde{\imath}=1}^{\mathfrak{q}}\frac{1}{\mathfrak{q}}\breve{t}_{\tilde{\imath}}^{\lambda}\right)^{1/\lambda}=\begin{pmatrix}\sqrt[\mathfrak{k}]{\begin{pmatrix}\left(1-\left(\prod_{\tilde{\imath}=1}^{\mathfrak{q}}\left(1-\left(1-d_{\tilde{\imath}}^{\mathfrak{k}}\right)^{\lambda}+\left(1-\left(\nu_{\tilde{\imath}}^{\mathfrak{k}}+d_{\tilde{\imath}}^{\mathfrak{k}}\right)\right)^{\lambda}\right)^{\frac{1}{\mathfrak{q}}}-\prod_{\tilde{\imath}=1}^{\mathfrak{q}}\left(1-\left(\nu_{\tilde{\imath}}^{\mathfrak{k}}+d_{\tilde{\imath}}^{\mathfrak{k}}\right)\right)^{\frac{\lambda}{\mathfrak{q}}}\right)\right)^{1/\lambda}\\ -\prod_{\tilde{\imath}=1}^{\mathfrak{q}}\left(1-\left(\nu_{\tilde{\imath}}^{\mathfrak{k}}+d_{\tilde{\imath}}^{\mathfrak{k}}\right)\right)^{\frac{1}{\mathfrak{q}}}\end{pmatrix}},\\ \sqrt[\mathfrak{k}]{1-\left(1-\left(\prod_{\tilde{\imath}=1}^{\mathfrak{q}}\left(1-\left(1-u_{\tilde{\imath}}^{\mathfrak{k}}\right)^{\lambda}+\left(1-\left(\nu_{\tilde{\imath}}^{\mathfrak{k}}+u_{\tilde{\imath}}^{\mathfrak{k}}\right)\right)^{\lambda}\right)^{\frac{1}{\mathfrak{q}}}-\prod_{\tilde{\imath}=1}^{\mathfrak{q}}\left(1-\left(\nu_{\tilde{\imath}}^{\mathfrak{k}}+u_{\tilde{\imath}}^{\mathfrak{k}}\right)\right)^{\frac{\lambda}{\mathfrak{q}}}\right)\right)^{1/\lambda}},\\ \sqrt[\mathfrak{k}]{1-\left(1-\left(\prod_{\tilde{\imath}=1}^{\mathfrak{q}}\left(1-\left(1-d_{\tilde{\imath}}^{\mathfrak{k}}\right)^{\lambda}+\left(1-\left(\nu_{\tilde{\imath}}^{\mathfrak{k}}+d_{\tilde{\imath}}^{\mathfrak{k}}\right)\right)^{\lambda}\right)^{\frac{1}{\mathfrak{q}}}-\prod_{\tilde{\imath}=1}^{\mathfrak{q}}\left(1-\left(\nu_{\tilde{\imath}}^{\mathfrak{k}}+d_{\tilde{\imath}}^{\mathfrak{k}}\right)\right)^{\frac{\lambda}{\mathfrak{q}}}\right)\right)^{1/\lambda}}\end{pmatrix}. \quad (18)$$

Property 1: (*Idempotency*) Let $\breve{t}_{\mathfrak{i}} = (\nu_{\mathfrak{i}}, u_{\mathfrak{i}}, d_{\mathfrak{i}})$ and $\breve{t}_{\mathfrak{i}} = \breve{t}$, then idempotency for GTSFIWA is defined as follows:

$$GTSFIWA_\lambda(\breve{t}_1, \breve{t}_2, \ldots, \breve{t}_q) = \breve{t}. \tag{19}$$

Furthermore, by setting the value of $\lambda = 1$ in Eq. 17, then we obtain the TSFIWA operator, such as

$$TSFIWA(\breve{t}_1, \breve{t}_2, \ldots, \breve{t}_q) = \left(\begin{array}{c} \sqrt[t]{1 - \prod_{\mathfrak{i}=1}^{q}\left(1 - \nu_{\mathfrak{i}}^{t}\right)^{\omega_{\mathfrak{i}}}}, \sqrt[t]{\prod_{\mathfrak{i}=1}^{q}\left(1 - \nu_{\mathfrak{i}}^{t}\right)^{\omega_{\mathfrak{i}}} - \prod_{\mathfrak{i}=1}^{q}\left(1 - \left(\nu_{\mathfrak{i}}^{t} + u_{\mathfrak{i}}^{t}\right)\right)^{\omega_{\mathfrak{i}}}}, \\ \sqrt[t]{\prod_{\mathfrak{i}=1}^{q}\left(1 - \nu_{\mathfrak{i}}^{t}\right)^{\omega_{\mathfrak{i}}} - \prod_{\mathfrak{i}=1}^{q}\left(1 - \left(\nu_{\mathfrak{i}}^{t} + d_{\mathfrak{i}}^{t}\right)\right)^{\omega_{\mathfrak{i}}}} \end{array} \right). \tag{20}$$

Finally, for $\lambda = 1$, we use the value of the weight vector $(\frac{1}{q}, \frac{1}{q}, \ldots, \frac{1}{q})$ in Eq. 17, and then we obtain the TSFIA operator, such as

$$TSFIA(\breve{t}_1, \breve{t}_2, \ldots, \breve{t}_q) = \frac{1}{q} \oplus_{j=1}^{q} \breve{t}_j = \left(\begin{array}{c} \sqrt[t]{1 - \prod_{\mathfrak{i}=1}^{q}\left(1 - \nu_{\mathfrak{i}}^{t}\right)^{\frac{1}{q}}}, \sqrt[t]{\prod_{\mathfrak{i}=1}^{q}\left(1 - \nu_{\mathfrak{i}}^{t}\right)^{\frac{1}{q}} - \prod_{\mathfrak{i}=1}^{q}\left(1 - \left(\nu_{\mathfrak{i}}^{t} + u_{\mathfrak{i}}^{t}\right)\right)^{\frac{1}{q}}}, \\ \sqrt[t]{\prod_{\mathfrak{i}=1}^{q}\left(1 - \nu_{\mathfrak{i}}^{t}\right)^{\frac{1}{q}} - \prod_{\mathfrak{i}=1}^{q}\left(1 - \left(\nu_{\mathfrak{i}}^{t} + d_{\mathfrak{i}}^{t}\right)\right)^{\frac{1}{q}}} \end{array} \right). \tag{21}$$

Definition 6: A GTSFIWGA operator is particularized by

$$GTSFIWGA_\lambda(\breve{t}_1, \breve{t}_2, \ldots, \breve{t}_q) = \frac{1}{\lambda}\left(\otimes_{\mathfrak{i}=1}^{q} (\lambda \breve{t}_{\mathfrak{i}})^{\omega_{\mathfrak{i}}}\right) = \frac{1}{\lambda}\left((\lambda \breve{t}_1)^{\omega_1} \otimes (\lambda \breve{t}_2)^{\omega_2} \otimes \ldots \otimes (\lambda \breve{t}_q)^{\omega_q}\right). \tag{22}$$

Theorem 2: Using the information in Eq. 22, we derive the following theory, such as

$$GTSFIWGA_\lambda(\breve{t}_1, \breve{t}_2, \ldots, \breve{t}_q) = \left(\begin{array}{c} \sqrt[t]{1 - \left(1 - \prod_{\mathfrak{i}=1}^{q}\left(1 - \left(1 - \nu_{\mathfrak{i}}^{t}\right)^{\lambda} + \left(1 - \left(\nu_{\mathfrak{i}}^{t} + d_{\mathfrak{i}}^{t}\right)\right)^{\lambda}\right)^{\omega_{\mathfrak{i}}} + \prod_{\mathfrak{i}=1}^{q}\left(1 - \left(\nu_{\mathfrak{i}}^{t} + d_{\mathfrak{i}}^{t}\right)\right)^{\lambda\omega_{\mathfrak{i}}}\right)^{1/\lambda}}, \\ \sqrt[t]{\left(\begin{array}{c} \left(1 - \prod_{\mathfrak{i}=1}^{q}\left(1 - \left(1 - \nu_{\mathfrak{i}}^{t}\right)^{\lambda} + \left(1 - \left(\nu_{\mathfrak{i}}^{t} + u_{\mathfrak{i}}^{t}\right)\right)^{\lambda}\right)^{\omega_{\mathfrak{i}}} + \prod_{\mathfrak{i}=1}^{q}\left(1 - \left(\nu_{\mathfrak{i}}^{t} + u_{\mathfrak{i}}^{t}\right)\right)^{\lambda\omega_{\mathfrak{i}}}\right)^{1/\lambda} \\ -\prod_{\mathfrak{i}=1}^{q}\left(1 - \left(\nu_{\mathfrak{i}}^{t} + u_{\mathfrak{i}}^{t}\right)\right)^{\omega_{\mathfrak{i}}} \end{array} \right)}, \\ \sqrt[t]{\left(\begin{array}{c} \left(1 - \prod_{\mathfrak{i}=1}^{q}\left(1 - \left(1 - \nu_{\mathfrak{i}}^{t}\right)^{\lambda} + \left(1 - \left(\nu_{\mathfrak{i}}^{t} + d_{\mathfrak{i}}^{t}\right)\right)^{\lambda}\right)^{\omega_{\mathfrak{i}}} + \prod_{\mathfrak{i}=1}^{q}\left(1 - \left(\nu_{\mathfrak{i}}^{t} + d_{\mathfrak{i}}^{t}\right)\right)^{\lambda\omega_{\mathfrak{i}}}\right)^{1/\lambda} \\ -\prod_{\mathfrak{i}=1}^{q}\left(1 - \left(\nu_{\mathfrak{i}}^{t} + d_{\mathfrak{i}}^{t}\right)\right)^{\omega_{\mathfrak{i}}} \end{array} \right)} \end{array} \right). \tag{23}$$

Proof: Omitted.

Example 3: Suppose five TSFVs be, $\breve{t}_1 = (0.80, 0.50, 0.30), \breve{t}_2 = (0.40, 0.60, 0.70)$, $\breve{t}_3 = (0.85, 0.45, 0.25), \breve{t}_4 = (0.50, 0.30, 0.00)$, and $\breve{t}_5 = (0.55, 0.55, 0.55)$ and $\omega = (0.25, 0.20, 0.15, 0.10, 0.30)$, by using the suggested operators GTSFIWGA and GTSFIWA and the existing operator TSF Frank weighted averaging (TSFFWA) to assemble these TSFVs for various λ. There are some current approaches to determine the entire values. Table 1 illustrates the results.

Table 1 is the aggregated matrix of the given TSFVs, which shows that if the NMD of any one of the TSFVs is 0, then by using the TSFFWA operator, the aggregated value of NMD becomes 0 through all the various values of λ as one NMD affect all the other values of NMD. While utilizing GTSFIWGA and GTSFIWA operators, the aggregated values of NMD through various λ are different as it does not affect any other value of NMD. Table 2 shows the score value by utilizing Table 1.

TABLE 1 Aggregated matrix of TSFVs.

λ	TSFFWA	GTSFIWGA	GTSFIWA
2	$(0.97029, 0.40294, 0)$	$(0.84321, 0.62488, 0.24674)$	$(0.84321, 0.67802, 0.2384)$
4	$(0.92712, 0.40478, 0)$	$(0.84321, 0.59212, 0.25256)$	$(0.84321, 0.70648, 0.23307)$
6	$(0.89103, 0.40553, 0)$	$(0.84321, 0.5633, 0.25863)$	$(0.84321, 0.72196, 0.2279)$
10	$(0.81677, 0.4063, 0)$	$(0.84321, 0.52194, 0.27174)$	$(0.84321, 0.72487, 0.21797)$
12	$(0.76813, 0.4065, 0)$	$(0.84321, 0.51233, 0.27889)$	$(0.84321, 0.71986, 0.21319)$
16	$(0.6283, 0.40692, 0)$	$(0.84321, 0.51694, 0.29485)$	$(0.84321, 0.70641, 0.20398)$

TABLE 2 Matrix of score value $s(\breve{t})$.

λ	TSFFWA	GTSFIWGA	GTSFIWA
2	0.76224	0.2043	0.19265
4	0.505932	0.20855	0.18302
6	0.353838	0.21104	0.17652
10	0.161517	0.21324	0.1752
12	0.092817	0.21358	0.17746
16	−0.015571	0.21341	0.18305

4 Interaction MSM operators for TSF information

In the section on the development of green manufacturing engineering, we concentrate on utilizing the IMSM theory in the environment of TSF set theory, such as TSFIMSM, TSFIBM, and TSFIWMSM operators. Some dominant and reliable properties are also invented for evaluated work.

Definition 7: (Maclaurin, 1729) Suppose $\breve{t}_{\tilde{\mathrm{l}}}$ $(\tilde{\mathrm{l}} = 1, 2, \ldots, \mathrm{q})$ and $\zeta = 1, 2, \ldots, И$ be the collection of positive real numbers. The MSM operator is stated by

$$MSM^{(\zeta)}(\breve{t}_1, \breve{t}_2, \ldots, \breve{t}_{\mathrm{q}}) = \left(\frac{\sum_{1 \le \tilde{\mathrm{l}}_1 < \tilde{\mathrm{l}}_2 < \ldots < \tilde{\mathrm{l}}_\zeta \le \mathrm{q}} \prod_{j=1}^{\zeta} \breve{t}_{\tilde{\mathrm{l}}_j}}{C_{\mathrm{q}}^{\zeta}} \right)^{\frac{1}{\zeta}}. \tag{24}$$

With various valuable properties, such as

$$MSM^{(\zeta)}(0, 0, \ldots, 0) = 0, \tag{25}$$
$$MSM^{(\zeta)}(1, 1, \ldots, 1) = 1, \tag{26}$$
$$MSM^{(\zeta)}(\breve{t}_1, \breve{t}_2, \ldots, \breve{t}_{\mathrm{q}}) \le MSM^{(\zeta)}(\hat{t}_1, \hat{t}_2, \ldots, \hat{t}_{\mathrm{q}}), \text{if } \breve{t}_{\tilde{\mathrm{l}}} \le \hat{t}_{\tilde{\mathrm{l}}} \forall \tilde{\mathrm{l}}, \tag{27}$$
$$min_{\tilde{\mathrm{l}}}\{\breve{t}_{\tilde{\mathrm{l}}}\} \le MSM^{(\zeta)}(\breve{t}_1, \breve{t}_2, \ldots, \breve{t}_{\mathrm{q}}) \le max_{\tilde{\mathrm{l}}}\{\breve{t}_{\tilde{\mathrm{l}}}\}. \tag{28}$$

Furthermore, we expose some special cases of the evaluated theory, such as
While adding $\zeta = 1$ in Eq. 24, we obtain the AM operator, such as:

$$MSM^{(1)}(\breve{t}_1, \breve{t}_2, \ldots, \breve{t}_{\mathrm{q}}) = \left(\frac{\sum_{1 \le \tilde{\mathrm{l}} \le \mathrm{q}} \breve{t}_{\tilde{\mathrm{l}}}}{C_{\mathrm{q}}^{1}} \right)^{1} = \frac{1}{\mathrm{q}} \sum_{\tilde{\mathrm{l}}=1}^{\mathrm{q}} \breve{t}_{\tilde{\mathrm{l}}}. \tag{29}$$

While adding $\zeta = 2$ in Eq. 24, then we obtain the BM operator for $p = \mathrm{ł} = 1$, such as

$$MSM^{(2)}(\breve{t}_1, \breve{t}_2, \ldots, \breve{t}_{\mathrm{q}}) = \left(\frac{\sum_{1 \le \tilde{\mathrm{l}}_1 < \tilde{\mathrm{l}}_2 \le \mathrm{q}} \prod_{j=1}^{2} \breve{t}_{\tilde{\mathrm{l}}_j}}{C_{\mathrm{q}}^{2}} \right)^{\frac{1}{2}} = \left(\frac{1}{\mathrm{q}(\mathrm{q}-1)} \sum_{\tilde{\mathrm{l}}_1, \tilde{\mathrm{l}}_2 = 1, \tilde{\mathrm{l}}_1 \neq \tilde{\mathrm{l}}_2}^{\mathrm{q}} \breve{t}_{\tilde{\mathrm{l}}_1} \breve{t}_{\tilde{\mathrm{l}}_2} \right)^{\frac{1}{2}}. \tag{30}$$

