INTERTEMPORAL CHOICE AND ITS ANOMALIES

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INTERTEMPORAL CHOICE AND ITS ANOMALIES

Topic Editors:
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This Research Topic of the journals *Frontiers in Applied Mathematics and Statistics* and *Frontiers in Psychology* on "Intertemporal choice and its anomalies" has collected ten manuscripts on several fields, which demonstrates that this topic is of interest for many important research activities outside of traditional domains of economics and finance. In effect, the concepts of time preference, rationality and time-inconsistency in intertemporal choice can be applied to a wide variety of social problems such as addictions, retirement plans, and health, among others. On the other hand, the main anomalies or paradoxes in intertemporal choice (such as delay effect, sign effect, magnitude effect, delay-speedup asymmetry, and sequence effect) have been considered, as manifestations of many problematic behaviors of self-control in intertemporal choice.

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EDITORIAL
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Editorial: Intertemporal Choice and Its Anomalies

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Keywords: intertemporal choice, anomaly, behavioral economics, time preference, time-consistency

Editorial on the Research Topic

Intertemporal Choice and Its Anomalies

This special issue on "Intertemporal choice and its anomalies" collected up-to-date papers on the topic from various research disciplines: behavioral economics, computer science, mathematical, cognitive and cultural psychology, behavioral ecology, and econophysics. This issue demonstrates that the studies of intertemporal choice and its anomalies have been extended into important research activities outside of traditional domains of economics.

In economics, intertemporal choice has been modeled with the concept of time preference, and rationality (i.e., time-consistency) in intertemporal choice has also been assumed. This conceptualization was shown to be useful to investigate a wide variety of social problems such as addiction, retirement plans, and health-maintaining behaviors even when the violation of the time-inconsistency was not taken into account. Recent developments in behavioral economics suggest that once irrationality in intertemporal choice (anomalies such as hyperbolic discounting or delay effect, sign effect, magnitude effect, delay-speedup asymmetry, and sequence effects) were seriously considered, many problematic behaviors can be considered as manifestations of self-control problems in intertemporal choice.

This issue demonstrates that behavioral economic approaches to self-control problems in intertemporal choice have been attracting wide attention from diverse disciplines of science. The editors are pleased to witness this spread of interest into broad research areas and we hope the movement speed up without temporal discounting over time.

Ogura et al. introduce a model in the ambit of foraging ecology which predicts that the profitability of a smaller-sooner (SS) food option can be higher than that of the larger-later (LL) alternative, depending on the duration in which the producer can monopolize a food patch. Their paper includes numerical simulations on the assumption of variable food amount in each patch involving realistic behavioral parameters.

Cruz Rambaud and Sánchez Pérez reveal the existence of several anomalies or paradoxes in the context of EU and DU models affecting the amount of the reward: the peanuts and the magnitude effects, respectively, which seem go in opposite directions. Their paper jointly analyses both effects in a wide setting involving choices under risk and over time, searching implications between both anomalies.

On the other hand, Cruz Rambaud et al. formalize mathematically the concept of improving sequence effect. More specifically, they prove that the improving sequence effect for monetary rewards must be necessarily rationalized by using a non-separable discount function. Moreover, under certain conditions, they demonstrate that the delay and the magnitude effects are necessary (but not sufficient) conditions for the existence of the improving sequence effect.

Tsuruta and Inukai aim to assess the role of group intertemporal decision-making. They experimentally investigate how to aggregate individual time preferences by clarifying who has the most influence on group decisions among heterogeneous group members. They formulate two hypotheses: the multilateral bargaining hypothesis, which is based on the multilateral bargaining model, and the median voter hypothesis.

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The main contribution of this paper is that the median patient member in a group has a significant impact on group decisions in an unstructured bargaining situation.

dos Santos and Martinez derive inconsistency as the result of a subjective time dilation perception effect, inspired by the special theory of relativity. They focus on a generalized model which encompasses psychophysical effects on time perception, by proposing a transformation of the time interval between the pay times of two rewards. As a result, they present a generalized two-argument hyperbolic utility function for the Bernoulli (logarithmic) one.

Takemura and Murakami present a test to examine various models of probability weighting functions which are considered non-linear functions of probability in behavioral decision theory. On the other hand, they propose some axiomatic properties and a test to specifically examine the generalized hyperbolic logarithmic model, power model, and exponential power model of the probability weighting functions.

Kim and Takahashi state that, in computational theory of learning, the multi-armed bandit problem is one of the most intensively studied decision problems with unknown rewards and probability distributions. In economic decision theory, two types of uncertainty, i.e., risk and ambiguity (also referred to as Knightian uncertainty) have been distinctly formulated. Risk is uncertainty with known probability distributions, while ambiguity is uncertainty with unknown distributions. The latter is closely linked with the multi-armed bandit problem. In the study, we employed several state-of-the-art computational learning algorithms to resolve the optimization problem of decision under ambiguity. This computational approach may be unified with behavioral economic approaches to ambiguity problem in the future.

Ishii and Eisen start from the idea of social discounting. They carry out two studies. The first one showed that compared to North Americans, Japanese discount more steeply a partner's outcomes compared to their own future outcomes, whilst the decrease in the subjective value of the partner's outcomes accelerates less as a function of social distance. The second study tested Japanese and Germans and found that the hyperbolic with exponent model fitted the participants' discounting behaviors better than the other models.

Attema and Lipman implement a recently introduced measure of deviations from constant impatience, called the "Decreasing Impatience (DI)-index," to estimate the degree to which people deviate from constant impatience. They observe that decreasing impatience is the modal preference, although constant and increasing impatience are no exceptions. Furthermore, the DI-index is higher for individual health outcomes than for societal health outcomes, but is not distributed differently among the three classes of discounters.

Li et al. analyse how future time perspective (FTP) affects intertemporal choice. To do this, the conducted an experiment where all participants completed a series of intertemporal choice tasks, in which they chose from gain- and loss-related choices occurring at various time points. Results showed that the participants who received the future-imagining manipulation had more limited FTP. The participants in the limited FTP condition had higher discount rates on gain-related choices but showed no difference on loss-related choices.

AUTHOR CONTRIBUTIONS

All authors listed have made a substantial, direct and intellectual contribution to the work, and approved it for publication.

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ORIGINAL RESEARCH
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Future Time Perspective Impacts Gain-Related but Not Loss-Related Intertemporal Choice

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Future time perspective (FTP) modulates individuals' temporal orientation in selecting their motivations and goals, which widely influences their cognitions and behaviors. However, it remains unclear how FTP exactly affects intertemporal choice. To clarify the effect of FTP on intertemporal choice, 90 college students ($M_{\text{age}} = 21.70$, $SD = 1.23$) were randomly assigned to the limited FTP condition (16 males, 29 females) and the open-ended FTP condition (17 males, 28 females). In the limited FTP condition, participants were instructed to imagine their states of being 70 years old, whereas in the open-ended FTP condition, they were instructed to describe their current states. All participants then completed a series of intertemporal choice tasks, in which they chose from gain- and loss-related choices occurring at various time points. Results showed that the participants who received the future-imagining manipulation had more limited FTP compared with those who did not receive the manipulation, which confirmed the validity of the FTP manipulation. A 2 (FTP: limited vs. open-ended) × 2 (type of choice: gain vs. loss) repeated measures ANOVA on discount rate revealed a significant interaction between these two factors. The participants in the limited FTP condition had higher discount rates on gain-related choices but showed no difference on loss-related choices compared with the participants under the open-ended FTP condition. The results suggest that limited FTP could lower individuals' future orientation (i.e., willingness to delay an outcome) on gain-related, but not on loss-related, intertemporal decision-making.

Keywords: future time perspective, imagine future, intertemporal choice, gain-related choice, loss-related choice, discount rate

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INTRODUCTION

Intertemporal choice involves tradeoffs among costs and benefits occurring at different time points (Frederick et al., 2002). A typical paradigm on intertemporal choice is to ask people to choose between sooner and later gains. People tend to choose the sooner gains, although the later gains are larger in size (Frederick et al., 2002; Green and Myerson, 2004; Berns et al., 2007).

A series of elegant mathematical models have been proposed by economists and psychologists to interpret such a preference for sooner gains (Frederick et al., 2002), such as the exponential discounting model (Samuelson, 1937) and hyperbolic discounting model (Ainslie, 1975). One common idea in these models is that the subjective value or utility of an outcome would be mentally discounted by decision-makers when the outcome is delayed. The degree of discounting can be indexed by a discount rate—a larger discount rate indicates a higher degree of discounting, which means that a sooner gain is more preferred over a later gain (Frederick et al., 2002).

Delay discounting happens not only to gains but also to losses. Discounting of future losses and gains could be described in similar discounting functions (Loewenstein, 1987; Estle et al., 2006). However, losses are usually discounted at lower rates compared with gains, which is termed the sign effect or gain-loss asymmetry (Loewenstein and Prelec, 1992; Frederick et al., 2002; Xu et al., 2009). Loss aversion from the prospect theory (Kahneman and Tversky, 1979) has been applied to interpret such an effect. Loss aversion illustrates that losses have a larger psychological impact compared with gains of the same size, which means that the psychological impact of delayed losses is also larger than delayed gains of the same size (Loewenstein and Prelec, 1992; Frederick et al., 2002). The sign effect also suggests that subjective values of losses are less influenced by delay compared with gains.

As delay discounting involves evaluation and choice of outcomes that will happen in the future, perception of future time is particularly relevant to intertemporal decision-making. Investigations on how perception of speed, length, concomitant cost, and risk of time delay influence intertemporal choice have shown that the temporal discount rate would be higher when a same period of delay is perceived to be slower, longer, more costly, or more risky (Frederick et al., 2002; Löckenhoff et al., 2011).

Future time perspective (FTP), as a critical component of time perception, can also influence delay discounting (Guo et al., 2017). The socioemotional selectivity theory (SST; Carstensen et al., 1999) asserts that individuals' orientation of life goals is associated with their FTP. In the context of SST, FTP specifically refers to individuals' subjective perception of the open-endedness of their future time. According to SST, people prioritize future-oriented goals (e.g., acquisition of knowledge) and distal outcomes (e.g., a bright future) more when they perceive their future time as open-ended, whereas people prioritize present-oriented goals and immediate outcomes (e.g., fulfillment of emotional satisfaction) more when they perceive their future time as limited. As people grow older, they perceive future time as increasingly limited, and thus, they gradually change their life goals from future- to present-oriented. The age-related transition in goal orientation resulting from FTP change has been verified, and moreover, it brings widespread and pervasive effects onto cognitions and behaviors, such as attention, memory, social interaction, and decision-making (e.g., Carstensen et al., 1999; Reed and Carstensen, 2012).

According to SST, FTP can affect intertemporal decision-making such that older adults are more present oriented compared with younger adults when making intertemporal choices. Empirical studies have shown that older adults had lower discount rates (i.e., more future oriented) compared with younger adults did (Green et al., 1999; Harrison, 2002; Read and Read, 2004; Reimers et al., 2009; Simon et al., 2010; Jimura et al., 2011; Löckenhoff et al., 2011), which seems to contradict the prediction of SST. One possibility is that age difference in discount rate might be confounded by multiple factors. Indeed, the psychological motives underlying intertemporal choice are complex, including not only perception of time but also factors related to intelligence (Shamosh and Gray, 2007), personality (Wittmann and Paulus, 2008), and sensitivity to rewards (Samanez-Larkin et al., 2011). All these factors could be related to age difference in discount rate. To clarify the effect of FTP on discount rate, the effects of these age-related confounding factors need to be controlled. To achieve the purpose, the present study experimentally manipulated younger adults' FTP to examine its effect on intertemporal choice.

To control for age-related confounding factors when examining the effect of FTP on intertemporal choice, the present study recruited younger adults only and experimentally manipulated their FTP to see how FTP manipulation alters their discount rate during intertemporal choice. Empirical studies have demonstrated that participants' FTP could be manipulated by asking them to imagine different scenarios relevant to the open-endedness of future time, such as to imagine a limited or expansive future (Fredrickson and Carstensen, 1990; Fung et al., 1999; Valero et al., 2015). To foreshorten participants' FTP in the current study, we instructed them to imagine and describe their states of themselves being 70 years old (Ye, 2014).

As limited FTP leads to more focus on present-oriented outcomes, and open-ended FTP leads to more future-oriented outcomes (Carstensen et al., 1999; Freund and Baltes, 2008), we postulated that

Hypothesis 1. Participants with foreshortened FTP (in the limited FTP condition) would have higher temporal discount rates compared with participants who received no FTP manipulation (in the open-ended FTP condition).

As described above, the sign effect, or say, gain-loss asymmetry, in intertemporal choice (e.g., Thaler, 1981; Loewenstein and Prelec, 1992) suggests that loss may be less affected by time perception. We thus expected that

Hypothesis 2. The effect of FTP on temporal discount rate would be smaller for losses than for gains.

MATERIALS AND METHODS

Participants

Participants were 90 college students from Hubei University in China (M_{age} = 21.70, SD = 1.23). They were randomly assigned to the limited FTP condition (16 males, 29 females, M_{age} = 21.49, SD = 1.06) and open-ended FTP condition (17 males, 28 females, M_{age} = 21.84, SD = 1.36). Eight additional participants were excluded, including five who failed in following the instructions and three who did not complete the experiment. The present study was approved by the Ethics Committee of the Faculty of Education in Hubei University in terms of the ethics and safety of psychological experiments. Written informed consent was obtained from all participants. Each participant was paid ¥20 (∼$3.1) at the end of the experiment.

Materials

FTP Scale

The Chinese version of the FTP scale (Fung et al., 2001; Cronbach's alpha = 0.76) was used to measure subjective perception of future time. The scale consists of 10 items (an example item is "Many opportunities await me in the future"). Participants rated the items on a five-point Likert scale (from 1 = "very untrue" to 5 = "very true"). A higher total score indicates that future time is perceived as more open-ended. The Cronbach's alpha was 0.72 in the present study.

Guidance for Imagination of Future

To make FTP limited, participants were asked to imagine and describe their states of health, cognition, and emotion at the age of 70 years according to the guidance developed by Ye (2014). The guidance includes four open-end questions: (1) "Please imagine and describe your health status when you are 70 years old"; (2) "Please imagine and describe your daily life when you are 70 years old"; (3) "Please imagine and describe your emotional changes when you are 70 years old"; (4) "Please imagine and describe changes in your abilities of cognition, memory, and thinking." Participants' answer to each question should consist of 50 words at least. FTP was measured by the Chinese version of the FTP scale after the imagination to check validity of the manipulation.

Intertemporal Decision-Making Task

Participants needed to make a series of choices between an immediate gain (or loss) of ¥1000 ($157.7) and a delayed (i.e., 2 months later) gain (or loss) of ¥1050, 1100, 1150, 1250, 1350, 1500, 1700, 1950 ($165.5, 173.4, 181.3, 197.0, 212.8, 236.4, 268.0). The amounts of gain and loss, as well as the length of time interval, were determined according to Tao et al. (2015), which reported that these were sensitive for the detection of experimental effects among young Chinese participants.

Experimental Design and Procedure

The current study adopted a 2 (FTP: limited vs. open-ended) × 2 (type of choice: gain vs. loss) experimental design, with FTP as a between-subject variable and type of choice as a within-subject variable.

Participants were randomly assigned to either the limited FTP or open-ended FTP condition. Participants in the former condition received FTP manipulation (i.e., imagining their future states), whereas those in the latter condition were asked to describe their current states by four questions similar to the guidance for imagination: (1) "Please describe your current states of health"; (2) "Please describe your daily life"; (3) "Please describe your emotional states"; (4) "Please describe your abilities in cognition, memory, and thinking." Then, all participants completed the FTP scale. They then turned to 12 gain-related and 12 loss-related intertemporal choice tasks, which were presented on a computer screen by E-Prime 2.0.

In the gain-related intertemporal-choice tasks, the description of the situation reads:

"Suppose that you have participated in a rewarding social activity, and you have two options to get your monetary reward: (1) receive it now; (2) receive it 2 months later. The amounts of money are different in these two options. Please make a choice that you prefer in each of the follow-up pairs of options."

In the loss-related intertemporal-choice tasks, the description of the situation reads:

"Suppose that you have made a serious mistake in a group activity, and you have to compensate for it by paying money. You have two options to pay: (1) pay it now; (1) pay it 2 months later. The amounts of money are different in these two options. Please make a choice that you prefer in each of the follow-up pairs of options."

The immediate and delayed options were presented on the left or right side of the computer screen randomly. The order of gain and loss was counterbalanced across subjects (the procedure details are given in **Figure 1**).

Data Analyses

Participants' preference for immediate or delayed gain/loss was indexed by the temporal discount rate originated from the hyperbolic discounting function: $V_d = V/(1+kd)$, where V_d is the subjective value after discounting, V is the objective value without discounting, k is the discount rate, and d is the length of delay (Mazur and Coe, 1987; Frederick et al., 2002; Kazuhisa and Hajime, 2016). To obtain the discount rate (k) for each participant, we first identified his/her switching point in the series of intertemporal choice tasks: the point where he/she changed

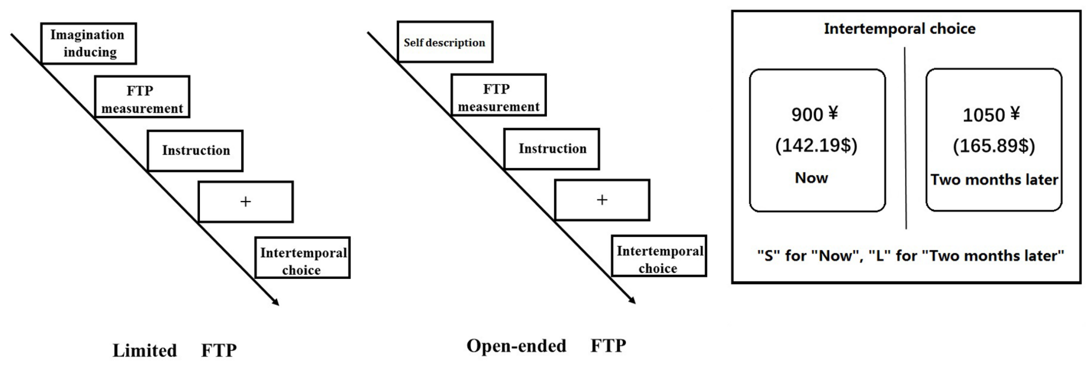

FIGURE 1 | The procedure of the experiment under open-ended vs. limited FTP condition. The choice pairs were presented in random order within the gain/loss condition. The positions (left or right) of the immediate choices and delayed choices were pseudo-randomized, such that the immediate choices were presented on the left side in half of the trials but on the right side in the other half of the trials.

choice from the immediate to a delayed option, or from a delayed to the immediate option. At this switching point, the immediate (representing V_d in the hyperbolic discounting function) and delayed outcomes (representing V) had the same subjective value for the certain participants. With these values, the discount rate (k) could be calculated for each participant based on the hyperbolic discounting function. All discount rates (ks) were then submitted to SPSS 22 for a 2 (FTP: limited vs. open-ended) × 2 (type of choice: gain vs. loss) mixed design, repeated measures ANOVA.

The discount rate is often not normally distributed (e.g., Jones and Rachlin, 2006; Margittai et al., 2015), which violates the assumption of ANOVA. To confirm the reliability of results, we repeated the ANOVA for discount factor f (i.e., the immediate value divided by future value at the switching point), which is usually normal distributed.

RESULTS

Demographics

Independent t-tests showed no significant difference in the participants' age between the limited FTP and open-ended FTP conditions, $t = -1.38$, $p = 0.17$, Cohen's $d = 0.29$. No significant difference was found in the level of monthly living consumption between conditions [for the limited FTP condition, M = ¥1,153.33 ($181.74), SD = ¥209.54 ($33.02); for the open-ended FTP condition, M = ¥1,235.56 ($194.70), SD = ¥295.54 ($46.57); $t = -1.52$, $p = 0.13$, Cohen's $d = 0.32$].

FTP Manipulation Check

The mean score of FTP measured after manipulation was 28.64 ($SD = 6.51$) in the limited FTP condition and 35.11 ($SD = 4.29$) in the open-ended FTP condition. Independent t-tests showed that the latter condition had significantly higher FTP scores compared with the former condition ($t = -5.56$, $p < 0.01$, Cohen's $d = 1.19$), indicating that the limited FTP group had more limited FTP. The results confirmed the validity of the manipulation.

Analyses of Discount Rate

The discount rates (ks) for the different experimental conditions are as follows: for the limited FTP condition, mean $k_{\text{gain}} = 0.15$ ($SD = 0.12$) and mean $k_{\text{loss}} = 0.05$ ($SD = 0.09$); for the open-ended FTP condition, mean $k_{\text{gain}} = 0.08$ ($SD = 0.07$) and mean $k_{\text{loss}} = 0.03$ ($SD = 0.04$).

A 2 (FTP: limited vs. open-ended) × 2 (type of choice: gain vs. loss) repeated measures ANOVA on discount rates showed that the main effect of FTP was significant (i.e., higher in the limited FTP condition than in the open-ended FTP condition), $F_{(1,88)} = 11.68$, $p < 0.01$, $\eta_p^2 = 0.12$; the main effect of types of choice was significant (i.e., higher for gains than for losses), $F_{(1,88)} = 35.53$, $p < 0.01$, $\eta_p^2 = 0.30$; and the interaction effect was significant, $F_{(1,88)} = 4.31$, $p = 0.05$, $\eta_p^2 = 0.05$. As shown in **Figure 2**, simple effect analyses showed that the participants with limited FTP had higher discount rates for gains than those with open-ended FTP, $F_{(1,88)} = 11.39$, $p < 0.01$, $\eta_p^2 = 0.11$; the discount rates for loss between these two FTP conditions had no significant difference, $F_{(1,88)} = 2.11$, $p = 0.15$, $\eta_p^2 = 0.02$.

To confirm the reliability of results, we repeated the ANOVA for discount factor f (i.e., the immediate value divided by a future value at the switching point), which is usually normal distributed. Discount rates (fs) for different experimental conditions are as follows: for the limited FTP condition, mean $f_{\text{gain}} = 0.79$ ($SD = 0.02$) and mean $f_{\text{loss}} = 0.92$ ($SD = 0.02$); for the open-ended FTP condition, mean $f_{\text{gain}} = 0.87$ ($SD = 0.01$) and mean $f_{\text{loss}} = 0.95$ ($SD = 0.01$).

A 2 (FTP: limited vs. open-ended) × 2 (type of choice: gain vs. loss) repeated measures ANOVA on discount rates (fs) showed similar results with abovementioned: the main effect of FTP was significant, $F_{(1,88)} = 8.40$, $p < 0.01$, $\eta_p^2 = 0.09$; the main effect of types of choice was significant, $F_{(1,88)} = 46.75$, $p < 0.01$, $\eta_p^2 = 0.35$; the interaction effect was marginally significant, $F_{(1,88)} = 3.33$, $p = 0.07$, $\eta_p^2 = 0.04$. As shown in **Figure 2**, simple effect analyses showed that participants with limited FTP had a lower f value for gains than those with open-ended FTP, $F_{(1,88)} = 9.03$, $p < 0.01$, $\eta_p^2 = 0.10$. The discount (fs) for loss between these two FTP conditions had no significant difference, $F_{(1,88)} = 1.47$, $p = 0.23$, $\eta_p^2 = 0.01$.

DISCUSSION

By manipulating younger participants' FTP, the current study revealed that FTP modulates discount rates for gains, but not for losses. To be specific, participants under the limited FTP condition discounted gains more than their counterparts under the open-ended FTP condition, but no significant difference was seen in discounting of loss between these two conditions. The results partly confirm our hypotheses that limited FTP would lead to higher discount rates, and that the impact of FTP on discount rate would be higher for gains than for losses.

The results that limited FTP contributes to higher discount rates on gains indicate that perceived open-endedness of future time remarkably affects participants' intertemporal choice. The finding is consistent with the suggestion of SST (Carstensen et al., 1999) that open-ended FTP makes people focus more on future-oriented motivations, goals, and outcomes. When people have a long period of time ahead to live for, they tend to be more willing to delay gains to achieve more.

Older adults (with limited FTP) have been shown to have lower discount rates than younger adults (with open-ended FTP) (e.g., Harrison, 2002; Jimura et al., 2011), which seems to contradict the speculation derived from SST (Carstensen et al., 1999). We argue that these studies did not well control for other age-related confounding variables, and thus could not be used to infer the effect of FTP on intertemporal choice. The current study manipulated the FTP of younger adults to examine its effect on intertemporal choice, such that the confounding effects of age-related factors were clearly excluded. By doing so, we found that FTP indeed significantly influences participants' intertemporal choice. Moreover, emerging literature has explained the changes in FTP in different ages from the perspective of "psychological connectedness to the future self"

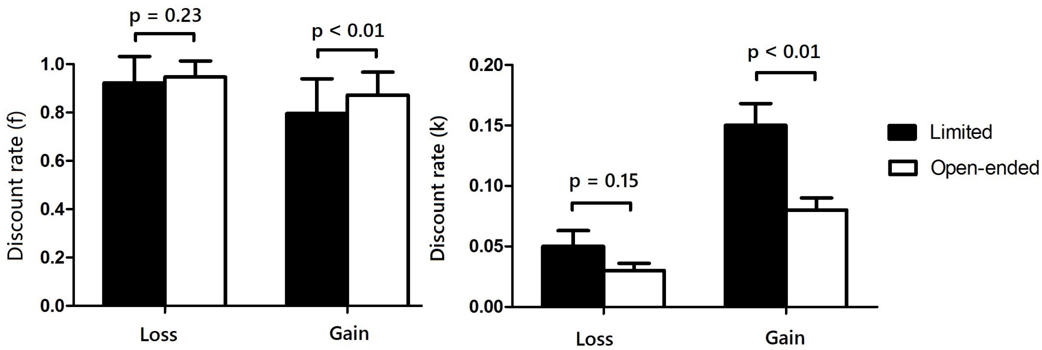

FIGURE 2 | Discount rates of intertemporal choices for gains and losses under limited vs. open-ended FTP condition.

(Urminsky, 2017). People are more likely to be "impatient" and prioritize the present over the future when they perceive a weak link between current and future self, compared with those who perceive a close link between current and future (Hershfield et al., 2011; Urminsky, 2017). In the present study, participants in the imagination group had more limited FTP compared with the control group, and thus, they might perceive a weaker connection between their present and future self so that they preferred instant rewards in decision-making.

We found no significant difference in the discount rates for losses between the limited and open-ended FTP conditions. This result did not verify our hypothesis that limited FTP would increase the discount rates for both gains and losses but supported the hypothesis that FTP impacts discounting of losses less than that of gains. The finding is compatible with the sign effect: people discount losses at lower rates compared with gains in intertemporal choice (Thaler, 1987; Loewenstein and Prelec, 1992). A small increase in loss might bring a psychological impact comparable in terms of size to a psychological impact brought by a larger increase in gain, as asserted by the prospect theory (Kahneman and Tversky, 1979).

It might be arbitrary to conclude that FTP did not impact the discount rate for losses based on our results. Although our settings for the intertemporal decision tasks (i.e., amount of gains/losses and length of delay) were found to be proper in the former studies (Liu et al., 2015; Tao et al., 2015), they may be not sensitive enough to catch the effects of FTP on discount rates for losses. To address this limitation in the current study, future studies may systematically change these settings to verify the impact of FTP on discounting of losses.

Future studies could manipulate older adults' FTP to examine the effect of FTP on intertemporal choice. Although manipulation of FTP results in similar patterns of cognitive and behavioral changes among younger and older adults (Fredrickson and Carstensen, 1990; Fung et al., 1999; Valero et al., 2015), it is unsure whether this is also the case for intertemporal choice. Future studies could also explore the effect of FTP in different domains. Most studies on intertemporal choice have focused on monetary gains and/or losses, whereas a few studies have revealed that people may discount other items (e.g., food) differently than money (e.g., Frederick et al., 2002). It is thus important to examine the robustness of the FTP effect on intertemporal choice across domains.