While adding $\zeta = 2$ in Eq. 24, then we obtain the GBM operator for $p = ꝁ = r = 1$, such as

$$MSM^{(3)}(\breve{t}_1, \breve{t}_2, \ldots, \breve{t}_{\mathfrak{q}}) = \left(\frac{\sum_{1 \le \check{\imath}_1 < \check{\imath}_2 < \check{\imath}_3 \le \mathfrak{q}} \prod_{\jmath=1}^{3} \breve{t}_{\check{\imath}_\jmath}}{C_{\mathfrak{q}}^{3}}\right)^{\frac{1}{3}} = \left(\frac{1}{\mathfrak{q}(\mathfrak{q}-1)(\mathfrak{q}-2)} \sum_{\check{\imath}_1,\check{\imath}_2,\check{\imath}_3=1, \check{\imath}_1 \neq \check{\imath}_2 \neq \check{\imath}_3}^{\mathfrak{q}} \breve{t}_{\check{\imath}_1} \breve{t}_{\check{\imath}_2} \breve{t}_{\check{\imath}_3}\right)^{\frac{1}{3}}. \tag{31}$$

While adding $\zeta = \mathfrak{q}$ in Eq. 24, then we obtain the GM operator, such as

$$MSM^{(\mathfrak{q})}(\breve{t}_1, \breve{t}_2, \ldots, \breve{t}_{\mathfrak{q}}) = \left(\frac{\sum_{1 \le \check{\imath}_1 < \check{\imath}_2 < \ldots < \check{\imath}_{\mathfrak{q}} \le \mathfrak{q}} \prod_{\jmath=1}^{\mathfrak{q}} \breve{t}_{\check{\imath}_\jmath}}{C_{\mathfrak{q}}^{\mathfrak{q}}}\right)^{\frac{1}{\mathfrak{q}}} = \left(\prod_{\check{\imath}=1}^{\mathfrak{q}} \breve{t}_{\check{\imath}}\right)^{\frac{1}{\mathfrak{q}}}. \tag{32}$$

Definition 8: A TSFIMSM operator is stated by

$$TSFIMSM^{(\zeta)}(\breve{t}_1, \breve{t}_2, \ldots, \breve{t}_{\mathfrak{q}}) = \left(\frac{\sum_{1 \le \check{\imath}_1 < \check{\imath}_2 < \ldots < \check{\imath}_\zeta \le \mathfrak{q}} \prod_{\jmath=1}^{\zeta} \breve{t}_{\check{\imath}_\jmath}}{C_{\mathfrak{q}}^{\zeta}}\right)^{\frac{1}{\zeta}}. \tag{33}$$

Furthermore, we derive that the following theory is valid, such as

$$\prod_{\jmath=1}^{\zeta} \breve{t}_{\check{\imath}_\jmath} = \begin{pmatrix} \sqrt[ꝁ]{\prod_{\jmath=1}^{\zeta}\left(1 - d_{\check{\imath}_\jmath}^{ꝁ}\right) - \prod_{\jmath=1}^{\zeta}\left(1 - \left(\nu_{\check{\imath}_\jmath}^{ꝁ} + d_{\check{\imath}_\jmath}^{ꝁ}\right)\right)}, \\ \sqrt[ꝁ]{1 - \prod_{\jmath=1}^{\zeta}\left(1 - u_{\check{\imath}_\jmath}^{ꝁ}\right)}, \sqrt[ꝁ]{1 - \prod_{\jmath=1}^{\zeta}\left(1 - d_{\check{\imath}_\jmath}^{ꝁ}\right)} \end{pmatrix}. \tag{34}$$

Then, we obtain

$$\oplus_{1 \le \check{\imath}_1 < \check{\imath}_2 < \ldots < \check{\imath}_\zeta \le \mathfrak{q}} \otimes_{\jmath=1}^{\zeta} \breve{t}_{\check{\imath}_\jmath} = \begin{pmatrix} \sqrt[ꝁ]{1 - \prod_{1 \le \check{\imath}_1 < \check{\imath}_2 < \ldots < \check{\imath}_\zeta \le \mathfrak{q}} \left(1 - \prod_{\jmath=1}^{\kappa}\left(1 - d_{\check{\imath}_\jmath}^{ꝁ}\right) - \prod_{\jmath=1}^{\kappa}\left(1 - \left(\nu_{\check{\imath}_\jmath}^{ꝁ} + d_{\check{\imath}_\jmath}^{ꝁ}\right)\right)\right)}, \\ \sqrt[ꝁ]{\begin{pmatrix} \left(\prod_{1 \le \check{\imath}_1 < \check{\imath}_2 < \ldots < \check{\imath}_\zeta \le \mathfrak{q}} \left(\prod_{\jmath=1}^{\kappa}\left(1 - u_{\check{\imath}_\jmath}^{ꝁ}\right) - \prod_{\jmath=1}^{\kappa}\left(1 - \left(\nu_{\check{\imath}_\jmath}^{ꝁ} + u_{\check{\imath}_\jmath}^{ꝁ}\right)\right)\right)\right) \\ - \prod_{1 \le \check{\imath}_1 < \check{\imath}_2 < \ldots < \check{\imath}_\zeta \le \mathfrak{q}} \prod_{\jmath=1}^{\kappa}\left(1 - \left(\nu_{\check{\imath}_\jmath}^{ꝁ} + u_{\check{\imath}_\jmath}^{ꝁ}\right)\right) \end{pmatrix}}, \\ \sqrt[ꝁ]{\begin{pmatrix} \left(\prod_{1 \le \check{\imath}_1 < \check{\imath}_2 < \ldots < \check{\imath}_\zeta \le \mathfrak{q}} \left(\prod_{\jmath=1}^{\kappa}\left(1 - d_{\check{\imath}_\jmath}^{ꝁ}\right) - \prod_{\jmath=1}^{\kappa}\left(1 - \left(\nu_{\check{\imath}_\jmath}^{ꝁ} + d_{\check{\imath}_\jmath}^{ꝁ}\right)\right)\right)\right) \\ - \prod_{1 \le \check{\imath}_1 < \check{\imath}_2 < \ldots < \check{\imath}_\zeta \le \mathfrak{q}} \prod_{\jmath=1}^{\kappa}\left(1 - \left(\nu_{\check{\imath}_\jmath}^{ꝁ} + d_{\check{\imath}_\jmath}^{ꝁ}\right)\right) \end{pmatrix}} \end{pmatrix}. \tag{35}$$

According to the presence of the theory of the TSFIWA operator, we have

$$\otimes_{\jmath=1}^{\zeta} \breve{t}_{\check{\imath}_\jmath} = \begin{pmatrix} \sqrt[ꝁ]{\prod_{\jmath=1}^{\kappa}\left(1 - d_{\check{\imath}_\jmath}^{ꝁ}\right) - \prod_{\jmath=1}^{\kappa}\left(1 - \left(\nu_{\check{\imath}_\jmath}^{ꝁ} + d_{\check{\imath}_\jmath}^{ꝁ}\right)\right)}, \\ \sqrt[ꝁ]{1 - \prod_{\jmath=1}^{\kappa}\left(1 - u_{\check{\imath}_\jmath}^{ꝁ}\right)}, \sqrt[ꝁ]{1 - \prod_{\jmath=1}^{\kappa}\left(1 - d_{\check{\imath}_\jmath}^{ꝁ}\right)} \end{pmatrix}.$$

For $\zeta = 2$, we have

$$\oplus_{1\le\tilde{\imath}_1<\tilde{\imath}_2\le q}\otimes_{\jmath=1}^{\zeta}\breve{t}_{\tilde{\imath}_\jmath}=\oplus_{1\le\tilde{\imath}_1<\tilde{\imath}_2\le q}\left(\sqrt[q]{\prod_{\jmath=1}^{2}\left(1-d_{\tilde{\imath}_\jmath}^{ŀ}\right)-\prod_{\jmath=1}^{2}\left(1-\left(\nu_{\tilde{\imath}_\jmath}^{ŀ}+u_{\tilde{\imath}_\jmath}^{ŀ}\right)\right)},\sqrt[ŀ]{1-\prod_{\jmath=1}^{2}\left(1-u_{\tilde{\imath}_\jmath}^{ŀ}\right)},\sqrt[ŀ]{1-\prod_{\jmath=1}^{2}\left(1-d_{\tilde{\imath}_\jmath}^{ŀ}\right)}\right),$$

$$=\begin{pmatrix}\sqrt[ŀ]{1-\prod_{1\le\tilde{\imath}_1<\tilde{\imath}_2\le q}\left(1-\prod_{\jmath=1}^{2}\left(1-d_{\tilde{\imath}_\jmath}^{ŀ}\right)-\prod_{\jmath=1}^{2}\left(1-\left(\nu_{\tilde{\imath}_\jmath}^{ŀ}+d_{\tilde{\imath}_\jmath}^{ŀ}\right)\right)\right)},\\ \sqrt[ŀ]{\prod_{1\le\tilde{\imath}_1<\tilde{\imath}_2\le q}\left(1-\prod_{\jmath=1}^{2}\left(1-u_{\tilde{\imath}_\jmath}^{ŀ}\right)-\prod_{\jmath=1}^{2}\left(1-\left(\nu_{\tilde{\imath}_\jmath}^{ŀ}+u_{\tilde{\imath}_\jmath}^{ŀ}\right)\right)\right)-\prod_{1\le\tilde{\imath}_1<\tilde{\imath}_2\le q}\prod_{\jmath=1}^{2}\left(1-\left(\nu_{\tilde{\imath}_\jmath}^{ŀ}+u_{\tilde{\imath}_\jmath}^{ŀ}\right)\right)},\\ \sqrt[ŀ]{\prod_{1\le\tilde{\imath}_1<\tilde{\imath}_2\le q}\left(1-\prod_{\jmath=1}^{2}\left(1-d_{\tilde{\imath}_\jmath}^{ŀ}\right)-\prod_{\jmath=1}^{2}\left(1-\left(\nu_{\tilde{\imath}_\jmath}^{ŀ}+d_{\tilde{\imath}_\jmath}^{ŀ}\right)\right)\right)-\prod_{1\le\tilde{\imath}_1<\tilde{\imath}_2\le q}\prod_{\jmath=1}^{2}\left(1-\left(\nu_{\tilde{\imath}_\jmath}^{ŀ}+d_{\tilde{\imath}_\jmath}^{ŀ}\right)\right)}\end{pmatrix}.$$

For $\zeta=2$, we obtain the correct result. Furthermore, for we assume that the theory in Eq. 35 also holds for $\zeta=\kappa$, we have

$$\oplus_{1\le\tilde{\imath}_1<\tilde{\imath}_2<\ldots<\tilde{\imath}_\kappa\le q}\otimes_{\jmath=1}^{\kappa}\breve{t}_{\tilde{\imath}_\jmath}=\begin{pmatrix}\sqrt[ŀ]{1-\prod_{1\le\tilde{\imath}_1<\tilde{\imath}_2<\ldots<\tilde{\imath}_\kappa\le q}\left(1-\prod_{\jmath=1}^{\kappa}\left(1-d_{\tilde{\imath}_\jmath}^{ŀ}\right)-\prod_{\jmath=1}^{\kappa}\left(1-\left(\nu_{\tilde{\imath}_\jmath}^{ŀ}+d_{\tilde{\imath}_\jmath}^{ŀ}\right)\right)\right)},\\ \sqrt[ŀ]{\begin{pmatrix}\left(\prod_{1\le\tilde{\imath}_1<\tilde{\imath}_2<\ldots<\tilde{\imath}_\kappa\le q}\left(1-\prod_{\jmath=1}^{\kappa}\left(1-u_{\tilde{\imath}_\jmath}^{ŀ}\right)-\prod_{\jmath=1}^{\kappa}\left(1-\left(\nu_{\tilde{\imath}_\jmath}^{ŀ}+u_{\tilde{\imath}_\jmath}^{ŀ}\right)\right)\right)\right)\\ -\prod_{1\le\tilde{\imath}_1<\tilde{\imath}_2<\ldots<\tilde{\imath}_\kappa\le q}\prod_{\jmath=1}^{\kappa}\left(1-\left(\nu_{\tilde{\imath}_\jmath}^{ŀ}+u_{\tilde{\imath}_\jmath}^{ŀ}\right)\right)\end{pmatrix}},\\ \sqrt[ŀ]{\begin{pmatrix}\left(\prod_{1\le\tilde{\imath}_1<\tilde{\imath}_2<\ldots<\tilde{\imath}_\kappa\le q}\left(1-\prod_{\jmath=1}^{\kappa}\left(1-d_{\tilde{\imath}_\jmath}^{ŀ}\right)-\prod_{\jmath=1}^{\kappa}\left(1-\left(\nu_{\tilde{\imath}_\jmath}^{ŀ}+d_{\tilde{\imath}_\jmath}^{ŀ}\right)\right)\right)\right)\\ -\prod_{1\le\tilde{\imath}_1<\tilde{\imath}_2<\ldots<\tilde{\imath}_\kappa\le q}\prod_{\jmath=1}^{\kappa}\left(1-\left(\nu_{\tilde{\imath}_\jmath}^{ŀ}+d_{\tilde{\imath}_\jmath}^{ŀ}\right)\right)\end{pmatrix}}\end{pmatrix}$$

$$=\nu^{ŀ}_{\oplus_{1\le\tilde{\imath}_1<\tilde{\imath}_2<\ldots<\tilde{\imath}_\kappa\le q}\otimes_{\jmath=1}^{\kappa}\breve{t}_{\tilde{\imath}_\jmath}}+u^{ŀ}_{\oplus_{1\le\tilde{\imath}_1<\tilde{\imath}_2<\ldots<\tilde{\imath}_\kappa\le q}\otimes_{\jmath=1}^{\kappa}\breve{t}_{\tilde{\imath}_\jmath}}+d^{ŀ}_{\oplus_{1\le\tilde{\imath}_1<\tilde{\imath}_2<\ldots<\tilde{\imath}_\kappa\le q}\otimes_{\jmath=1}^{\kappa}\breve{t}_{\tilde{\imath}_\jmath}},$$

$$=\begin{pmatrix}1-\prod_{1\le\tilde{\imath}_1<\tilde{\imath}_2<\ldots<\tilde{\imath}_\kappa\le q}\left(1-\prod_{\jmath=1}^{\kappa}\left(1-u_{\tilde{\imath}_\jmath}^{ŀ}\right)-\prod_{\jmath=1}^{\kappa}\left(1-\left(\nu_{\tilde{\imath}_\jmath}^{ŀ}+u_{\tilde{\imath}_\jmath}^{ŀ}\right)\right)\right)\\ -\prod_{1\le\tilde{\imath}_1<\tilde{\imath}_2<\ldots<\tilde{\imath}_\kappa\le q}\prod_{\jmath=1}^{\kappa}\left(1-\left(\nu_{\tilde{\imath}_\jmath}^{ŀ}+u_{\tilde{\imath}_\jmath}^{ŀ}\right)\right)\\ -\prod_{1\le\tilde{\imath}_1<\tilde{\imath}_2<\ldots<\tilde{\imath}_\kappa\le q}\prod_{\jmath=1}^{\kappa}\left(1-\left(\nu_{\tilde{\imath}_\jmath}^{ŀ}+d_{\tilde{\imath}_\jmath}^{ŀ}\right)\right)\end{pmatrix},$$

$$=\left(1+\prod_{1\le\tilde{\imath}_1<\tilde{\imath}_2<\ldots<\tilde{\imath}_\kappa\le q}\prod_{\jmath=1}^{\kappa}\left(u_{\tilde{\imath}_\jmath}^{ŀ}+d_{\tilde{\imath}_\jmath}^{ŀ}\right)-\prod_{1\le\tilde{\imath}_1<\tilde{\imath}_2<\ldots<\tilde{\imath}_\kappa\le q}\left(1-\prod_{\jmath=1}^{\kappa}\left(1-u_{\tilde{\imath}_\jmath}^{ŀ}\right)-\prod_{\jmath=1}^{\kappa}\left(1-\left(\nu_{\tilde{\imath}_\jmath}^{ŀ}+u_{\tilde{\imath}_\jmath}^{ŀ}\right)\right)\right)\right).$$