CONCLUSION

By experimentally manipulating younger participants' FTP, the current study found that limited FTP led to stronger temporal discounting on gains, but not on losses, compared with open-ended FTP. The finding suggests that FTP is more likely to impact intertemporal decisions on gains than on losses: people are less willing to delay gains when they perceive their future life time is limited. The study provides direct evidence on the relationship between FTP and discount rate in intertemporal choice. This finding contributes to reconciling the contradiction in the literature and supports SST, which asserts a strong relation between FTP and the temporal orientation of motivations and goals.

AUTHOR CONTRIBUTIONS

TL, XG, and SY designed the study. FQ participated in the data collection. TL, YT, and XG carried out the statistical analysis and wrote the paper. SY is the principal investigator of this project, and supervised the statistical analysis and the manuscript writing and revision. XH assisted with writing the article. All authors read and approved the final manuscript.

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Decreasing Impatience for Health Outcomes and Its Relation With Healthy Behavior

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There is a growing amount of literature suggesting people tend to behave inconsistently over time, which is driven by decreasing impatience. In addition, many studies have found relations between discounting estimates from experiments and field behavior, such as smoking cessation and dieting. However, these studies often did not separate time inconsistency from other factors such as utility curvature or the level of discounting. In order to establish the relation between field behavior and the degree of time inconsistency, it is therefore necessary to obtain a pure measure of the latter that is not distorted by these other factors. The present study implements a recently introduced measure of deviations from constant impatience, called the "Decreasing Impatience (DI)-index," to estimate the degree to which people deviate from constant impatience. We provide the first extension of DI to health outcomes, both for individual and societal discounting using three different starting points. Moreover, we include a survey gathering information about several health-related behaviors, in order to test for the relationship between the amount of decreasing impatience and healthy behavior. We observe that decreasing impatience is the modal preference, although constant and increasing impatience are no exceptions, and, hence, these types of discounters should not be neglected. Furthermore, the DI-index is higher for individual health outcomes than for societal health outcomes, but is not distributed differently among the three classes of discounters. The DI-index decreases with starting period for individual health outcomes, but not for societal health outcomes. Very few significant relations between time inconsistency and self-reported health-related behavior were found.

Keywords: decreasing impatience, health, increasing impatience, time inconsistency, time preference

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INTRODUCTION

Many daily decisions require an intertemporal trade-off between earlier and later consequences. These vary from savings for pensions, to learning for exams, to more exercise now to reduce the chance of becoming obese later. In these decisions, agents' discount rates play an important role. Economic theory predicts that the more agents discount the future, the less they will engage in future-oriented behavior, such as saving. During the last few decades it has become clear that besides the discount rate, the amount of time inconsistency is also highly relevant for many decisions. For example, heterogeneity in time inconsistency may explain why agents with the same absolute discount rate differ in their tendency to postpone an annoying task [1].

Because of their differential impact on intertemporal choices, it is crucial to disentangle time inconsistency and discount rates in empirical studies. Furthermore, these two factors may both be confused with utility curvature, which also affects most elicitations of discounting parameters [2, 3]. A recent study proposed a way to disentangle these three fundamentally different concepts, including a first empirical test demonstrating its feasibility [1]. In particular, Rohde [1] introduced the Decreasing Impatience (DI)-Index, which is a summary measure of the degree to which an agent deviates from constant discounting; i.e., the degree to which the agent is time inconsistent. She showed that this measure is neither affected by the *level* of impatience, nor by the shape of the agent's utility function. The experimental results reported in her paper indicated that, for monetary outcomes, decreasing impatience was the modal type of discounting, confirming most of the previous literature. However, it also became clear that a sizable minority of subjects was increasingly impatient, highlighting the need to account for this type of preferences as well. Finally, Rohde's [1] experiment was complemented by a survey asking several questions about health-related behavior and found no significant associations between those and the DI-index.

This lack of association is surprising, given the perceived importance of decreasing impatience, and may have several explanations. One of them is simply a lack of power, but another one may be the use of money as a stimulus used in the elicitation of the DI-index, to predict health-related behavior. Earlier work, however, has demonstrated that deviations from constant discounting are more pronounced for health outcomes compared to monetary outcomes [4]. As such, the lack of association reported in Rohde [1] may be explained by the disparity between elicitation and outcome. To test this hypothesis, we elicit four DI-indices using health outcomes in this study: two in an individual context and two in a societal context. In addition, we implement the same survey as Rohde to see if the use of health outcomes allows us to observe a significant association between time consistency and health behaviors, such as smoking and alcohol consumption.

This paper is organized as follows. We introduce the theoretical background in section Theoretical background. After that, we describe the experimental design in section Experiment, followed by the results in section Results. Finally, in section Discussion, we discuss the results, and conclude.

THEORETICAL BACKGROUND

Notation

In our experiment, we will consider indifferences between timed outcomes (t:x), where x denotes a health improvement and t denotes its time of onset. We consider the usual preference relation $\succcurlyeq$ over these outcomes. A weak [strict] preference is denoted by $\succcurlyeq$ [$\succ$] and indifference by $\sim$. Throughout the paper we assume the discounted utility (DU) model to hold:

$$DU\,(x, t) = \delta\,(t)\,U(x). \tag{1}$$

Here, DU denotes discounted utility, δ(t) is the discount function, and U(x) is the instantaneous utility of outcome x. The standard DU model is the constant discounting model [5], which models intertemporal outcome profiles by the following formula:

$$DU\,(x_t, x_{t+1}, \ldots, x_T) = \sum\nolimits_{t=0}^{T} \gamma^t U(x_t), \tag{2}$$

where γ represents the discount factor. One of the axioms of this model is stationarity, which causes agents to always act time consistently; i.e., they stick to their plans [6]. However, many empirical studies have demonstrated that agents often will not behave this way, with a tendency to postpone annoying tasks (e.g., doing homework, stopping smoking) and to indulge in activities giving immediate benefits (e.g., purchasing a car, eating candy) [7, 8]. This kind of behavior can often be explained by hyperbolic discounting models that incorporate decreasing impatience. The most popular hyperbolic discounting function is quasi-hyperbolic discounting [9, 10]:

$$DU\,(x_t, x_{t+1}, \ldots, x_T) = U\,(x_0) + \sum\nolimits_{t=1}^{T} \beta\delta^t U(x_t). \tag{3}$$

Here, $0 \leq \beta \leq 1$ represents a measure of the "immediacy effect" or "present bias," giving a penalty to all outcomes occurring in the future, but not discriminating between the amounts of the delay for $t > 0$. This model reduces to constant discounting for the special case where $\beta = 1$. In other words, the quasi-hyperbolic model assumes that agents are decreasingly impatient only when the present is involved, and constantly impatient when only comparing future outcomes. By contrast, alternative models allow for universally decreasing impatience, i.e., even if the present is not involved (e.g., power discounting [11], proportional discounting [12], generalized hyperbolic discounting [13], and nonlinear time perception [14]).

Related Literature

Some previous studies have investigated the measurement of decreasing impatience. First, Prelec [15] proposed to assess the degree of Pratt-Arrow convexity of the logarithm of the discount function. However, this method is hard to implement in practice because it requires assumptions about or measurement of the utility function, and also specifying a parametric form of the discount function, for example assuming constant discounting. There is overwhelming empirical evidence of violations of constant discounting in the monetary domain [16], but more recently such violations have also been widely documented for health outcomes [17, 18]. In both domains, there is increasing evidence that a substantial minority of subjects is increasingly impatient, both for money [19] and for health [4], highlighting the need to accommodate this behavior as well [20]. However, to the best of our knowledge, no studies have been performed yet that measure the degree of time

Abbreviations: DBI, Duration before implementation; DI, Decreasing impatience; DU, Discounted utility; EQ-5D, EuroQol Five-Dimension; ERIM, Erasmus Research Institute of Management.

inconsistency for health outcomes, be it decreasing or increasing impatience.

As can be seen from Equation 1, intertemporal choices are not only affected by time preferences, but also by utility curvature. Therefore, when attempting to measure discount functions, it is important to assure they are separated from any effects of utility on intertemporal choices. Rohde [21] developed the hyperbolic factor to simplify this practical implementation, which was measured by Attema et al. [19], who found that a majority of choices satisfied increasing impatience. A few years later, Bleichrodt et al. [4] used the method of Attema et al. in the health domain, where they observed decreasing impatience to be the dominant pattern, but also a substantial minority who were increasingly impatient. However, as explained by Rohde [1], the hyperbolic factor can only be computed for modest amounts of decreasing and increasing impatience. This led her to propose the DI-index, which does not suffer from this drawback, and she measured it for monetary outcomes. Our study is the first to measure the DI-indices for health outcomes. As such, in this study we are able to measure the degree of time inconsistency without any distortion caused by utility curvature.

EXPERIMENT

Subjects and Design

Our subject pool consisted of a sample of 99 university students (47 female, 50 male, 2 unknown), with a mean age of 19.3 (SD = 1.6). Subjects were recruited by the Erasmus Research Participation System and the experiment was administered in the Erasmus Behavioral Lab. The subjects received course credits for their participation. This study was carried out in accordance with the recommendations of the Erasmus Research Institute of Management (ERIM) Internal Review Board. The protocol was approved by the ERIM Internal Review Board, Section Experiments. All subjects gave written informed consent in accordance with the Declaration of Helsinki. This study used a within-subjects design, to determine DI-indices for both individual and societal decision-making for health outcomes. All subjects completed the individual task first, after which they completed the societal task. Our stimuli-durations for the both tasks were chosen to maximize comparability to Rohde's [1] study. Hence, we set these durations to 0, 2, and 4 months, with these stimuli-durations being presented in increasing order in the survey.

Procedure

Subjects received paper-and-pencil instructions (see Appendix A) for this study as part of a larger survey on health-related decision-making, which is not discussed in this paper. For the individual task, subjects had to assume they were experiencing chronic back pain, as described by the following problems:

- You have moderate problems in walking about.
- You have moderate problems performing your usual activities (e.g., work, study, housework, family or leisure activities).
- You have moderate pain or discomfort.

Subjects were instructed that two treatments (A and B) were available to relieve this chronic back pain. The health improvements of treatment A and B were based on the description suggested by Bleichrodt et al. [4], and consisted of an improvement in some dimensions of the EuroQol Five-Dimension (EQ-5D) classification system. The descriptions of these two treatments were presented to subjects on a separate paper, which was put on subjects' desks. In all cases, Treatment B was more effective than Treatment A. Both treatments removed the pain, but B also improved the problems with walking and usual activities. The effects of the treatments started immediately at its onset and lasted for exactly one week. The amount of time before the treatments occur will be referred to as "duration before implementation" (DBI). In all cases, chronic back pain would return after 1 week. Such questions are common in elicitations of time preferences for health outcomes, except that usually only one change in health is studied (e.g., Treatment A), with its duration being varied [22, 23]. As explained by Bleichrodt et al. [4], the advantages of keeping the duration of change fixed are that the utility for time duration can be entirely general, without having to impose simplifying assumptions to enable the analysis of responses, and that subjects will more likely concentrate on the time point at which the change occurs (i.e., the DBI). The latter is desirable if one is interested in the properties of the discount function and not in those of the utility function.

The societal task used a similar set-up, except that in this task the health improvements did not accrue to the subjects personally but to a group of students (similar to them), who were suffering from chronic back pain. The description of this chronic health state was identical to the description in the individual task. Subjects had to trade off a higher amount of people receiving the same health improvement to a delay of the realization of this health improvement. More specifically, they faced a trade-off between treating 40 students (Option A) or incurring a delay to treat 50 students (Option B). These numbers were also used by Rohde [1], in terms of monetary outcomes (i.e., €40 and €50). By using these numbers, our ratio of the earlier to the later outcomes was the same, facilitating comparison of the discount rates. The treatment was the same in both conditions (individual vs. societal), being Treatment B of the individual task, i.e., alleviating the pain and problems on other dimensions for 1 week.

In both the societal and individual task, subjects faced a choice list (see Appendix A for an example), with Treatment A on the left hand side, given a fixed DBI (t_{0-2}), and Treatment B on the right hand side with a monotonically increasing DBI. We elicit indifferences at $s_0 = 0$, $s_1 = 2$ months, and $s_2 = 4$ months. These numbers were also used by Rohde [1], except that she used weeks instead of months. We did so because our health improvement lasted 1 week and it might have caused confusion if both this duration and the delay were expressed in months. Before subjects started working on these choice lists, they faced several questions aimed at determining comprehension. We infer indifference at the DBI where subjects switch from B to A, in agreement with the conventional use of choice lists in experimental economics [24].

Calculation of DI-Indices

Summarizing, we obtain three indifferences for both individual and societal health outcomes. Indifferences of the form $(s_i : x) \sim (t_i : y)$ can be evaluated by the following equation under DU (Equation 1):

$$\mathrm{D}(s_i)\mathrm{U}(\mathrm{x}) = \mathrm{D}(t_i)\mathrm{U}(\mathrm{y}) \text{ with i } = 0, \ 1 \text{ or } 2. \tag{4}$$

As is described by Rohde [1], the DI-index can be computed from every two indifferences of the form (s:x)∼(t:y) and (s+σ:x)∼(t+τ:y) [1]. In our experiment we obtain s, t, τ and σ as follows: for any s_i, with i = 0, 1 or 2, we set $s_i = s$. We then obtain t by the elicited indifference $(s_i : x) \sim (t_i : y)$, with $t = t_i$. Next, we set σ, which corresponds here to setting $\sigma = s_{i+1} - s_i$, and elicit the indifference $(s_{i+1} : x) \sim (t_{i+1} : y)$. We proceed by finding τ by determining $t_{i+1} - t_i$. The DI-index is then given by:

$$DI = \frac{\tau - \sigma}{\sigma \, (t - s)}, \tag{5}$$

where (as shown by Rohde [1]) constant [decreasing, increasing] impatience corresponds to a DI-index of 0 [>0, <0].

In the case of societal discounting, we elicit similar indifferences to $(s_i : x) \sim (t_i : y)$ where x and y are replaced by m and n, where m and n are the number of patients treated at time period, e.g., $(s_i : 40) \sim (t_i : 50)$. The derivation of DI-indices does not change.

Because two questions are needed to elicit one DI-index, our design enabled us to elicit two DI-indices for both the individual and the societal task, where we will indicate individual DI-indices by DI-I and societal DI-indices by DI-S. We furthermore add to these the pre-set durations used to derive them, yielding DI-I-02 and DI-I-24 (and DI-S-02 and DI-S-24 for societal).

To give an example, imagine that for the individual task a subject in our study has the following indifferences:

$$(0 : x) \sim (5 : y);$$
$$(2 : x) \sim (8 : y);$$
$$(4 : x) \sim (13 : y).$$

This means that we have s = 0 and σ = 2 for the first two indifferences, and s = 2 and σ = 2 for the last two indifferences. We elicited indifferences at t's = 5, 8, 13. This gives t = 5 and $\tau = 3$ when the DI-index is calculated based on the first two indifferences, yielding DI-I-02 = 0.1. Similarly, when constructing the DI-index of the second and third indifferences, this yields DI-I-24 = 0.25. Hence, this subject would be classified as decreasingly impatient.

Survey Questions

In order to maximize comparability with the study of Rohde [1], we implemented the same questions in an accompanying survey. This consisted of a number of demographic and behavioral characteristics and the self-control questions of Ameriks et al. [25]. The survey developed by Ameriks et al. [25] aims to measure self-control problems with a self-reported questionnaire. Additional self-awareness questions concerning sports, study, and class preparation were asked [1, 26], which were administered on an 8-point Likert scale. In this paper we focus on the role of DI-indices in health behavior. As such, our results for self-control problems and self-awareness can be found in Appendix B. The health behavior variables measured [using identical questions as in Rohde [1]] include the number of hours of sports per week, smoking behavior, amount of alcohol consumption per week, and length and weight (out of which the body mass index was computed). The following demographics were obtained: age, gender, whether or not respondents live with their parents, nationality (Dutch or non-Dutch), and whether and how much money they saved. In addition, we measured subjects' health status on a 10-point scale and their subjective life expectancy.

RESULTS

Five subjects did not complete the societal task, while one did not complete the individual task. As in Rohde [1], the number of subjects who always chose the patient option was quite high, especially for the societal task (12 never switched in both tasks, and 26 subjects did not switch in the societal task). These subjects had to be excluded from the analysis of decreasing impatience, since it was not possible to compute a DI-index for them (14 subjects who did not switch in the societal task, did switch in the individual task and could, thus, be included in the analysis of the DI-I. All analyses were also performed excluding the respondents who did not switch in the societal task, which did not yield different results. The results are available from the authors upon request). Another 9 subjects indirectly violated impatience by having $s_i > t_i$ for at least one indifference in DI-I, and 9 subjects (not necessarily the same) had $s_i > t_i$ in DI-S. These subjects were also dropped from the sample, although we could analyze the subparts including the subjects who did not violate impatience for one of the two tasks. Finally, two subjects had multiple switching points and were also removed from our sample. This resulted in 99-1-12-9-2 = 75 included subjects for the individual task, and 99-5-26-9-2 = 57 included subjects for the societal task. **Figure 1** plots the distributions of the DI-indices for both tasks.

Table 1 classifies the subjects into increasing, decreasing, and constant impatience for each of the four choices (i.e., two for the individual task and two for the societal task). A comparison of DI-I-02 and DI-I-24 reveals evidence of quasi-hyperbolic discounting: DI-I-02 is significantly higher than DI-I-24 according to a Wilcoxon signed ranks test ($p < 0.01$). However, we did not find such a difference for the societal task ($p = 0.81$). Spearman rank correlation analyses showed a significant correlation between the two indices for both tasks ($p < 0.01$).

Table 2 shows summary measures of the DI-indices. These indicate a difference between the two tasks, with those of the individual task being higher than those of the societal task. Wilcoxon signed ranks tests confirm the significance of these differences at the 10% level ($p < 0.09$ for DI-02 and $p < 0.02$ for DI-24). However, comparing the percentages of DI, CI, and II (as derived from **Table 1**), it turns out that such differences are not present for the discounting classifications (χ^2-tests, $p > 0.44$).

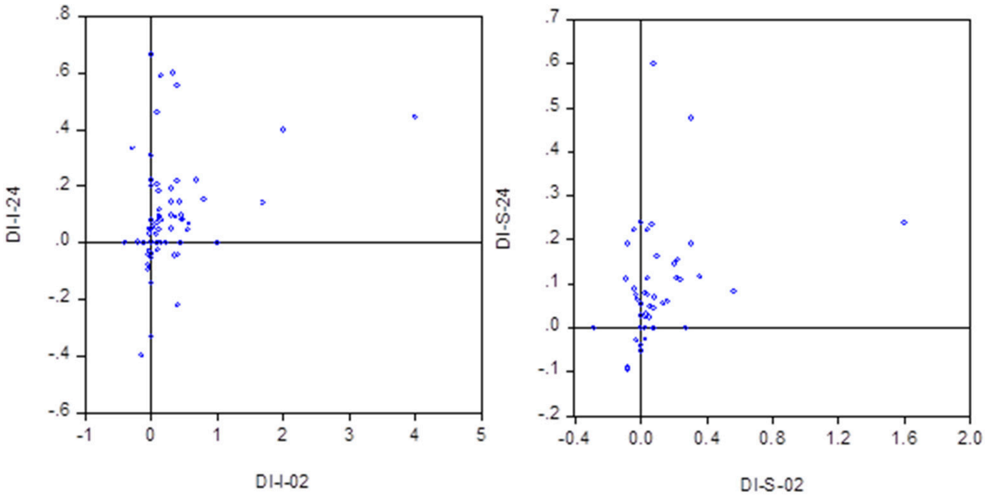

FIGURE 1 | Distributions individual DI indices **(left)** and societal DI-indices **(right)**.

TABLE 1 | Classification of degree of impatience.

	DI-I-02	DI-I-24	Total	DI-S-02	DI-S-24	Total
Constant impatience	21 (28%)	19 (25.3%)	40 (26.7%)	13 (22.8%)	13 (22.8%)	26 (22.8%)
Decreasing impatience	42 (56%)	42 (56%)	84 (56%)	30 (52.6%)	34 (59.6%)	64 (56.1%)
Increasing impatience	12 (16%)	14 (18.7%)	26 (17.3%)	14 (24.6%)	10 (17.5%)	24 (21.1%)
Total	75	75	150	57	57	114

Discount Factors

Because we used a qualitative health state improvement in the individual task, it was not possible to estimate discount factors for this task. For the societal task, on the other hand, this was possible, when assuming a linear societal utility function over number of patients. In the first choice list, the indifference under constant discounting is evaluated as in Equation (2), by:

$$40 = 50^{*}\gamma^{t} \leftrightarrow \gamma = 0.8^{1/t}, \qquad (6)$$

with t in months and γ the monthly discount factor. This resulted in average annual discount rates close to 30%. However, it should be kept in mind that this is an overestimate of the true discount rate since the non-switchers are excluded. If we include them with the conservative assumption that these subjects have a discount rate of 0%, the average discount rates reduce to rates around 10%. We also performed the analysis assuming all these subjects would have switched at the next possible switching point, not included in the choice list. This gives their maximum possible discount rate (i.e., 5.68%). The average discount rates are around 20% in that case. All these analyses did not show any differences between the three choice lists.

The social discount rates were not correlated with age or gender (Spearman test), but one of the behavioral variables was correlated with the discount rates. We found a negative correlation between living at home and the discount rate ($p < 0.02$ for all three discount factors). All other correlations were not significant.

Relationship DI-Index With Demographics and Health Behavior

Ninety-one subjects completed the survey. **Table 3** provides summary statistics of the survey responses. None of the DI-indices were correlated with age and gender, except for DI-S-12, which was correlated with gender ($p < 0.05$). In particular, women were found to have a lower DI-index than men. The regressions of each variable on the DI-indices also did not yield any significant coefficients, except for hours of sports, where the coefficient of DI-I-23 was positive and significant ($p = 0.05$). The direction of the latter correlation is counterintuitive.

DISCUSSION

This study has been the first to quantify the amount of time consistency for health outcomes, without distortions caused by utility curvature or the level of discounting. We find that the majority of subjects are decreasingly impatient for both individual and societal choices, but still there is a sizable minority with either constant or increasing impatience. Furthermore, the *amount* of decreasing impatience is higher for individual choices than for societal choices, although there is no such difference for the *degree* of decreasingly impatient choices. Hence, it seems

TABLE 2 | Summary statistics DI-indices.

	DI-I-02	DI-I-24	DI-S-02	DI-S-24
Mean (s.d.)	0.24 (0.57)	0.08 (0.19)	0.08 (0.24)	0.07 (0.13)
Median	0.09	0.05	0.03	0.05
Interquartile range	0–0.35	0–0.14	0–0.08	0–0.11

that those subjects who are decreasingly impatient, are so more strongly for individual choices than for societal choices, but the qualitative distribution of discount types does not differ between tasks. We also observe evidence in favor of quasi-hyperbolic discounting, with more decreasing impatience when the present is involved than when both outcomes occur in the future. Finally, we do not find many significant associations between the DI-indices and the demographic and health-related data obtained from the survey.

Our results are largely similar to those of Rohde [1], indicating that both the frequency and the amount of decreasing impatience are similar for health and money. In particular, the data for both domains clearly reveal decreasing impatience, but at the same time highlight the necessity to also allow for agents with increasing impatience, which constitute a non-negligible part of the population. The results presented in this study also confirm the conclusion drawn by Bleichrodt et al. [4] with regard to the amount of decreasing and increasing impatience.

The reported study has a number of drawbacks. First, the experiment always started with the individual task. Future work could randomize these to control for order effects. A second drawback is the high amount of subjects for whom the DI-index could not be calculated, because they did not switch between the smaller-sooner outcome and the larger-later outcome. This problem may be addressed by extending the choice list, so that the DI-index can also be calculated for subjects with a low, but positive, time preference. Another possibility is to use a larger time unit, such as years instead of months. However, this will come at the expense of the precision of time preference estimates for early switchers. A third limitation is that, as in most time preference measurements, our design did not allow for negative discounting. Future work could therefore extend this study to allow for the assessment of negative discount rates, since there is some evidence for this, especially for outcomes framed as losses [27, 28], also in the health domain [29–31]. However, it is important to be aware that a violation of constant discounting then has a different interpretation, since time inconsistency for patient subjects means that they are either decreasingly or increasingly *patient*, instead of *impatient*. Moreover, the theoretical derivation of the DI-index [1] was only performed for the case of impatience, and, hence, it is not yet clear if the same results also apply to the case of patience. Fourth, our instructions told subjects to adopt chronic back pain as their neutral level of health. Because most subjects were healthy, chronic back pain could have been perceived as a loss and not as neutral. However, empirical evidence suggests that the reference point or neutral level of health can be manipulated and even healthy subjects usually adopt a health state which is worse than their current health if instructed to do so [32–35].

TABLE 3 | Summary statistics of demographic and behavioral variables.

Variable	Description	Mean (SD)
Gender	Male/female	51.5% male
Age	Age in years	19.3 (1.6)
Health	Health on a 10-point Likert scale	8.4 (1.1)
SLE	Subjective life expectancy	84.1 (9.9)
Sports	Number of hours of sports per week	4.5 (2.9)
Smoke	Daily smoker/Smokes every now and then/No-smoker	27.5% smokes every day or every now and then
Alcoholdays	Average number of days drinking alcohol per week	1.6 (1.1)
Alcoholglasses	Average number of glasses of alcohol on drinking days	4.5 (3.9)
Alcoholweek	Average number of glasses of alcohol per week (Alcoholdays*Alcoholglasses)	9.2 (11.8)
BMI	Body Mass Index (Weight in kg divided by length in meters squared)	21.6 (2.2)
Parents	Dummy for subjects living with their parents	38%
Nationality	Dummy for Dutch subjects	62.6% Dutch
Saving	Dummy for saving money (1) or not (0)	76%
Monthly savings	Average monthly savings in euros	€390.80 (€2262.55)

Another future research avenue would be to extend the measurement of the DI-index to a more representative sample of the general public. This may also shed more light on the relationship between the amount of time inconsistency and health-related behaviors such as exercising, smoking, alcohol consumption, and body-mass index. That is, it may unravel if the lack of association in our study is the result of the low sample size or an inherent result, indicating that previously observed relations between time preference and healthy behavior are fully attributable to the level of impatience and risk aversion, instead of time inconsistency. Another explanation could be that this lack of association is related to our measurements, as we obtained estimates for health behavior through self-report, while DI-indices are obtained through revealed preference. Future work could attempt to test the association between DI and observed behaviors such as real-life exercise. Finally, future research can measure the DI-index for others' monetary outcomes (i.e., at the societal level for money).

Several implications arise from our study. First, health outcomes have similar characteristics as money with respect to degrees of time inconsistency. This holds both for the amounts of decreasing and increasing impatience, and for the amount of very patient choices. Second, the significant amount of increasing impatience implies that several common hyperbolic discount functions are not suitable to capture everyone's time preferences; hence, attention should be directed toward more general models such as the constant sensitivity model to accommodate increasing impatience for health outcomes [20, 36]. Third, there is a difference between individual and societal time inconsistency in that the DI-index is higher for individual choices than for societal choices. However, the distribution of decreasingly, constantly, and increasingly impatient subjects does not differ between individual and societal health choices.

It can be concluded from our results that health and money show similar amounts of decreasing and increasing impatience. This highlights the need to look more deeply into discounting models that accommodate increasing impatience, instead of only focusing on the modeling of decreasing impatience. In addition, we have shown both similarities and discrepancies in time consistency between individual and societal tasks in the health domain. Finally, we did not find robust evidence of relations between the amount of time inconsistency and demographic characteristics or health-related behavior, although more research is needed to explore this relationship in more detail.

AUTHOR CONTRIBUTIONS

AA designed the experiment. AA and SL jointly administered the experiment and analyzed the data. AA wrote a first draft of the manuscript, which was then complemented by SL. In the last phase of the study, AA and SL together finalized the submitted version of the manuscript.

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SUPPLEMENTARY MATERIAL

The Supplementary Material for this article can be found online at: https://www.frontiersin.org/articles/10.3389/fams.2018.00016/full#supplementary-material

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Performance in Multi-Armed Bandit Tasks in Relation to Ambiguity-Preference Within a Learning Algorithm

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Ellsberg paradox in decision theory posits that people will inevitably choose a known probability of winning over an unknown probability of winning even if the known probability is low [1]. One of the prevailing theories that addresses the Ellsberg paradox is known as "ambiguity-aversion." In this study, we investigated the properties of ambiguity-aversion in four distinct types of reinforcement learning algorithms: ucb1-tuned [2], modified ucb1-tuned, softmax [3], and tug-of-war [4, 5]. We took the following scenario as our sample, in which there were two slot machines and each machine dispenses a coin according to a probability that is generated by its own probability density function (PDF). We then investigated the choices of a learning algorithm in such multi-armed bandit tasks. There were different reactions in multi-armed bandit tasks, depending on the ambiguity-preference in the learning algorithms. Notably, we discovered a clear performance enhancement related to ambiguity-preference in a learning algorithm. Although this study does not directly address the issue of ambiguity-aversion theory highlighted in Ellsberg paradox, the differences among different learning algorithms suggest that there is room for further study regarding the Ellsberg paradox and the decision theory.

Keywords: decision making, Ellsberg paradox, ambiguity aversion, reinforcement learning, machine learning, artificial intelligence, natural computing, neuroeconomics

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1. INTRODUCTION

Recently, neuroeconomics has been developing into an increasingly important academic discipline that helps to explain human behavior. Ellsberg paradox is a crucial topic in neuroeconomics, and researchers have employed various theories to approach and to resolve the paradox. The basic concept behind the Ellsberg paradox is that people will always choose a known probability of winning over an unknown probability of winning, even if the known probability is low and the unknown probability could be a near guarantee of winning.

Let us start with an example. Suppose we have an urn that contains 30 red balls and 60 other balls that are either black or yellow. We do not know how many black or yellow balls are there, but we know that the total number of black balls plus the total number of yellow balls equals 60. The balls are well mixed so that each individual ball is as likely to be drawn as any other.

You are now given a choice between two gambles:

[Gamble A] You receive \$100 if you draw a red ball,
[Gamble B] You receive \$100 if you draw a black ball.

In addition, you are given the choice between these two gambles (about a different draw from the same urn):

[Gamble C] You receive \$100 if you draw a red or yellow ball,
[Gamble D] You receive \$100 if you draw a black or yellow ball.

Participants are tempted to choose [Gamble A] and [Gamble D]. However, these choices violate the postulates of subjective expected utility [1].

It is well known that ambiguity-aversion property of decision-making is one of the prevailing theories advanced to explain this paradox. On the other hand, reinforcement learning algorithms, such as ucb1-tuned [2], modified ucb1-tuned, softmax [3], and tug-of-war dynamics [4, 5], have been employed in multiple approaches in artificial intelligence (AI) applications. There is tremendous potential for neuroeconomic studies to investigate the properties of decision-making through the use of AI (learning) algorithms. This study is the first attempt to investigate the properties of learning algorithms with regards to the ambiguity-preference point of view.

In this study, we took a multi-armed bandit problem (MAB) as a decision-making problem. We considered two slot machines A and B. Each machine gave rewards with individual probability density function (PDF) whose mean and standard deviations were μ_A (μ_B) and σ_A (σ_B), respectively. The player makes a decision on which machine to play at each trial, trying to maximize the total reward obtained after repeating several trials. The MAB is used to determine the optimal strategy for finding the machine with the highest rewards as accurately and quickly as possible by referring to past experiences. The MAB is related to many application problems in diverse fields, such as communications (cognitive networks [6, 7]), commerce (internet advertising [8]), and entertainment (Monte Carlo tree search techniques in computer game programs [9, 10]).

In this study, we focused on limited MAB cases. Machine A has constant probability 1/3, and machine B has probabilities generated by normal distribution $N(\frac{1}{3} + \Delta\mu, \sigma^2)$. Here, we hypothesize that the total rewards from probabilities generated by a PDF is the same as the total rewards directly from the same PDF if we only focus on the average rewards using $1,000$ samples. On the basis of this hypothesis, we consider MABs, where PDFs are $\delta(\frac{1}{3})$ and $N(\frac{1}{3} + \Delta\mu, \sigma^2)$. Here, $\delta(x)$ is a delta function. In this study, "ambiguity" is expressed by σ although "ambiguity" becomes "risk" if our hypothesis does not hold.

2. LEARNING ALGORITHMS

2.1. Ambiguity-Neutral: SOFTMAX Algorithm

SOFTMAX algorithm is a well-known algorithm for solving MABs [3]. In this algorithm, the selecting probability of A or B, $P'_A(t)$ or $P'_B(t)$, is given by the following Boltzmann distributions:

$$P'_A(t) = \frac{\exp[\beta \cdot Q_A(t)]}{\exp[\beta \cdot Q_A(t)] + \exp[\beta \cdot Q_B(t)]}, \quad (1)$$

$$P'_B(t) = \frac{\exp[\beta \cdot Q_B(t)]}{\exp[\beta \cdot Q_A(t)] + \exp[\beta \cdot Q_B(t)]}, \quad (2)$$

where $Q_k(t)$ ($k \in \{A, B\}$) is given by $\frac{\sum_{j=1}^{N_k(t)} R_k(j)}{N_k(t)}$. Here, $N_k(t)$ is the number of selections of machine k until time t and $R_k(j)$ is the reward from machine k at time j. β is a time-dependent form in our study, which is as follows:

$$\beta(t) = \tau \cdot t. \quad (3)$$

$\beta = 0$ corresponds to a random selection and $\beta \to \infty$ corresponds to a greedy action. The SOFTMAX algorithm is "ambiguity-neutral" because "ambiguity" σ is not used in the algorithm.

2.2. Ambiguity-Neutral: Tug-Of-War Dynamics

In the tug-of-war (TOW) dynamics, a machine that has larger X_k ($k \in \{A, B\}$) is played in each time [4, 5]. Displacement X_A ($= -X_B$) is determined by the following equations:

$$X_A(t+1) = Q_A(t) - Q_B(t) + \xi(t), \quad (4)$$

$$Q_k(t) = \sum_{j=1}^{N_k(t)} (R_k(j) - K). \quad (5)$$

Here, $Q_k(t)$ is an "estimate" of information of past experiences accumulated from the initial time 1 to the current time t, $N_k(t)$ is the number of selections of machine k until time t, $R_k(j)$ is the reward from machine k at time j, $\xi(t)$ is an arbitrary fluctuation to which the body is subjected, and K is a parameter. Consequently, the TOW evolves according to a simple rule: in addition to the fluctuation, if machine k is played at each time t, $R_k - K$ is added to $X_k(t)$. The TOW is also "ambiguity-neutral" because "ambiguity" σ is not used in the algorithm.

2.3. Ambiguity-Preference: UCB1-Tuned Algorithm

In the UCB1-tuned algorithm, a machine that has larger "index" is played in each time [2].

Initialization: Play each machine once.

Loop: Play machine j that maximizes following index,

$$\overline{x}_j(t) + \sqrt{\frac{ln(n)}{n_j} min(\frac{1}{4}, V_j(n_j))}, \quad (6)$$

$$V_j(s) = (\frac{1}{s}\sum_{\tau=1}^{s} x_{j,\tau}^2) - \overline{x}_{j,s}^2 + \sqrt{\frac{2ln(t)}{s}}, \quad (7)$$

where $\overline{x}_j(t)$ is the average reward obtained from machine j, n_j is the number of times machine j has been played so far, and

n is the overall number of plays done so far. The UCB1-tuned algorithm has "ambiguity-preference" property because it selects high variance ("ambiguity") machines in the early stage.

2.4. Ambiguity-Aversion: Modified UCB1-Tuned Algorithm

In the modified UCB1-tuned algorithm, a machine that has larger "index" is played in each time. Compared to UCB1-tuned algorithm, the sign of the second term in the index becomes minus.

Initialization: Play each machine once.

Loop: Play machine j that maximizes following index,

$$\bar{x}_j(t) - \sqrt{\frac{ln(n)}{n_j} min(\frac{1}{4}, V_j(n_j))}, \tag{8}$$

$$V_j(s) = (\frac{1}{s}\sum_{\tau=1}^{s} x_{j,\tau}^2) - \bar{x}_{j,s}^2 + \sqrt{\frac{2ln(t)}{s}}, \tag{9}$$

where $\bar{x}_j(t)$ is the average reward obtained from machine j, n_j is the number of times machine j has been played so far, and n is the overall number of plays done so far. The UCB1-tuned algorithm has "ambiguity-aversion" property because it selects low variance ("ambiguity") machines in the early stage.

3. RESULTS

In this study, we focused on the following limited MAB cases. On the basis of the hypothesis, we considered MABs where PDF of machine A is $\delta(\frac{1}{3})$, and PDF of machine B is $N(\frac{1}{3} + \Delta\mu, \sigma^2)$, respectively. "Ambiguity" is expressed by σ.

For positive $\Delta\mu$, we investigate 30 cases where $\Delta\mu = 0.00$, 0.05, 0.10, 0.15, and 0.20, and $\sigma = 0.05$, 0.10, 0.15, 0.20, 0.25, and 0.30, respectively. **Figure 1** shows performance comparison between four learning algorithms for the MABs. The horizontal axis denotes $\Delta\mu$ (6 different σ cases for each either $\Delta\mu$). The vertical axis denotes total rewards (average of 1,000 samples) until time $t = 1{,}000$ (also see Appendix in Supplementary Material).

For positive $\Delta\mu$ cases, machine B is the correct selection because expected value of machine B is higher than A. This means that ambiguity-preference is needed for correct selections. The UCB1-tuned algorithm (ambiguity-preference) has higher performance than the modified UCB1-tuned algorithm (ambiguity-aversion) in the positive $\Delta\mu$ cases. Performance of the UCB1-tuned algorithms (ambiguity-preference) slightly increases as the ambiguity (σ) of the problems increases, whereas performance of the modified UCB1-tuned algorithms (ambiguity-aversion) largely decreases as ambiguity (σ) of the problems increases.

Performances of TOW and SOFTMAX are higher than those of UCB1-tuned and modified UCB1-tuned algorithms because each of the former two algorithms has a parameter that optimized the problems. That is, each of the two algorithms has an advantage over the latter two algorithms that have no parameter. Performances of the former two algorithms (ambiguity-neutral) slightly decrease as ambiguity (σ) of the problems increases. This is because incorrect decisions are slightly increased as estimation for mean value of rewards becomes largely fluctuated.

For negative $\Delta\mu$, we also investigated 30 cases where $\Delta\mu = 0.00$, 0.05, 0.10, 0.15, and 0.20, and $\sigma = 0.05$, 0.10, 0.15, 0.20, 0.25, and 0.30, respectively. **Figure 2** shows the performance comparison between four learning algorithms for the MABs. The horizontal axis denotes $\Delta\mu$ (6 different σ cases for each $\Delta\mu$). The vertical axis denotes total rewards (average of 1,000 samples) until time $t = 1{,}000$ (also see Appendix in Supplementary Material).

For negative $\Delta\mu$ cases, machine A is correct selection because expected value of machine A is higher than B. This means that ambiguity-aversion is needed for correct selections. The modified UCB1-tuned algorithm (ambiguity-aversion)

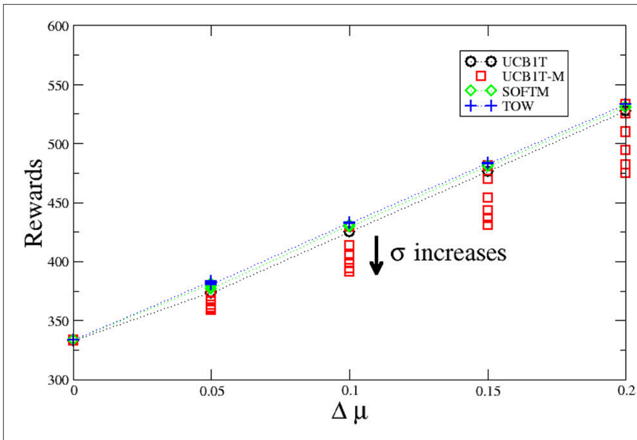

FIGURE 1 | Performance comparison between four learning algorithms for MAB where PDFs are $\delta(\frac{1}{3})$ and $N(\frac{1}{3} + \Delta\mu, \sigma^2)$. $\Delta\mu$ is positive (cases where machine B is correct decision).

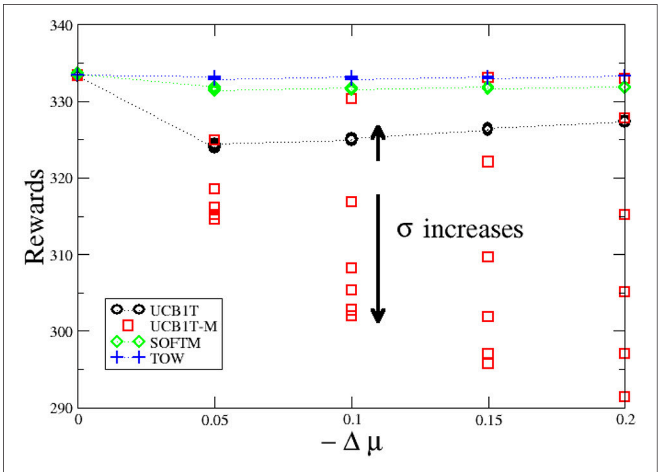

FIGURE 2 | Performance comparison between four learning algorithms for MAB where PDFs are $\delta(\frac{1}{3})$ and $N(\frac{1}{3} + \Delta\mu, \sigma^2)$. $\Delta\mu$ is negative (cases where machine A is correct decision).

has higher performance than the UCB1-tuned algorithm (ambiguity-preference) in the negative $\Delta\mu$ cases only in $\sigma = 0.05$. Performance of the UCB1-tuned algorithms (ambiguity-preference) slightly increases as ambiguity (σ) of the problems increases, whereas performance of the modified UCB1-tuned algorithms (ambiguity-aversion) largely decreases as ambiguity (σ) of the problems increases.

Performances of TOW and SOFTMAX are higher than those of UCB1-tuned and modified UCB1-tuned algorithms because each of the former two algorithms has a parameter that optimized the problems as well as the positive $\Delta\mu$ cases. Performances of the former two algorithms (ambiguity-neutral) also slightly decrease as the ambiguity (σ) of the problems increases because of the same reason as the positive $\Delta\mu$ cases.

4. CONCLUSION AND DISCUSSION

In both cases (positive $\Delta\mu$ and negative $\Delta\mu$), performance of the UCB1-tuned algorithms (ambiguity-preference) slightly increases as the ambiguity (σ) of the problems increases, whereas performance of the modified UCB1-tuned algorithms (ambiguity-aversion) largely decreases as the ambiguity (σ) of the problems increases. This means that ambiguity-aversion property of learning algorithm has a negative contribution to its performances for MABs, whereas ambiguity-preference has a positive contribution.

From these limited computer simulation results, we conclude that ambiguity-aversion property does not work for efficient decision-making in the learning point of view (repeated decision-making situations). Another point of view will be necessary for justification of ambiguity-aversion property. We suggest that the differences among learning algorithms require further study on the Ellsberg paradox and decision theory.

AUTHOR CONTRIBUTIONS

S-JK and TT designed research. S-JK performed computer simulations. S-JK and TT analyzed the data. S-JK wrote the manuscript. All authors reviewed the manuscript.

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SUPPLEMENTARY MATERIAL

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Cultural Similarities and Differences in Social Discounting: The Mediating Role of Harmony-Seeking

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One's generosity to others declines as a function of social distance, which is known as social discounting. We examined cultural similarities and differences in social discounting and the mediating roles of the two aspects of interdependence (self-expression and distinctiveness of the self) as well as the two aspects of independence (harmony-seeking and rejection avoidance). Using the same procedure that previous researchers used to test North Americans, Study 1 showed that compared to North Americans, Japanese discount more steeply a partner's outcomes compared to their own future outcomes, whereas the decrease in the subjective value of the partner's outcomes accelerates less as a function of social distance. To examine the cultural similarities and differences in social discounting in more detail, Study 2 tested Japanese and Germans and found that the hyperbolic with exponent model fitted the participants' discounting behaviors better than the other models, except for the loss condition in Germans where the utility of the *q*-exponential model was indicated. Moreover, although the social discounting rate was higher in Japanese than in Germans, the cultural difference was limited to the gain frame. However, the decline in a person's generosity accelerated less as a function of social distance in Japanese than in Germans. Furthermore, the cultural difference in the social discounting in gains was mediated by the level of harmony-seeking, which was higher in Germans than in Japanese. Implications for individuals' generosity against the backdrop of cultural characteristics are discussed.

Keywords: social discounting, culture, harmony-seeking, hyperbolic with exponent model, gains and losses

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INTRODUCTION

Humans are unique in their formation of cooperative relationships with unrelated individuals and living within a group. Specifically, other-regarding motives are crucial, as they promote cooperative relationships and altruistic behaviors. However, the motives are likely influenced by social distance, and therefore people are not always generous to everyone. Whereas people generally behave generously to close others, the tendency to generosity decreases for distant others (Hoffman et al., 1996; Jones and Rachlin, 2006; Rachlin and Jones, 2008; Goeree et al., 2010). Moreover, the variation in the social orientation of independence and interdependence across cultures also influences other-regarding motives (Yamagishi, 1988; Buchan et al., 2006; Strombach et al., 2014). In the present research, we examined the influence of social distance and cultures on other-regarding motives in the social discounting framework by comparing mathematical models. We also tested a hypothesis that generosity toward others increases as a function of harmony-seeking orientation, which differs across cultures.

Social Discounting

People generally tend to discount future outcomes in exchange for immediate but smaller gains. Previous research examined the tendency of delay discounting by asking people to choose to either receive a stable amount of money with a specified delay or to receive a smaller amount of money immediately. Empirical evidence has suggested that the decline in the subjective value of future outcomes is likely steeper in the early delay phase and becomes more gradual as the delay gets longer (Ainslie, 1975; Mazur, 1987). Suppose that there are two alternatives. One is to receive \$450 immediately, whereas the other is to receive \$500 after 1 week. In this case, people will tend to prefer to receive \$450 immediately. On the other hand, if people are asked to choose either to receive \$450 after 5 years or to receive \$500 after 5 years and 1 week, they will tend to prefer to receive \$500 after 5 years and 1 week. People's preferences for the two options are thus reversed in spite of the same length of delay (i.e., 1 week). Given such time-inconsistent choice behavior, it has been pointed out that a hyperbolic function better describes an individual's delay discounting, compared to an exponential model assuming that the subjective value of future outcomes declines in a time-consistent manner (Kirby, 1997). Moreover, previous research (e.g., Laibson, 1997) has also proposed a quasi-hyperbolic discount model (sometimes called a quasi-exponential discount model) explaining such time-inconsistent choice behavior based on internal conflicts between "selves" having two or more exponential discount rates stocked within a single individual. Nevertheless, it has been suggested that models assuming a hyperbolic function (e.g., the q-exponential discount model and the hyperbolic with exponent model) fit individuals' discounting behaviors better than the quasi-hyperbolic discount model (Takahashi, 2008; Ishii et al., 2018).

The social discounting framework focuses on choices between individuals who differ in social distance (i.e., a person and another person at a given social distance), instead of intertemporal choices. Jones and Rachlin (2006) assumed that social discounting would be related to delay discounting based on an expected positive association between other-regarding motives and self-control. Adopting the delay-discounting framework, Jones and Rachlin (2006) asked participants to choose that either another person receives a stable amount of money or they receive a smaller amount of money by manipulating the distances between the giver and the receiver. The scholars then compared an exponential discounting function and a hyperbolic discounting function to see which fits the behavioral data of social discounting better. The exponential discounting function and the hyperbolic discounting function are written, respectively, as follows:

$$V(N) = \frac{V(0)}{\exp(kN)} \tag{1}$$

$$V(N) = \frac{V(0)}{1 + kN} \tag{2}$$

where N is the social distance, $V(N)$ is the subjective value of a reward (or payment) when the receiver is a person at N, and k is a free parameter that represents the discount rate. The findings indicate that as in the case of delay discounting, the hyperbolic discount function is more suitable for explaining the tendency that an individual's generosity to others is discounted by social distance. Concretely, the decline in the subjective value of the outcomes that another person receives is inconsistent across people who differ in social distance: The decline is likely steeper in early phase of social distance, whereas it becomes more gradual as social distance grows larger.

After Jones and Rachlin's (2006) study, Rachlin and Jones (2008) adopted a more general form of the hyperbolic equation to explain the influence of social distance on an individual's generosity to others. The equation is

$$V(N) = \frac{V(0)}{1 + kN^{S}} \tag{3}$$

where an exponent s is added to the social distance. s is a power-function parameter that suggests individual differences in the sensitivity of $V(N)/V(0)$ to N. When $s = 1$, the equation is identical to the hyperbolic equation (2). However, when s is less than 1, the decrease in $V(N)/V(0)$ over the course of the social distance diminishes faster than in the (simple) hyperbolic model. In Rachlin and Jones's (2008) study (Experiment 1), s was 1.03, and the hyperbolic function was almost congruent with that reported in Jones and Rachlin (2006), which was based on the hyperbolic equation (2).

Takahashi (2010) proposed a q-exponential social discounting model based on Tsallis' statistics. The equation is

$$V(N) = \frac{V(0)}{\exp_q(kN)} = V(0)/[1 + (1-q)kN]^{1/(1-q)} \tag{4}$$

where $\exp_q(x)$, which is equal to $[1 + (1-q)x]^{\frac{1}{1-q}}$, is a q-exponential function. When a parameter $q = 1$, Equation (4) expresses $V(N) = V(0)^{*} \exp(-kN)$, which is equal to exponential model (1). When $q = 0$, Equation (4) expresses $V(N) = V(0)/(1 + kN)$, which is equal to hyperbolic model (2). As a result, $1-q$ indicates the extent to which a person discounts another person's reward (or payment) inconsistently depending on different social distances. This means that the decrease in the subjective value of another person's reward (or payment) is more inconsistent as $1-q$ becomes larger (i.e., q becomes smaller). Takahashi (2013) examined social discounting in gain and loss and found that social loss is discounted less than social gain as the social distance increases. When he compared exponential, hyperbolic, and q-exponential models, he found that whereas the hyperbolic model fitted best to social gain, the q-exponential model fitted best to social loss. In the gain and loss frames, the q-values were smaller than 1, suggesting inconsistent social discounting across people differing in social distance.

Cultural Differences in Social Discounting and Other-Regarding Motives

Previous researchers have suggested that culture influences interpersonal choices, which have not been fully investigated, however (Weber and Morris, 2010). In terms of delay discounting, it is known that East Asians are less likely than North Americans to discount future rewards (Du et al., 2002; Takahashi et al., 2009; Kim et al., 2012; Ishii et al., 2017). Wang et al. (2016) conducted a large-scale international survey of time preference and suggested the influences of cultural differences in uncertainty avoidance and long-term orientation (Hofstede et al., 2010) on intertemporal choices.

As in the case of delay discounting, cultural differences in social discounting have been shown. Strombach et al. (2014) tested Germans and Chinese for social discounting. The hyperbolic discount function was fitted to the outcomes another person received regardless of cultures. Moreover, although the degree to which the participants discounted another person's reward did not vary between the cultures, the decrease in the subjective value of another person's reward was steeper among Germans than among Chinese. Thus, compared to Germans, Chinese were less generous to their closer friends but more generous to distant others. Such a weak effect of social distance in East Asians has also been reported by Buchan et al. (2006), who examined the extent to which participants have other-regarding motives (i.e., trusting others) in an investment game. Whereas North Americans trusted in-group members more than outgroup members, the effect of group membership was weak among Chinese, Japanese, and Koreans. Ito et al. (2011) examined social discounting rates among Japanese and North Americans and found that the discount rate was higher among Japanese than among North Americans not only when the receiver was a relative but also when the receiver was a stranger. In Ito et al. (2011), the hyperbolic discount function was also fitted to another person's reward in both cultures. Furthermore, a recent study by Ma et al. (2015) showed that in addition to the utility of the hyperbolic discounting model, Chinese who were raised in rural areas were more generous to another person than Chinese who were raised in urban areas. This finding suggests that individuals' orientation toward others, which is fostered by socioecological environments and not the cultural category, influences an individual's interpersonal choices.

The previous findings on social discounting are inconsistent with the expected cultural differences based on the well-known cultural dimension of independence and interdependence (or individualism vs. collectivism; Triandis, 1989; Markus and Kitayama, 1991). It is assumed that the self has been characterized as relatively independent and separate from other people in Western cultural contexts and as more interdependent and connected with others in East Asian cultural contexts, and that the cultural dimension fosters psychological processes that vary across different cultures. Based on the cultural dimension, reflecting the interdependence emphasized in East Asian cultures, East Asians are expected to be more generous than are Westerners even in the context of social discounting. However, the previous findings contradict this expectation. Why does such a contradiction occur?

We suggest that the duality of interdependence proposed by Hashimoto and Yamagishi (2016) might be a possible clue to solve the contradiction. They proposed that interdependence consists of one's tendency to seek social harmony with others to achieve mutual social relationships by considering and responding to others' feelings and needs (called *harmony-seeking*) as well as one's sensitivity to negative perceptions and the feelings of others resulting from constraints based on social relationships, where people depend closely on each other (called *rejection avoidance*). It is crucial not to be rejected and ostracized from others in a collective culture in which members are connected with strong ties. Hashimoto and Yamagishi (2016) found that despite the traditional idea that harmony-seeking is a main feature of interdependence that differs across cultures, interestingly, harmony-seeking was higher in North Americans than in Japanese. In contrast, Japanese perceived higher rejection-avoidance than did North Americans. Given that harmony-seeking is crucial for forming mutually cooperative relationships with others, the significance should be universal. Nevertheless, the finding that North Americans perceived higher harmony-seeking than did Japanese suggests that having other-regarding motives is more useful for Westerners than for East Asians with regard to living in their sociocultural environments. In particular, compared to the collective environment in East Asia where interpersonal relationships are relatively stable and fixed so that the cost paid to form new relationships is relatively high, in the sociocultural environment surrounding Westerners which is characterized as mobile and fosters the formation of new relationships, generosity signaling an individual's good intentions and trustworthiness would be more effective. Accordingly, as generous behaviors attract another person's attention and enhance the possibility of being chosen as a partner, the utility of generosity should be higher in Western cultures than in East Asian cultures.

The Present Study

This research sought to examine cultural similarities and differences in social discounting and the role of harmony-seeking in the differences. In Study 1, to find initial evidence for large social discounting among Japanese, we tested Japanese utilizing the same procedure used by Rachlin and Jones (2008, Experiment 3). In Study 2, testing Japanese and Germans and estimating the parameters computed by four social discounting models corresponding to Equations (1)–(4), that is, the exponential, hyperbolic, hyperbolic with exponent, and q-exponential models, we examined cultural differences in social discounting and orientation toward harmony-seeking. This examination is novel in terms of two issues. First, we examined the social discounting of not only future gains but also future losses. In delay discounting, the tendency of people to discount future gains more than future losses is called the sign effect (e.g., Frederick et al., 2002). To our knowledge, no study has examined cultural differences in the social discounting of loss and the sign effect. Second, although previous research has suggested the utility of the hyperbolic discounting model across cultures, no study has shown the utility by comparing the hyperbolic discounting model with related models, such as the hyperbolic with exponent model and the q-exponential model. Although Takahashi (2013) compared the exponential, hyperbolic, and q-exponential models based on social discounting behaviors collected from Japanese, it is unclear whether the findings could apply to social discounting behaviors in another culture. Further, it is unclear whether the utility of the q-exponential model could be confirmed by comparison to the hyperbolic with exponent model. Taken together, regardless of cultures, social loss should be discounted less than social gain as the social distance increases. In addition, the subjective value of another person's outcome should decline inconsistently across people differing in social distance. The decrease should be steeper in the choice for close friends, whereas the decrease should become more moderate in the choice for distant others. In addition to these cultural similarities, we expected that cultural differences in other-regarding motives expressed by harmony-seeking would manifest as a person's generosity to others in social discounting.