Then, we prove that the information in Eq. 35 is also valid for $\zeta=\kappa+1$; we have

$$\oplus_{1\le\tilde{\imath}_1<\tilde{\imath}_2<\ldots<\tilde{\imath}_{\kappa+1}\le q}\otimes_{\jmath=1}^{\kappa+1}\breve{t}_{\tilde{\imath}_\jmath}=\oplus_{1\le\tilde{\imath}_1<\tilde{\imath}_2<\ldots<\tilde{\imath}_{\kappa+1}\le q}\left(\otimes_{\jmath=1}^{\kappa}\breve{t}_{\tilde{\imath}_\jmath}\otimes\breve{t}_{\tilde{\imath}_{\kappa+1}}\right),$$

$$=\oplus_{1\le\tilde{\imath}_1<\tilde{\imath}_2<\ldots<\tilde{\imath}_{\kappa+1}\le q}\left(\sqrt[q]{\prod_{\jmath=1}^{\kappa}\left(1-d_{\tilde{\imath}_\jmath}^{ŀ}\right)-\prod_{\jmath=1}^{\kappa}\left(1-\left(\nu_{\tilde{\imath}_\jmath}^{ŀ}+d_{\tilde{\imath}_\jmath}^{ŀ}\right)\right)},\sqrt[q]{1-\prod_{\jmath=1}^{\kappa}\left(1-u_{\tilde{\imath}_\jmath}^{ŀ}\right)},\sqrt[q]{1-\prod_{\jmath=1}^{\kappa}\left(1-d_{\tilde{\imath}_\jmath}^{ŀ}\right)}\right)\otimes\breve{t}_{\tilde{\imath}_{\kappa+1}},$$

$$=\oplus_{1\le\tilde{\imath}_1<\tilde{\imath}_2<\ldots<\tilde{\imath}_{\kappa+1}\le q}\left(\sqrt[q]{\prod_{\jmath=1}^{\kappa}\left(1-d_{\tilde{\imath}_\jmath}^{ŀ}\right)\left(1-d_{\tilde{\imath}_{\kappa+1}}^{ŀ}\right)-\prod_{\jmath=1}^{\kappa}\left(1-\left(\nu_{\tilde{\imath}_\jmath}^{ŀ}+d_{\tilde{\imath}_\jmath}^{ŀ}\right)\right)\left(1-\left(\nu_{\tilde{\imath}_{\kappa+1}}^{ŀ}+d_{\tilde{\imath}_{\kappa+1}}^{ŀ}\right)\right)},\sqrt[ŀ]{1-\prod_{\jmath=1}^{\kappa}\left(1-u_{\tilde{\imath}_\jmath}^{ŀ}\right)\left(1-u_{\tilde{\imath}_{\kappa+1}}^{ŀ}\right)},\sqrt[ŀ]{1-\prod_{\jmath=1}^{\kappa}\left(1-d_{\tilde{\imath}_\jmath}^{ŀ}\right)\left(1-d_{\tilde{\imath}_{\kappa+1}}^{ŀ}\right)}\right),$$

$$=\oplus_{1\le\tilde{\imath}_1<\tilde{\imath}_2<\ldots<\tilde{\imath}_{\kappa+1}\le q}\begin{pmatrix}\sqrt[ŀ]{\prod_{\jmath=1}^{\kappa+1}\left(1-d_{\tilde{\imath}_\jmath}^{ŀ}\right)-\prod_{\jmath=1}^{\kappa+1}\left(1-\left(\nu_{\tilde{\imath}_\jmath}^{ŀ}+d_{\tilde{\imath}_\jmath}^{ŀ}\right)\right)},\\ \sqrt[q]{1-\prod_{\jmath=1}^{\kappa+1}\left(1-u_{\tilde{\imath}_\jmath}^{ŀ}\right)},\sqrt[q]{1-\prod_{\jmath=1}^{\kappa+1}\left(1-d_{\tilde{\imath}_\jmath}^{ŀ}\right)}\end{pmatrix},$$

$$= \left(\sqrt[q]{1 - \prod_{1 \le \tilde{\imath}_1 < \tilde{\imath}_2 < \ldots < \tilde{\imath}_{\kappa+1} \le \mathfrak{q}} \left(1 - \left(\prod_{j=1}^{\kappa+1} \left(1 - d_{\tilde{\imath}_j}^{\natural} \right) - \prod_{j=1}^{\kappa+1} \left(1 - \left(\nu_{\tilde{\imath}_j}^{\natural} + d_{\tilde{\imath}_j}^{\natural} \right) \right) \right) \right)}, \sqrt[\natural]{\left(\begin{array}{c} \left(\prod_{1 \le \tilde{\imath}_1 < \tilde{\imath}_2 < \ldots < \tilde{\imath}_{\kappa+1} \le \mathfrak{q}} \left(1 - \left(\prod_{j=1}^{\kappa+1} \left(1 - u_{\tilde{\imath}_j}^{\natural} \right) - \prod_{j=1}^{\kappa+1} \left(1 - \left(\nu_{\tilde{\imath}_j}^{\natural} + u_{\tilde{\imath}_j}^{\natural} \right) \right) \right) \right) \right) \\ - \prod_{1 \le \tilde{\imath}_1 < \tilde{\imath}_2 < \ldots < \tilde{\imath}_{\kappa+1} \le \mathfrak{q}} \prod_{j=1}^{\kappa+1} \left(1 - \left(\nu_{\tilde{\imath}_j}^{\natural} + u_{\tilde{\imath}_j}^{\natural} \right) \right) \end{array} \right)}, \right.$$

$$\left. \sqrt[\natural]{\left(\begin{array}{c} \left(\prod_{1 \le \tilde{\imath}_1 < \tilde{\imath}_2 < \ldots < \tilde{\imath}_{\kappa+1} \le \mathfrak{q}} \left(1 - \left(\prod_{j=1}^{\kappa+1} \left(1 - d_{\tilde{\imath}_j}^{\natural} \right) - \prod_{j=1}^{\kappa+1} \left(1 - \left(\nu_{\tilde{\imath}_j}^{\natural} + d_{\tilde{\imath}_j}^{\natural} \right) \right) \right) \right) \right) \\ - \prod_{1 \le \tilde{\imath}_1 < \tilde{\imath}_2 < \ldots < \tilde{\imath}_{\kappa+1} \le \mathfrak{q}} \prod_{j=1}^{\kappa+1} \left(1 - \left(\nu_{\tilde{\imath}_j}^{\natural} + d_{\tilde{\imath}_j}^{\natural} \right) \right) \end{array} \right)} \right).$$

Hence, we obtain our required result.

Theorem 3: Here, we derive the theory of the TSFIMSM operator in the presence of TSF information, such as

$$TSFIMSM^{(\zeta)}\left(\breve{t}_1, \breve{t}_2, \ldots, \breve{t}_{\mathfrak{q}}\right) = \left(\frac{\sum_{1 \le \tilde{\imath}_1 < \tilde{\imath}_2 < \ldots \le \tilde{\imath}_\zeta \le \mathfrak{q}} \prod_{j=1}^{\zeta} \breve{t}_{\tilde{\imath}_j}}{C_{\mathfrak{q}}^{\zeta}} \right)^{\frac{1}{\zeta}},$$

$$= \left(\begin{array}{c} \sqrt[\natural]{\left(1 - \left(\prod_{1 \le \tilde{\imath}_1 < \tilde{\imath}_2 < \ldots < \tilde{\imath}_\zeta \le \mathfrak{q}} \left(\begin{array}{c} \left(1 - \left(\prod_{j=1}^{\zeta} \left(1 - d_{\tilde{\imath}_j}^{\natural} \right) - \prod_{j=1}^{\zeta} \left(1 - \left(\nu_{\tilde{\imath}_j}^{\natural} + d_{\tilde{\imath}_j}^{\natural} \right) \right) \right) \right)^{\frac{1}{C_{\mathfrak{q}}^{\zeta}}} - \\ \left(\prod_{1 \le \tilde{\imath}_1 < \tilde{\imath}_2 < \ldots < \tilde{\imath}_\zeta \le \mathfrak{q}} \prod_{j=1}^{\zeta} \left(1 - \left(\nu_{\tilde{\imath}_j}^{\natural} + d_{\tilde{\imath}_j}^{\natural} \right) \right) \right)^{\frac{1}{C_{\mathfrak{q}}^{\zeta}}} \\ - \left(\left(\prod_{1 \le \tilde{\imath}_1 < \tilde{\imath}_2 < \ldots < \tilde{\imath}_\zeta \le \mathfrak{q}} \prod_{j=1}^{\zeta} \left(1 - \left(\nu_{\tilde{\imath}_j}^{\natural} + d_{\tilde{\imath}_j}^{\natural} \right) \right) \right)^{\frac{1}{C_{\mathfrak{q}}^{\zeta}}} \right) \end{array} \right)^{\frac{1}{\zeta}} \right)^{\frac{1}{\zeta}} \right)}, \\ \sqrt[\natural]{\left(1 - \left(1 - \left(\prod_{1 \le \tilde{\imath}_1 < \tilde{\imath}_2 < \ldots < \tilde{\imath}_\zeta \le \mathfrak{q}} \left(\begin{array}{c} \left(1 - \left(\prod_{j=1}^{\zeta} \left(1 - u_{\tilde{\imath}_j}^{\natural} \right) - \prod_{j=1}^{\zeta} \left(1 - \left(\nu_{\tilde{\imath}_j}^{\natural} + u_{\tilde{\imath}_j}^{\natural} \right) \right) \right) \right)^{\frac{1}{C_{\mathfrak{q}}^{\zeta}}} - \\ \prod_{1 \le \tilde{\imath}_1 < \tilde{\imath}_2 < \ldots < \tilde{\imath}_\zeta \le \mathfrak{q}} \prod_{j=1}^{\zeta} \left(1 - \left(\nu_{\tilde{\imath}_j}^{\natural} + u_{\tilde{\imath}_j}^{\natural} \right) \right) \end{array} \right) \right)^{\frac{1}{C_{\mathfrak{q}}^{\zeta}}} \right)^{\frac{1}{\zeta}} \right)}, \\ \sqrt[\natural]{\left(1 - \left(1 - \left(\prod_{1 \le \tilde{\imath}_1 < \tilde{\imath}_2 < \ldots < \tilde{\imath}_\zeta \le \mathfrak{q}} \left(\begin{array}{c} \left(1 - \left(\prod_{j=1}^{\zeta} \left(1 - d_{\tilde{\imath}_j}^{\natural} \right) - \prod_{j=1}^{\zeta} \left(1 - \left(\nu_{\tilde{\imath}_j}^{\natural} + d_{\tilde{\imath}_j}^{\natural} \right) \right) \right) \right)^{\frac{1}{C_{\mathfrak{q}}^{\zeta}}} - \\ \prod_{1 \le \tilde{\imath}_1 < \tilde{\imath}_2 < \ldots < \tilde{\imath}_\zeta \le \mathfrak{q}} \prod_{j=1}^{\zeta} \left(1 - \left(\nu_{\tilde{\imath}_j}^{\natural} + d_{\tilde{\imath}_j}^{\natural} \right) \right) \end{array} \right) \right)^{\frac{1}{C_{\mathfrak{q}}^{\zeta}}} \right)^{\frac{1}{\zeta}} \right)} \end{array} \right). \quad (36)$$

Proof: Consider

$$\oplus_{1\le\tilde{\imath}_1<\tilde{\imath}_2<\ldots\le\tilde{\imath}_\zeta\le\mathfrak{q}}\otimes_{\jmath=1}^{\zeta}\breve{t}_{\tilde{\imath}_\jmath}=\left(\begin{array}{c}\sqrt[\text{ƚ}]{1-\prod_{1\le\tilde{\imath}_1<\tilde{\imath}_2<\ldots<\tilde{\imath}_\zeta\le\mathfrak{q}}\left(1-\left(\prod_{\jmath=1}^{\zeta}\left(1-d_{\tilde{\imath}_\jmath}^{\text{ƚ}}\right)-\prod_{\jmath=1}^{\zeta}\left(1-\left(\nu_{\tilde{\imath}_\jmath}^{\text{ƚ}}+d_{\tilde{\imath}_\jmath}^{\text{ƚ}}\right)\right)\right)\right)},\\ \sqrt[\text{ƚ}]{\left(\begin{array}{c}\prod_{1\le\tilde{\imath}_1<\tilde{\imath}_2<\ldots<\tilde{\imath}_\zeta\le\mathfrak{q}}\left(1-\left(\prod_{\jmath=1}^{\zeta}\left(1-u_{\tilde{\imath}_\jmath}^{\text{ƚ}}\right)-\prod_{\jmath=1}^{\zeta}\left(1-\left(\nu_{\tilde{\imath}_\jmath}^{\text{ƚ}}+u_{\tilde{\imath}_\jmath}^{\text{ƚ}}\right)\right)\right)\right)\\ -\prod_{1\le\tilde{\imath}_1<\tilde{\imath}_2<\ldots<\tilde{\imath}_\zeta\le\mathfrak{q}}\prod_{\jmath=1}^{\zeta}\left(1-\left(\nu_{\tilde{\imath}_\jmath}^{\text{ƚ}}+u_{\tilde{\imath}_\jmath}^{\text{ƚ}}\right)\right)\end{array}\right)},\\ \sqrt[\text{ƚ}]{\left(\begin{array}{c}\prod_{1\le\tilde{\imath}_1<\tilde{\imath}_2<\ldots<\tilde{\imath}_\zeta\le\mathfrak{q}}\left(1-\left(\prod_{\jmath=1}^{\zeta}\left(1-d_{\tilde{\imath}_\jmath}^{\text{ƚ}}\right)-\prod_{\jmath=1}^{\zeta}\left(1-\left(\nu_{\tilde{\imath}_\jmath}^{\text{ƚ}}+d_{\tilde{\imath}_\jmath}^{\text{ƚ}}\right)\right)\right)\right)\\ -\prod_{1\le\tilde{\imath}_1<\tilde{\imath}_2<\ldots<\tilde{\imath}_\zeta\le\mathfrak{q}}\prod_{\jmath=1}^{\zeta}\left(1-\left(\nu_{\tilde{\imath}_\jmath}^{\text{ƚ}}+d_{\tilde{\imath}_\jmath}^{\text{ƚ}}\right)\right)\end{array}\right)}\end{array}\right).$$

Then, we have

$$\nu^{\text{ƚ}}_{\oplus_{1\le\tilde{\imath}_1<\tilde{\imath}_2<\ldots\le\tilde{\imath}_\zeta\le\mathfrak{q}}\otimes_{\jmath=1}^{\zeta}\breve{t}_{\tilde{\imath}_\jmath}}+u^{\text{ƚ}}_{\oplus_{1\le\tilde{\imath}_1<\tilde{\imath}_2<\ldots\le\tilde{\imath}_\zeta\le\mathfrak{q}}\otimes_{\jmath=1}^{\zeta}\breve{t}_{\tilde{\imath}_\jmath}}+d^{\text{ƚ}}_{\oplus_{1\le\tilde{\imath}_1<\tilde{\imath}_2<\ldots\le\tilde{\imath}_\zeta\le\mathfrak{q}}\otimes_{\jmath=1}^{\zeta}\breve{t}_{\tilde{\imath}_\jmath}}=\left(\begin{array}{c}1+\prod_{1\le\tilde{\imath}_1<\tilde{\imath}_2<\ldots<\tilde{\imath}_\zeta\le\mathfrak{q}}\left(1-\left(\prod_{\jmath=1}^{\zeta}\left(1-u_{\tilde{\imath}_\jmath}^{\text{ƚ}}\right)-\prod_{\jmath=1}^{\zeta}\left(1-\left(\nu_{\tilde{\imath}_\jmath}^{\text{ƚ}}+u_{\tilde{\imath}_\jmath}^{\text{ƚ}}\right)\right)\right)\right)\\ -\prod_{1\le\tilde{\imath}_1<\tilde{\imath}_2<\ldots<\tilde{\imath}_\zeta\le\mathfrak{q}}\prod_{\jmath=1}^{\zeta}\left(\left(u_{\tilde{\imath}_\jmath}^{\text{ƚ}}+d_{\tilde{\imath}_\jmath}^{\text{ƚ}}\right)\right)\end{array}\right),$$