STUDY 1

Materials and Methods

Ethics Statement

The study was reviewed and approved by the Experimental Research Ethics Committee at the Graduate School of Humanities, Kobe University. The participants provided a written informed consent at the beginning of the study. All responses were confidential.

Participants and Procedure

Ninety-two Japanese undergraduate students (55 females and 37 males, M_{age} = 18.78 years, SD = 0.78) at a Japanese University participated in this study. They were recruited through a psychology subject pool in the university.

First, following the procedure used in Jones and Rachlin (2006) and Rachlin and Jones (2008), participants were asked to imagine that they created a list of 100 people who were closest to them in the world and placed the people in social distance so that their dearest friend was ranked 1 whereas a mere acquaintance was ranked 100. The participants were then asked to make a series of hypothetical binary choices under the assumption that their choices involved real money. Each choice consisted of two alternatives: (a) The participants themselves would receive a fixed amount of 7,500 yen (about US$75) after a certain period of delay, or (b) a partner at some specified social distance from the participant on the list would receive the fixed amount of 7,500 yen immediately. Option (a) for the participant's delayed receipt was always presented in the left column, and option (b) for the partner's immediate receipt was always presented in the right column. The participants were asked to choose whether they preferred option (a) or (b). In option (a), there were 11 delay periods: immediately, 2 days, 5 days, 10 days, 1 month, 2 months, 6 months, 1 year, 2 years, 5 years, and 10 years. For each partner, the 11 delay options for the participant's receipt of 7,500 yen were compared with the immediate option for the partner's receipt of 7,500 yen. It was expected that when the participants perceived the value of their delayed receipt of 7,500 yen as small, at some point, they would switch their choices from their delayed receipt to their partner's immediate receipt. For each partner, the point at which the participant was indifferent between his or her delayed receipt and the partner's immediate receipt was obtained by averaging the delay of his or her receipt just before he or she switched the choice to the partner's immediate receipt and the delay of his or her receipt compared with the partner's immediate receipt immediately after his or her switching of the choice. For example, if a participant preferred the receipt of 7,500 yen with a 5-day delay to the partner's immediate receipt of 7,500 yen, whereas the participant's preferred the partner's immediate receipt of 7,500 yen to a 10-day delay of the participant's receipt of 7,500 yen, the indifference point was a delay of 7.5 days. The partners varied based on six types of social distance: 1 (i.e., a dearest friend), 2, 10, 20, 50, and 100 (i.e., a mere acquaintance). Thus, six indifference points indicating the length of the delay were computed for each participant. The order of the 11 delay options (ascending or descending) for each partner and the order of the six types of social distance (ascending vs. descending) were counterbalanced across the participants. Accordingly, there were 66 choices in total.

Rachlin and Jones (2008, Experiment 3) proposed an equation regarding the positive association between social distance and the delay of the participant's reward:

$$D = cN^{1/s}, \tag{5}$$

where D is the length of the delay, N is the social distance, and c is substituted for $(k_{\text{social}}/k_{\text{delay}})^{1/s}$. k is a discount rate under the assumption of hyperbolic delay discounting and social discounting below:

$$\text{Delaydiscounting} : \mathrm{V(D)} = \frac{\mathrm{V(0)}}{1 + \mathrm{k_{delay}D^S}}$$

$$\text{Socialdiscounting} : \mathrm{V(N)} = \frac{\mathrm{V(0)}}{1 + \mathrm{k_{social}N}}$$

where $V(D)$ is the subjective value of a reward at delay D, and s is a power-function parameter and suggests individual differences in the sensitivity of $V(D)/V(0)$ to D.

In Rachlin and Jones (2008, Experiment 3), c was 1.6, and the exponent of Equation (5) (i.e., $1/s$) was 1.5. These values suggest that the discount rate of social discounting is greater than that of delay discounting, and that the equivalent length of the delay accelerates as the social distance increases.

Results and Discussion

Following Rachlin and Jones (2008, Experiment 3), the median length of delay across all the participants was computed for each varied partner in the six types of social distance.

By performing a nonlinear regression with R, we fitted a model corresponding to Equation (5) to the median length of delay. Accordingly, c was 7.09, and the exponent of Equation (5) (i.e., $1/s$) was 1.35. Thus, compared to the values reported by Rachlin and Jones (2008, Experiment 3), c was larger whereas the exponent of Equation (5) was smaller. This larger c value suggests that in spite of the common tendency for people to more steeply discount their generosity to their partner compared to their generosity to "their future selves," Japanese showed a more prominent tendency to do this, compared to the American participants in Rachlin and Jones (2008, Experiment 3). However, the smaller value of the exponent of Equation (5) suggests that the length of the delay accelerated less as a function of social distance in Japanese than in Americans in Rachlin and Jones (2008, Experiment 3).

STUDY 2

Materials and Methods

Ethics Statement

The study was reviewed and approved by the Experimental Research Ethics Committee at the Graduate School of Humanities, Kobe University. The participants provided written informed consent at the beginning of the study. All responses were confidential.

Participants and Procedure

One hundred twenty-seven German undergraduate students at a German University and 121 Japanese undergraduate students at a Japanese University participated in this study. Because 26 German participants and 2 Japanese participants lacked at least one indifference point due to their misunderstanding of the instruction, their data were excluded from the analyses. Thus, data from 101 Germans (83 females and 18 males, M_{age} = 22.71 years, SD = 3.16) and 119 Japanese (69 females and 50 males, M_{age} = 19.54 years, SD = 1.10) were analyzed[1].

As in Study 1, the participants were initially asked to imagine that they created a list of 100 people who were closest to them in the world and placed the people in various social distances so that their dearest friend was ranked 1 whereas a mere acquaintance was ranked 100 on the list. They were then asked to make a series of hypothetical binary choices on future gains under the assumption that their choices involved real money. Each choice consisted of two alternatives: (a) The participants themselves received a certain amount of money immediately, or (b) a partner at some specified social distance from the participant on the list received the fixed amount of 7,500 yen (or 60 euro) immediately. Option (a) for the participant's receipt was always presented in the left column, and option (b) for the partner's receipt was always presented in the right column. The participants were asked to choose whether they preferred option (a) or (b). In option (a), the immediate options varied from 500 to 8,500 yen (or from 4 to 68 euro), in increments of 1,000 yen (or 8 euro). Thus, nine options were prepared for the participant's receipt. For each partner, the nine options for the participant's receipt were compared with the immediate option for the partner's receipt of 7,500 yen (or 60 euro). The point at which the participant was indifferent between his or her receipt and the partner's receipt was obtained by averaging the amount of the participant's receipt just before he or she switched the choice to the partner's receipt and the amount of the participant's receipt compared with the partner's receipt immediately after his or her switching of the choice. For example, if a participant preferred to receive 4,500 yen immediately instead of the partner receiving 7,500 yen immediately, whereas the participant preferred the partner receiving 7,500 yen immediately rather than the participant himself or herself receiving 3,500 yen immediately, the indifference point was 4,000 yen. The partners varied based on seven types of social distance: 1 (i.e., a dearest friend), 2, 5, 10, 20, 50, and 100 (i.e., a mere acquaintance). Thus, seven indifference points indicating the amount of receipt were computed for each participant. The order of the nine options for the participant's receipt (ascending or descending) for each partner and the order of the seven types of social distance (ascending vs. descending) were counterbalanced across the participants. After the participants were asked to choose options regarding future gains for all the types of partners, they were also asked to choose options regarding future losses in the same manner, except that each choice consisted of two alternatives: (a) a certain amount of money was taken away from the participant immediately, or (b) the fixed amount of 7,500 yen (or 60 euro) was taken away from a partner at some specified social distance from the participant on the list immediately. The domain of the choice (gain vs. loss) was thus a within-participant factor. Accordingly, there were 126 choices in total. Future gains and losses were expressed in yen for Japanese participants and converted into euros for German participants, with 1 yen equaling 0.008 euro.

Finally, the participants responded on four independence and interdependence scales (Hashimoto and Yamagishi, 2016), which consist of rejection avoidance (e.g., I find myself being concerned about what others think of me), self-expression (e.g., I always express my opinions in a straightforward manner), harmony-seeking (e.g., I try to respect the feelings of others), and distinctiveness of the self (e.g., I want to live my life differently from others). There were eight statements for each scale. The participants indicated how well each of the statements described them on 7-point Likert-type scales (1 = *doesn't describe me at all*; 7 = *describes me very much*). The four scales (i.e., rejection avoidance, self-expression, harmony-seeking, and distinctiveness of the self) had reasonable reliabilities in Japan (αs = 0.83, 0.86, 0.66, and 0.66) and Germany (αs = 0.78, 0.82, 0.53, and 0.70).

After performing a nonlinear regression with R for each culture, we then fitted the exponential, hyperbolic, hyperbolic with exponent, and q-exponential models, which correspond to Equations (1)–(4), respectively, to the mean indifference points across all participants for gain and loss, respectively. We further fitted these models to the mean indifference points in each genotype. The Akaike information criterion (AIC) was used to estimate the goodness of fit. We performed model selection based on the AIC. A smaller AIC indicates a better model fit.

In addition, we computed the area under the curve (AUC) for the gain or loss conditions separately to estimate the extent to which the participants discounted gains or losses. For each culture, social distance and indifference points were standardized by dividing them by the maximum values so that they varied between 0 and 1. Instead of fitting a curve, we connected adjacent delay points by straight lines and computed the area under these lines. Each line made a trapezoid; thus, the total area

[1] The proportion of female participants was significantly higher in Germans than in Japanese, χ^2 = 14.98, $P < 0.001$. However, no significant effect of gender was found in the analysis of the AUC (Fs < 2.44, Ps > 0.11). Thus, gender was excluded in the following analyses.

could be computed by summing the sizes of the trapezoids: $(y_{i+1} + y_i) \times (x_{i+1} - x_i)/2$, where x_i and x_{i+1} indicate social distance (x_{i+1} is a one-unit farther partner compared to x_i) and y_i and y_{i+1} are the subjective values of a gain or loss corresponding to these partners. A smaller AUC indicated greater social discounting.

Results and Discussion

The AIC and the parameters were estimated for each of the four models. **Table 1** summarizes the results. The AIC values showed that the hyperbolic with exponent model fitted the observed data better than the other three models, except for the loss condition in Germans where the q-exponential model fitted the observed data better than the other three models. Overall, Japanese discounted their generosity to their partner more than did Germans in the gain frame, whereas the cultural difference disappeared in the loss frame. In addition, Japanese discounted their generosity to their partner more in the gain frame than in the loss frame, whereas the difference between the two conditions almost disappeared among Germans. **Figure 1** plots the means of subjective value being equivalent to the partner's fixed amount of gain and loss among Germans and Japanese, which were fitted with the hyperbolic with exponent model. Moreover, the parameter s was smaller among Japanese than among Germans for gains and losses. This result suggests that the decrease in a person's generosity to his or her partner accelerated less as a function of social distance among Japanese than among Germans.

Next, the AUC was submitted to a mixed-model analysis of variance (ANOVA) with one between-subject variable (culture: Germany and Japan) and one within-subject variable (outcome: gain and loss). The results showed a statistically significant main effect of outcome, $F(1,218) = 5.43$, $P = 0.02$, $\eta_p^2 = 0.02$. The AUC was statistically significantly smaller for gains ($M = 0.31$, $SD = 0.22$) than for losses ($M = 0.36$, $SD = 0.23$). The interaction between culture and outcome was also statistically significant, $F(1,218) = 6.06$, $P = 0.01$, $\eta_p^2 = 0.03$. **Table 2** presents the relevant means. For gains, the main effect of culture was statistically significant, $F(1,218) = 5.87$, $P = 0.02$, $\eta_p^2 = 0.03$. The AUC was statistically significantly smaller for Japanese ($M = 0.29$, $SD = 0.23$) than for Germans ($M = 0.36$, $SD = 0.20$). In contrast, for losses, the main effect of culture was not statistically significant, $F(1,218) = 0.00$, $P = 0.95$ (Germans: $M = 0.36$, $SD = 0.20$; Japanese: $M = 0.36$, $SD = 0.25$). In Japanese, the

TABLE 1 | AIC and parameters for four models.

	Hyperbolic with exponent	Exponential	Hyperbolic	q-Exponential
Gain				
Germany				
AIC	−45.92	−4.20	−12.89	−34.64
Parameter	$k_{he} = 0.05$	$k_e = 0.03$	$k_h = 0.05$	$k_q = 0.11$
	$s = 0.66$			$q = -1.73$
Japan				
AIC	−26.62	0.32	−5.13	−19.00
Parameter	$k_{he} = 0.10$	$k_e = 0.05$	$k_h = 0.09$	$k_q = 0.45$
	$s = 0.48$			$q = -3.01$
Loss				
Germany				
AIC	−25.95	−4.69	−14.50	−37.58
Parameter	$k_{he} = 0.05$	$k_e = 0.03$	$k_h = 0.05$	$k_q = 0.10$
	$s = 0.71$			$q = -1.42$
Japan				
AIC	−43.26	−5.21	−13.42	−28.27
Parameter	$k_{he} = 0.04$	$k_e = 0.02$	$k_h = 0.04$	$k_q = 0.09$
	$s = 0.66$			$q = -1.74$

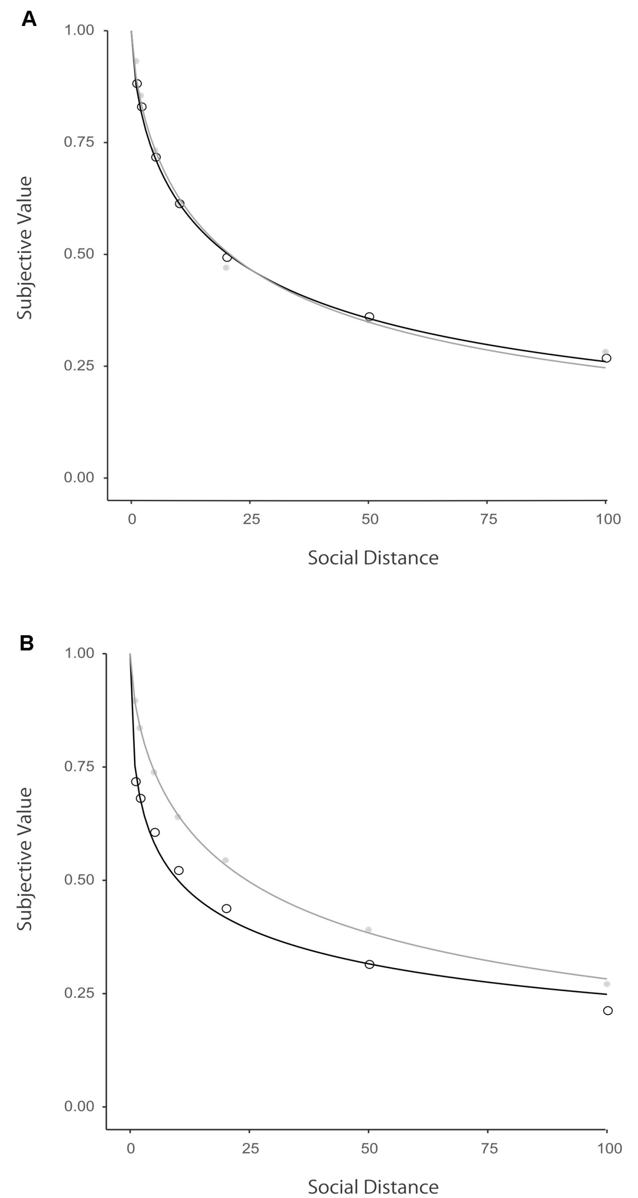

FIGURE 1 | Hyperbolic with exponent functions with social distance for all the participants (German: **A**, Japan: **B**). Mean indifference points were plotted in circle. The curved lines were illustrated in black for gains and in gray for losses.

AUC was statistically significantly smaller for gains than for losses, $F(1,118) = 10.74$, $P = 0.001$, $\eta_p^2 = 0.08$. However, in Germans, there was no difference in the AUC between the two outcomes, $F(1,100) = 0.01$, $P = 0.92$. Moreover, the mean AUCs for gains and losses were highly correlated regardless of culture [Germans: $r(99) = 0.52$, Japanese: $r(117) = 0.52$, $Ps < 0.001$].

As for independence and interdependence (**Table 3**), Germans ($M = 4.70$, $SD = 1.06$) were statistically significantly higher in self-expression than were Japanese ($M = 4.09$, $SD = 1.20$), $t(218) = 3.96$, $P < 0.001$, whereas Japanese ($M = 5.33$, $SD = 1.09$) were statistically significantly higher in rejection avoidance than were Germans ($M = 3.97$, $SD = 1.13$), $t(218) = 9.07$, $P < 0.001$. There was no cultural difference in the distinctiveness of the self (Germans: $M = 4.50$, $SD = 1.01$, Japanese: $M = 4.59$, $SD = 1.07$), $t(218) = 0.61$, $P = 0.54$. Importantly, Germans ($M = 5.88$, $SD = 0.60$) showed higher harmony-seeking than did Japanese ($M = 5.29$, $SD = 0.79$), $t(218) = 6.11$, $P < 0.001$. These patterns were identical to those reported by Hashimoto and Yamagishi (2016) testing North Americans and Japanese. The AUCs for gains and losses were statistically significantly positively correlated with harmony-seeking in Germans [gain: $r(99) = 0.28$, $P = 0.005$. loss: $r(99) = 0.30$, $P = 0.002$] and Japanese [gain: $r(117) = 0.25$, $P = 0.005$. loss: $r(117) = 0.30$, $P = 0.003$). Regardless of culture, those who are higher in harmony-seeking discount their generosity to their partner less in gain and loss frames. However, the AUCs were not statistically significantly correlated with any of the other three scales of independence and interdependence either in the gain frame [for Germans, self-expression: $r(99) = 0.13$, rejection avoidance: $r(99) = -0.05$, distinctiveness of the self: $r(99) = 0.15$; for Japanese, self-expression: $r(117) = 0.08$, rejection avoidance: $r(117) = -0.08$, distinctiveness of the self: $r(117) = -0.17$) or in the loss frame (for Germans, self-expression: $r(99) = 0.01$, rejection avoidance: $r(99) = -0.03$, distinctiveness of the self: $r(99) = 0.03$; for Japanese, self-expression: $r(117) = 0.14$, rejection avoidance: $r(117) = -0.03$, distinctiveness of the self: $r(117) = -0.14$].

We then examined whether harmony-seeking, which varied across cultures, mediates the cultural difference in the AUC for gains. A multiple regression analysis was conducted, in which culture (0 = Japan, 1 = Germany) and harmony-seeking were entered to predict the AUC for gains. Culture is associated with harmony-seeking [$b = 0.59$, $SE = 0.10$, $t(218) = 6.11$, $P < 0.001$] and the AUC for gains [$b = 0.07$, $SE = 0.03$, $t(218) = 2.42$, $P = 0.02$]. When both culture and harmony-seeking were entered simultaneously to predict the AUC for gains, harmony-seeking significantly predicted the AUC for gains [$b = 0.08$, $SE = 0.02$, $t(217) = 4.00$, $P < 0.001$]. On the other hand, the direct path between culture and the AUC for gains was no longer significant, $b = 0.02$, $SE = 0.03$, $t(217) = 0.79$, $P = 0.43$. A bootstrap analysis with a 95% confidence interval (CI; bootstrap sample = 5,000), which was conducted following Preacher and Hayes (2008), revealed a statistically significant indirect effect [CI = (0.02, 0.08)].

TABLE 2 | Mean AUCs and standard deviations in gain and loss frames in Germans and Japanese.

Frame	Germans		Japanese	
	M	***SD***	***M***	***SD***
Gain	0.36	0.20	0.29	0.23
Loss	0.36	0.20	0.36	0.25

TABLE 3 | Mean ratings and standard deviations of the aspects of independence and interdependence in Germans and Japanese.

Aspect	Germans		Japanese	
	M	***SD***	***M***	***SD***
Independence				
Self-expression	4.70	1.06	4.09	1.20
The distinctiveness of the self	4.50	1.01	4.59	1.07
Interdependence				
Harmony-seeking	5.88	0.60	5.29	0.79
Rejection avoidance	3.97	1.13	5.33	1.09

GENERAL DISCUSSION

We examined cultural similarities and differences in social discounting and the mediating role of harmony-seeking. Using the same procedure as Rachlin and Jones (2008, Experiment 3), Study 1 showed that Japanese discounted their partner's outcomes more steeply compared to their own future outcomes. Although the tendency for Japanese was similar to that found for North Americans by Rachlin and Jones (2008, Experiment 3), the former was more obvious than the latter. However, the decrease in the subjective value of the partner's outcomes, which corresponds to the length of delay to the participants' receipt of outcomes, accelerated less as a function of social distance among Japanese than among American participants in Rachlin and Jones's (2008), study (Experiment 3). These patterns imply that compared to Westerners, Japanese show larger social discounting, but their social discounting behaviors are less influenced by the increase in social distance. To examine the cultural similarities and differences in social discounting in more detail, Study 2 tested Japanese and Germans and found that the hyperbolic with exponent model fitted the participants' discounting behaviors better than the other models, except for the loss condition in Germans, where the utility of the q-exponential model was indicated. Moreover, although the social discounting rate was higher in Japanese than in Germans, the cultural difference was limited to the gain frame. Furthermore, we found that in the gain frame an individual's generosity to another person increased as a function of harmony-seeking, which differs cross-culturally.

Whereas previous research suggested the usefulness of the q-exponential model (Takahashi, 2013), the present research indicated the usefulness of the hyperbolic with exponent model cross-culturally. This result advances our understanding of a generalized mathematical model accounting for social discounting behaviors. To verify the validity of the current finding, the usefulness of the hyperbolic with exponent model should be examined further in the future. That said, we should hasten to add that consistent with the findings by Takahashi (2013), the hyperbolic with exponent model and the q-exponential model in this research suggest interpersonal inconsistency in social discounting regardless of cultures and frames, which is an important feature for understanding the nature of an individual's social choice.

Previous research suggested that East Asians are not more generous than Westerners, which is inconsistent with the shared idea that interdependence is more emphasized by East Asians than by Westerners. The current findings are congruent with those from previous research. To solve this inconsistency, we focused on the duality of interdependence proposed by Hashimoto and Yamagishi (2016), according to which interdependence consists of harmony-seeking and rejection avoidance. They found that the latter is higher but the former is lower in Japanese compared to North Americans. As expected, in testing Germans and Japanese, we found that Germans perceive harmony-seeking higher than do Japanese. This finding suggests that Hashimoto and Yamagishi's (2016) finding could be generalized to another Western culture (Germany). In addition, one advantage of the present research is that we demonstrated that the cultural difference in harmony-seeking could account for the cultural difference in the social discounting of gain. Examination of such an underlying mechanism will contribute to our understating of an individual's social choice reflecting her or his premise constructed and acquired through her or his living in a given sociocultural environment.

The decrease in a person's generosity to others was less obvious in Japanese than in Germans. This pattern was similar to that in Strombach et al.'s (2014) comparison of Chinese and Germans. It is also congruent with the findings of Buchan et al. (2006), who showed that the effect of group membership was weaker in Chinese, Japanese, and Koreans than in North Americans in terms of trustworthiness. This result suggests that although Germans are more generous to others in gains than Japanese overall, the cultural difference in generosity is more pronounced among closer friends than among distant others. Thus, whereas Germans might behave generously to specific others to whom they think they can pay a cost (i.e., forego a reward) to maintain friendships with others, the generosity of Japanese might be moderate but relatively fair to acquaintances.

Whereas Japanese discounted social gain more than social loss, which is consistent with Takahashi's (2013) finding, no difference in social discounting between gain and loss was found among Germans. The Japanese pattern is consistent with the sign effect in delay discounting whereby people are more likely to behave impulsively in gains than in losses. In contrast, as the German participants in this study might have sufficiently high other-regarding motives, place great weight on others' outcomes, and likely dismiss their own outcome regardless of the frames, a difference between gain and loss might not appear. As this research provided the first evidence on cultural similarities and differences in the social discounting of losses and the sign effect, future work should examine the validity of the current findings in different cultures.

To understand cultural differences in social discounting, it would be informative to explore what socioecological factors influence costly generosity to others. A person's generosity to others based on sacrificing his or her reward might be more useful in a mobile environment where interpersonal relationships consist of relatively weak ties and showing one's goodness and trustworthiness through one's prosocial and generous behaviors are crucial to form and maintain relationships, compared to in a stable environment, where interpersonal relationships are fixed and taken for granted. Moreover, it will also be important to look at whether and to what extent socioecological factors moderate the asymmetry between the social discounting of gains and losses. Residential mobility (Oishi, 2010) and relational mobility (Schug et al., 2010) might be useful for explaining the cultural differences in social discounting.

The present research has several limitations. First, although it followed the manipulation of social distance used in previous research (Jones and Rachlin, 2006; Rachlin and Jones, 2008) and assumed that the representations of social distance do not vary across cultures, we did not verify whether this assumption was correct. Thus, the present research cannot deny the possibility that the relationship type the participants imagined corresponding to a given social distance might differ across cultures, or that the cultural difference in the imagined relationship type might influence the cultural difference in social discounting. Given that in social discounting, a person's generosity increases when his or her partners are members of mates and genetic kinships (Hackman et al., 2015), the current finding that Germans are more generous than Japanese to others might be because Germans are more likely than Japanese to imagine members of mates and genetic kinships as close friends. Moreover, even if the imagined relationship type does not differ across cultures, perceived emotional closeness to a person at a given social distance might differ between cultures. Hackman et al. (2015) demonstrated that independently of the effect of the relationship type, emotional closeness increases a person's generosity. This result suggests a possibility that compared to Japanese, Germans perceived greater emotional closeness to others, particularly to closer friends, and that cultural difference led to the Germans' greater generosity. Future work should focus on these issues. Second, we used hypothetical gains and losses in this study, although Locey et al. (2011) demonstrated that there is no difference in social discounting between real and hypothetical rewards. Another limitation is that we did not consider how the amount of the gains and losses influences social discounting. Rachlin and Jones (2008, Experiment 2) indicated that people discount another person's reward more as the amount increases. The possibility that the cultural effect in social discounting might change as a function of the amount of gains could be tested in future work.

Despite these limitations, we believe that the present research showing the usefulness of the hyperbolic with exponent model cross-culturally and the mediating role of harmony-seeking in the cultural differences in social discounting in gains contributes to our understanding of the mechanism of individuals' generosity against the backdrop of the characteristics of sociocultural environments. We hope that future research addresses the generalizability of our findings in divergent cultural contexts.

AUTHOR CONTRIBUTIONS

KI designed the research and wrote the paper. KI and CE performed the research and analyzed the data.

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The Magnitude and "Peanuts" Effects: Searching Implications

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The framework of this paper is the field of decision-making processes in which people face the choice between probabilistic and dated rewards. Traditionally, the preferences for probabilistic outcomes have been analyzed by the Expected Utility (EU) model whilst the preferences for dated rewards have been studied by the Discounted Utility (DU) model. Nevertheless, recent empirical findings have revealed the existence of several anomalies or paradoxes in both contexts. Specifically, EU and DU models exhibit an anomaly affecting the amount of the reward, viz the "peanuts" and the magnitude effects, respectively, which seem to go in opposite directions. The aim of this paper is to analyze both effects jointly in a wide setting involving choices subject to risk and over a period of time, and thereby identify and consider the implications of one anomaly on the other.

Keywords: intertemporal choice, discounted utility model, expected utility model, magnitude effect, "peanuts" effect

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1. INTRODUCTION

This manuscript deals with the interaction between magnitude effect and "peanuts" effect, a term used by Prelec and Loewenstein [1], Weber and Chapman [2], and Haisley et al. [3] to describe the effect of a decreasing risk-aversion with decreasing monetary rewards. This is of particular importance since these two effects move in opposite directions, and the analysis of their interactions can be useful to validate any decision-making model which takes into account the elements of time and risk. The first part of the paper considers some implications of peanuts effect, magnitude effect, subendurance, and their reverse versions. The second part of the paper focuses on discount functions V given by the product of a utility function u and a function g depending on p and t, and goes on to analyze the implications which the magnitude effect and peanuts effect have on the utility function u.