$$\frac{1}{C_{\mathfrak{q}}^{\zeta}}\left(\oplus_{1\le\tilde{\imath}_1<\tilde{\imath}_2<\ldots\le\tilde{\imath}_\zeta\le\mathfrak{q}}\otimes_{\jmath=1}^{\zeta}\breve{t}_{\tilde{\imath}_\jmath}\right)=\left(\begin{array}{c}\sqrt[\text{ƚ}]{1-\prod_{1\le\tilde{\imath}_1<\tilde{\imath}_2<\ldots<\tilde{\imath}_\zeta\le\mathfrak{q}}\left(1-\left(\prod_{\jmath=1}^{\zeta}\left(1-d_{\tilde{\imath}_\jmath}^{\text{ƚ}}\right)-\prod_{\jmath=1}^{\zeta}\left(1-\left(\nu_{\tilde{\imath}_\jmath}^{\text{ƚ}}+d_{\tilde{\imath}_\jmath}^{\text{ƚ}}\right)\right)\right)\right)^{\frac{1}{C_{\mathfrak{q}}^{\zeta}}}},\\ \sqrt[\text{ƚ}]{\left(\begin{array}{c}\left(\prod_{1\le\tilde{\imath}_1<\tilde{\imath}_2<\ldots<\tilde{\imath}_\zeta\le\mathfrak{q}}\left(1-\left(\prod_{\jmath=1}^{\zeta}\left(1-u_{\tilde{\imath}_\jmath}^{\text{ƚ}}\right)-\prod_{\jmath=1}^{\zeta}\left(1-\left(\nu_{\tilde{\imath}_\jmath}^{\text{ƚ}}+u_{\tilde{\imath}_\jmath}^{\text{ƚ}}\right)\right)\right)\right)^{\frac{1}{C_{\mathfrak{q}}^{\zeta}}}\right)\\ -\left(\prod_{1\le\tilde{\imath}_1<\tilde{\imath}_2<\ldots<\tilde{\imath}_\zeta\le\mathfrak{q}}\prod_{\jmath=1}^{\zeta}\left(1-\left(\nu_{\tilde{\imath}_\jmath}^{\text{ƚ}}+u_{\tilde{\imath}_\jmath}^{\text{ƚ}}\right)\right)\right)^{\frac{1}{C_{\mathfrak{q}}^{\zeta}}}\end{array}\right)},\\ \sqrt[\text{ƚ}]{\left(\begin{array}{c}\left(\prod_{1\le\tilde{\imath}_1<\tilde{\imath}_2<\ldots<\tilde{\imath}_\zeta\le\mathfrak{q}}\left(1-\left(\prod_{\jmath=1}^{\zeta}\left(1-d_{\tilde{\imath}_\jmath}^{\text{ƚ}}\right)-\prod_{\jmath=1}^{\zeta}\left(1-\left(\nu_{\tilde{\imath}_\jmath}^{\text{ƚ}}+d_{\tilde{\imath}_\jmath}^{\text{ƚ}}\right)\right)\right)\right)^{\frac{1}{C_{\mathfrak{q}}^{\zeta}}}\right)\\ -\left(\prod_{1\le\tilde{\imath}_1<\tilde{\imath}_2<\ldots<\tilde{\imath}_\zeta\le\mathfrak{q}}\prod_{\jmath=1}^{\zeta}\left(1-\left(\nu_{\tilde{\imath}_\jmath}^{\text{ƚ}}+d_{\tilde{\imath}_\jmath}^{\text{ƚ}}\right)\right)\right)^{\frac{1}{C_{\mathfrak{q}}^{\zeta}}}\end{array}\right)}\end{array}\right).$$

Therefore,

$$\begin{aligned}&=\nu^{\text{ƚ}}_{\frac{1}{C_{\mathfrak{q}}^{\zeta}}\left(\oplus_{1\le\tilde{\imath}_1<\tilde{\imath}_2<\ldots\le\tilde{\imath}_\zeta\le\mathfrak{q}}\otimes_{\jmath=1}^{\zeta}\breve{t}_{\tilde{\imath}_\jmath}\right)}+u^{\text{ƚ}}_{\frac{1}{C_{\mathfrak{q}}^{\zeta}}\left(\oplus_{1\le\tilde{\imath}_1<\tilde{\imath}_2<\ldots\le\tilde{\imath}_\zeta\le\mathfrak{q}}\otimes_{\jmath=1}^{\zeta}\breve{t}_{\tilde{\imath}_\jmath}\right)}+d^{\text{ƚ}}_{\frac{1}{C_{\mathfrak{q}}^{\zeta}}\left(\oplus_{1\le\tilde{\imath}_1<\tilde{\imath}_2<\ldots\le\tilde{\imath}_\zeta\le\mathfrak{q}}\otimes_{\jmath=1}^{\zeta}\breve{t}_{\tilde{\imath}_\jmath}\right)},\\ &=\left(\begin{array}{c}1+\prod_{1\le\tilde{\imath}_1<\tilde{\imath}_2<\ldots<\tilde{\imath}_\zeta\le\mathfrak{q}}\left(1-\left(\prod_{\jmath=1}^{\zeta}\left(1-u_{\tilde{\imath}_\jmath}^{\text{ƚ}}\right)-\prod_{\jmath=1}^{\zeta}\left(1-\left(\nu_{\tilde{\imath}_\jmath}^{\text{ƚ}}+u_{\tilde{\imath}_\jmath}^{\text{ƚ}}\right)\right)\right)\right)^{\frac{1}{C_{\mathfrak{q}}^{\zeta}}}\\ -\left(\prod_{1\le\tilde{\imath}_1<\tilde{\imath}_2<\ldots<\tilde{\imath}_\zeta\le\mathfrak{q}}\prod_{\jmath=1}^{\zeta}\left(u_{\tilde{\imath}_\jmath}^{\text{ƚ}}+d_{\tilde{\imath}_\jmath}^{\text{ƚ}}\right)\right)^{\frac{1}{C_{\mathfrak{q}}^{\zeta}}}\end{array}\right),\end{aligned}$$

$$\left(\frac{\sum_{1\le\breve{\imath}_1<\breve{\imath}_2<\ldots\le\breve{\imath}_\zeta\le q}\prod_{\jmath=1}^{\zeta}\breve{t}_{\breve{\imath}_\jmath}}{C_q^\zeta}\right)^{\frac{1}{\zeta}} = \left(\begin{array}{c} \sqrt[ł]{\left(1-\left(\prod_{1\le\breve{\imath}_1<\breve{\imath}_2<\ldots<\breve{\imath}_\zeta\le q}\left(\begin{array}{c}\left(1-\left(\prod_{\jmath=1}^{\zeta}\left(1-d^{ł}_{\breve{\imath}_\jmath}\right)-\prod_{\jmath=1}^{\zeta}\left(1-\left(\nu^{ł}_{\breve{\imath}_\jmath}+d^{ł}_{\breve{\imath}_\jmath}\right)\right)\right)\right)^{\frac{1}{C_q^\zeta}}- \\ \left(\prod_{1\le\breve{\imath}_1<\breve{\imath}_2<\ldots<\breve{\imath}_\zeta\le q}\prod_{\jmath=1}^{\zeta}\left(1-\left(\nu^{ł}_{\breve{\imath}_\jmath}+d^{ł}_{\breve{\imath}_\jmath}\right)\right)\right)^{\frac{1}{C_q^\zeta}}\end{array}\right)^{\frac{1}{\zeta}} \\ -\left(\left(\prod_{1\le\breve{\imath}_1<\breve{\imath}_2<\ldots<\breve{\imath}_\zeta\le q}\prod_{\jmath=1}^{\zeta}\left(1-\left(\nu^{ł}_{\breve{\imath}_\jmath}+d^{ł}_{\breve{\imath}_\jmath}\right)\right)\right)^{\frac{1}{C_q^\zeta}}\right)\right)^{\frac{1}{\zeta}}\right)}, \\ \sqrt[ł]{\left(1-\left(1-\left(\prod_{1\le\breve{\imath}_1<\breve{\imath}_2<\ldots<\breve{\imath}_\zeta\le q}\left(\begin{array}{c}\left(1-\left(\prod_{\jmath=1}^{\zeta}\left(1-u^{ł}_{\breve{\imath}_\jmath}\right)-\prod_{\jmath=1}^{\zeta}\left(1-\left(\nu^{ł}_{\breve{\imath}_\jmath}+u^{ł}_{\breve{\imath}_\jmath}\right)\right)\right)\right)^{\frac{1}{C_q^\zeta}}- \\ \prod_{1\le\breve{\imath}_1<\breve{\imath}_2<\ldots<\breve{\imath}_\zeta\le q}\prod_{\jmath=1}^{\zeta}\left(1-\left(\nu^{ł}_{\breve{\imath}_\jmath}+u^{ł}_{\breve{\imath}_\jmath}\right)\right)\end{array}\right)\right)^{\frac{1}{C_q^\zeta}}\right)^{\frac{1}{\zeta}}\right)}, \\ \sqrt[ł]{\left(1-\left(1-\left(\prod_{1\le\breve{\imath}_1<\breve{\imath}_2<\ldots<\breve{\imath}_\zeta\le q}\left(\begin{array}{c}\left(1-\left(\prod_{\jmath=1}^{\zeta}\left(1-d^{ł}_{\breve{\imath}_\jmath}\right)-\prod_{\jmath=1}^{\zeta}\left(1-\left(\nu^{ł}_{\breve{\imath}_\jmath}+d^{ł}_{\breve{\imath}_\jmath}\right)\right)\right)\right)^{\frac{1}{C_q^\zeta}}- \\ \prod_{1\le\breve{\imath}_1<\breve{\imath}_2<\ldots<\breve{\imath}_\zeta\le q}\prod_{\jmath=1}^{\zeta}\left(1-\left(\nu^{ł}_{\breve{\imath}_\jmath}+d^{ł}_{\breve{\imath}_\jmath}\right)\right)\end{array}\right)\right)^{\frac{1}{C_q^\zeta}}\right)^{\frac{1}{\zeta}}\right)} \end{array}\right).$$

Property 4: When $\breve{t}_{\breve{\imath}} = \breve{t} = (\nu, u, d)$, then we have

$$TSFIMSM^{(\zeta)}(\breve{t}_1, \breve{t}_2, \ldots, \breve{t}_q) = \breve{t}. \tag{37}$$

Proof: Assume that $\breve{t}_{\breve{\imath}} = \breve{t} = (\nu, u, d)$, then we have

$$TSFIMSM^{(\zeta)}(\breve{t}_1, \breve{t}_2, \ldots, \breve{t}_q) = \left(\begin{array}{c} \sqrt[ł]{\left(1-\left(\prod_{1\le\breve{\imath}_1<\breve{\imath}_2<\ldots<\breve{\imath}_\zeta\le q}\left(\begin{array}{c}\left(1-\left(\prod_{\jmath=1}^{\zeta}(1-d^{ł})-\prod_{\jmath=1}^{\zeta}(1-(\nu^{ł}+d^{ł}))\right)\right)^{\frac{1}{C_q^\zeta}}- \\ \left(\prod_{1\le\breve{\imath}_1<\breve{\imath}_2<\ldots<\breve{\imath}_\zeta\le q}\prod_{\jmath=1}^{\zeta}(1-(\nu^{ł}+d^{ł}))\right)^{\frac{1}{C_q^\zeta}}\end{array}\right)^{\frac{1}{\zeta}} \\ -\left(\left(\prod_{1\le\breve{\imath}_1<\breve{\imath}_2<\ldots<\breve{\imath}_\zeta\le q}\prod_{\jmath=1}^{\zeta}(1-(\nu^{ł}+d^{ł}))\right)^{\frac{1}{C_q^\zeta}}\right)\right)^{\frac{1}{\zeta}}\right)}, \\ \sqrt[ł]{\left(1-\left(1-\left(\begin{array}{c}\prod_{1\le\breve{\imath}_1<\breve{\imath}_2<\ldots<\breve{\imath}_\zeta\le q}\left(\left(1-\left(\prod_{\jmath=1}^{\zeta}(1-u^{ł})-\prod_{\jmath=1}^{\zeta}(1-(\nu^{ł}+u^{ł}))\right)\right)^{\frac{1}{C_q^\zeta}}\right)- \\ \left(\left(\prod_{1\le\breve{\imath}_1<\breve{\imath}_2<\ldots<\breve{\imath}_\zeta\le q}\prod_{\jmath=1}^{\zeta}(1-(\nu^{ł}+u^{ł}))\right)^{\frac{1}{C_q^\zeta}}\right)\end{array}\right)\right)^{\frac{1}{\zeta}}\right)}, \\ \sqrt[ł]{\left(1-\left(1-\left(\begin{array}{c}\prod_{1\le\breve{\imath}_1<\breve{\imath}_2<\ldots<\breve{\imath}_\zeta\le q}\left(\left(1-\left(\prod_{\jmath=1}^{\zeta}(1-d^{ł})-\prod_{\jmath=1}^{\zeta}(1-(\nu^{ł}+d^{ł}))\right)\right)^{\frac{1}{C_q^\zeta}}\right)- \\ \left(\left(\prod_{1\le\breve{\imath}_1<\breve{\imath}_2<\ldots<\breve{\imath}_\zeta\le q}\prod_{\jmath=1}^{\zeta}(1-(\nu^{ł}+d^{ł}))\right)^{\frac{1}{C_q^\zeta}}\right)\end{array}\right)\right)^{\frac{1}{\zeta}}\right)} \end{array}\right),$$

$$=\left(\begin{array}{c}\sqrt[\text{ł}]{\left(1-\left(\prod_{1\le \check{i}_1<\check{i}_2<\ldots<\check{i}_\zeta\le q}\left(\left(\left(\left(1-(1-d^{\text{ł}})^{\zeta}-(1-(\nu^{\text{ł}}+d^{\text{ł}}))^{\zeta}\right)^{\frac{1}{C_q^{\zeta}}}\right)-\left(\left(\prod_{1\le \check{i}_1<\check{i}_2<\ldots<\check{i}_\zeta\le q}(1-(\nu^{\text{ł}}+d^{\text{ł}}))^{\zeta}\right)^{\frac{1}{C_q^{\zeta}}}\right)\right)^{\frac{1}{\zeta}}\right)^{\frac{1}{\zeta}}-\left(\prod_{1\le \check{i}_1<\check{i}_2<\ldots<\check{i}_\zeta\le q}(1-(\nu^{\text{ł}}+d^{\text{ł}}))^{\zeta}\right)^{\frac{1}{C_q^{\zeta}}}\right)\right)},\\ \sqrt[\text{ł}]{\left(1-\left(1-\left(\prod_{1\le \check{i}_1<\check{i}_2<\ldots<\check{i}_\zeta\le q}\left(\left(\left(1-\left((1-u^{\text{ł}})^{\zeta}-(1-(\nu^{\text{ł}}+u^{\text{ł}}))^{\zeta}\right)\right)^{\frac{1}{C_q^{\zeta}}}\right)-\left(\prod_{1\le \check{i}_1<\check{i}_2<\ldots<\check{i}_\zeta\le q}(1-(\nu^{\text{ł}}+u^{\text{ł}}))^{\zeta}\right)\right)^{\frac{1}{C_q^{\zeta}}}\right)^{\frac{1}{\zeta}}\right)\right)},\\ \sqrt[\text{ł}]{\left(1-\left(1-\left(\prod_{1\le \check{i}_1<\check{i}_2<\ldots<\check{i}_\zeta\le q}\left(\left(\left(1-\left((1-d^{\text{ł}})^{\zeta}-(1-(\nu^{\text{ł}}+d^{\text{ł}}))^{\zeta}\right)\right)^{\frac{1}{C_q^{\zeta}}}\right)-\left(\prod_{1\le \check{i}_1<\check{i}_2<\ldots<\check{i}_\zeta\le q}(1-(\nu^{\text{ł}}+d^{\text{ł}}))^{\zeta}\right)\right)^{\frac{1}{C_q^{\zeta}}}\right)^{\frac{1}{\zeta}}\right)\right)}\end{array}\right),$$

$$=\left(\begin{array}{c}\sqrt[\text{ł}]{\left(1-\left(\left(\left(\left(\left(1-\left((1-d^{\text{ł}})^{\zeta}-(1-(\nu^{\text{ł}}+d^{\text{ł}}))^{\zeta}\right)^{C_q^{\zeta}}\right)^{\frac{1}{C_q^{\zeta}}}\right)-\left(\left((1-(\nu^{\text{ł}}+d^{\text{ł}}))^{\zeta}\right)^{C_q^{\zeta}}\right)^{\frac{1}{C_q^{\zeta}}}\right)^{\frac{1}{\zeta}}\right)^{\frac{1}{\zeta}}-\left(\left((1-(\nu^{\text{ł}}+d^{\text{ł}}))^{\zeta}\right)^{C_q^{\zeta}}\right)^{\frac{1}{C_q^{\zeta}}}\right)\right)},\\ \sqrt[\text{ł}]{\left(1-\left(1-\left(\left(\left(1-\left((1-u^{\text{ł}})^{\zeta}-(1-(\nu^{\text{ł}}+u^{\text{ł}}))^{\zeta}\right)\right)^{C_q^{\zeta}}\right)^{\frac{1}{C_q^{\zeta}}}-\left((1-(\nu^{\text{ł}}+u^{\text{ł}}))^{\zeta}\right)^{C_q^{\zeta}}\right)^{\frac{1}{C_q^{\zeta}}}\right)^{\frac{1}{\zeta}}\right)},\\ \sqrt[\text{ł}]{\left(1-\left(1-\left(\left(\left(1-\left((1-d^{\text{ł}})^{\zeta}-(1-(\nu^{\text{ł}}+d^{\text{ł}}))^{\zeta}\right)\right)^{C_q^{\zeta}}\right)^{\frac{1}{C_q^{\zeta}}}-\left((1-(\nu^{\text{ł}}+d^{\text{ł}}))^{\zeta}\right)^{C_q^{\zeta}}\right)^{\frac{1}{C_q^{\zeta}}}\right)^{\frac{1}{\zeta}}\right)}\end{array}\right),$$

$$=\sqrt[\text{ł}]{(1-d^{\text{ł}})-(1-(\nu^{\text{ł}}+d^{\text{ł}}))},\sqrt[\text{ł}]{(1-(1-(1-(1-u^{\text{ł}}))))},\sqrt[\text{ł}]{(1-(1-(1-(1-d^{\text{ł}}))))},$$
$$=(\nu,u,d)=\check{t}.$$

Property 5: Suppose $\check{t}=(1,0)$ and $\check{t}=(0,1)$ then

$$\check{t}\le TSFIMSM^{(\zeta)}(\check{t}_1,\check{t}_2,\ldots,\check{t}_q)\le\check{t}.$$

Proof: Using Theorem 6, the property of boundedness can be proven.

Few specific cases of the $TSFIMSM^{(\zeta)}$ operator are discussed as follows:

When $\zeta=1$, then $TSFIMSM^{(\zeta)}$ the operator becomes the TSF interaction averaging (TSFIA) operator in Eq.36. When $\zeta=2$, then $TSFIMSM^{(\zeta)}$ the operator becomes the special TSF interaction BM (TSFIBM) operator $(p=\text{ł}=1)$ as follows:

$$TSFIBM(\check{t}_1,\check{t}_2,\ldots,\check{t}_q)=\sqrt[\text{ł}]{\left(\frac{1}{q(q-1)}\oplus_{\check{i}_1,\check{i}_2=1,\check{i}_1\ne\check{i}_2}^{q}\left(t_{\check{i}_1}\otimes t_{\check{i}_2}\right)\right)},\tag{37a}$$

$$
= \left(\begin{array}{c}
\sqrt[k]{\left(\begin{array}{c} 1 - \left(\left(1 - \prod_{\breve{\imath}_1, \breve{\imath}_2 = 1, \breve{\imath}_1 \neq \breve{\imath}_2}^{q} \left(\prod_{j=1}^{2} \left(1 - d^{k}_{\breve{\imath}_j} \right) - \prod_{j=1}^{2} \left(1 - \left(\nu^{k}_{\breve{\imath}_j} + d^{k}_{\breve{\imath}_j} \right) \right) \right) \right)^{\frac{1}{q(q-1)}} \right) \\ - \left(\left(\prod_{\breve{\imath}_1, \breve{\imath}_2 = 1, \breve{\imath}_1 \neq \breve{\imath}_2}^{q} \prod_{j=1}^{2} \left(1 - \left(\nu^{k}_{\breve{\imath}_j} + d^{k}_{\breve{\imath}_j} \right) \right) \right)^{\frac{1}{q(q-1)}} \right) - \\ \left(\left(\prod_{\breve{\imath}_1, \breve{\imath}_2 = 1, \breve{\imath}_1 \neq \breve{\imath}_2}^{q} \prod_{j=1}^{2} \left(1 - \left(\nu^{k}_{\breve{\imath}_j} + d^{k}_{\breve{\imath}_j} \right) \right) \right)^{\frac{1}{2q(q-1)}} \right) \end{array} \right)^{\frac{1}{2}}}, \\
\sqrt[k]{\left(1 - \left(1 - \left(\begin{array}{c} \left(1 - \prod_{\breve{\imath}_1, \breve{\imath}_2 = 1, \breve{\imath}_1 \neq \breve{\imath}_2}^{q} \left(\prod_{j=1}^{2} \left(1 - u^{k}_{\breve{\imath}_j} \right) - \prod_{j=1}^{2} \left(1 - \left(\nu^{k}_{\breve{\imath}_j} + u^{k}_{\breve{\imath}_j} \right) \right) \right) \right)^{\frac{1}{q(q-1)}} \\ - \left(\prod_{\breve{\imath}_1, \breve{\imath}_2 = 1, \breve{\imath}_1 \neq \breve{\imath}_2}^{q} \prod_{j=1}^{2} \left(1 - \left(\nu^{k}_{\breve{\imath}_j} + u^{k}_{\breve{\imath}_j} \right) \right) \right)^{\frac{1}{q(q-1)}} \end{array} \right) \right)^{\frac{1}{2}} \right)}, \\
\sqrt[k]{\left(1 - \left(1 - \left(\begin{array}{c} \left(1 - \prod_{\breve{\imath}_1, \breve{\imath}_2 = 1, \breve{\imath}_1 \neq \breve{\imath}_2}^{q} \left(\prod_{j=1}^{2} \left(1 - d^{k}_{\breve{\imath}_j} \right) - \prod_{j=1}^{2} \left(1 - \left(\nu^{k}_{\breve{\imath}_j} + d^{k}_{\breve{\imath}_j} \right) \right) \right) \right)^{\frac{1}{q(q-1)}} \\ - \left(\prod_{\breve{\imath}_1, \breve{\imath}_2 = 1, \breve{\imath}_1 \neq \breve{\imath}_2}^{q} \prod_{j=1}^{2} \left(1 - \left(\nu^{k}_{\breve{\imath}_j} + d^{k}_{\breve{\imath}_j} \right) \right) \right)^{\frac{1}{q(q-1)}} \end{array} \right) \right)^{\frac{1}{2}} \right)}
\end{array} \right),
$$

where $\breve{t}_{\breve{\imath}_1} = (\nu_{\breve{\imath}_1}, u_{\breve{\imath}_1}, d_{\breve{\imath}_1}), \breve{t}_{\breve{\imath}_2} = (\nu_{\breve{\imath}_2}, u_{\breve{\imath}_2}, d_{\breve{\imath}_2}),$

Proof:

$$
\breve{t}_{\breve{\imath}_1} \otimes \breve{t}_{\breve{\imath}_2} = \left(\begin{array}{c} \sqrt[k]{\left(\left(\left(1 - d^{k}_{\breve{\imath}_1} \right) * \left(1 - d^{k}_{\breve{\imath}_2} \right) \right) - \left(\left(1 - \left(\nu^{k}_{\breve{\imath}_1} + d^{k}_{\breve{\imath}_1} \right) \right) * \left(1 - \left(\nu^{k}_{\breve{\imath}_2} + d^{k}_{\breve{\imath}_2} \right) \right) \right) \right)}, \\ \sqrt[k]{\left(1 - \left(\left(1 - u^{k}_{\breve{\imath}_1} \right) * \left(1 - u^{k}_{\breve{\imath}_2} \right) \right) \right)}, \sqrt[k]{\left(1 - \left(\left(1 - d^{k}_{\breve{\imath}_1} \right) * \left(1 - d^{k}_{\breve{\imath}_2} \right) \right) \right)} \end{array} \right),
$$

$$
= \sqrt[k]{\prod_{j=1}^{2} \left(1 - d^{k}_{\breve{\imath}_j} \right) - \prod_{j=1}^{2} \left(1 - \left(\nu^{k}_{\breve{\imath}_j} + d^{k}_{\breve{\imath}_j} \right) \right)}, \sqrt[k]{1 - \prod_{j=1}^{2} \left(1 - u^{k}_{\breve{\imath}_j} \right)}, \sqrt[k]{1 - \prod_{j=1}^{2} \left(1 - d^{k}_{\breve{\imath}_j} \right)},
$$

$$
\nu^{k}_{\breve{t}_{\breve{\imath}_1} \otimes \breve{t}_{\breve{\imath}_2}} + u^{k}_{\breve{t}_{\breve{\imath}_1} \otimes \breve{t}_{\breve{\imath}_2}} + d^{k}_{\breve{t}_{\breve{\imath}_1} \otimes \breve{t}_{\breve{\imath}_2}} = 2 - \prod_{j=1}^{2} \left(1 - u^{k}_{\breve{\imath}_j} \right) - \prod_{j=1}^{2} \left(1 - \left(\nu^{k}_{\breve{\imath}_j} + d^{k}_{\breve{\imath}_j} \right) \right),
$$

$$
\oplus^{q}_{\breve{\imath}_1, \breve{\imath}_2 = 1, \breve{\imath}_1 \neq \breve{\imath}_2} \left(\breve{t}_{\breve{\imath}_1} \otimes \breve{t}_{\breve{\imath}_2} \right) = \left(\begin{array}{c}
\sqrt[k]{\left(1 - \left(1 - \prod_{\breve{\imath}_1, \breve{\imath}_2 = 1, \breve{\imath}_1 \neq \breve{\imath}_2}^{q} \left(\prod_{j=1}^{2} \left(1 - d^{k}_{\breve{\imath}_j} \right) - \prod_{j=1}^{2} \left(1 - \left(\nu^{k}_{\breve{\imath}_j} + d^{k}_{\breve{\imath}_j} \right) \right) \right) \right) \right)}, \\
\sqrt[k]{\left(\begin{array}{c} 1 - \left(\prod_{\breve{\imath}_1, \breve{\imath}_2 = 1, \breve{\imath}_1 \neq \breve{\imath}_2}^{q} \left(\prod_{j=1}^{2} \left(1 - u^{k}_{\breve{\imath}_j} \right) \right) - \prod_{j=1}^{2} \left(1 - \left(\nu^{k}_{\breve{\imath}_j} + u^{k}_{\breve{\imath}_j} \right) \right) \right) - \\ \left(\prod_{\breve{\imath}_1, \breve{\imath}_2 = 1, \breve{\imath}_1 \neq \breve{\imath}_2}^{q} \prod_{j=1}^{2} \left(1 - \left(\nu^{k}_{\breve{\imath}_j} + u^{k}_{\breve{\imath}_j} \right) \right) \right) \end{array} \right)}, \\
\sqrt[k]{\left(\begin{array}{c} 1 - \left(\prod_{\breve{\imath}_1, \breve{\imath}_2 = 1, \breve{\imath}_1 \neq \breve{\imath}_2}^{q} \left(\prod_{j=1}^{2} \left(1 - d^{k}_{\breve{\imath}_j} \right) \right) - \prod_{j=1}^{2} \left(1 - \left(\nu^{k}_{\breve{\imath}_j} + d^{k}_{\breve{\imath}_j} \right) \right) \right) - \\ \left(\prod_{\breve{\imath}_1, \breve{\imath}_2 = 1, \breve{\imath}_1 \neq \breve{\imath}_2}^{q} \prod_{j=1}^{2} \left(1 - \left(\nu^{k}_{\breve{\imath}_j} + d^{k}_{\breve{\imath}_j} \right) \right) \right) \end{array} \right)}
\end{array} \right),
$$

$$
\nu^{k}_{\oplus^{q}_{\breve{\imath}_1, \breve{\imath}_2 = 1, \breve{\imath}_1 \neq \breve{\imath}_2} \left(\breve{t}_{\breve{\imath}_1} \otimes \breve{t}_{\breve{\imath}_2} \right)} + u^{k}_{\oplus^{q}_{\breve{\imath}_1, \breve{\imath}_2 = 1, \breve{\imath}_1 \neq \breve{\imath}_2} \left(\breve{t}_{\breve{\imath}_1} \otimes \breve{t}_{\breve{\imath}_2} \right)} + d^{k}_{\oplus^{q}_{\breve{\imath}_1, \breve{\imath}_2 = 1, \breve{\imath}_1 \neq \breve{\imath}_2} \left(\breve{t}_{\breve{\imath}_1} \otimes \breve{t}_{\breve{\imath}_2} \right)}, = \left(\begin{array}{c} 2 - \left(\prod_{\breve{\imath}_1, \breve{\imath}_2 = 1, \breve{\imath}_1 \neq \breve{\imath}_2}^{q} \left(\prod_{j=1}^{2} \left(1 - u^{k}_{\breve{\imath}_j} \right) \right) - \prod_{j=1}^{2} \left(1 - \left(\nu^{k}_{\breve{\imath}_j} + u^{k}_{\breve{\imath}_j} \right) \right) \right) \\ - \left(\prod_{\breve{\imath}_1, \breve{\imath}_2 = 1, \breve{\imath}_1 \neq \breve{\imath}_2}^{q} \prod_{j=1}^{2} \left(1 - \left(\nu^{k}_{\breve{\imath}_j} + u^{k}_{\breve{\imath}_j} \right) \right) \right) - \left(\prod_{\breve{\imath}_1, \breve{\imath}_2 = 1, \breve{\imath}_1 \neq \breve{\imath}_2}^{q} \prod_{j=1}^{2} \left(1 - \left(\nu^{k}_{\breve{\imath}_j} + d^{k}_{\breve{\imath}_j} \right) \right) \right) \end{array} \right),
$$

$$= 2 - \left(\prod_{\tilde{\imath}_1,\tilde{\imath}_2=1,\tilde{\imath}_1 \neq \tilde{\imath}_2}^{q} \left(\prod_{j=1}^{2} \left(1 - u_{\tilde{\imath}_j}^{k}\right) \right) - \prod_{j=1}^{2} \left(1 - \left(\nu_{\tilde{\imath}_j}^{k} + u_{\tilde{\imath}_j}^{k}\right)\right) \right) + \left(\prod_{\tilde{\imath}_1,\tilde{\imath}_2=1,\tilde{\imath}_1 \neq \tilde{\imath}_2}^{q} \prod_{j=1}^{2} \left(u_{\tilde{\imath}_j}^{k} - d_{\tilde{\imath}_j}^{k}\right) \right),$$

$$\frac{1}{q(q-1)} \left(\oplus_{\tilde{\imath}_1,\tilde{\imath}_2=1,\tilde{\imath}_1 \neq \tilde{\imath}_2}^{q} \left(\breve{t}_{\tilde{\imath}_1} \otimes \breve{t}_{\tilde{\imath}_2} \right) \right) = \left(\begin{array}{c} \sqrt[k]{\left(1 - \left(1 - \prod_{\tilde{\imath}_1,\tilde{\imath}_2=1,\tilde{\imath}_1 \neq \tilde{\imath}_2}^{q} \left(\prod_{j=1}^{2} \left(1 - d_{\tilde{\imath}_j}^{k}\right) \right) - \prod_{j=1}^{2} \left(1 - \left(\nu_{\tilde{\imath}_j}^{k} + d_{\tilde{\imath}_j}^{k}\right)\right) \right)^{\frac{1}{q(q-1)}} \right)}, \\ \sqrt[k]{\left(\begin{array}{c} \left(1 - \prod_{\tilde{\imath}_1,\tilde{\imath}_2=1,\tilde{\imath}_1 \neq \tilde{\imath}_2}^{q} \left(\prod_{j=1}^{2} \left(1 - u_{\tilde{\imath}_j}^{k}\right) \right) - \prod_{j=1}^{2} \left(1 - \left(\nu_{\tilde{\imath}_j}^{k} + u_{\tilde{\imath}_j}^{k}\right)\right) \right)^{\frac{1}{q(q-1)}} \\ - \left(\prod_{\tilde{\imath}_1,\tilde{\imath}_2=1,\tilde{\imath}_1 \neq \tilde{\imath}_2}^{q} \prod_{j=1}^{2} \left(1 - \left(\nu_{\tilde{\imath}_j}^{k} + u_{\tilde{\imath}_j}^{k}\right)\right) \right)^{\frac{1}{q(q-1)}} \end{array} \right)}, \\ \sqrt[k]{\left(\begin{array}{c} \left(1 - \prod_{\tilde{\imath}_1,\tilde{\imath}_2=1,\tilde{\imath}_1 \neq \tilde{\imath}_2}^{q} \left(\prod_{j=1}^{2} \left(1 - d_{\tilde{\imath}_j}^{k}\right) \right) - \prod_{j=1}^{2} \left(1 - \left(\nu_{\tilde{\imath}_j}^{k} + d_{\tilde{\imath}_j}^{k}\right)\right) \right)^{\frac{1}{q(q-1)}} \\ - \left(\prod_{\tilde{\imath}_1,\tilde{\imath}_2=1,\tilde{\imath}_1 \neq \tilde{\imath}_2}^{q} \prod_{j=1}^{2} \left(1 - \left(\nu_{\tilde{\imath}_j}^{k} + d_{\tilde{\imath}_j}^{k}\right)\right) \right)^{\frac{1}{q(q-1)}} \end{array} \right)} \end{array} \right),$$