Individual decision-making has been studied within disciplines which range from economics to psychology, passing through areas such as neuroscience [4]. However, these studies have always been carried out using two principal models:

1. The Discounted Utility (DU) model which is employed to assess a stream of rewards with different maturities [5]. In this way, individuals try to maximize their discounted payoff or "utility" which is given by:

$$U_0 = \sum_{t=0}^{T} \delta^t u_t, \tag{1}$$

where U_0 is the present value of the sequence, u_t is the utility obtained from the outcome available at instant t ($t = 1, 2, \ldots, T$) and δ represents the discount factor (which is lower than or equal to 1).

2. The Expected Utility (EU) theory which is employed to model those decision processes involving risky choices [6], that is to say, when the amounts have been specified in terms of probability. In the same way, individuals try to maximize their expected utility:

$$U_0 = \sum_{k=0}^{n} p_k u_k, \tag{2}$$

where U_0 is the present value of the experience, u_k the utility obtained from the k-th experience and p_k the probability associated with this experience (where obviously the n associated probabilities amount to 1: $\sum_{k=0}^{n} p_k = 1$).

The DU and EU models have a similar structure given that they are based on the same theoretical principles, since alternatives are assessed by taking into account the sum of their utilities [7]. Moreover, DU and EU are general models with the power to predict, and their high level of acceptance is explained by their simplicity and the fact that they are based on the traditional systems of calculation of the present and actuarial values, respectively.

Nowadays, the DU and EU models are the standard theories of rational choice over time and under risk, respectively, in many social and behavioral sciences [8]. However, it is well known that models are simplified representations of reality, and so this is a limitation when trying to describe the actual behavior of people [9]. In effect, several anomalies have recently been detected which must be taken into account when analyzing a real situation.

Despite their similarity, decisions involving intertemporal choices and uncertainty have traditionally been studied in different research areas, given that delayed and risky rewards do not require the same treatment. Some cautious attempts to integrate DU and EU models, such as the Discounted Expected Utility (DEU) [10, 11], have been made in order to analyze individual behavior in decisions involving time delay and risk [11, 12]. In the same vein, in psychology, it is usual to interpret time in a probabilistic way [13], or to translate risks into delays when facing risky choices [14].

Other scholars [1, 7, 15] have studied the analogies and the anomalies observed in DU and EU models by arguing that they start from certain fundamental psychological properties of multidimensional prospect valuation. In the following paragraph, some anomalies of the DU (intertemporal choices) and EU (risky choices) models are jointly presented:

1. The common difference effect (DU anomaly) and the common ratio effect (EU anomaly) [1].
2. The immediacy effect (DU anomaly) and the certainty effect (EU anomaly) [16].
3. The magnitude effect (DU anomaly) and the peanuts effect (EU anomaly).

In spite of the fact that the magnitude and the peanuts effects are embedded in different frameworks, intuitively they seem to move in opposite ways because "the peanuts effect seems to reveal decreasing sensitivity to payoffs at larger stakes, while the magnitude effect seems to reveal increasing sensitivity to payoffs at large stakes" [11]. In effect, whilst the magnitude effect occurs in the DU model, the peanuts effect makes sense in the EU model. For this reason, in this paper we will follow a joint model in which time and risk preferences are integrated, in the same line as Schneider [11] and Baucells and Heukamp [17].

In this paper, we will focus on the study of the magnitude effect and its relation with the peanuts effect both depending on the amounts of reward. The importance of this research is obvious: "Prelec and Loewenstein could not explain both effects, and this challenge has remained unresolved over the subsequent twenty-five years, posing an apparent impossibility result that no common approach to modeling risk and time preferences can capture both of these basic behaviors" [11].

The DU and EU models propose that relative preference between two options is consistent, even if their amounts of reward are increased by a constant factor [18, 19]. However, some authors [20–23] have demonstrated that in decisions with delayed rewards, the preference increases as its amount increases. This magnitude effect is based on the premise that the patience of individuals is directly related to the reward amount and that individuals are more patient for large rewards than for those of smaller amounts, leading to an increased preference for the larger later outcomes [24, 25]. Following Schneider [11], the magnitude effect may be represented as follows:

$$(x, p, s) \sim (y, p, t) \text{ implies } (kx, p, s) \prec (ky, p, t), \tag{3}$$

where $x < y$, $s < t$ and $k > 1$, where p represents the probability of occurrence which is considered constant in both alternatives. This paradox explains how large outcomes are discounted at a lower rate than smaller ones, the discount rate being a decreasing function of the size of the reward [7].

On the other hand, in choices involving risk, the amount of the reward has the opposite effect on the decision-making process. As pointed out by some scholars [26, 27], the peanuts effect has not yet been addressed as thoroughly as the magnitude effect. Only some recent studies [11] have mathematically analyzed this effect. It may be defined as follows ($x < y$):

$$(x, p, t) \sim (y, q, t) \text{ implies } (kx, p, t) \succ (ky, q, t), \tag{4}$$

the probabilities of occurrence being p and q, with $p > q$, and $k > 1$. Another definition of the peanuts effect was provided by Schneider and Day [28], as follows. Consider a lottery $f:(x, p; 0, 1-p)$, with $x > 0$. Let $c := \{f, E(f)\}$. Then, the peanuts effect holds if $f \succ_c E(f)$, for sufficiently small x, and $E(f) \succ_c f$, for sufficiently large x, for every $p \in (0, 1)$.

Weber and Chapman [2] revealed that "a utility function cannot account for the decrease in the size of the peanuts effect for smaller probabilities". In the same way, Schneider and Day [28] proved that the peanuts effect cannot hold for Cumulative Prospect Theory (CPT) or any non-choice-set-dependent EU model. Finally, Leland and Schneider [8] define the indifference relationship $SS \widetilde{\approx}_t LL$ in the framework of the so-called Salience Weighted Utility over Presentations (SWUP) model in the following way:

$$\mu(x, y)[u(y) - u(x)]\frac{\delta^s + \delta^t}{2} = \pi(s, t)(\delta^s - \delta^t)\frac{u(y) + u(x)}{2}$$

and the preference relationship $SS' \precsim_t LL'$ by the following inequality:

$$\mu(x,y)[u(y)-u(x)]\frac{\delta^s+\delta^t}{2} > \pi(s,t)(\delta^s-\delta^t)\frac{u(y)+u(x)}{2},$$

where $\mu(x,y)$ and $\pi(s,t)$ are the so-called "salience functions". In this way, they characterize the magnitude and the peanuts effects by imposing some conditions on the utility and the salience functions.

As indicated in the previous paragraphs, the magnitude and peanuts effects seem to move in opposite directions, and can be explained by two *a priori* psychological principles:

1. The magnitude effect may be explained by the psychology of perception, where individuals are more sensitive to absolute than to relative differences in magnitude [1, 29]. It should be borne in mind that decision-makers see larger delayed rewards as investments. In this way, the potential of earning a bigger amount of money leads to choosing the delayed reward with its corresponding increase.
2. The risk-seeking in the peanuts effect may be explained by the anticipated emotion of disappointment [16]. Disappointment is an emotion which is experienced when perceiving that a different situation would have led to a better result [30]. As some scholars [31, 32] suggest, the effect of anticipated emotions may influence individual decisions. According to the perceived level of this disappointment, the utility is a function of the difference between the actual outcome and the expected value of the gamble. In order to avoid the feeling of disappointment, a person prefers a less risky gamble over a riskier one. Because of this, the decision-maker becomes risk-seeking for potential smaller rewards given that losing them invokes less disappointment than losing a bigger one.

The influence of negative emotion is likely to be greater in risky choice than in intertemporal choice because the possibility of gaining a reward smaller than expected is implicit in risky choices. The presented studies confirm that, from a psychological point of view, probabilistic discounting is not identical to that which underlies temporal discounting.

The methodology employed in this paper is the interaction between time and risk preferences. Its main contribution is the way by which time and risk preferences interact given certain assumptions. As a result, a wide variety of mathematical relationships arise by using the concepts of regularity and subendurance, and an important representation of the discount function $V(x,p,t)$ as $u(x)g(p,t)$.

After describing the state of the art concerning the mathematical treatment in the existing literature of both the magnitude and the peanuts effects, this paper is organized as follows. In section 2, the possible relationships between the magnitude and the peanuts effects are analyzed by using a very general definition of discount function involving time delay and risk, and the presence or absence of the so-called subendurance. In section 3, the general expression of the discount function $V(x,p,t)$ is restricted to the functional form $u(x)g(p,t)$, where u is a utility function. Under these circumstances, some significant relationships can be obtained between the reverse magnitude and the peanuts effects in the presence of a regular discount function. Finally, section 4 summarizes and concludes.

2. FRAMEWORK AND GENERAL RESULTS

2.1. Preliminaries

It will prove useful to begin with some definitions [17].

Definition 1. Consider the set $\mathcal{M} = X \times P \times T$, where $X = [0,+\infty)$, $P = [0,1]$, and $T = [0,+\infty)$. A *discount function* is a continuous real-valued function $V(x,p,t)$ defined on $\mathcal{M}$ which is strictly increasing in the first and second components, and strictly decreasing in the third.

However, Baucells and Heukamp [17] require that V tends to zero whenever xpe^{-t} tends to zero, but in this paper this restriction will be relaxed, and we will assume that V converges to zero only when $x \to 0$ or $p \to 0$, by allowing that

$$\lim_{t\to+\infty} V(x,p,t) := L(x,p) \geq 0.$$

In the first case, V is said to be *regular* whilst, if $L(x,p) > 0$, V is said to be *singular*. The paper by Baucells and Heukamp [17] implicitly includes the regularity of V. In some further results we will require this condition but in others this condition will not be necessary. The following definitions have been obtained from Schneider [11].

Definition 2. The *peanuts effect* (resp. *reverse peanuts effect*) is said to hold if, for every $0 < x < y$, $p > q$ and $k > 1$, $(x,p,t) \sim (y,q,t)$ implies $(kx,p,t) \succ (ky,q,t)$ (resp. $(kx,p,t) \prec (ky,q,t)$).

Definition 3. The *magnitude effect* (resp. *reverse magnitude effect*) is said to hold if, for every $0 < x < y$, $s < t$ and $k > 1$, $(x,p,s) \sim (y,p,t)$ implies $(kx,p,s) \prec (ky,p,t)$ (resp. $(kx,p,s) \succ (ky,p,t)$).

The following definition reflects the idea that the larger the reward, the more subjects are willing to wait in exchange for improved probabilities [17].

Definition 4. *Subendurance* (resp. *reverse subendurance*) is said to hold if, for every $0 < x < y$, $s < t$ and $p < q$, $(y,p,s) \sim (y,q,t)$ implies $(x,p,s) \succ (x,q,t)$ (resp. $(x,p,s) \prec (x,q,t)$).

2.2. General Results

Lemma 1. Given an $x \in X$, let $V_x : P \times T \to \mathbb{R}$ be the real function defined by $V_x(p,t) := V(x,p,t)$. If V is regular, then, for every $(p,t) \in P \times T$ and every $q > p$, there exists a $\Delta = \Delta(x,p,q,t) > 0$ such that $V_x(q,t+\Delta) = V_x(p,t)$.

Proof. Given an $x \in X$, for every $(p,t) \in P \times T$ and every $q > p$, let us consider the following real-valued function:

$$V_{x,q} : T \to \mathbb{R}$$

defined as:

$$V_{x,q}(r) := V_x(q,r).$$

Taking into account the definition of V, the inequality $V_{x,q}(t) > V_x(p,t)$ holds. Moreover, as V is regular, when $r \to +\infty$, $V_{x,q}(r) \to 0$. Therefore, there exists a r_0, large enough, such that $V_{x,q}(r_0) \leq V_x(p,t) < V_{x,q}(t)$. As V is continuous and decreasing in t, by the Intermediate Value Theorem there exists a value $\Delta > 0$ such that $V_{x,q}(t+\Delta) = V_x(p,t)$, from where $V_x(q, t+\Delta) = V_x(p,t)$. □

Corollary 1. Let $V_p : X \times T \to \mathbb{R}$ be the real function defined by $V_p(x,t) := V(x,p,t)$. If V is regular, then, for every $(x,t) \in X \times T$ and every $y > x$, there exists a $\Delta = \Delta(x,y,p,t) > 0$ such that $V_p(y, t+\Delta) = V_p(x,t)$.

Proof. The proof is analogous to that of Lemma 1 because the amount x and the probability p play similar rôles. □

Observe that the next two results do not require the condition of regularity of V.

Lemma 2. Let $V_t : X \times P \to \mathbb{R}$ be the real function defined by $V_t(x,p) := V(x,p,t)$. For every $(x,p) \in X \times P$ and every $q > p$, there exists a $0 < k = k(x,p,q,t) < 1$ such that $V_t(kx,q) = V_t(x,p)$.

Proof. Given a $t \in T$, for every $(x,p) \in X \times P$ and every $q > p$, let us consider the following real-valued function:

$$V_{q,t} : X \to \mathbb{R}$$

defined as:

$$V_{q,t}(z) := V_t(z,q).$$

Taking into account the definition of V, the inequality $V_{q,t}(x) > V_t(x,p)$ holds. Moreover, as $V_{q,t}(0) = 0$, then $V_{q,t}(0) \leq V_t(x,p) < V_{q,t}(x)$. As V is continuous and increasing in x, by the Intermediate Value Theorem there exists a value $0 < k = k(x,p,q,t) < 1$ such that $V_{q,t}(kx) = V_t(x,p)$, from where $V_t(kx,q) = V_t(x,p)$. □

Corollary 2. Let $V_t : X \times P \to \mathbb{R}$ be the real function defined by $V_t(x,p) := V(x,p,t)$. For every $(x,p) \in X \times P$ and every $y > x$, there exists a $0 < k = k(x,p,q,t) < 1$ such that $V_t(y,kp) = V_t(x,p)$.

Proof. The proof is analogous to that of Lemma 1 because the amount x and the probability p play similar rôles. □

Observation 1. It can be shown that the peanuts effect holds if and only if, for every $0 < x < y$, $p > q$ and $0 < h < 1$, $(x,p,t) \sim (y,q,t)$ implies $(hx,p,t) \prec (hy,q,t)$. In effect, assume that $(x,p,t) \sim (y,q,t)$, with $0 < x < y$ and $p > q$. If, for a given $0 < h < 1$, $(hx,p,t) \succeq (hy,q,t)$, then $V_{p,t}(0) \leq V(hy,q,t) \leq V_{p,t}(hx)$. As $V_{p,t}$ is continuous and increasing, by the Intermediate Value Theorem, there would be a $h_0 \leq h$ such that $(h_0x,p,t) \sim (hy,q,t)$. As $1/h_0 > 1$, by hypothesis,

$$\left(\frac{1}{h_0}h_0x, p, t\right) \succ \left(\frac{1}{h_0}hy, q, t\right) \succeq (y,q,t),$$

which is a contradiction. Therefore, $(hx,p,t) \prec (hy,q,t)$. The proof of the reciprocal implication is analogous.

Observation 2. Analogously to Observation 1, it can be shown that the magnitude effect holds if and only if, for every $0 < x < y$, $s < t$ and $0 < h < 1$, $(x,p,s) \sim (y,p,t)$ implies $(hx,p,s) \succ (hy,p,t)$.

Observation 3. Finally, observe that it can be shown that subendurance holds if and only if, for every $0 < x < y$, $s < t$ and $p < q$, $(x,p,s) \sim (x,q,t)$ implies $(y,p,s) \prec (y,q,t)$. The proof is analogous to that of observations 1 and 2.

Theorem 1. The magnitude and the peanuts effects imply subendurance.

Proof. In effect, assume that the magnitude and the peanuts effects hold. In order to show that subendurance is satisfied, we are going to start from the following indifference relation:

$$(y,p,s) \sim (y,q,t), \tag{5}$$

where $s < t$, and $p < q$. By Lemma 2, there is a "trade-off" between y and q, in the way that the amount y can be increased until z, in exchange for diminishing the probability up to the value p, keeping the former indifference between the involved rewards, that is to say, such that:

$$(y,p,s) \sim (z,p,t), \tag{6}$$

where $z > y$. Therefore, as the magnitude effect holds, one has:

$$(ky,p,s) \prec (kz,p,t), \tag{7}$$

with $k > 1$. On the other hand, we can apply transitivity to equivalences (5) and (6), and so derive that:

$$(y,q,t) \sim (z,p,t). \tag{8}$$

By applying now the peanuts effect to the former equivalence and for the same value of k, one has:

$$(ky,q,t) \succ (kz,p,t). \tag{9}$$

Finally, the transitivity applied again to preferences (7) and (9) results in:

$$(ky,p,s) \prec (ky,q,t), \tag{10}$$

from where subendurance holds (see Observation 3). □

Theorem 2. The following statements hold[1]:

1. The magnitude effect and the reverse subendurance imply the reverse peanuts effect.
2. The reverse magnitude effect and the subendurance imply the peanuts effect.
3. The reverse magnitude effect and the reverse peanuts effect imply the reverse subendurance.

[1]The additional condition of the discount function being regular will be taken into account when a trade-off between reward amount (or probability) and time is necessary in order to apply Lemma 1 and Corollary 1.

4. The peanuts effect and the reverse subendurance imply the reverse magnitude effect .
5. The reverse peanuts effect and the subendurance imply the magnitude effect.

Proof. The proof is analogous to that of Theorem 1. □

All the implications included in theorems 1 and 2 can be schematized in **Figure 1**[2]. Observe that:

1. Only in the context of subendurance, the magnitude and the peanuts effects are compatible. That is to say, in the context of reverse subendurance, these effects are incompatible.
2. In both contexts (subendurante and reverse subendurance), there are some implications of the magnitude for the reverse peanuts effects, and of the reverse magnitude for the peanuts effects.

3. SEARCHING PARTICULAR IMPLICATIONS

3.1. Introducing a Discount Function

Assume that the discount function has the functional form $V(x,p,t) := u(x)g(p,t)$, where u is a utility function and $g(p,t)$ is a non-negative continuous function defined in $P \times [0, t_0)$ (t_0 can even be $+\infty$, in which case $[0, t_0) = T$) satisfying the following conditions:

1. $g(p,t) > 0$.
2. $g(p,t)$ is increasing with respect to p, and
3. $g(p,t)$ is decreasing with respect to t.

The discount function is said to be [33]

1. *Regular* if $t_0 = +\infty$ and $\lim_{t\to+\infty} g(p,t) = 0$, for every $p \in P$ (see **Figure 2**).
2. *Singular* if $t_0 = +\infty$ and $\lim_{t\to+\infty} g(p,t) := L(p) > 0$, for some $p \in P$ (see **Figure 3**).
3. *With bounded domain* if $t_0 \in \mathbb{R}$.

3.2. Particular Results

According to Baucells and Heukamp [17], this model is not compatible with subendurance and reverse subendurance. In these conditions, we can put forward the following propositions.

Proposition 1. A sufficient condition for the magnitude effect (resp. reverse magnitude effect) is that, for every $0 < x < y, s < t$, $p \geq q$ and $k > 1$, $(x,p,s) \sim (y,q,t)$ implies $(kx,p,s) \prec (ky,q,t)$ (resp. $(kx,p,s) \succ (ky,q,t)$). If V is regular, this condition is also necessary.

Proof. Obviously, the condition is sufficient (it suffices to take $p = q$). Let us see the necessity. In effect, assume that the magnitude effect holds. In order to show that the aforementioned condition holds, we are going to start from the following indifference relation:

$$(x,p,s) \sim (y,q,t), \tag{11}$$

where $0 < x < y, s < t$ and $p \geq q$. From the former equivalence, it can be deduced that

$$u(x)g(p,s) = u(y)g(q,t). \tag{12}$$

By Lemma 1, the next step is to propose a "trade-off" between q and t, in the way that the instant t can be delayed until t', in exchange for increasing the probability up to p, keeping the indifference between the involved rewards, that is to say, such that:

$$(x,p,s) \sim (y,p,t').$$

In effect, it suffices to define (taking into account that $g(p,\cdot)$ is continuous and decreasing)

$$t' := g(p,\cdot)^{-1}\left[\frac{u(x)}{u(y)}g(p,s)\right] > t. \tag{13}$$

The existence of such t' is guaranteed because V is regular. Therefore, as the magnitude effect holds, one has:

$$(kx,p,s) \prec (ky,p,t'), \tag{14}$$

for every $k > 1$.

Finally, observe that, from the former equalities and inequalities, the following inequality is satisfied:

$$\frac{u(kx)}{u(ky)} < \frac{g(p,t')}{g(p,s)} = \frac{u(x)}{u(y)} = \frac{g(q,t)}{g(p,s)},$$

which obviously derives in:

$$(kx,p,s) \prec (ky,q,t)$$

and so the condition is necessary. The reasoning for the reverse magnitude effect is analogous. □

Proposition 2. The peanuts effect (resp. reverse peanuts effect) holds if and only if, for every $0 < x < y, s \leq t, p > q$ and $k > 1$, $(x,p,s) \sim (y,q,t)$ implies $(kx,p,s) \succ (ky,q,t)$ (resp. $(kx,p,s) \prec (ky,q,t)$).

Proof. Obviously, the condition is sufficient (it suffices to take $s = t$). Let us see the necessity. In effect, assume that the peanuts effect holds. In order to show that the aforementioned condition holds, we are going to start from the following indifference relation:

$$(x,p,s) \sim (y,q,t), \tag{15}$$

where $0 < x < y, s \leq t$ and $p > q$. From the former equivalence, it can be deduced that

$$u(x)g(p,s) = u(y)g(q,t). \tag{16}$$

[2]In order to interpret this chart and the next one correctly, it is necessary to take into account the following criterium: if a cell can be vertically embedded in a contiguous cell, the property indicated in the first (smaller) cell implies the property enclosed in the second (bigger) cell.

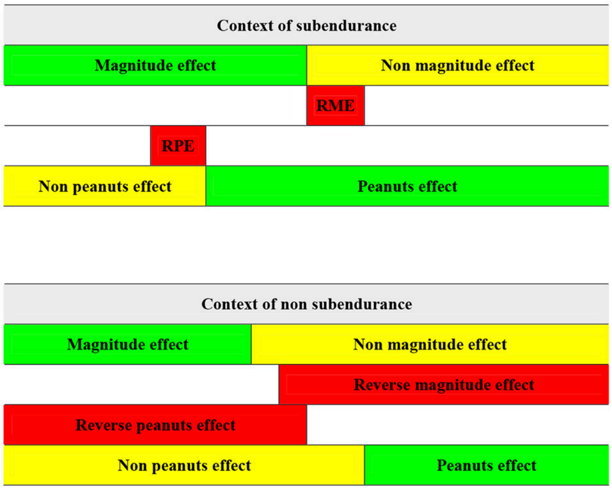

FIGURE 1 | Implications between the magnitude and the peanuts effects. Green denotes de presence of the mentioned effect. Yellow denotes de absence of the indicated effect. Red represents the existence of the reverse of the mentioned effect. Source: own elaboration.

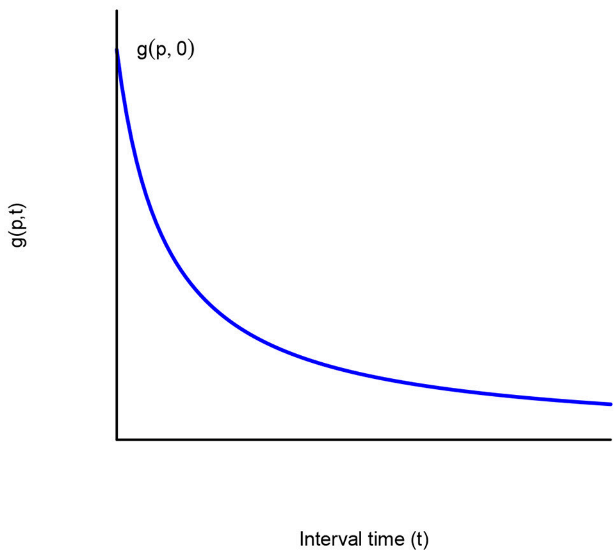

FIGURE 2 | A regular discount function. Source: own elaboration.

The next step is to propose a "trade-off" between q and t, in the way that the probability q can be reduced until q', in exchange for anticipating the availability of the second reward up to s, keeping the indifference between the involved rewards, that is to say, such that:

$$(x, p, s) \sim (y, q', s).$$

In effect, it suffices to define (taking into account that $g(\cdot, s)$ is continuous and decreasing, and $g(0, s) = 0$)

$$q' := g(\cdot, s)^{-1}\left[\frac{u(x)}{u(y)} g(p, s)\right] < q. \tag{17}$$

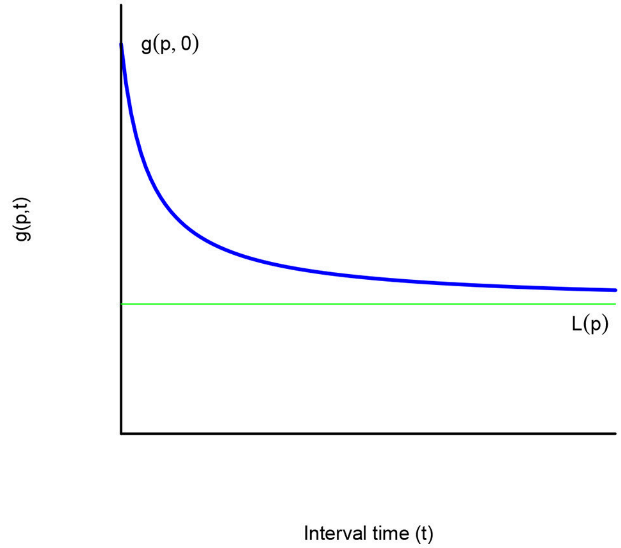

FIGURE 3 | A singular discount function. Source: own elaboration.

Therefore, as the peanuts effect holds, one has:

$$(kx, p, s) \succ (ky, q', s), \tag{18}$$

for every $k > 1$.

However, observe that, from the former equalities and inequalities, the following inequality is satisfied:

$$\frac{u(kx)}{u(ky)} > \frac{g(q', s)}{g(p, s)} = \frac{u(x)}{u(y)} = \frac{g(q, t)}{g(p, s)},$$

which obviously derives in:

$$(kx, p, s) \succ (ky, q, t)$$

and so the condition is necessary. The reasoning for the reverse peanuts effect is analogous. □

The following results are a direct consequence of propositions 1 and 2 in the framework defined at the beginning of section 3.

Corollary 3. The peanuts effect implies the reverse magnitude effect. Moreover, if the discount function involved in the intertemporal choice is regular, then the reverse magnitude effect implies the peanuts effect. □

Corollary 4. If the model involved in the intertemporal choice is the q-exponential discounting, then the reverse magnitude effect is equivalent to the peanuts effect.

Proof. In effect, take into account that the q-exponential (in particular, the exponential, the hyperbolic and the linear discounting) discount function [34] is regular, as required by Corollary 3. □

The following proposition provides a characterization of the utility function u involved in the context of the peanuts effect.

Proposition 3. The peanuts effect holds if and only if the elasticity of the utility function

$$\epsilon_u(z) := z[\ln u(z)]'$$

is decreasing.

Proof. Let us assume that the peanuts effect holds and that $0 < x < y$. Let p and q ($p > q$) be two probabilities such that $(x, p, t) \sim (y, q, t)$. In such a case,

$$u(x)g(p, t) = u(y)g(q, t).$$

By the peanuts effect, $(kx, p, t) \succ (ky, q, t)$ for every $k > 1$, and so

$$u(kx)g(p, t) > u(ky)g(q, t).$$

By dividing the left and the right-hand sides of the former expressions, one has:

$$\frac{u(kx)}{u(x)} > \frac{u(ky)}{u(y)}$$

and so

$$\ln u(kx) - \ln u(x) > \ln u(ky) - \ln u(y).$$

As $k > 1$, we can write $k := 1 + h$, with $h > 0$. Therefore,

$$\ln u(x + hx) - \ln u(x) > \ln u(y + hy) - \ln u(y).$$

Dividing both sides of the former inequality by h and letting $h \to 0$, we obtain:

$$x \left. \frac{\mathrm{d} \ln u(z)}{\mathrm{d}z} \right|_{z=x} \geq y \left. \frac{\mathrm{d} \ln u(z)}{\mathrm{d}z} \right|_{z=y}.$$

Consequently, the elasticity of the utility function, $z[\ln u(z)]'$, is decreasing. The converse implication can be easily shown. □

This result coincides with that provided by [35].

Corollary 5. If the peanuts effect holds then u is ln-concave.

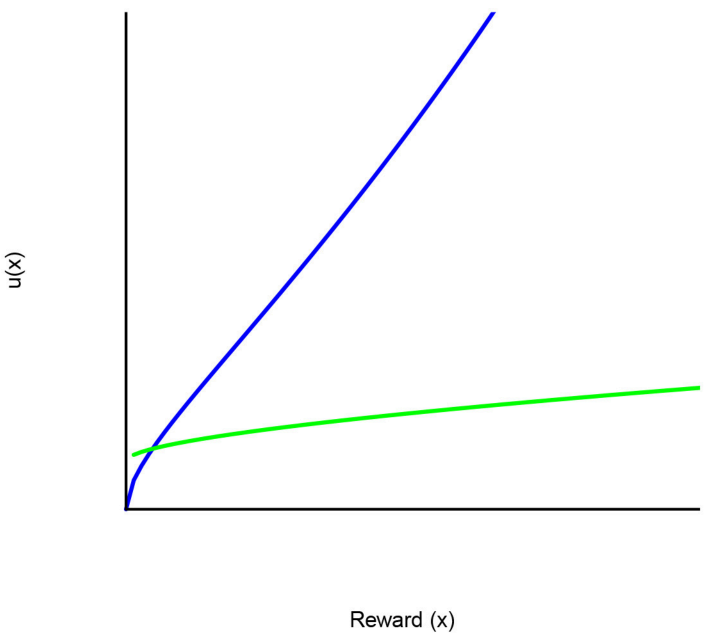

FIGURE 4 | Counterexample of corollary 5. Source: own elaboration.

Intertemporal choice model: $V(x, p, t) = u(x)g(p, t)$		
***u* ln-convex**	***u* ln-concave**	**ln *u* non-monotonic**
ε_u **increasing**	ε_u **decreasing**	ε_u **non-monotonic**
Magnitud effect	**Reverse magnitude effect**	**Non magnitude effect**
Reverse peanuts effect	**Peanuts effect**	**Non peanuts effect**

FIGURE 5 | Convexity, and peanuts and magnitude effects Green denotes de presence of the mentioned effect. Yellow denotes de absence of the indicated effect. Red represents the existence of the reverse of the mentioned effect. Source: own elaboration.

Proof. In effect, if the peanuts effect holds, by Proposition 3, $z[\ln u(z)]'$ is decreasing. As z is increasing, then $[\ln u(z)]'$ is decreasing. Therefore, u is ln-concave. □

The converse statement is not true. In effect, the utility function $u(z) = \exp\{\sqrt{z}\} - 1$ is ln-concave (see **Figure 4**, line in blue) but

$$[z[\ln u(z)]']' = \frac{1}{2}\frac{\sqrt{z}\exp\{\sqrt{z}\}}{\exp\{\sqrt{z}\} - 1}$$

is increasing (see **Figure 4**, line in green).

Analogously, we can show the following statements.

Proposition 4. The magnitude effect holds if and only if the elasticity of the utility function is increasing. □

Corollary 6. If u is ln-convex then the magnitude effect holds. □

All the implications between the analyzed effects and the ln-convexity of u can be seen in **Figure 5**.

4. CONCLUSIONS

In behavioral finance, the decision-making processes should be explained through appropriate theoretical models and taking into account the possible anomalies that human behaviors entail. The treatment of these paradoxes can improve the explanatory power of the economic models by involving some suitable tools from the fields of economics and psychology.

Although sometimes time and risky preferences may be considered as analogous concepts, the temporal and probabilistic discount functions are not identical. To illustrate the difference between intertemporal and probabilistic choices, we refer to two anomalies of the discounted and expected utility models: the magnitude and the peanuts effects, respectively.

The magnitude effect occurs in intertemporal choices where the larger reward is usually related to a longer waiting time and a lower discount rate. Given that these choices occur free of negative feelings such as disappointment, the decision-maker, in search of greater profits, may prefer to wait. However, the peanuts effect occurs in uncertain choices in which the disappointment experienced is directly related to the amount and probability of the result. Specifically, the decision-maker is more prone to make risky decisions when a small amount is involved in the experiment.

This paper has presented a model simultaneously applied to time and risk which could explain both the magnitude effect in choices over time and the peanuts effect in choices under risk. In pursuit of establishing the relationship between both effects, we have obtained some implications in a broad framework by considering the presence or absence of subendurance. In the particular case in which $V(x, p, t)$ adopts the expression $u(x)g(p, t)$, where u is a utility function, the reverse magnitude and the peanuts effects are equivalent when the discount function is regular. Finally, some implications have been deduced involving the ln-convexity of $u(x)$.

AUTHOR CONTRIBUTIONS

SC and AS contributed to the design and implementation of the research, and to the writing of the manuscript.

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How Are Individual Time Preferences Aggregated in Groups? A Laboratory Experiment on Intertemporal Group Decision-Making

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The study of intertemporal decision-making is an interdisciplinary scientific topic of economics, psychology, and neuroscience. Most of these studies focus on individual intertemporal decisions, but little is known about the relationship between groups and individual time preferences. As a result, we intend to assess the role of group intertemporal decision-making. We experimentally investigate how to aggregate individual time preferences by clarifying who has the most influence on group decisions among heterogeneous group members. We formulate two hypotheses. The first is the multilateral bargaining hypothesis, which is based on the multilateral bargaining model. If people employ this model to reach agreement, the most patient member in a group has the greatest impact on group choices. The second is the median voter hypothesis, which is based on the median voter model. When people employ this model to reach agreement, the median patient member in a group has the greatest impact on group choices. Here, we find that the median patient member in a group has a significant impact on group decisions in an unstructured bargaining situation. This finding suggests that people use the majority voting rule during group intertemporal decision-making. Thus, our findings support the median voter hypothesis. Furthermore, the results of a chat analysis show that this result is partially due to people's conformity with the majority opinion.

Keywords: intertemporal choice, impatience, group decision-making, multilateral bargaining, median voter model, laboratory experiment, chatting, conformity with majority

INTRODUCTION

Many essential economic decisions are made by groups such as companies and households. For example, investment plans in companies are decided in meetings with multiple people, and saving plans in households are decided by partners. Environmental decisions are also intertemporal choices. For example, we are faced with a decision between consuming ecological resources today vs. enjoying a rich environment later. These types of decisions are mainly made by the society, such as local communities, governments, and so on.

In general, economists describe a group choice as a summation of individual choices or representative individuals. Do real-life group decisions over time work in this way? Although it is important that we understand how intertemporal group decision are made, there is little empirical evidence on the relationship between group and individual time preferences. Very few studies have used experiments to examine groups' intertemporal choices[1]. Yang and Carlsson [2] investigated whether a group consensus related to intertemporal decision-making was derived from individual time preferences. In their research, participants were wives and husbands in rural China, where each couple answered intertemporal questions. They found that 11% of the consensus decisions made by couples were more patient than both the wife's and husband's individual choices and that 9% of the consensus decisions were more impatient than the individuals' choices. Interestingly, their result suggested that some proportion of group consensus decisions were not simply a summation of individuals' choices. Nevertheless, they failed to reveal why couples' consensus decisions were beyond the individuals' time preferences. Furthermore, they focused on family financial decisions made by spouses. Thus, we cannot apply their findings to groups' intertemporal choices in general. Yang and Carlsson [2] and Carlsson et al. [3] also investigated who in the family had the most influence on group consensus decisions, identifying that husbands had a greater impact than wives did in joint choices. However, this result could be culture specific and, thus, might not be generalizable to other regions or countries.

The purpose of this study is to investigate the mechanism behind groups' intertemporal decision-making in heterogeneous time preferences by clarifying who has a significant impact on groups' intertemporal choices. As such, we contribute to the literature by investigating the difference between group preferences and individual preferences. Charness and Sutter [4] indicate that groups make more self-interested decisions than individuals. He and Villeval [5] show that group decisions reflect the same level of inequality aversion as individual decisions. Many studies investigated the difference between group and individual risk preferences [6–12]. Although an increasing number of studies are comparing groups' and individuals' decision-making, few studies focus on groups' intertemporal decision-making [2, 3]. Our study is the first to empirically examine the mechanism behind how groups reach intertemporal decisions. Our second contribution to the literature is that we use anonymous experimental protocols to eliminate various unobservable effects. In previous studies [2, 3], the participants are families, who use face-to-face dialogue to make decisions. Here, we gather unrelated participants, who then communicate with each other via text messages on a computer. The third contribution of our study is that our groups contain more than two members (i.e., three members). This makes it possible to generalize the results to real-life group settings. Our fourth contribution is that we analyze the chat messages during group decision-making to establish how the group forms a consensus. The fifth contribution of our study is to show how individual time preferences are aggregated in groups. The mechanism behind time preference aggregation has not been studied previously. In summary, the goal of this study is to determine the processes behind group decisions in an intertemporal context.

We formulate two hypotheses. First, group intertemporal choices are decided based on the multilateral bargaining model; that is, the most patient member has a significant impact on group choices (multilateral bargaining hypothesis). Second, group intertemporal decisions are determined based on the median voter model; that is, the median patient member has a substantial impact on group choices (median voter hypothesis). To test these hypotheses, we conduct laboratory experiments. Here, participants make intertemporal choices individually in individual conditions. In addition, participants make intertemporal choices in a group (three people) through discussion to reach group decisions in group conditions. We analyze the individual discount factors elicited in the individual conditions and the group discount factors elicited in the group conditions. Groups consist of a most patient member, a median patient member, and a least patient member[2]. Here, we examine whose discount factor is the closest to the group discount factor and interpret this member as having the greatest impact on a group decision. Consequently, we check the results using a regression analysis, post-estimation analysis, and Bayesian analysis.

Based on the results of the regression and post-estimation analyses, we reject the multilateral bargaining hypothesis. However, rejecting the multilateral bargaining hypothesis and not rejecting the median voter hypothesis does not guarantee support for the median voter hypothesis [13]. To resolve this problem, we conduct a Bayesian analysis and calculate the Bayes factors. The results support the median voter hypothesis. Finally, we examine the text data generated by the participants' computer-based chats to investigate why the median voter hypothesis is supported. Based on this analysis, we find that the third person to express an opinion (i.e., the last person) tends to follow the majority opinion. This phenomenon might lead to the observed result, even though the experimental group setting is a free discussion and follows the unanimity rule.

The rest of the paper is organized as follows. Section Methods describes our experimental design and procedures. Section Hypotheses presents the two hypotheses. Section Results analyzes the results using regression, post-estimation, Bayesian, and chat analyses. Section Conclusion concludes the paper.

METHODS

The experiment took place at Osaka University in Japan. We conducted six sessions with a total of 105 student participants, with each session comprising 15 or 18 participants and lasting

[1]Bixter et al. [1] also studied group and individual time preferences. They investigated whether individual choices after group decisions are affected by group agreement.

[2]Of course, there are cases where all the members' individual discount factors are same or two of the three members' individual discount factors are same. We explain how we handled these cases in section Regression Analysis and Post Estimation (Regression analysis and post estimation).

for approximately 1 h. All participants gave written informed consent according to the Declaration of Helsinki and the guidelines approved by the ethical committee (Institute of Social and Economic Research Ethical Committee at Osaka University) prior to the experiment. The experiment was computerized using z-Tree [14].

Participants expressed their time preferences by making a series of choices between early and delayed options of different denominations (i.e., using choice titration [15]). For example, participants were asked whether they would prefer JPY 1,750 today or JPY 2,000 1 month later. After making the choice, the amount of the delayed option was changed, which meant participants faced new options. We explained how to obtain the amount of the delayed option to which participants were indifferent between the delayed option and the early option in a simplified manner. For example, person X was indifferent between receiving JPY 1,750 today and JPY 2,100 1 month later. Thus, the amount of the delayed option to which the participant was indifferent to receiving the early option was JPY 2,100. We tested what amount of the delayed option was indifferent to JPY 1,750 by asking the question, "Do you prefer receiving JPY 1,750 today or JPY 2,000 1 month later?" When person X chose JPY 1,750 today, we then asked, "Would you prefer to receive JPY 1,750 today or JPY 2,120 1 month later?" In this way, we obtained an approximate indifference amount for the delayed option, relative to the amount of the early option. We repeated this process four times in order to obtain the indifference amount of the delayed option. All alternatives are shown in **Figure 1**. We calculated the indifference amount of the delayed option as follows. We used two values: "High up," which was the highest amount of the delayed option a participant did not choose; and "Lowdown," which was the lowest amount of the delayed option that a participant did choose. We defined the mean of "High up" and "Lowdown" $\left(\left\{Highup + Lowdown\right\}/2\right)$ as the indifference amount of the delayed option, relative to the amount of the early option. All indifference values are shown in the **Supplementary Material**. For example, when a participant reached C in **Figure 1** and (early date, later date) = (today, 1 month), the indifference amount of the delayed option (1 month) relative to the amount (JPY 1,750) of the early option (today) was JPY 2,165. In this case, "High up" was JPY 2,150 and "Lowdown" was JPY 2,180. The participants were not informed of all the available alternatives or the titration rule before the task. The combinations of dates for the early and delayed options were categorized into two types: (early date, later date) = (today, 1 month) and (1 month, 2 months). The order of the two types of dates was determined randomly.

After obtaining the indifference amount of the delayed option relative to the amount of the early option, we calculated the discount factor. In this study, we referred to the discount factors as time preferences. We used a linear utility function: $u(x) = x$. We assumed the indifference amount of the delayed option relative to the amount of the early option to be $x_{delayed}$, and that of the early option to be x_{early}. Later, we calculated the discount factor δ as $x_{early} = \delta^t x_{delayed}$ $\left(i.e.,\ \delta = \left(x_{early}/x_{delayed}\right)^{\frac{1}{t}}\right)$, for year t. Because there were two types of dates for the two options, (early date, later date) = (today, 1 month) and (1 month, 2 months), the values of t were 1/12 and 1/12, respectively. We referred to δ as the patience. A high value of δ denoted patience and a low value denoted impatience. Thus, the most impatient preference was A, and the most patient preference was P in **Figure 1** because x_{early} was always 1,750. The impatience preference ranking was A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P.

There were three choice conditions: an individual condition, a different condition ("for another"), and a group condition. All participants were tested for all three conditions. To control the order effect of the choice conditions, we conducted various combinations of all three conditions. In other words, we conducted six sessions. For the individual condition, participants chose alternatives for themselves; for the "for another" condition, participants chose alternatives for other members of the same group on an individual basis. Here, the chosen option was paid to the other members of their group. The payment mechanism is described in the next paragraph. We failed to analyze the "for another" condition here, because it was described in detail by Truruta et al. [16][3]. For the group condition, three participants per group discussed the options using text messages on a computer. As a result, participants could not identify the other group members' gender, visual aspect, race, and so on. We did this to eliminate unobservable effects on decision-making that would result from a face-to-face discussion. Moreover, this enabled us to analyze the text data for each group discussion: for example, the extent of discussion, who expresses an opinion first, and whether persuasion occurs. Subsequently, we investigated our finding that median voter hypothesis was supported (i.e., the median patient member had the greatest impact on group choices). In the group condition, the amount of the reward shown was per person: for example, "Each member is going to receive JPY 1,750 today or JPY 2,000 one month later." Participants, then, needed to make a group choice through discussion. Hence, the decision rule was unanimity. No time limits were imposed in any of the conditions.

The amount of the reward per person was the same in all three choice conditions. One of the options that the participants chose was selected randomly by the computer, and the selected option was paid to participants. The option that was paid was selected from all group choices a person's group chose under the group condition, all individual choices s/he chose under the individual condition, and all individual choices the other group members (two persons) chose under the "for another" condition. The reward was an Amazon gift card, which was e-mailed to the participant. The advantage of this reward was that we could make the transaction cost the same between the immediate reward and the delayed reward. The average reward was approximately JPY 1,890.

After participants arrived at the laboratory, the instructions were distributed. The instructions[4] are included in the **Supplementary Material**. Participants sat in cubicles and

[3]We do not include the "for another" condition in our analysis because we are not interested in this category in this study. See Tsuruta [16] for a discussion of this condition. Thus, we analyze only the group and individual conditions.

[4]The instruction is written in Japanese.

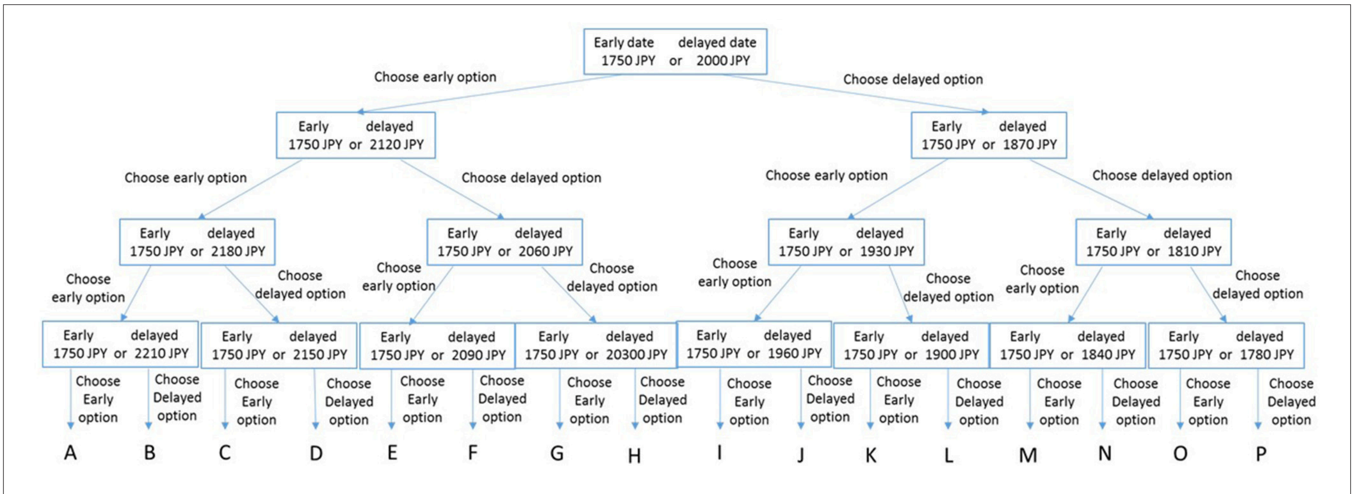

FIGURE 1 | Tree diagram of alternatives. Participants choose whether to receive a lower amount of money at an early date or a higher amount at a later date. After choosing an option, the amount of the delayed option is changed, and the participants face new options. After four iterations, the indifference amount for the delayed option relative to the amount of the early option can be determined. The indifference amount for the delayed option is shown for 16 categories (A, B, …, P). The most impatient preference is A, and the most patient preference is P. The impatience preference ranking is A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P.

chose the alternatives using a computer. After three choice conditions, we informed participants about the reward they were to be paid, and they completed a post-experimental questionnaire that inquired about their demographic data.

HYPOTHESES

We formulate two hypotheses. The first hypothesis is based on the multilateral bargaining model. According to this hypothesis, the most patient member in a group has the greatest impact on group choices (multilateral bargaining hypothesis). The second hypothesis is based on the median voter model. According to this hypothesis, the median patient member has the greatest impact on group choices (median voter hypothesis).

Multilateral Bargaining Hypothesis

This subsection describes the multilateral bargaining hypothesis, which is based on the multilateral bargaining model. This hypothesis predicts that the most patient member has the greatest impact on group choices. Our experimental setting is close to multilateral bargaining, in that members of a group negotiate with each other over group choices.

The bilateral bargaining model is well known as a result of Rubinstein's pioneering work [17]. The multilateral bargaining model was developed by Baron and Ferejohn [18] and Banks and Duggan [19] and had been studied mainly in the fields of political science and economics. In the popular model setting of multilateral bargaining [18], a proposer is selected from all members with some probability, who then proposes the allocation of the surplus. Then, the members vote on the proposals (i.e., accept or not accept). Under the majority voting rule, the proposal is implemented and the game ends when the majority of the members vote to accept. Otherwise, the procedure is repeated, including the selection of a proposer [18]. Under the unanimity voting rule, the proposal is implemented and the game ends when all members vote to accept. Otherwise, the procedure is repeated by selecting another proposer [20].

Our experiment differs from these bargaining models, where members decide how to allocate the resource among themselves, because members decide on a common group discount factor (i.e., time preferences) in our experimental setting. Here, we follow Ambrus et al. [6], who studied how individual risk preferences were aggregated in groups and also employed the multilateral bargaining model as a theoretical background. We do so because we also investigate how individual preferences are aggregated in groups. Moreover, applying the multilateral bargaining model is plausible, because people have heterogeneous opinions and make group decisions through discussion in our experimental setting.

The main difference between our hypothesis and that of Ambrus et al. [6] is that we assume heterogeneous individual discount factors, whereas they assume common discount factors among members of a group. Our assumption is closer to how group decisions are made in real-life. Another difference is that we employ the unanimity rule; that is, group members' choices must be the same after the discussion[5].

Several theoretical studies based on the multilateral bargaining model assume heterogeneity of individual discount factors and employ the unanimity rule [21–23]. According to these studies, a more patient member receives a higher payoff and has stronger bargaining power. This is because patient members can reject unfavorable proposals more easily than impatient members. The unfavorable option refers to an objective option (i.e., a lower reward). For example, suppose we have a proposer A, a patient member B ($\delta_B = 0.9$), and an impatient member C ($\delta_C = 0.2$), when proposer A proposes that A receives 60, B receives 20, and

[5]In their experimental setting, Ambrus et al. [6] employed the unanimity rule, as we did. However, they built their hypothesis using both the unanimity rule and the majority rule.

C receives 20 at time t, the patient member B compares 20 to δ_B $(= 0.9) \times$ (the expected allocation of B at $t + 1$) to decide whether or not to accept the proposal. Similarly, the impatient member C compares 20 to δ_C $(= 0.2) \times$ (the expected allocation of C at $t + 1$) to decide whether or not to accept the proposal. Here, the impatient member C's expected payoff at $t + 1$ is likely to be lower than that of the patient member owing to the difference in the discount factors. When B and C's allocations at $t + 1$ are both 40, the patient member B rejects the proposal at t ($20 < 0.9 \times 40$), but the impatient member C accepts the proposal at t ($20 > 0.2 \times 40$).

We apply this logic to our experimental setting. Thus, the multilateral bargaining hypothesis states that the most patient member has the greatest impact on group choices.

Median Voter Hypothesis

This subsection describes the median voter hypothesis, which is based on the median voter model. This hypothesis predicts that the median patient member has the greatest impact on group choices in our experiment.

The median voter model is well known and is applied in various academic fields [24]. Here, the median voter's choices are selected as the group choices under majority rule if all voters have a single peaked preference [25]. We apply this logic to our experimental setting. Thus, we hypothesize that the median patient member has a significant impact on group choices. According to this hypothesis, we predict that group decisions are conducted under the majority voting rule, not bargaining, even though they can discuss the options freely.

In our experimental setting, the median patient member's final choices are the same as the final group choices when the group members employ the majority voting rule. The explanation is as follows (see **Figure 1**). First, we consider the situation in which all the members' preferences are different. For example, there are three members of the same group, each of whom prefer a different option (D, E, and F). For the first question (JPY 1,750 at an early date or JPY 2,000 at a later date), all three members select the early option. For the second question (JPY 1,750 at an early date or JPY 2,120 at a later date), the person who likes D the best selects the early option, but the other members select the delayed option. Consequently, the group chooses the delayed option because they employ majority voting. For the third question (JPY 1,750 at an early date or JPY 2,060 at a later date), all three members select the early option. For the fourth question (JPY 1,750 at an early date or JPY 2,090 at a later date), the person who likes F the best selects the delayed option, but the other members select the early option. In this case, the group chooses the early option because they employ majority voting. Therefore, the final group choice is E, which is the same as the median patient member's final choice. The results for the other cases are the same. Second, we consider the situation in which two of the members' final choices are the same. The median patient member always belongs to the majority, because there are three group members. For example, if one member prefers C but the other two prefer D, then the median patient member is the person who prefers D. If one member prefers I but two prefer E, then the median patient member is the person who prefers E. Therefore, the median patient member's final choice is also the final group choice under the majority voting rule, because the median patient member always belongs to the majority. Finally, we consider the situation in which all the members' final choices are the same. Evidently, the median patient member's final choice is the same as the final group choice in this situation.

Consequently, the median voter hypothesis states that the median patient member has the greatest impact on group choices.

TABLE 1 | Discount factors for each rank under each combination of dates.

	Rank	*N*	Mean	Median	S.D.	Min	Max
Today vs. 1 month	1	35	0.777	0.815	0.119	0.184	0.815
	2	35	0.695	0.815	0.203	0.109	0.815
	3	35	0.448	0.497	0.277	0.061	0.815
	Group	35	0.683	0.815	0.196	0.129	0.815
1 month vs. 2 month	1	35	0.770	0.815	0.128	0.184	0.815
	2	35	0.701	0.815	0.208	0.092	0.815
	3	35	0.586	0.737	0.268	0.061	0.815
	Group	35	0.708	0.815	0.186	0.184	0.815

Here, (today vs. 1 month) and (1 month vs. 2 months) refer to the combinations of dates for the early option and the delayed option. Using (today vs. 1 month) as an example, participants choose to receive a small reward today or a larger reward one month later. Each rank refers to the order of the discount factor under the individual condition in the same group. Rank 1 refers to the most patient member (the highest discount factor) in a group, rank 2 refers to the median member (the second highest discount factor), and rank 3 refers to the most impatient member (the third highest discount factor, i.e., the lowest discount factor). Group refers to the discount factor for the group condition.

RESULTS

Regression Analysis and Post-estimation Analysis

We classify group members into three ranks, as follows. Rank 1, rank 2, and rank 3 represent the order of the discount factor under the individual condition in the same group[6]. Rank 1 refers to the most patient member (the highest discount factor) in a group, rank 2 refers to the median member (the second highest discount factor), and rank 3 refers to the most impatient member (the third highest discount factor: i.e., the lowest discount factor). If two members in the same group have the same value, we handle it as follows. For example, when the value of member A's discount factor is 0.8, that of member B's discount factor is 0.8, and that of member C's discount factor is 0.7, we set rank 1 and rank 2 to 0.8 and rank 3 to 0.7. If all members of the group have the same value, we set all three rank values to the same value.

Table 1 shows the descriptive statistics for each rank and the group discount factors for each date. **Figure 2** shows the mean discount factors of the (today, 1 month) condition. **Figure 3** shows the mean discount factors of the (1 month, 2 months) condition. In both cases, rank 2 seems to be close to the group discount factor.

[6]We explain how to calculate discount factor (i.e., time preferences) in section Methods.

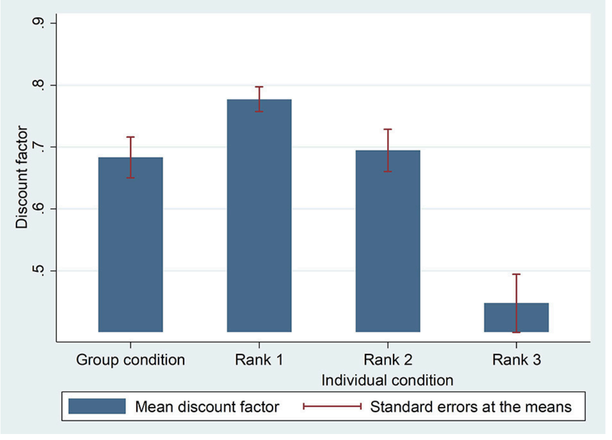

FIGURE 2 | Mean discount factors for the group condition and the individual conditions (Rank 1, Rank 2, and Rank 3). The combination of dates for the early option and the delayed option is (today vs. 1 month). Participants choose to receive a smaller reward today or a larger reward one month later. Each rank refers to the order of the discount factor under the individual condition in the same group. Rank 1 refers to the most patient member (the highest discount factor) in a group, rank 2 refers to the median member (the second highest discount factor), and rank 3 refers to the most impatient member (the third highest discount factor, i.e., the lowest discount factor). Group refers to the discount factor for the group condition.