$$\nu^{k}_{\frac{1}{q(q-1)}\left(\oplus_{\tilde{\imath}_1,\tilde{\imath}_2=1,\tilde{\imath}_1 \neq \tilde{\imath}_2}^{q} \left(\breve{t}_{\tilde{\imath}_1} \otimes \breve{t}_{\tilde{\imath}_2} \right) \right)} + u^{k}_{\frac{1}{q(q-1)}\left(\oplus_{\tilde{\imath}_1,\tilde{\imath}_2=1,\tilde{\imath}_1 \neq \tilde{\imath}_2}^{q} \left(\breve{t}_{\tilde{\imath}_1} \otimes \breve{t}_{\tilde{\imath}_2} \right) \right)} + d^{k}_{\frac{1}{q(q-1)}\left(\oplus_{\tilde{\imath}_1,\tilde{\imath}_2=1,\tilde{\imath}_1 \neq \tilde{\imath}_2}^{q} \left(\breve{t}_{\tilde{\imath}_1} \otimes \breve{t}_{\tilde{\imath}_2} \right) \right)},$$

$$= \left(\begin{array}{c} 1 - \left(1 - \prod_{\tilde{\imath}_1,\tilde{\imath}_2=1,\tilde{\imath}_1 \neq \tilde{\imath}_2}^{q} \left(\prod_{j=1}^{2} \left(1 - u_{\tilde{\imath}_j}^{k}\right) \right) - \prod_{j=1}^{2} \left(1 - \left(\nu_{\tilde{\imath}_j}^{k} + u_{\tilde{\imath}_j}^{k}\right)\right) \right)^{\frac{1}{q(q-1)}} \\ - \left(\prod_{\tilde{\imath}_1,\tilde{\imath}_2=1,\tilde{\imath}_1 \neq \tilde{\imath}_2}^{q} \prod_{j=1}^{2} \left(1 - \left(\nu_{\tilde{\imath}_j}^{k} + u_{\tilde{\imath}_j}^{k}\right)\right) \right)^{\frac{1}{q(q-1)}} \\ - \left(\prod_{\tilde{\imath}_1,\tilde{\imath}_2=1,\tilde{\imath}_1 \neq \tilde{\imath}_2}^{q} \prod_{j=1}^{2} \left(1 - \left(\nu_{\tilde{\imath}_j}^{k} + d_{\tilde{\imath}_j}^{k}\right)\right) \right)^{\frac{1}{q(q-1)}} \end{array} \right),$$

$$= \left(1 - \left(\begin{array}{c} 1 - \prod_{\tilde{\imath}_1,\tilde{\imath}_2=1,\tilde{\imath}_1 \neq \tilde{\imath}_2}^{q} \left(\prod_{j=1}^{2} \left(1 - u_{\tilde{\imath}_j}^{k}\right) \right) - \\ \prod_{j=1}^{2} \left(1 - \left(\nu_{\tilde{\imath}_j}^{k} + u_{\tilde{\imath}_j}^{k}\right)\right) \end{array} \right)^{\frac{1}{q(q-1)}} + \left(\prod_{\tilde{\imath}_1,\tilde{\imath}_2=1,\tilde{\imath}_1 \neq \tilde{\imath}_2}^{q} \prod_{j=1}^{2} \left(u_{\tilde{\imath}_j}^{k} - d_{\tilde{\imath}_j}^{k}\right) \right)^{\frac{1}{q(q-1)}} \right),$$

$$\left(\frac{1}{q(q-1)} \left(\oplus_{\tilde{\imath}_1,\tilde{\imath}_2=1,\tilde{\imath}_1 \neq \tilde{\imath}_2}^{q} \left(\breve{t}_{\tilde{\imath}_1} \otimes \breve{t}_{\tilde{\imath}_2} \right) \right) \right)^{\frac{1}{2}} = \left(\begin{array}{c} \sqrt[k]{\left(\begin{array}{c} \left(\begin{array}{c} 1 - \left(\left(1 - \prod_{\tilde{\imath}_1,\tilde{\imath}_2=1,\tilde{\imath}_1 \neq \tilde{\imath}_2}^{q} \left(\prod_{j=1}^{2} \left(1 - u_{\tilde{\imath}_j}^{k}\right) \right) - \prod_{j=1}^{2} \left(1 - \left(\nu_{\tilde{\imath}_j}^{k} + u_{\tilde{\imath}_j}^{k}\right)\right) \right)^{\frac{1}{q(q-1)}} \right) \\ - \left(\prod_{\tilde{\imath}_1,\tilde{\imath}_2=1,\tilde{\imath}_1 \neq \tilde{\imath}_2}^{q} \prod_{j=1}^{2} \left(1 - \left(\nu_{\tilde{\imath}_j}^{k} + d_{\tilde{\imath}_j}^{k}\right)\right) \right)^{\frac{1}{q(q-1)}} \end{array} \right)^{\frac{1}{2}} \\ - \left(\prod_{\tilde{\imath}_1,\tilde{\imath}_2=1,\tilde{\imath}_1 \neq \tilde{\imath}_2}^{q} \prod_{j=1}^{2} \left(1 - \left(\nu_{\tilde{\imath}_j}^{k} + d_{\tilde{\imath}_j}^{k}\right)\right) \right)^{\frac{1}{2q(q-1)}} \end{array} \right)}, \\ \sqrt[k]{\left(1 - \left(\begin{array}{c} 1 - \left(\left(1 - \prod_{\tilde{\imath}_1,\tilde{\imath}_2=1,\tilde{\imath}_1 \neq \tilde{\imath}_2}^{q} \left(\prod_{j=1}^{2} \left(1 - u_{\tilde{\imath}_j}^{k}\right) \right) - \prod_{j=1}^{2} \left(1 - \left(\nu_{\tilde{\imath}_j}^{k} + u_{\tilde{\imath}_j}^{k}\right)\right) \right)^{\frac{1}{q(q-1)}} \right) \\ - \left(\prod_{\tilde{\imath}_1,\tilde{\imath}_2=1,\tilde{\imath}_1 \neq \tilde{\imath}_2}^{q} \prod_{j=1}^{2} \left(1 - \left(\nu_{\tilde{\imath}_j}^{k} + u_{\tilde{\imath}_j}^{k}\right)\right) \right)^{\frac{1}{q(q-1)}} \end{array} \right)^{\frac{1}{2}} \right)}, \\ \sqrt[k]{\left(1 - \left(\begin{array}{c} 1 - \left(\left(1 - \prod_{\tilde{\imath}_1,\tilde{\imath}_2=1,\tilde{\imath}_1 \neq \tilde{\imath}_2}^{q} \left(\prod_{j=1}^{2} \left(1 - d_{\tilde{\imath}_j}^{k}\right) \right) - \prod_{j=1}^{2} \left(1 - \left(\nu_{\tilde{\imath}_j}^{k} + d_{\tilde{\imath}_j}^{k}\right)\right) \right)^{\frac{1}{q(q-1)}} \right) \\ - \left(\prod_{\tilde{\imath}_1,\tilde{\imath}_2=1,\tilde{\imath}_1 \neq \tilde{\imath}_2}^{q} \prod_{j=1}^{2} \left(1 - \left(\nu_{\tilde{\imath}_j}^{k} + d_{\tilde{\imath}_j}^{k}\right)\right) \right)^{\frac{1}{q(q-1)}} \end{array} \right)^{\frac{1}{2}} \right)} \end{array} \right),$$

When $\zeta = 3$, then the $TSFIMSM^{(\zeta)}$ operator becomes the special generalized TSFIBM operator $(p = 1, \text{ƚ} = 1, r = 1)$ ĩs:

$$TSFIBM\left(\breve{t}_1, \breve{t}_2, \ldots, \breve{t}_q\right) = \left(\frac{1}{q(q-1)(q-2)} \oplus^{q}_{\tilde{\imath}_1,\tilde{\imath}_2,\tilde{\imath}_3=1,\tilde{\imath}_1 \neq \tilde{\imath}_2 \neq \tilde{\imath}_3}\left(t_{\tilde{\imath}_1} \otimes t_{\tilde{\imath}_2} \otimes t_{\tilde{\imath}_3}\right)\right)^{\frac{1}{3}},$$

$$= \left(\begin{array}{c} \sqrt[\text{ƚ}]{\left(\left(1 - \left(\left(1 - \prod_{\tilde{\imath}_1,\tilde{\imath}_2,\tilde{\imath}_3=1,\tilde{\imath}_1 \neq \tilde{\imath}_2 \neq \tilde{\imath}_3}^{q}\left(\prod_{\jmath=1}^{3}\left(1 - d_{\tilde{\imath}_\jmath}^{\text{ƚ}}\right)\right) - \prod_{\jmath=1}^{3}\left(1 - \left(\nu_{\tilde{\imath}_\jmath}^{\text{ƚ}} + d_{\tilde{\imath}_\jmath}^{\text{ƚ}}\right)\right)\right)^{\frac{1}{q(q-1)(q-2)}}\right)\right)^{\frac{1}{3}} - \left(\prod_{\tilde{\imath}_1,\tilde{\imath}_2,\tilde{\imath}_3=1,\tilde{\imath}_1 \neq \tilde{\imath}_2 \neq \tilde{\imath}_3}^{q}\prod_{\jmath=1}^{2}\left(1 - \left(\nu_{\tilde{\imath}_\jmath}^{\text{ƚ}} + d_{\tilde{\imath}_\jmath}^{\text{ƚ}}\right)\right)\right)^{\frac{1}{q(q-1)(q-2)}}\right) - \left(\prod_{\tilde{\imath}_1,\tilde{\imath}_2,\tilde{\imath}_3=1,\tilde{\imath}_1 \neq \tilde{\imath}_2 \neq \tilde{\imath}_3}^{q}\prod_{\jmath=1}^{2}\left(1 - \left(\nu_{\tilde{\imath}_\jmath}^{\text{ƚ}} + d_{\tilde{\imath}_\jmath}^{\text{ƚ}}\right)\right)\right)^{\frac{1}{3q(q-1)(q-2)}}\right)}, \\ \sqrt[\text{ƚ}]{\left(1 - \left(\left(1 - \left(\left(1 - \prod_{\tilde{\imath}_1,\tilde{\imath}_2,\tilde{\imath}_3=1,\tilde{\imath}_1 \neq \tilde{\imath}_2 \neq \tilde{\imath}_3}^{q}\left(\prod_{\jmath=1}^{3}\left(1 - u_{\tilde{\imath}_\jmath}^{\text{ƚ}}\right)\right) - \prod_{\jmath=1}^{3}\left(1 - \left(\nu_{\tilde{\imath}_\jmath}^{\text{ƚ}} + u_{\tilde{\imath}_\jmath}^{\text{ƚ}}\right)\right)\right)^{\frac{1}{q(q-1)(q-2)}}\right)\right)^{\frac{1}{3}} - \left(\prod_{\tilde{\imath}_1,\tilde{\imath}_2,\tilde{\imath}_3=1,\tilde{\imath}_1 \neq \tilde{\imath}_2 \neq \tilde{\imath}_3}^{q}\prod_{\jmath=1}^{2}\left(1 - \left(\nu_{\tilde{\imath}_\jmath}^{\text{ƚ}} + u_{\tilde{\imath}_\jmath}^{\text{ƚ}}\right)\right)\right)^{\frac{1}{q(q-1)(q-2)}}\right)\right)}, \\ \sqrt[\text{ƚ}]{\left(1 - \left(\left(1 - \left(\left(1 - \prod_{\tilde{\imath}_1,\tilde{\imath}_2,\tilde{\imath}_3=1,\tilde{\imath}_1 \neq \tilde{\imath}_2 \neq \tilde{\imath}_3}^{q}\left(\prod_{\jmath=1}^{3}\left(1 - d_{\tilde{\imath}_\jmath}^{\text{ƚ}}\right)\right) - \prod_{\jmath=1}^{3}\left(1 - \left(\nu_{\tilde{\imath}_\jmath}^{\text{ƚ}} + d_{\tilde{\imath}_\jmath}^{\text{ƚ}}\right)\right)\right)^{\frac{1}{q(q-1)(q-2)}}\right)\right)^{\frac{1}{3}} - \left(\prod_{\tilde{\imath}_1,\tilde{\imath}_2,\tilde{\imath}_3=1,\tilde{\imath}_1 \neq \tilde{\imath}_2 \neq \tilde{\imath}_3}^{q}\prod_{\jmath=1}^{2}\left(1 - \left(\nu_{\tilde{\imath}_\jmath}^{\text{ƚ}} + d_{\tilde{\imath}_\jmath}^{\text{ƚ}}\right)\right)\right)^{\frac{1}{q(q-1)(q-2)}}\right)\right)} \end{array} \right),$$

where $\breve{t}_{\tilde{\imath}_1} = (\nu_{\tilde{\imath}_1}, u_{\tilde{\imath}_1}, d_{\tilde{\imath}_1}), \breve{t}_{\tilde{\imath}_2} = (\nu_{\tilde{\imath}_2}, u_{\tilde{\imath}_2}, d_{\tilde{\imath}_2})$ and $\breve{t}_{\tilde{\imath}_3} = (\nu_{\tilde{\imath}_3}, u_{\tilde{\imath}_3}, d_{\tilde{\imath}_3})$.

When $\zeta = q$, then the $TSFIMSM^{(\zeta)}$ operator becomes the T-spherical fuzzy interaction geometric averaging (TSFIGA) operator in Eq. 11.

When every $\breve{t}_{\tilde{\imath}} = (\nu_{\tilde{\imath}}, u_{\tilde{\imath}}, d_{\tilde{\imath}})$ $(\tilde{\imath} = 1, 2, \ldots, q)$ becomes $\breve{t}_{\tilde{\imath}} = (1, 0)$ $(\tilde{\imath} = 1, 2, \ldots, q)$ then

$$TSFIMSM^{(\zeta)}\left(\breve{t}_1, \breve{t}_2, \ldots, \breve{t}_q\right) = (1, 0).$$

When every $\breve{t}_{\tilde{\imath}} = (\nu_{\tilde{\imath}}, u_{\tilde{\imath}}, d_{\tilde{\imath}})$ $(\tilde{\imath} = 1, 2, \ldots, q)$ becomes $\breve{t}_{\tilde{\imath}} = (0, 1)$ $(\tilde{\imath} = 1, 2, \ldots, q)$, then

$$TSFIMSM^{(\zeta)}\left(\breve{t}_1, \breve{t}_2, \ldots, \breve{t}_q\right) = (0, 1).$$

Definition 9: A TSFIWMSM operator is particularized by

$$TSFIWMSM^{(\zeta)}\left(\breve{t}_1, \breve{t}_2, \ldots, \breve{t}_q\right) = \left(\frac{\oplus_{1 \leq \tilde{\imath}_1 < \tilde{\imath}_2 < \ldots < \tilde{\imath}_\zeta \leq q} \otimes_{\jmath=1}^{\zeta}\left(\omega_{\tilde{\imath}_\jmath} \breve{t}_{\tilde{\imath}_\jmath}\right)}{C_q^\zeta}\right)^{\frac{1}{\zeta}}. \tag{38}$$

Theorem 5: Using the theory in Eq. 38, we derive the following information, such as

$$TSFIWMSM^{(\zeta)}\left(\breve{t}_1, \breve{t}_2, \ldots, \breve{t}_q\right) = \left(\frac{\oplus_{1 \leq \tilde{\imath}_1 < \tilde{\imath}_2 < \ldots < \tilde{\imath}_\zeta \leq q} \otimes_{\jmath=1}^{\zeta}\left(\omega_{\tilde{\imath}_\jmath} \breve{t}_{\tilde{\imath}_\jmath}\right)}{C_q^\zeta}\right)^{\frac{1}{\zeta}}.$$

TABLE 3 Aggregated results by using different methods.

ζ	TSFMSM	TSFWMSM	TSFIMSM	TSFIWMSM
1	$(0.55, 0.4267, 0.1654)$	$(0.8760, 0.00021, 0)$	$(0.919, 0.545, 0.3004)$	$(0.771, 0.701, 0.354)$
2	$(0.69509, 0.4572, 0.17732)$	$(0.8981, 0.000124, 0)$	$(0.765, 0.469, 0.2537)$	$(0.666, 0.6, 0.298)$
3	$(0.8181, 0.29851, 0.07312)$	$(0.95686, 0.00007, 0)$	$(0.766, 0.418, 0.2284)$	$(0.608, 0.545, 0.269)$
4	$(0.94578, 0.020886, 0.0)$	$(0.9990, 0.000014, 0)$	$(0.921, 0.191, 0.1167)$	$(0.569, 0.509, 0.251)$

$$= \left(\begin{array}{c} \sqrt[\natural]{\left(1 - \left(\left(\left(\prod_{1 \le i_1 < i_2 < \ldots < i_\zeta \le \mathfrak{q}} \left(1 - \left(\begin{array}{c} \prod_{j=1}^{\zeta} \left(1 - \left(\left(1 - \nu_{i_j}^{\natural} \right)^{\omega_{i_j}} - \left(1 - \left(\nu_{i_j}^{\natural} + d_{i_j}^{\natural} \right) \right)^{\omega_{i_j}} \right) \right) \\ - \prod_{j=1}^{\zeta} \left(1 - \left(\nu_{i_j}^{\natural} + d_{i_j}^{\natural} \right) \right)^{\omega_{i_j}} \end{array} \right) \right) \right)^{\frac{1}{C_{\mathfrak{q}}^{\zeta}}} \right)^{\frac{1}{\zeta}} - \left(\prod_{1 \le i_1 < i_2 < \ldots < i_\zeta \le \mathfrak{q}} \prod_{j=1}^{\zeta} \left(1 - \left(\nu_{i_j}^{\natural} + d_{i_j}^{\natural} \right) \right)^{\omega_{i_j}} \right)^{\frac{1}{C_{\mathfrak{q}}^{\zeta}}} \right) - \left(\prod_{1 \le i_1 < i_2 < \ldots < i_\zeta \le \mathfrak{q}} \prod_{j=1}^{\zeta} \left(1 - \left(\nu_{i_j}^{\natural} + d_{i_j}^{\natural} \right) \right)^{\omega_{i_j}} \right)^{\frac{1}{\zeta C_{\mathfrak{q}}^{\zeta}}} \right)}, \\ \sqrt[\natural]{\left(1 - \left(1 - \left(\left(\prod_{1 \le i_1 < i_2 < \ldots < i_\zeta \le \mathfrak{q}} \left(1 - \left(\begin{array}{c} \prod_{j=1}^{\zeta} \left(1 - \left(\left(1 - \nu_{i_j}^{\natural} \right)^{\omega_{i_j}} - \left(1 - \left(\nu_{i_j}^{\natural} + u_{i_j}^{\natural} \right) \right)^{\omega_{i_j}} \right) \right) \\ - \prod_{j=1}^{\zeta} \left(1 - \left(\nu_{i_j}^{\natural} + u_{i_j}^{\natural} \right) \right)^{\omega_{i_j}} \end{array} \right) \right) \right)^{\frac{1}{C_{\mathfrak{q}}^{\zeta}}} \right)^{\frac{1}{\zeta}} - \left(\prod_{1 \le i_1 < i_2 < \ldots < i_\zeta \le \mathfrak{q}} \prod_{j=1}^{\zeta} \left(1 - \left(\nu_{i_j}^{\natural} + u_{i_j}^{\natural} \right) \right)^{\omega_{i_j}} \right)^{\frac{1}{C_{\mathfrak{q}}^{\zeta}}} \right) \right)}, \\ \sqrt[\natural]{\left(1 - \left(1 - \left(\left(\prod_{1 \le i_1 < i_2 < \ldots < i_\zeta \le \mathfrak{q}} \left(1 - \left(\begin{array}{c} \prod_{j=1}^{\zeta} \left(1 - \left(\left(1 - \nu_{i_j}^{\natural} \right)^{\omega_{i_j}} - \left(1 - \left(\nu_{i_j}^{\natural} + d_{i_j}^{\natural} \right) \right)^{\omega_{i_j}} \right) \right) \\ - \prod_{j=1}^{\zeta} \left(1 - \left(\nu_{i_j}^{\natural} + d_{i_j}^{\natural} \right) \right)^{\omega_{i_j}} \end{array} \right) \right) \right)^{\frac{1}{C_{\mathfrak{q}}^{\zeta}}} \right)^{\frac{1}{\zeta}} - \left(\prod_{1 \le i_1 < i_2 < \ldots < i_\zeta \le \mathfrak{q}} \prod_{j=1}^{\zeta} \left(1 - \left(\nu_{i_j}^{\natural} + d_{i_j}^{\natural} \right) \right)^{\omega_{i_j}} \right)^{\frac{1}{C_{\mathfrak{q}}^{\zeta}}} \right) \right)} \end{array} \right). \quad (39)$$

Example 4: Suppose $\breve{t}_1 = (0.8, 0.5, 0.3), \breve{t}_2 = (0.9, 0.7, 0.0), \breve{t}_3 = (0.5, 0.2, 0.1), \breve{t}_4 = (0.4, 0.3, 0.2)$ be four TSFVs and $\omega = (0.15, 0.40, 0.25, 0.20)$. Some methods are utilized to calculate the aggregated values. Table 3 and Table 4 show the results.

From the aforementioned information, we noticed that the derived theory has a lot of benefits because the proposed theory is the modified theory of a bundle of existing knowledge. Furthermore, we aim to prove the supremacy and effectiveness of the derived idea with the help of comparative analysis.

5 Application in MADM technique

In this part, we describe the exact procedures for a novel MADM method using the suggested operators in a TSF environment. Suppose $\{F_1, F_2, \ldots, F_m\}$ be a collection of alternatives, the collection of attributes be $\{C_1, C_2, \ldots, C_{\mathfrak{q}}\}$ associated with the weight vector $(\omega_1, \omega_2, \ldots, \omega_{\mathfrak{q}})$, satisfying $\omega_{\check{\mathfrak{i}}} \ge 0$ and $\sum_{\check{\mathfrak{i}}=1}^{\mathfrak{q}} \omega_{\check{\mathfrak{i}}} = 1$. Utilizing TSFV $\breve{t}_{\mathfrak{l}_j} = (\nu_{\mathfrak{l}_j}, u_{\mathfrak{l}_j}, d_{\mathfrak{l}_j})$, the decision maker calculates alternatives concerning

TABLE 4 Aggregated matrix of score values $\mathfrak{s}(\breve{t})$ by using the information in Table 3.

ζ	TSFMSM	TSFWMSM	TSFIMSM	TSFIWMSM
1	0.059	0.802	0.69562	0.25813
2	0.107	0.827	0.31279	0.1112
3	0.219	0.862	0.32463	0.068703
4	0.582	0.927	0.7201	0.04928

attribute to form $D = (\breve{t}_{\tilde{\imath}_j})_{m\times n}$. The proposed multiple attribute decision-making method's particular steps are listed as follows, along with the given aggregation operators.

Step 1: While calculating alternatives $F_{\tilde{\imath}}$ concerning attributes C_j, the decision makers give TSFV $\breve{t}_{\tilde{\imath}_j} = (\nu_{\tilde{\imath}_j}, u_{\tilde{\imath}_j}, d_{\tilde{\imath}_j})$ and formed a decision matrix as $D = (\breve{t}_{\tilde{\imath}_j})_{m\times n}$.The normalized decision matrix is created by transforming the TSF matrix. The higher attribute values are better if the attributes are beneficial whereas lower attribute values are better if the attributes are cost attributes. The normalized TSF decision matrix is obtained by converting the cost-type attribute value into benefit-type value, which results in $D' = (\breve{t}'_{\tilde{\imath}_j})_{m\times n}$.

$$\breve{t}'_{\tilde{\imath}_j} = \left\{ \begin{array}{ll} \breve{t}_{\tilde{\imath}_j} & for\ benifit\ attribute\ C_j \\ \breve{t}^{c}_{\tilde{\imath}_j} & for\ cost\ attribute\ C_j \end{array} \right\},$$

where , $\breve{t}^{c}_{\tilde{\imath}_j} = (d_{\tilde{\imath}_j}, u_{\tilde{\imath}_j}, \nu_{\tilde{\imath}_j})$.

TABLE 5 T-spherical fuzzy decision matrix.

	C_1	C_2	C_3	C_4
F_1	(0.80, 0.50, 0.30)	(0.90, 0.70, 0.00)	(0.50, 0.20, 0.10)	(0.40, 0.30, 0.20)
F_2	(0.70, 0.40, 0.20)	(0.50, 0.10, 0.30)	(0.60, 0.50, 0.40)	(0.80, 0.70, 0.10)
F_3	(0.50, 0.30, 0.40)	(0.80, 0.60, 0.20)	(0.40, 0.20, 0.30)	(0.90, 0.50, 0.20)
F_4	(0.65, 0.50, 0.30)	(0.60, 0.50, 0.10)	(0.80, 0.45, 0.30)	(0.75, 0.65, 0.40)
F_5	(0.50, 0.40, 0.60)	(0.40, 0.30, 0.20)	(0.70, 0.60, 0.20)	(0.60, 0.10, 0.40)

As every attribute belongs to the same kind, moving to the next step without transforming is possible.

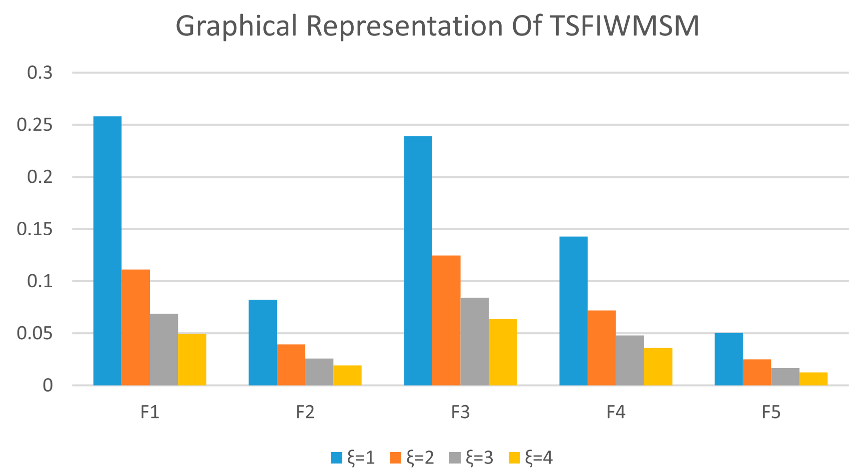

FIGURE 1
Demonstration of various ζ using TSFIWMSM in Table 6.

TABLE 6 Results of various ζ in the $TSFIWMSM^{(\zeta)}$ operator.

	$s(\breve{t}_1)$	$s(\breve{t}_2)$	$s(\breve{t}_3)$	$s(\breve{t}_4)$	$s(\breve{t}_5)$	Ranking	Optimal
$\zeta = 1$	0.25813	0.08209	0.23918	0.14275	0.05022	$F_1 > F_3 > F_4 > F_2 > F_5$	F_1
$\zeta = 2$	0.11112	0.03928	0.12458	0.07182	0.02493	$F_3 > F_1 > F_4 > F_2 > F_5$	F_3
$\zeta = 3$	0.06870	0.02573	0.08416	0.04793	0.01657	$F_3 > F_1 > F_4 > F_2 > F_5$	F_3
$\zeta = 4$	0.04928	0.01912	0.06353	0.03596	0.01241	$F_3 > F_1 > F_4 > F_2 > F_5$	F_3

Step 2: Add alternative ranking values $\breve{t}_{\tilde{\imath}_j} = (v_{\tilde{\imath}_j}, u_{\tilde{\imath}_j}, d_{\tilde{\imath}_j})$ ($\tilde{\imath} = 1, 2, \ldots, q$, $j = 1, 2, \ldots, m$) that have been normalized into collaborative units $\breve{t}_{\tilde{\imath}} = (v_{\tilde{\imath}}, u_{\tilde{\imath}}, d_{\tilde{\imath}})$. If the attribute weights are known, $\breve{t}_{\tilde{\imath}}$ are used to aggregate by using Eqs 21, 26.

Step 3: Evaluate the score value of $\breve{t}_{\tilde{\imath}}$.

Step 4: Utilizing the score function according to the technique in definition (3), rank $\breve{t}_{\tilde{\imath}}$.

Step 5: The optimal alternative should be chosen after ranking the other options using $\breve{t}_{\tilde{\imath}}$.

5.1 Application in digital green innovation

Green innovation is a process that involves developing green technologies, products, and processes as well as the accompanying companies, management, and systems. Digital innovation creates new possibilities for climate change mitigation and adaptation. Based on the Paris Agreement and the United Nations Framework Convention on Climate Change, a fundamental framework for the world's response to climate change has been formed. Green innovation" is technological innovation that involves energy conservation, pollution prevention, waste recycling, green product design, or corporate environmental management. These innovations can significantly lessen the negative effects on the environment while also generating value for the company and its stakeholders. How to improve the green competitiveness and profit of enterprises in the environment of climate change is an important issue. In this case, we need to choose the most talented

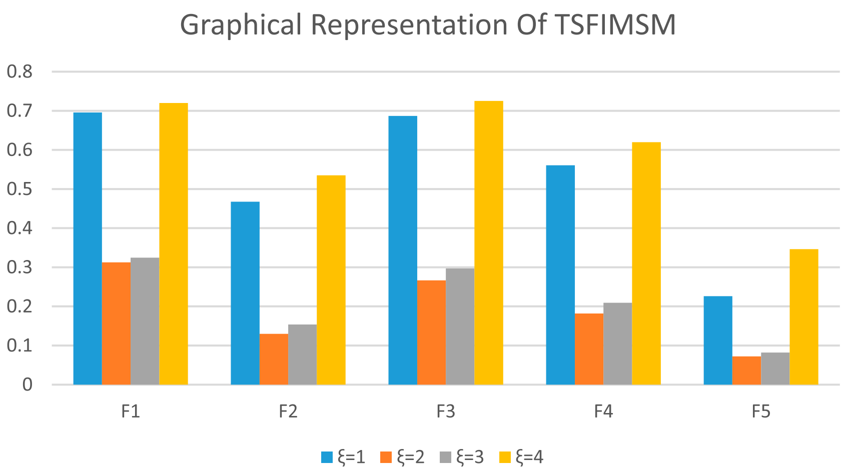

FIGURE 2
Graphical demonstration of various ζ by using the data in Table 7.

TABLE 7 Results of various ζ in the $TSFIMSM^{(\zeta)}$ operator.

	$s(\breve{t}_1)$	$s(\breve{t}_2)$	$s(\breve{t}_3)$	$s(\breve{t}_4)$	$s(\breve{t}_5)$	Ranking	Optimal
$\zeta = 1$	0.69562	0.46773	0.68705	0.56086	0.22611	$F_1 > F_3 > F_4 > F_2 > F_5$	F_1
$\zeta = 2$	0.31279	0.12983	0.26658	0.1819	0.07227	$F_1 > F_3 > F_4 > F_2 > F_5$	F_1
$\zeta = 3$	0.32463	0.15368	0.29714	0.20925	0.08197	$F_1 > F_3 > F_4 > F_2 > F_5$	F_1
$\zeta = 4$	0.7201	0.53496	0.72523	0.61968	0.34646	$F_3 > F_1 > F_4 > F_2 > F_5$	F_3

TABLE 8 Aggregated matrix of the score value using Tables 6, 7.

Operator	$s(\breve{t}_1)$	$s(\breve{t}_2)$	$s(\breve{t}_3)$	$s(\breve{t}_4)$	$s(\breve{t}_5)$
$TSFIWMSM^{(3)}$	0.0687	0.0257	0.08416	0.04793	0.01657
$TSFIMSM^{(3)}$	0.32463	0.15368	0.29714	0.20925	0.08197

TABLE 9 Ranking of scores utilizing the aforementioned operators.