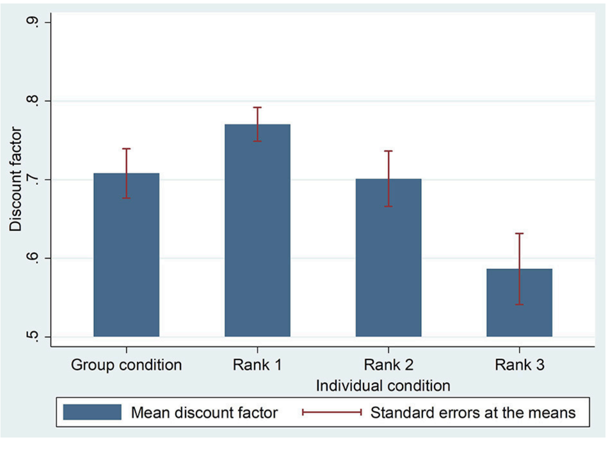

FIGURE 3 | Mean discount factors for the group condition and individual conditions (Rank 1, Rank 2, and Rank 3). The combination of dates for the early option and the delayed option is (1 month vs. 2 months). Participants choose to receive a smaller reward one month later or a larger reward two months later. Each rank refers to the order of the discount factor under the individual condition in the same group. Rank 1 refers to the most patient member (the highest discount factor) in a group, rank 2 refers to the median member (the second highest discount factor), and rank 3 refers to the most impatient member (the third highest discount factor. i.e., the lowest discount factor). Group refers to the discount factor for the group condition.

For a more detailed analysis, we conduct a regression analysis and a post-estimation analysis. These analyses are partially based on the work of Ambrus et al. [6], who also analyze how individual preferences are aggregated at the group level. They investigated risk preferences using a lottery task and selfishness using a gift exchange game. We focus on the model in which the group decision is a linear function of $\left(\delta_g^{(rank\ i)}\right)_{i=1,2,3}$:

$$\delta_g^{group} = constant + \alpha_1\delta_g^{(rank1)} + \alpha_2\delta_g^{(rank2)} + \alpha_3\delta_g^{(rank3)} + \epsilon_g, (1)$$

where δ_g^{group} denotes group g's elicited group discount factor, and $\delta_g^{(rank\ i)}$ denotes rank i's elicited individual discount factor in group g. We use $\delta_g^{(rank\ j)}$ to refer to the jth highest discount factor among the individuals in group g (in particular, $\delta_g^{(rank\ 1)}$ refers to the highest and $\delta_g^{(rank\ 3)}$ refers to the lowest discount factors). We interpret the coefficients (i.e., α_1, α_2, α_3) as the influence of each rank on the group discount factors.

Next, we conduct a post-estimation analysis using our two hypotheses (i.e., the multilateral bargaining hypothesis and the median voter hypothesis): see section Hypotheses. Later, we conducted post-estimation analyses, as follows. First, we test whether the most patient member has the greatest impact on group choices by analyzing whether we can reject $\alpha_2 = \alpha_3 = 0$. Subsequently, we test this more strictly by analyzing whether we can reject $\alpha_2 = \alpha_3 = 0$ and $\alpha_1 = 1$. If these tests are rejected, we can interpret that the multilateral bargaining hypothesis is rejected. Second, we test whether the median patient member has the greatest impact on group choices by analyzing whether we can reject$\alpha_1 = \alpha_3 = 0$. We also test this more strictly by analyzing whether we can reject $\alpha_1 = \alpha_3 = 0$ and $\alpha_2 = 1$. If these tests are rejected, we can interpret that the median voter hypothesis is rejected. Third, we test whether the most impatient member has an impact on group choices. Here, we test whether we can reject $\alpha_1 = \alpha_2 = 0$, and we test this more strictly by analyzing whether we can reject $\alpha_1 = \alpha_2 = 0$ and $\alpha_3 = 1$. If these tests are rejected, we can interpret from the results that the most impatient member does not have the greatest impact on group choices.

Table 2 shows the results of the regression of the group discount factors on the ordered individual discount factors. The coefficient of the median member's discount factor, α_2, is positive and significant for both combinations of dates. **Table 3** shows the results of the post-estimation analyses. Here, we reject that "the most patient member has the greatest impact on group choices," "the most patient member strongly has the greatest impact on group choices," "the most impatient member has the greatest impact on group choices," and "the most impatient member strongly has the greatest impact on group choices" for both combinations of dates at the 1% level. We cannot reject "the median patient member has the greatest impact on group choices" and "the median patient member strongly has the greatest impact on group choices" for both combinations of dates at the 1% level. Therefore, we reject the multilateral bargaining hypothesis. In addition, the most impatient member does not have the greatest impact on group choices.

Bayesian Analysis

In this subsection, we conduct a Bayesian analysis. As mentioned in the previous subsection, we reject the multilateral bargaining hypothesis. However, rejecting this hypothesis and not rejecting the median voter hypothesis does not guarantee support for the median voter hypothesis. In other words, not rejecting a hypothesis is not the same as supporting the hypothesis [13]. To resolve this problem, we conduct a Bayesian analysis and calculate the Bayes factors. Then, we analyze which hypothesis best fits the experimental data.

TABLE 2 | Ordinary least squares (OLS) regression of group discount factors on individual discount factors.

	(1) Today vs. 1 month	(2) 1 month vs. 2 months
$\delta^{(rank\ 1)}$	−0.0750	0.110
	(0.257)	(0.314)
$\delta^{(rank\ 2)}$	0.581***	0.688**
	(0.175)	(0.270)
$\delta^{(rank\ 3)}$	0.179	0.0124
	(0.107)	(0.0726)
Constant	0.258	0.134
	(0.156)	(0.138)
Observations	35	35
R-squared	0.556	0.700

Robust standard errors in parentheses.
, **, * denote significance at the 1%, 5%, 10% level.*
Here, (today vs. 1 month) and (1 month vs. 2 months) refer to the combinations of dates for the early option and the delayed option. Using (today vs. 1 month) as an example, participants choose to receive a small reward today or a larger reward one month later. Each rank refers to the order of the discount factor under the individual condition in the same group. Rank 1 refers to the most patient member (the highest discount factor) in a group, rank 2 refers to the median member (the second highest discount factor), and rank 3 refers to the most impatient member (the third highest discount factor, i.e., the lowest discount factor). Group refers to the discount factor for the group condition.

TABLE 3 | Results from the post-estimation analysis (*p*-values).

	Today vs. 1 month	1 month vs. 2 months
The most patient member has the greatest impact on group choices ($\alpha_2 = \alpha_3 = 0$)	0.0000***	0.0033***
The most patient member strongly has the greatest impact on group choices ($\alpha_1 = 1$) and ($\alpha_2 = \alpha_3 = 0$)	0.0000***	0.0088***
Median patient member has the greatest impact on group choices ($\alpha_1 = \alpha_3 = 0$)	0.1954	0.9403
Median patient member strongly has the greatest impact on group choices ($\alpha_2 = 1$) and ($\alpha_1 = \alpha_3 = 0$)	0.0128**	0.2182
The most impatient member has the greatest impact on group choices ($\alpha_1 = \alpha_2 = 0$)	0.0010***	0.0001***
The most impatient member strongly has the greatest impact on the group choices ($\alpha_3 = 1$) and ($\alpha_1 = \alpha_2 = 0$)	0.0000***	0.0000***

, **, * denote significance at the 1%, 5%, 10% level.*
In this table reports the post-estimation analysis results from ***Table 2****. Statistical significance means that the hypothesis is rejected. For example, the result of first row and column (0.0000***) means that $\alpha_2 = \alpha_3 = 0$ is rejected with a p-value of 0.0000.*

We first conduct the following four linear regressions using Bayesian methods and, then, calculate the Bayes factors to judge which hypothesis best fits the experimental data.

$$\delta_g^{group} = constant + b_1\delta_g^{(rank\ 1)} + b_2\delta_g^{(rank\ 2)} + b_3\delta_g^{(rank\ 3)} + \epsilon_g \tag{2}$$

$$\delta_g^{group} = constant + c_1\delta_g^{(rank\ 1)} + \epsilon_g \tag{3}$$

$$\delta_g^{group} = constant + c_2\delta_g^{(rank\ 2)} + \epsilon_g \tag{4}$$

$$\delta_g^{group} = constant + c_3\delta_g^{(rank\ 3)} + \epsilon_g, \tag{5}$$

where δ_g^{group} denotes group g's elicited group discount factor, and $\delta_g^{(rank\ i)}$ denotes rank i's elicited individual discount factor in group g. We use $\delta_g^{(rank\ j)}$ to refer to the jth highest discount factor among the individuals in group g (in particular, $\delta_g^{(rank\ 1)}$ refers to the highest and $\delta_g^{(rank\ 3)}$ refers to the lowest discount factors). Model (2) includes all the members' discount factors as explanatory variables. Model (3) includes only the most patient member's discount factor (i.e., $\delta_g^{(rank\ 1)}$) as an explanatory variable. Model (4) includes only the median patient member's discount factor (i.e., $\delta_g^{(rank\ 2)}$) as an explanatory variable. Model (5) includes only the most impatient member's discount factor (i.e., $\delta_g^{(rank\ 3)}$). If the median voter hypothesis is supported, Model (4) will fit the experimental data better than Models (3) and (5).

To fit a Bayesian parametric model, we need to specify the likelihood function or the distribution of the data and the prior distributions for all model parameters. In Model (2), the Bayesian linear model has five parameters: four regression coefficients (i.e., a constant, $\delta^{(rank\ 1)}$, $\delta^{(rank\ 2)}$, and $\delta^{(rank\ 3)}$) and the variance of the data. We assume a normal distribution for the dependent variable (i.e., δ^{group}) and start with a non-informative Jeffreys prior [26] for the parameters. Under the Jeffreys prior, the joint prior distribution of the coefficients and the variance is proportional to the inverse of the variance. In Models (3), (4), and (5), we also assume a normal distribution for the dependent variable (*i.e.*, δ^{group}) and start with a non-informative Jeffreys prior for the three parameters (i.e., a constant, $\delta^{(rank\ R)}$, and the variance of the data). Here, R is set to 1 in Model (3), 2 in Model (4), and 3 in Model (5).

We can write model (2) as follows:

$$\delta^{group} \sim N\left(X\beta,\ \sigma^2\right)$$
$$\left(\beta,\ \sigma^2\right) \sim 1/\sigma^2,$$

where X is design matrix, and $\beta = \left(constant, \delta^{(rank\ 1)}, \delta^{(rank\ 2)}, \delta^{(rank\ 3)}\right)'$, which is a vector

TABLE 4 | Results of Bayesian linear regression.

		Today vs. 1 month				1 month vs. 2 months			
Model		Mean	Std. Dev.	[95% Cred. Interval]		Mean	Std. Dev.	[95% Cred. Interval]	
(2)	$\delta^{(rank\ 1)}$	−0.064	0.277	−0.644	0.449	0.086	0.206	−0.361	0.465
	$\delta^{(rank\ 2)}$	0.570	0.185	0.203	0.940	0.692	0.168	0.384	1.022
	$\delta^{(rank\ 3)}$	0.180	0.105	−0.038	0.380	0.013	0.109	−0.193	0.225
	Constant	0.255	0.164	−0.049	0.603	0.148	0.118	−0.068	0.396
(3)	$\delta^{(rank\ 1)}$	0.718	0.271	0.172	1.227	0.883	0.214	0.475	1.308
	Constant	0.125	0.212	−0.278	0.550	0.028	0.167	−0.308	0.351
(4)	$\delta^{(rank\ 2)}$	0.699	0.118	0.471	0.935	0.755	0.088	0.582	0.931
	Constant	0.198	0.086	0.024	0.367	0.178	0.064	0.053	0.305
(5)	$\delta^{(rank\ 3)}$	0.405	0.108	0.193	0.611	0.431	0.096	0.239	0.615
	Constant	0.503	0.055	0.398	0.609	0.455	0.061	0.338	0.576

Here, (today vs. 1 month) and (1 month vs. 2 months) refer to the combinations of dates for the early option and the delayed option. Using (today vs. 1 month) as an example, participants choose to receive a small reward today or a larger reward one month later. Each rank refers to the order of the discount factor under the individual condition in the same group. Rank 1 refers to the most patient member (the highest discount factor) in a group, rank 2 refers to the median member (the second highest discount factor), and rank 3 refers to the most impatient member (the third highest discount factor, i.e., the lowest discount factor). Group refers to the discount factor for the group condition. Mean reports the estimates of the posterior means. Std. Dev. reports the estimates of the posterior standard deviations. The credible interval reports the 95% probability that the coefficient is in the described range.

of coefficients. In Models (3), (4), and (5), the vector of coefficients is $\beta = \left(constant, \delta^{(rank\ R)}\right)'$, where R is 1 in Model (3), 2 in Model (4), and 3 in Model (5). Next, we calculate the Bayes factors. The Bayes factors compute the relative probabilities of how well each model fits the data, as compared with the base model. We report the log Bayesian factors. If a log Bayesian factor is larger than zero, the reference model fits the data better than the base model and vice versa. We calculate two cases, namely, where the base model is Model (2) and the base model is Model (4). We use the Metropolis-Hastings method. All statistical analyses are performed using STATA 14.0.

Table 4 shows the results of Bayesian linear regression. The mean reports the estimates of posterior means, which are the means of the marginal posterior distributions of the parameters. Std. Dev. reports the estimates of the posterior standard deviations, which are the standard deviations of the marginal posterior distributions. The credible interval reports the 95% probability that the coefficient is in the described range. In the case of Model (2), only rank 2 has a lower bound of the 95% credible interval that is positive for both combinations of dates for the early option and the delayed option. Therefore, we surmise that rank 2 has a strong influence on group choices. **Table 5** shows the log Bayes factors. The first and third columns report the log Bayes factors using Model (2) as a base model. As shown, the value of the log Bayes factor is positive when the reference model is Model (4) for both combinations of dates. Thus, Model (4) fits the data better than Model (2). In other words, the model that includes only the constant and rank 2 is better than the model that includes all of the coefficients. The second and fourth columns report the log Bayes factors using Model (4) as a base model. As shown, the value of the log Bayes factor is negative when the reference model is Model (3) for both combinations of dates. Thus, Model (4) fits the data better than Model (3). In other words, the model that includes the constant and rank 2 is better than the model that includes the constant and rank 1. From these results, we conclude that the median voter hypothesis is supported.

TABLE 5 | Log Bayes factors.

		Today vs. 1 month		1 month vs. 2 months	
		Base model			
		Model (2)	Model (4)	Model (2)	Model (4)
Reference model	Model (2)		−0.015		−1.830
	Model (3)	−7.608	−7.623	−10.008	−11.838
	Model (4)	0.015		1.830	
	Model (5)	−5.292	−5.307	−10.035	−11.865

Here, (today vs. 1 month) and (1 month vs. 2 months) refer to the combinations of dates for the early option and the delayed option. Using (today vs. 1 month) as an example, participants choose to receive a small reward today or a larger reward one month later. Models (2), (3), (4), and (5) are described in the Bayes analysis subsection. If a log Bayesian factor is larger than zero, the reference model fits the data better than the base model and vice versa.

Chat Analysis

We reject the multilateral bargaining model using a regression analysis and a post-estimation analysis in section Regression Analysis and Post-estimation Analysis. In addition, we discover that the median voter hypothesis is supported using a Bayesian analysis in section Bayesian Analysis. In this subsection, we investigate why the median voter hypothesis is supported by analyzing the text data from the group chats.

Why does the median patient member have a strong impact on group choices, even though the experimental setting uses the unanimity rule? Here, we focus on the order of preference expression, which we label as follows. The person who expresses an opinion first is 1st, the person who expresses an opinion second is 2nd, and the person who expresses an opinion third is 3rd.

When people express an opinion, this opinion is considered to be closest to their own preferences[7]. They can discuss whether their opinions differ from those of the other group members. Here, we refer to the choices in the individual condition as a member's own preferences. However, the proportion (the number expressing the opinion closest to own preference) ÷ (the number of all expressions) is only 80%[8]. In other words, 20% of those expressing an opinion fail to express their individual conditions. Are the proportions of each order of preference expression the same? **Table 6** shows the results[9]. The data in **Table 6** show the sum of the two combinations of dates (i.e., {Early date, later date} = {today, 1 month} and {1 month, 2 months}). The proportion of expressions closer to the own preferences declines from 1st to 3rd (i.e., 84.05% in 1st, 79.77% in 2nd, 73.93% in 3rd). The proportions in each order of preference expressions are statistically different [$\chi^2_{(2)} = 8.0602$, $p = 0.018$]. Multiple comparisons show that the proportion between 1st and 3rd is significantly different ($p = 0.014$, Bonferroni test), and the proportion between 2nd and 3rd is not significantly different ($p = 0.306$, Bonferroni test). Thus, those who express their own opinion third fail to express own preferences less often than those who do so first.

Why do people who express 3rd fail to express their own preferences? There are two situations in which "3rd" express their own opinion. First, the first person and the second person express the same opinion. Second, the first person and the second person express different opinions[10]. If the first person and the second person express the same opinion, the third person's preference differs from theirs, but the third person follows the first two. We call this "dishonesty due to conformity with majority." For example, 1st chooses the early option and 2nd chooses the early option. Consequently, although 3rd prefers the delayed option, s/he chooses the early option. We call this "conformity with majority." In the chat data, the total number of dishonest choices by 3rd is 67, as shown in **Table 6**. Out of these, the number of "dishonesty due to conformity with the majority" is 63. Therefore, there might be a tendency for people to follow the majority, even though they have their own opinions.

TABLE 6 | Table of whether the first expressions in the group chats are dishonest.

The order of preference expression		Honest	Dishonest	Total
1st	*N*	216	41	257
	(%)	(84.05)	(15.95)	(100)
2nd	*N*	205	52	257
	(%)	(79.77)	(20.23)	(100)
3rd	*N*	190	67	257
	(%)	(73.93)	(26.07)	(100)
Total	*N*	611	160	771
	(%)	(79.25)	(20.75)	(100)

The values show the sums of the two combinations of dates (i.e., {Early date, later date} = {today, 1 month} and {1 month, 2 months}). The order of preference expression is as follows. The person who expresses an opinion first is 1st, the person who expresses an opinion second is 2nd, and the person who expresses an opinion third is 3rd. Honest means that the first expression in the group chat is the same as that of the individual condition. Dishonest means that the first expression in the group chat is not the same as that of the individual condition. We exclude cases where no group chat occurred and where the order of preference expression is ambiguous.

We guess that this tendency affected our main findings that the median patient member has the greatest impact on group choices. The "dishonesty due to conformity with majority" works to the median member's advantage, as follows. For example, suppose we have two options, A and B, and three members (x, y, z) who have single-peaked preferences; in this situation, there are four patterns of members' preferences (i.e., {x, y, z} = {A, A, A}, {A, A, B}, {A, B, B}, {B, B, B}). Taking {x, y, z} = {A, A, B} as an example, the number of combinations of the order of preference expressions and own opinions is six (i.e., {1st, 2nd, 3rd} = {Ax, Ay, Bz}, {Ay, Ax, Bz}, {Bz, Ax, Ay}, {Bz, Ay, Ax}, {Ax, Bz, Ay}, and {Ay, Bz, Ax}). Here, Ax indicates that person x chooses option A. It is possible that "dishonesty due to conformity with the majority" occurs when {1st, 2nd, 3rd} = {Ax, Ay, Bz} and {Ay, Ax, Bz}. When the third person z fails to express his/her own opinion B, but instead expresses A to follow the majority, "dishonesty due to conformity with the majority" occurs. Consequently, all three members choose option A. Thus, alternative A (i.e., median person y's preference) is selected as the group choice. We reach the same conclusion for {x, y, z} = {A, B, B}. These cases might increase the significance of the median patient member's effect on group choices.

As discussed above, we guess that the median patient members have a significant impact on group choices, partially because there are many cases of "dishonesty due to conformity with the majority." Naturally, there might be other reasons why median patient members have an effect on group choices. This is left for future research.

CONCLUSION

This study investigates how individual intertemporal preferences are aggregated in groups through deliberation, by clarifying who has a significant effect on group choices. We formulated two hypotheses. First, the multilateral bargaining hypothesis

[7]For example, the person who prefers D in **Figure 1** is considered to choose the early option for the first question (JPY 1,750 at an early date or JPY 2,000 at a later date).

[8]We check whether people's opinions are the same as those of the individual conditions. We do not count opinions that change during the discussion and count only the first expression in each question. There is one expression per person in each question. As mentioned in the Method section, there are four expressions for one combination of dates for the early option and the delayed option, there are two combinations of dates (i.e., {today, 1 month} and {1 month, 2 months}), and there are 105 participants. Therefore, the number of all first expressions is 840 ($4 \times 2 \times 105 = 840$). Out of these, the number of first expressions not close to the own preferences is 168. Therefore, (the number choosing an option NOT closer to own preference) ÷ (the number of all expressions) = 168/840 = 0.2.

[9]We exclude cases where no chat occurred or where the order of preference expression is ambiguous in **Table 6**. Therefore, the final sample is 771.

[10]As mentioned in the Method section, participants always face a choice between two options.

is based on the multilateral bargaining model, which predicts that the most patient member in a group has the greatest impact on group choices. Second, the median voter hypothesis is based on the median voter model, which predicts that the median patient member in a group has the greatest impact on group choices. We found that the median patient member has a substantial impact on group choices; that is, the median voter hypothesis is supported. Moreover, we examined the text data from the group chats to investigate why the median voter hypothesis is supported. According to the chat analysis, people who express their own opinion third (i.e., last) tend to follow the majority opinion. This may be one reason for our result (i.e., the median member has the greatest impact on group choices), even though the experimental setting is a free discussion and uses the unanimity rule.

Our results indicate that a median patient member in a group has the greatest impact on group intertemporal choices, even while using a free discussion and the unanimity rule. When workers make investment decisions under these conditions in a meeting, the median patient opinion might be accepted. This finding is surprising because many economists assume that the most patient member has the strongest bargaining power in group choices.

In our experiment, the groups are small and contain an odd number of members (i.e., three). Thus, we cannot generalize our results for an even number of members or for large groups. For an even number of members, we cannot determine whether the more patient median member or the less patient median member has a greater impact. Furthermore, median patient's power may decrease in large groups when compared with small groups. Median member becomes a pivotal member in group decision less often in a large group than in a small group. Thus, the answer to how much impact which median member has in a large group is unknown. Resolving these issues is left to future research.

DATA AVAILABILITY STATEMENT

The raw data supporting the conclusions of this manuscript will be made available by the authors, without undue reservation, to any qualified researcher.

AUTHOR CONTRIBUTIONS

MT developed the study concept and the experimental paradigm and conducted the experiment. MT performed the data analysis and interpretation under the supervision of KI. MT and KI drafted the manuscript. Both authors contributed to the discussion section of the manuscript and approved the work for publication.

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SUPPLEMENTARY MATERIAL

The Supplementary Material for this article can be found online at: https://www.frontiersin.org/articles/10.3389/fams.2018.00043/full#supplementary-material

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METHODS
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A Testing Method of Probability Weighting Functions From an Axiomatic Perspective

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This study presents a testing approach to examine various models of probability weighting functions that are considered nonlinear functions of probability in behavioral decision theory, such as prospect theory. Although there are several empirical psychometric tests to examine probability weighting functions, there is no concrete method to examine these functions' axiomatic properties. We propose axiomatic properties and a testing method to examine the generalized hyperbolic logarithmic model, power model, and exponential power model of the probability weighting functions, and provide an illustrative example of the testing method.

Keywords: axiomatic approach, decision under risk, hyperbolic logarithmic discounting, probability weighting function, time discounting

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INTRODUCTION

A probability weighting function $W(p)$ is a nonlinear function of an objective probability p, where p is determined primarily from the frequentist view. Recently, they have received substantial empirical and theoretical attention [1–3]. They are used in many fields, such as behavioral decision theory, behavioral economics and neuroscience [4].

Several psychometric models have been proposed to represent probability weighting functions (e.g., [1–3, 5, 6]). Some proposed probability weighting models derive from time discounting models [2, 3, 7]. Rachin et al. [7] derived the model from the original hyperbolic function. Takahashi [2] used a q-exponential time discount function [8] to derive Prelec's [6] probability weighting function and an exponential power model. Takemura and Murakami [3] used a more direct assumption of time discounting to derive the hyperbolic logarithmic function model and the generalized hyperbolic logarithmic model as probability weighting functions.

Takemura and Murakami [3] used a generalized hyperbolic time discounting model that assumes both Fechner's [9] psychophysical law of time and a geometric distribution of trials. From this, they derived hyperbolic logarithmic type models. They were then able to examine the generalized hyperbolic model in the context of an axiomatic system. They used Gonzalez and Wu's [10] procedure to estimate the function parameters. To investigate goodness of fit, they computed both the Akaike Information Criterion (AIC) and Bayes Information Criterion (BIC). The results indicated that the generalized hyperbolic logarithmic mode originally proposed by Prelec [6] fitted better than median time discounting models, the one-parameter Prelec model, and the Tversky and Kahneman model.

Takemura and Murakami [3] made a key contribution by supporting, both theoretically and empirically, a possible psychological interpretation of a probability weighting function in the context of time discounting. However, the empirical method used in the study was a psychometric nonlinear regression study. Although there are several empirical psychometric tests available to examine probability weighting functions, there is no concrete method to examine the axiomatic properties of the probability weighting functions. Prelec [6] had already proposed the axiomatic properties for some weighting functions. However, no concrete axiomatic properties distinguished the individual models he proposed, and no testing method was suggested. Based on their axiomatic considerations, we propose axiomatic properties and a testing method to examine the generalized hyperbolic logarithmic model, power model, and exponential power model of the probability weighting functions, and provide an illustrative example of the testing method.

AXIOMATIC SYSTEM OF GENERALIZED HYPERBOLIC LOGARITHMIC MODEL, EXPONENTIAL POWER MODEL, AND POWER MODEL FOR PROBABILITY WEIGHTING FUNCTIONS

Counterexamples, such as the Allais paradox [11] and the Ellsberg paradox [12], have been identified in earlier studies. These paradoxes are interpreted as deviations from the independence axiom. Recently, they have been explained using theory systems. More specifically, these systems include the nonlinear utility theory [13–15]—which does not require this independence axiom—as well as the generalized expected utility theory [16]. Prospect theory [5, 17] integrates knowledge and past findings in nonlinear utility theory (or generalized expected utility theory) and behavioral decision-making theory.

In prospect theory, we assume a non-additive probability function, where a non-additive probability is a set function $\pi : 2^{\Omega} \rightarrow [0,\ 1]$ from an aggregate of subsets of a nonempty set Ω to a closed interval [0, 1]. The non-additive probability function is a set function satisfying both a boundedness condition ($\pi(\phi) = 0$, $\pi(\Omega) = 1$) and a monotonicity condition [if $A \subseteq B$, then $\pi(A) \leq \pi(B)$, where A, B are subsets of Ω]. A non-additive probability does not necessarily satisfy additivity conditions. Prelec [6] showed psychometric functions of non-additive probability (probability weighting functions) and axiomatic properties of the probability weighting functions based on prospect theory.

Based on the theoretical work of Prelec [6], we show axiomatic properties of the generalized hyperbolic logarithmic model, exponential power model, and power model for probability weighting functions.

For the set A of probability distributions P, Q, … on $X = [x^-, x^+]$, where $x^- < 0 < x^+$, let $\succcurlyeq$ be a preference relation. Prospects are considered distributions with finite support. Then, we assume the following axioms [6].

W1. Weak ordering: $\succcurlyeq$ is complete and transitive.

W2. Strict stochastic dominance: $P > Q$ if both $P \neq Q$ and P is stochastically dominants over Q.

W3. Certainty equivalent condition: For every P, $\exists\ x$ such that $(x) \sim P$.