Operator	Ranking of the score
$TSFIWMSM^{(3)}$	$\breve{t}_3 > \breve{t}_1 > \breve{t}_4 > \breve{t}_2 > \breve{t}_5$
$TSFIMSM^{(3)}$	$\breve{t}_1 > \breve{t}_3 > \breve{t}_4 > \breve{t}_2 > \breve{t}_5$

entrepreneurs from a large number of alternative projects to invest in. The criteria and methods presented in this paper are applicable to the selection of DGI investment projects.

Example 1: Pakistan has been listed as one of the top 10 nations in the world most impacted by climate change. The country's environmental sustainability strategy is managed by the government. All people concerned must take responsibility for maintaining the ecosystem and its resources, both the governmental and private organizations and particular people. To encourage the development of ideas and solutions for some of the most serious issues we face, the National Incubation Center regularly hosts tech conferences and innovation challenges. It is an honor to have teamed up with Pak Mission Society to find solutions to issues related to climate change, given the situation of the environment at the moment. We will be asking Pakistan's most talented entrepreneurs $F_m = \{F_1, F_2, F_3, F_4, F_5\}$ for business ideas based on these attributes:

Based on the theory, this study constructs a framework system for enterprises to choose DGI investment projects.

C_1; Green entrepreneurs should also have perceived credibility, bravery, and intellectual abilities.
C_2**:** They also possess planning and time management abilities, which are essential for the profession because they must remember particular rules and knowledge.
C_3**:** They should have effective interpersonal and communication skills and good management and entrepreneurial abilities.
C_4**:** They must be able to think strategically and work for sustainability and the planet's future alongside our future generations.

Solving the numerical example using decision-making steps is the best way to get the optimal solution.

Step 1: The TSFV is used to calculate the alternatives concerning the attributes, and the decision matrix $D = (\breve{t}_{\tilde{\imath}_j})_{5\times 4}$ is created and shown in Table 5.
Step 2: The attribute weight vector is assumed to be. We utilize Eq. 26 to calculate each alternate collective evaluation value. In this case, we use $TSFIWMSM^{(\zeta)}$ operator with $\zeta = 3$.

Figure 1 is the graphical representation of the TSFIWMSM operator using Table 6, which shows that when $\zeta = 1$, the optimal alternative is F_1 and for all $\zeta = 2$, 3, and 4, the optimal alternative is F_3.

Figure 2 is the graphical representation of the TSFIMSM operator using Table 7, which shows that when $\zeta = 1$, 2, and 3, the optimal alternative is F_1, and when $\zeta = 4$, the optimal alternative is F_3.

Step 3: Eq. 2 can be used to determine the scores $s(\breve{t}_{\tilde{\imath}})$ $(\tilde{\imath} = 1, 2, \ldots, 5)$ of the TSFIWMSM and TSFIMSM operator to obtain Table 8.
Step 4: According to the ranking of scores, $\breve{t}_{\tilde{\imath}}$ $(\tilde{\imath} = 1, 2, \ldots, 5)$ can be ranked as Table 9.
Step 5: Ranking of the alternatives would be

5.2 Parameter sensitivity and comparative analysis

This section discusses the impact factor of various ζ using TSFIWMSM and TSFIMSM operators. Here, we also discuss the comparison of proposed operators with existing operators.

TABLE 10 Ranking of alternatives utilizing the aforementioned operators.

Operator	Ranking of alternatives	Optimal alternative
$\boldsymbol{TSFIWMSM^{(3)}}$	$F_3 > F_1 > F_4 > F_2 > F_5$	F_3
$\boldsymbol{TSFIMSM^{(3)}}$	$F_1 > F_3 > F_4 > F_2 > F_5$	F_1

TABLE 11 Influence of the parameter.

	Method	Ranking
$\boldsymbol{\zeta = 3}$, ƚ $\boldsymbol{= 4}$	TSFIWMSM	$F_3 > F_1 > F_4 > F_2 > F_5$
$\boldsymbol{\zeta = 3}$, ƚ $\boldsymbol{= 6}$	TSFIWMSM	$F_1 > F_3 > F_4 > F_2 > F_5$
$\boldsymbol{\zeta = 3}$, ƚ $\boldsymbol{= 4}$	TSFIMSM	$F_1 > F_3 > F_4 > F_2 > F_5$
$\boldsymbol{\zeta = 3}$, ƚ $\boldsymbol{= 6}$	TSFIMSM	$F_1 > F_3 > F_4 > F_2 > F_5$

TABLE 12 Comparison of various proposed and existing aggregation operators.

	$s(\breve{t}_1)$	$s(\breve{t}_2)$	$s(\breve{t}_3)$	$s(\breve{t}_4)$	$s(\breve{t}_5)$	Ranking	Optimal
TSFIA	0.1346	0.037	0.1444	0.055	0.0107	$F_3 > F_1 > F_4 > F_2 > F_5$	F_3
TSFIWA	0.1789	0.028	0.1406	0.052	0.01	$F_1 > F_3 > F_4 > F_2 > F_5$	F_1
TSFIGA	0.138	0.038	0.145	0.055	0.011	$F_3 > F_1 > F_4 > F_2 > F_5$	F_3
TSFIWGA	0.183	0.029	0.141	0.052	0.01	$F_1 > F_3 > F_4 > F_2 > F_5$	F_1
GTSFIA	0.1303	0.036	0.1442	0.055	0.0263	$F_3 > F_1 > F_4 > F_2 > F_5$	F_3
GTSFIWA	0.5475	0.091	0.4541	0.185	0.0384	$F_1 > F_3 > F_4 > F_2 > F_5$	F_1
TSFWA	0.2152	0.0376	0.15044	0.0559	0.0126	$F_1 > F_3 > F_4 > F_2 > F_5$	F_1
TSFWG Mahmood et al. (2019)	0.0015	−0.0006	0.01189	0.0201	−0.0008	$F_4 > F_3 > F_1 > F_2 > F_5$	F_4
GTSFWG Chen et al. (2021)	0.0197	0.00816	0.01698	0.02572	0.00197	$F_4 > F_1 > F_3 > F_5 > F_2$	F_4
TSFWMSM Garg et al. (2022)	0.44209	0.40728	0.42396	0.46983	0.30623	$F_4 > F_1 > F_3 > F_2 > F_5$	F_4
TSFMSM Garg et al. (2022)	0.09325	0.03232	0.09417	0.04896	0.00877	$F_3 > F_1 > F_4 > F_2 > F_5$	F_3

5.3 Impact of the parameter ζ

In the $TSFIWMSM^{(\zeta)}$ operator, other $\boldsymbol{\zeta}$ can also be taken into consideration. Table 6 shows the outcomes if other ζ is taken into consideration. The ranking for $\zeta = 2, 3, 4$ is similar and F_3 is the optimal choice. If $\zeta = 1$, then F_1 reduces to the optimal choice and F_3 reduces to the suboptimal alternative. In actuality, the $TSFIWMSM^{(\zeta)}$ The operator becomes the TSFIGA operator when $\zeta = 1$, and the interrelation among arguments has not been considered. Outcomes of different rankings are, therefore, valid.

Table 7 shows the outcomes of utilizing the $TSFIMSM^{(\zeta)}$ operator in the aggregation technique in Step 2. Utilizing the $TSFIWMSM^{(\zeta)}$ operator, except $\zeta = 1$, the optimal alternative is F_1, whereas by using the $TSFIMSM^{(\zeta)}$ operator, when $\zeta = 4$, the optimal alternative is F_3. As different attribute weights might reflect the significance of various attributes, the outcomes are valid. Following the actual requirements and the features of the decision problems, decision makers can choose the attribute weights.

5.4 Sensitivity of the parameter ƚ

This section examines the sensitivity of the relevant parameter and how it affects the aggregation outcomes. To see the effects, we change the variable parameter and present the ranking outcomes in Table 11.

The TSFIMSM and TSFIWMSM operators also allow for the consideration of other ƚ. Table 11 shows the outcomes when ƚ is taken into account. For $\zeta = 3$, if we take ƚ $= 4$, the optimal alternative of TSFIWMSM is F_3 and if we take ƚ $= 6$ and so on, the ranking value is the same and the optimal alternative is F_1. For the TSFIMSM operator when $\zeta = 3$ we take ƚ $= 4$ and so on, the ranking value is the same and the optimal solution is F_1.

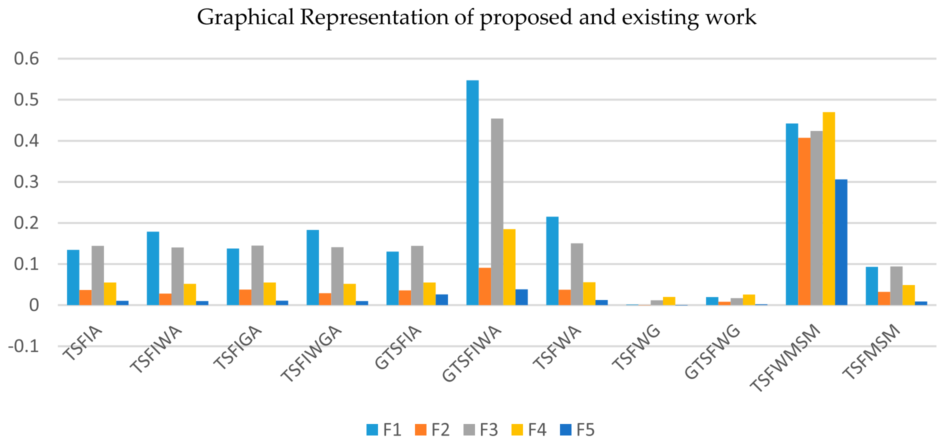

FIGURE 3
Graph illustrating the comparison between the proposed and existing operator utilizing the information in Table 12.

6 Comparison

The interrelation among membership, abstinence, and non-membership is not considered if operations of TSFVs in Definition 2 are utilized in the aggregation procedure. In Step 2, the aggregation operations mentioned previously are utilized, and the outcomes are shown in Table 12. It is clear from the outcomes that these differ from those of the TSFI operators, including the TSFIA, TSFIWA, TSFIGA, TSFIWGA, GTSFIA, and GTSFIWA operators, which are displayed in Table 12. With regards to the attribute C_2, the evaluation value of alternative F_1 is $(0.90, 0.70, 0.0)$, satisfying $\zeta = 2$ and $ɫ = 9$. If the operational laws in Definition 2 are utilized, all of the non-memberships in F_1 have no impact on the outcomes in the aggregated operators including the TSF weighted averaging (TSFWA) operator, TSF weighted geometric (TSFWG) operator (Mahmood et al., 2019), generalized TSFWA (GTSFWA) operator (Chen et al., 2021), generalized TSFWG (GTSFWG) operator, TSFMSM operator (Garg et al., 2022), and TSFWMSM operator (Garg et al., 2022), which are not reasonable. TSFI operators have been created to overcome the limitations. As a result, the outcomes obtained by utilizing TSFI operators are more logical. TSFVs are used to represent the evaluation values, which are more flexible than existing operators to describe uncertain and fuzzy information. Using the MSM, the relationship between more than two input arguments has been considered. The relationship between MD and NMD has been viewed by utilizing the new operation laws. Thus, the proposed method can produce more logical and scientific decision-making outcomes.

It is worth noticing that the results obtained using different operators are different. This is because of the nature of the operators and the parameters involved. For instance, the TSFIA operator has no associated weight vector, while the TSFIWA operator has a weight factor. Similar is the case with TSFIG and TSFWIG operators. However, the results of Tables 10–12 clearly show that the operators proposed in the paper have better results because of their advanced nature as the MSM operators proposed in Garg et al. (2022) have certain limitations which were discussed at the beginning of the paper. Furthermore, we have also compared the proposed work with Spearman's rank correlation coefficient measures, which was proposed by Sedgwick [Sedgwick, P. (2014) Spearman's rank correlation coefficient. *Bmj*, *349*.] in 2014, which is very valuable and dominant measures based on classical set theory. For these measures, it is not possible to evaluate the proposed types of data because the arranged information is given in the shape of T-spherical fuzzy numbers, and the existing measure was computed based on the crisp set which is the special case of the proposed theory, but in the future, we aim to derive it for T-spherical fuzzy sets to improve the worth of the proposed theory.

Figure 3 is the graphical representation of the proposed and existing operators utilizing Table 12, making it easy to understand the optimal alternative. Figure 3 graphically shows that F_3 is the optimal alternative to TSFIA, TSFIGA, GTSFIA, and TTSFMSM. F_1 is the optimal alternative of TSFIWA, TSFIWGA, GTSFIWA, and TSFWA, and F_4 is the optimal alternative to TSFWG, GTSFWG, and TSFWMSM.

6.1 Advantages of the proposed operator

The following section will elaborate on the method's superiority in dealing with situations of uncertainty by examining its properties in the context of some previous studies. As we all know, the traditional operator does not consider the interaction between MD, NMD, and AD with $ɫ \in \mathbb{Z}^+$ of different TSFVs, so they can get unreasonable results in some special cases, especially when abstinence and non-memberships are 0. On the contrary, the interaction operational rules can consider the interactions between MD, NMD, and AD sufficiently, so the method proposed in this paper is more reasonable to produce the ranking result because it can overcome the weakness when abstinence and non-memberships are 0. Hence, in today's complicated policymaking and risk management-related issues, data aggregation will be managed and useful for dealing with decision-making issues. The proposed operators reduce to earlier versions if we employ certain cases and are given in Remark 1:

Remark 1: The results obtained in Theorem 3 and Theorem 5 reduce to the framework of

1) SFSs if we place ƙ = 2.
2) PFSs if we place ƙ = 1.
3) qROFSs if we place $u(r) = 0$.
4) PyFSs if we place $u(r) = 0$ and ƙ = 2.
5) IFSs if we place $u(r) = 0$ and ƙ = 1.

Because of Remark 1, we claim that the proposed work is more generalized than the previous one and the other fuzzy frameworks cannot be applied to the current example because of the diverse nature of the information.

7 Conclusion

In this section, we will learn about the following operators and techniques, such as

1. Based on the interaction operations for TSFVs, this research suggested several interaction MSM operators for TSFVs and generalized the MSM operator to TSFVs. After that, we talked about several of their desirable traits, like idempotency and commutativity.
2. An approach for the MADM problems was also proposed under the consideration of derived operators.
3. We further examined some particular examples of these operators and compared the presented results with those of many existing operators to justify the validity and supremacy of the stated approaches.
4. While comparing the current operators, we know that the proposed method is more extensive than several other existent methods.

The primary benefit is that they can capture the relationships between several flexible input arguments due to the parameter ƙ. Moreover, they can consider how the MD, AD, and NMD of TSFVs interact to help solve various issues when one of the NMDs is 0. Using these operators' applications in recent studies is necessary to address decision-making issues. In future, we aim to target the derived theory and try to utilize them in the field of modern information fusion theory, artificial intelligence, machine learning, evaluations about resources and the environment, and decision-making analysis.

Data availability statement

The original contributions presented in the study are included in the article/Supplementary Material; further inquiries can be directed to the corresponding authors.

Author contributions

All authors listed have made a substantial, direct, and intellectual contribution to the work and approved it for publication.

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Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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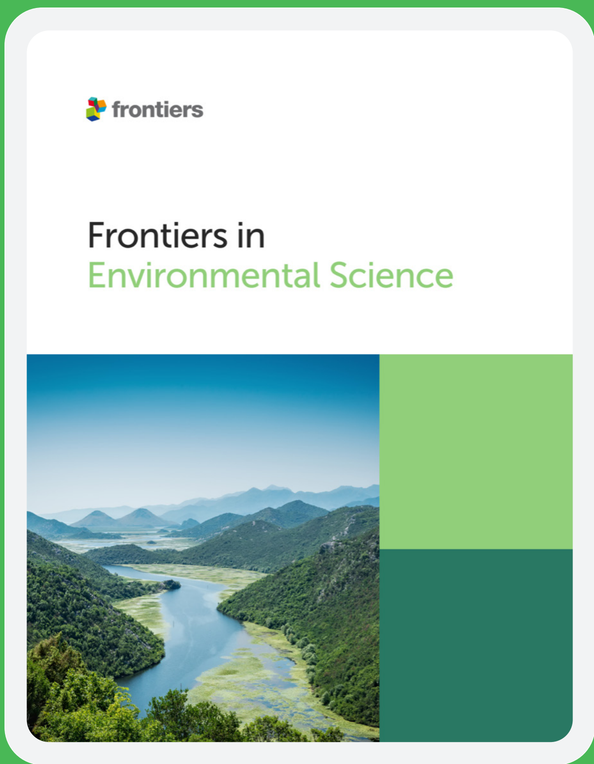

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