W4. Continuity in probabilities: If $(y,\ p) > (x)$ where $0 < p < 1$, then $\exists\ q, r$ such that $q < p < r$, $(y,q) > (x)$, and $(y,r) > (x)$. If $(y,\ p) < (x)$ where $0 < p < 1$, then $\exists\ q, r$ such that $q < p < r$, $(y,q) < (x)$ and $(y,r) < (x)$.

W5. Simple continuity: Let the set of all k nonpositive and $(n-k)$ nonnegative rank-ordered n-tuples from X be $S(k,n)$, where $0 \leq k \leq n$. If the preference relation induced on each set $S(k,n)$ is continuous for any probability vector $(p_1,\ p_2, \cdots,\ p_n)$, then there is simple continuity.

W6. Tradeoff consistency: Consider a prospect $(x,\ p_i;\ \mathbf{x_{-i}}, \mathbf{p_{-i}})$ with outcome c of rank i singled out and the set $R(k,n,p)$ of all sign-order and rank-order compatible prospects with a p-chance of a negative outcome. Assume there are not eight prospects, $(x,\ p_i;\ \mathbf{a_{-i}}, \mathbf{p_{-i}})$, $(y,\ p_i;\ \mathbf{b_{-i}}, \mathbf{p_{-i}})$, $(x',\ p_i;\ \mathbf{a_{-i}}, \mathbf{p_{-i}})$, $(y',\ p_i;\ \mathbf{b_{-i}}, \mathbf{p_{-i}})$, $(x',\ q_j;\ \mathbf{c_{-j}}, \mathbf{q_{-j}})$, $(y',\ q_j;\ \mathbf{d_{-j}}, \mathbf{q_{-j}})$, $(x,\ q_j;\ \mathbf{c_{-j}}, \mathbf{q_{-j}})$, and $(y,\ q_j;\ \mathbf{d_{-j}}, \mathbf{q_{-j}})$, such that the first and second groups of four belong to the same sign-order and rank-order compatible set, and

$$\begin{aligned}
\left(x,\ p_i;\ \mathbf{a_{-i}}, \mathbf{p_{-i}}\right) &\succcurlyeq \left(y,\ p_i;\ \mathbf{b_{-i}}, \mathbf{p_{-i}}\right), \\
\left(x',\ p_i;\ \mathbf{a_{-i}}, \mathbf{p_{-i}}\right) &\preccurlyeq \left(y',\ p_i;\ \mathbf{b_{-i}}, \mathbf{p_{-i}}\right), \\
\left(x',\ q_j;\ \mathbf{c_{-j}}, \mathbf{q_{-j}}\right) &\succcurlyeq \left(y',\ q_j;\ \mathbf{d_{-j}}, \mathbf{q_{-j}}\right), \\
\left(x,\ q_j;\ \mathbf{c_{-j}}, \mathbf{q_{-j}}\right) &< \left(y,\ q_j;\ \mathbf{d_{-j}}, \mathbf{q_{-j}}\right).
\end{aligned}$$

Then, tradeoff consistency holds.

The following assumptions are as described by Prelec [6].

Assumption 1: $\succcurlyeq$ satisfies axioms W1-W6, which support a sign-dependent and rank-dependent representation with a continuous and strictly increasing ratio scale $v(x)$, as well as a strictly increasing unique $w^-(p)$, $w^+(p)$ that is continuous on (0, 1), and satisfies $w^+(0) = w^-(0) = 0$, $w^+(1) = w^-(1) = 1$.

Assumption 2: There is a separable representation of the restriction of $\succcurlyeq$ to simple prospects, with $v(x)$, $w^-\left(p\right)$, and $w^+(p)$ satisfying the Assumption 1 conditions.

Definition 1 Conditional invariance [6]: $\succcurlyeq$ has **conditional invariance** if the following holds for any outcomes $x,\ y,\ x',\ y' \in X$, probabilities $q, p, r, s \in [0,1]$, and conditional probability λ, $0 < \lambda < 1$:

If $(x,\ p) \sim (y,q)$ and $(x,\ r) \sim (y,s)$, then $(x',\ \lambda p) \sim (y', \lambda q)$ implies $(x',\ \lambda r) \sim (y', \lambda s)$ or $(x',\ \lambda r) \sim (y', \lambda s)$.

Definition 2 Projection invariance [6]: $\succcurlyeq$ has **projection invariance** if the following holds for any outcomes $x,\ y \in X$, probabilities $q, p, r, s \in [0,1]$, and conditional probability λ, $0 < \lambda < 1$:

If $(x,\ p) \sim (y,q)$ and $\left(x,\ rp\right) \sim \left(y, sq\right)$, then $(x,\ r^2p) \sim (y, s^2q)$.

Proposition 1: The generalized hyperbolic logarithmic model proposition

Let $\succcurlyeq$ be a preference relation on R^+ where either Assumption 1 or 2 holds, conditional invariance (Definition 1) does not hold, and projection invariance (Definition 2) holds. Then, the probability weighting function $W(p)$ is a hyperbolic logarithm,

$$W(p) = (1 - k \log\ p)^{\beta},$$

where p is probability *(0<p)*, and k and β are positive constants, $k,\ \beta > 0$.

Proof

The proof of Proposition 1 is trivial and derived from a combination of Propositions 4 and 5 in the original theoretical work by Prelec [6]. Prelec [6] found that if $\succcurlyeq$ is a preference relation on R^+ where either Assumption 1 or 2 holds and conditional invariance (Definition 1) holds, then the weighting function *(0<p)* is either an exponential-power function or a power function (Proposition 4). Prelec [6] also found that if $\succcurlyeq$ is a preference relation on R^+ where either Assumption 1 or 2 holds and projection invariance (Definition 2) holds, then the weighting function *(0<p)* is either a hyperbolic logarithm or a power function (Proposition 5). Therefore, if $\succcurlyeq$ is a preference relation on R^+ satisfying Assumption 1 or 2, conditional invariance (Definition 1) does not hold, and projection invariance (Definition 2) holds, then the probability weighting function $W(p)$ is a hyperbolic logarithmic function.

Proposition 2: Proposition of the exponential power model

If $\succcurlyeq$ is a preference relation on R^+ satisfying Assumption 1 or 2, conditional invariance (Definition 1) holds, and projection invariance (Definition 2) does not hold, then the probability weighting function $W(p)$ is an exponential power function such as

$$W\left(p\right) = exp\left\{-k\left(1 - p^{\beta}\right)\right\},$$

where p is probability *(p>0)*, and k and β are positive constants, $k,\ \beta > 0$.

Proof

The proof of Proposition 2 is trivial and also derived from a combination of Propositions 4 and 5 in the original theoretical work by Prelec [6]. As in the same inference of Proposition 1, if $\succcurlyeq$ is a preference relation on R^+ satisfying Assumption 1 or 2, conditional invariance (Definition 1) holds and projection invariance (Definition 2) does not hold, then the probability weighting function $W(p)$ should be an exponential power function.

Proposition 3: Proposition of the power model

If $\succcurlyeq$ is a preference relation on R^+ satisfying Assumption 1 or 2, conditional invariance (Definition 1), and projection invariance (Definition 2), then the probability weighting function $W(p)$ is an exponential power

$$W\left(p\right) = p^{\beta},$$

where p is probability, and β is a positive constant, $\beta > 0$.

Proof

The proof of for this proposition is trivial and also derived from a combination of Propositions 4 and 5 in the original theoretical work by Prelec [6]. As in the same inference of Proposition 1, if $\succcurlyeq$ is a preference relation on R^+ satisfying Assumption 1 or 2, conditional invariance (Definition 1) holds and projection invariance (Definition 2) holds, then the probability weighting function $W(p)$ is a power function.

A TESTING METHOD TO EXAMINE AXIOMATIC PROPERTIES OF GENERALIZED HYPERBOLIC LOGARITHMIC MODEL, EXPONENTIAL POWER MODEL, AND POWER MODEL FOR PROBABILITY WEIGHTING

We propose a testing method to examine the generalized hyperbolic logarithmic model, power model, and exponential power model of the probability weighting functions, and provide an illustrative example of the testing method. First, we present the testing method using verification tasks of projection invariance and conditional invariance. We then give an example verifying the reliability and axioms and showing the goodness of fit of the models.

Figure 1 illustrates the testing experimental process, which had participants choose one option from two gambles. To assess reliability, projection invariance and conditional invariance should be examined at least twice. Additionally, trials are done at least 30 times to stabilize the responses.

The Projection Invariance Verification Process

Experimental screens and the task processes in the projection invariance verification process are shown in **Figure 2**. The task presented to the participants was to choose one option from two gambles as shown in **Table 1**. The participants were instructed to choose a preferred option from the experimenter.

The verification process of projection invariance is presented in **Table 1**. In the verification of projection invariance in **Table 1**, $y = 10{,}000$ yen, $p = 100\%$, $q = 50\%$, and $s = 50\%$ are given. In addition, **Table 1** presents values of the responses by participants shown in bold typeface.

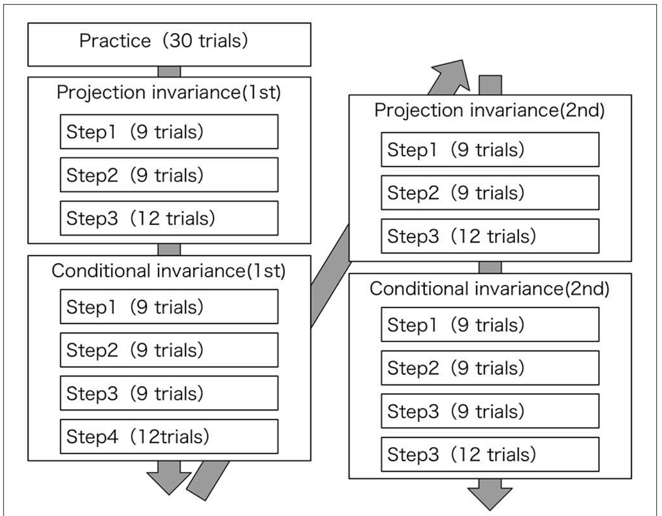

FIGURE 1 | Experimental process for testing axiomatic properties.

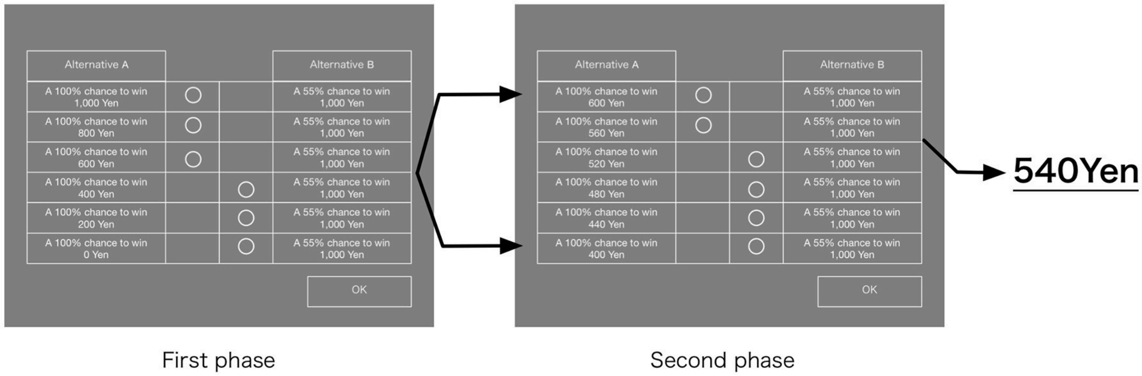

FIGURE 2 | Experimental screens and task process.

TABLE 1 | Verification process of projection invariance.

	Alternative A			Alternative B	
	Outcome	Probability		Outcome	Probability
Step 1	x	p	~	y	q
	5,000 yen	100%		10,000 yen	50%
Step 2	x	rp	~	y	sq
	5,000 yen	**50%**		10,000 yen	25%
Step 3	x	r^2p	~	$\hat{y}$	s^2q
	5,000 yen	25%		**10,000 yen**	12.5%

The values given in responses by the participants are shown in bold typeface.
y = 10,000 yen, p = 100%, q = 50%, s = 50%.

TABLE 2 | Stimulation sequences of projection invariance.

Sequence number	y (yen)	p (%)	q (%)	s (%)
1	10,000	100	10	50
2	10,000	100	20	50
3	10,000	100	30	50
4	10,000	100	40	50
5	10,000	100	50	50
6	10,000	100	60	50
7	10,000	100	70	50
8	10,000	100	80	50
9	10,000	100	90	50

The verification projection invariance tasks comprise three steps. To explain the verification process using the example presented in **Table 1**, in Step 1, x in the alternative A equivalent to the alternative B (to obtain 10,000 yen with 50%) is estimated from a pair comparison of the alternative A and alternative B in **Figure 2**. Next, in Step 2, r in the alternative A equivalent to the alternative B (to obtain 10,000 yen with 25%) is estimated. For x of the alternative A in Step 2, the x obtained in Step 1 is used. Finally, in Step 3, using x and r obtained in Step 1 and Step 2, the alternative A (to get 5,000 yen with 25%) is made. Then $\hat{y}$ is estimated (to obtain $\hat{y}$ yen with 12.5%). Here, when $\hat{y}$ obtained in Step 3 is 10,000 yen, which is the same as y, projection invariance is regarded as satisfied. Additionally, because $y =$ 10,000 yen is given in Step 3, to estimate $\hat{y}$, 20,000 yen and 0 yen, respectively, the maximum value and the minimum value of $\hat{y}$ were presented to the participants. Then they were asked to do the pair comparison, as shown in **Figure 2**.

Steps 1, 2, and 3 respectively comprise 9 trials, 9 trials, and 12 trials. The stimulation sequences used to verify projection invariance are shown in **Table 2**. Nine sequences of stimulation were prepared. Furthermore, because y is fixed at 10,000 yen in Step 3, when a participant gives a response to satisfy the axioms, the participant might continue to give the same response and is likely to change a response due to fluctuation of the psychological process. Therefore, with three sequences of dummy stimulation added to the nine sequences, Step 3 has 12 trials in all.

Verification Process of Conditional Invariance

Experimental screens and task processes were prepared in an identical form to that used for projection invariance in the verification process of conditional invariance. The participants were also instructed to choose a preferred option from two gambles, as shown in **Table 3**. The verification process of conditional invariance is presented in **Table 3**. For verification of the conditional invariance in **Table 3**, y = 20,000 yen, y' = 10,000 yen, p = 100%, q = 50%, s = 10%, and λ = 50% are given. Additionally, the values given in responses by the participants are shown in bold typeface in **Table 3**.

The verification tasks of conditional invariance comprise four steps. To explain the verification process using the example in **Table 3**, in Step 1, x of alternative A equivalent to alternative B (to get 20,000 yen with 50%) is estimated through a pair comparison between alternatives A and B as shown in **Figure 2**. Next, in Step 2, r of the alternative A equivalent to the alternative B (to get 20,000 yen with 10%) is estimated. For x of alternative A in Step 2, the x obtained in Step 1 is used. In Step 3, x' of the alternative A (to obtain x' yen with 50%) equivalent to the alternative B (to get 10,000 yen with 25%) is estimated. In Step 4, using r and x' obtained in Step 2 and Step 3, respectively, alternative A (to get 5,000 yen with 10%) is made. Then, $\hat{y}'$ of alternative B (to get $\hat{y}'$ yen with 5%) is estimated. Here, when $\hat{y}'$ obtained in Step

TABLE 3 | Verification process of conditional invariance.

	Alternative A			Alternative B	
	Outcome	**Probability**		**Outcome**	**Probability**
Step1	x	p	∼	y	q
	10,000 yen	100%		20,000 yen	50%
Step2	x	r	∼	y	s
	10,000 yen	**20%**		20,000 yen	10%
Step3	x'	λp	∼	y'	λq
	5,000 yen	50%		10,000 yen	25%
Step 4	x'	λr	∼	$\widehat{\boldsymbol{y}}'$	λs
	5,000 yen	10%		**10,000 yen**	5%

The values given in responses by the participants are shown in bold typeface.
y = 20,000 yen, y' = 10,000 yen, p = 100%, q = 50%, s = 10%, λ = 50%.

TABLE 4 | Stimulus sequences of conditional invariance (%).

Sequence number	y (yen)	y' (yen)	p (%)	q (%)	s (%)	λ (%)
1	20,000	10,000	100	10	2	50
2	20,000	10,000	100	20	4	50
3	20,000	10,000	100	30	6	50
4	20,000	10,000	100	40	8	50
5	20,000	10,000	100	50	10	50
6	20,000	10,000	100	60	12	50
7	20,000	10,000	100	70	14	50
8	20,000	10,000	100	80	16	50
9	20,000	10,000	100	90	18	50

4 is 10,000 yen, which is the same value as y', the conditional invariance is regarded as satisfied.

Steps 1, 2, and 3 are composed respectively of 9 trials, 9 trials, and 12 trials. The stimulus sequences used for verification of conditional invariance are presented in **Table 4**. Nine sequences of stimuli were prepared. In Step 4, although three dummy stimulus sequences were also prepared for the same reason as those for projection invariance, three stimuli from the sequences were randomly provided twice because of the experimental program's errors. As a result, Step 4 had 9 sequences plus 3 trials, i.e., 12 trials in total.

AN EXAMPLE OF THE TESTING METHOD

Participants

The participants were 14 undergraduate students (eleven female and three male) studying psychology at Waseda University aged between 21 and 25 years old. They were paid 1500 Japanese yen (about 15 dollars) to participate in a 1.5-h test. This study has ethical approval from the Academic Research Ethical Review Committee, Waseda University concerning Guidelines Regarding Academic Research Ethics, Waseda University. Participants provided written informed consent.

Materials and Procedure

We asked the participants to select their preferred option from two alternatives while watching the screen shown in **Figure 2**. The verification procedure of projection invariance and conditional invariance were as described above.

TABLE 5 | Reliability of projection invariance and conditional invariance (final step).

	Reliability	
Participant number	**Projection invariance**	**Conditional invariance**
1	0.043	−0.243
2	**0.871**	**0.668**
3	−0.324	0.264
4	−0.277	−0.181
5	**0.776**	−0.148
6	0.200	0.327
7	0.429	**0.637**
8	0.273	−0.114
9	0.432	−0.282
10	0.188	−0.204
11	0.198	0.225
12	0.231	−0.384
13	−0.154	−0.295
14	**0.856**	0.300

Intraclass coefficients that are 0.6 or greater presented in bold typeface.

Examination of Reliability

A participant was asked to work on the tasks twice in a row to assess the reliability. Intraclass correlation coefficients of projection invariance and conditional invariance by the participant was calculated. The intraclass correlation coefficients calculated from the first and second answers in the final step are shown in **Table 5**. Those calculated in all steps are shown in **Table 6**, with intraclass correlation coefficients that are 0.6 or greater presented in bold typeface. The medians of intraclass correlation coefficients calculated in the final step were 0.216 (maximum value, 0.871; minimum value, −0.324) in projection invariance and −0.131 (maximum value, 0.668; minimum value, −0.384) in conditional invariance. However, the medians of intraclass correlation coefficients calculated in all steps was 0.897 (maximum value, 0.992; minimum value, 0.401) in projection invariance and 0.765 (maximum value, 0.968; minimum value, 0.476) in conditional invariance.

As **Table 6** shows, the intraclass correlation coefficients are all 0.4 or greater for projection invariance and conditional invariance.

Verified Results of Projection Invariance and Conditional Invariance

Table 7 presents the numbers of sequences in which participants satisfied the axioms of projection invariance and conditional invariance. Because nine sequences were used to verify the axioms, when participants judge in accordance with the axioms in five sequences or more, the numbers are presented in bold typeface.

TABLE 6 | Reliability of projection invariance and conditional invariance (all steps).

Participant number	Reliability	
	Projection invariance	Conditional invariance
1	**0.963**	**0.814**
2	**0.969**	**0.968**
3	**0.906**	**0.861**
4	0.440	**0.747**
5	**0.888**	**0.774**
6	**0.753**	**0.709**
7	0.596	**0.816**
8	**0.812**	0.547
9	**0.919**	**0.801**
10	**0.992**	**0.964**
11	**0.947**	**0.756**
12	0.415	0.476
13	0.401	0.591
14	**0.936**	0.514

Intraclass coefficients that are 0.6 or greater presented in bold typeface.

TABLE 7 | Number of sequences satisfying the axiom (out of 9 sequences).

Participant number	Projection invariance	Conditional invariance
1	**6**	2
2	0	0
3	1	**5**
4	0	2
5	0	0
6	1	0
7	1	0
8	4	1
9	2	1
10	3	3
11	3	0
12	0	0
13	1	0
14	1	0

Participants judged in accordance with the axioms in five sequences or more, the numbers are presented in bold typeface.

Examination of Goodness of Fit of the Model

Free parameters, such as β and k, of the probability weighting function by the participant were estimated by the same experiment as in Gonzalez and Wu [10]. **Table 8** presents a list of examined models. In addition, **Table 9** shows models which are the fittest according to AIC. Twelve participants had the best fit with the hyperbolic logarithmic model. One participant had the best fit with the exponential power function. One participant had the best fit with the power function.

Relation Between Axioms and Goodness of Fit of Models

The correspondence between axioms and models is presented in **Table 10**. Relations between satisfied axioms and models

TABLE 8 | List of models.

MODELS	
Power function	$W(p) = p^{\beta},\ \beta > 0$
Exponential power function	$W(p) = exp\left\{-k(1-p^{\beta})\right\},\ k,\ \beta > 0$
Hyperbolic logarithmic	$W(p) = (1 - klog\ p)^{\beta},\ k,\ \beta > 0$

TABLE 9 | Models' AICs and the model with the smallest AIC by participant.

Participant number	Power function	Exponential power function	Hyperbolic logarithmic	Model with smallest AIC
1	0.147	−6.153	**−30.273**	Hyperbolic logarithmic
2	−20.817	**−39.206**	−35.191	Exponential power
3	−2.209	−6.616	**−15.394**	Hyperbolic logarithmic
4	0.839	−5.544	**−35.373**	Hyperbolic logarithmic
5	0.588	−2.453	**−20.864**	Hyperbolic logarithmic
6	−2.630	−7.651	**−31.070**	Hyperbolic logarithmic
7	−4.812	−16.459	**−33.081**	Hyperbolic logarithmic
8	−10.913	−17.567	**−21.558**	Hyperbolic logarithmic
9	−3.066	−7.844	**−20.826**	Hyperbolic logarithmic
10	**−35.174**	−34.653	−23.266	Power
11	−17.430	−23.191	**−28.357**	Hyperbolic logarithmic
12	−11.956	−15.919	**−20.930**	Hyperbolic logarithmic
13	−11.437	−20.500	**−41.932**	Hyperbolic logarithmic
14	−15.813	−23.401	**−24.774**	Hyperbolic logarithmic

The smallest AIC by participant presented in bold typeface.

TABLE 10 | Correspondence between axioms and models.

Name of models	Projection invariance	Conditional invariance
Power function	Yes	Yes
Exponential power function	No	Yes
Hyperbolic logarithmic	Yes	No

with goodness of fit are shown in **Table 11**. Because nine sequences were used to verify axioms, if axioms were satisfied in five and more sequences, then they are regarded as satisfied. The first participant satisfied projection invariance alone. The third participant satisfied conditional invariance. The other participants satisfied neither projection invariance nor conditional invariance. Results show that there is not a certain correspondence with normal quantitative psychometric methods that used the nonlinear regression method and the model fitting examination by AIC indicator.

CONCLUSION AND DISCUSSIONS

This study aimed to present a testing approach used to examine the generalized hyperbolic logarithmic model, power model, and exponential power model of the probability weighting functions that are considered nonlinear functions of probability in behavioral decision theory, for example, in prospect theory [5, 6]. Although many empirical psychometric tests are used to

TABLE 11 | Relations between satisfied axioms and models with goodness of fit.

Participant number	Satisfied axioms	Model with smallest AIC
1	Projection invariance	Hyperbolic Logarithmic
2	–	Exponential Power
3	Conditional invariance	Hyperbolic Logarithmic
4	–	Hyperbolic Logarithmic
5	–	Hyperbolic Logarithmic
6	–	Hyperbolic Logarithmic
7	–	Hyperbolic Logarithmic
8	–	Hyperbolic Logarithmic
9	–	Hyperbolic Logarithmic
10	–	Power
11	–	Hyperbolic Logarithmic
12	–	Hyperbolic Logarithmic
13	–	Hyperbolic Logarithmic
14	–	Hyperbolic Logarithmic

examine the probability weighting functions, there is no concrete method to examine the axiomatic properties of the probability weighting functions. Therefore, we propose axiomatic properties based on Prelec's [6] theory and a testing method to examine the generalized hyperbolic logarithmic model, power model, and exponential power model of the probability weighting functions, and provide an illustrative example of the testing method.

According to this result of the example experiment, the axiomatic properties of the probability weighting functions did not correspond to the psychometric fitting result of probability weighting functions. A similar result occurs in the additive conjoint systems in judgment and decision making. For example, empirical evaluations of double cancelation for the conjunctive measurement rejected the double cancelation axiom [18, 19]. However, psychometric studies have also indicated that the linear additive model fitted better [20]. There are some contradictions between psychometric studies and axiomatic studies. This case is the same as previous research. Further research is needed to identify why the discrepancies occur.

Luce and Steingrimsson [21] examined the Thomsen condition and the conjoint commutativity axiom, which they showed were equivalent. They also found that brightness and binaural loudness were supporting factors of conjoint commutativity. We must consider the reason for the unclear correspondence between the axiomatic testing and psychometric testing. One possibility is that the assumptions of the prospect theory did not hold in this experiment. Another is that the essential conditions, such as conditional invariance and projection invariance, did not hold in the experiment. Further research could investigate these possibilities.

In our study, the number of participants was limited and the participants were all trained psychology students. However, our sample size matches those in previous studies [5, 10], so we do not consider this to invalidate the results. Nevertheless, larger sample sizes in future experiments would be beneficial in examining the psychometric model of probability weighting functions.

Although we proposed an axiomatic testing method of Prelec's [6] probability weighting function, there other ways to interpret probability weighting, such as from the perspective of rational dynamic asset pricing theory. Rachev et al. [22] explained the main concepts of prospect theory and probability weighting functions within the framework of rational dynamic asset pricing theory. They derived a modified Prelec weighting function and introduced a new parametric class for weighting probability functions. We did not examine the theoretical notions proposed by Rachev et al. [22]. Further theoretical examinations are needed to seek an adequate probability weighting function.

AUTHOR CONTRIBUTIONS

KT and HM developed the theoretical formalism, performed the analytic calculations and performed data analysis. Both authors contributed to the final version of the manuscript. KT supervised the project.

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Ecological Validity of Impulsive Choice: Consequences of Profitability-Based Short-Sighted Evaluation in the Producer-Scrounger Resource Competition

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Results of intertemporal choice paradigm have been accounted for mostly by psychological terms such as temporal discounting of subjective value. Inability to wait for delayed gratification (choice impulsiveness, as opposed to self-control) is often taken to represent violated rationality. If viewed from foraging ecology, however, such impulsiveness can be accountable as adaptive adjustments to requirements in nature. First, under the circumstance where foragers stochastically encounter food items, the optimal diet-menu model suggests that each option must be evaluated by profitability (*e/h*), which is the ratio of energetic gain (*e*) per handling time (*h*), a short-sighted currency. As h includes the delay, profitability will be hyperbolically lower for long-delay food. Second, because of the resource competition between producing and scrounging foragers, profitability of the producer's gain will critically depend on the scrounger's behaviors. We first constructed an analytical model. The model predicted that the profitability of small and short-delay food option (*SS*) can be higher than that of the large and long-delay alternative (*LL*), depending on the duration in which the producer can monopolize a food patch (finder's share). Next, we conducted numerical simulations on the assumption of variable food amount in each patch with realistic set of behavioral parameters. Although non-linearity of profitability function largely reduced profitability for variable amount of food, *SS* still can have a higher profitability than *LL* when the finder's share is small. Because *SS* is consumed more quickly, it is more resistant against scrounging than *LL*. In good accordance, foraging domestic chicks form a synchronized flock and show socially-facilitated investment of effort. If raised in competition, chicks develop a higher degree of choice impulsiveness.

Keywords: foraging theory, social foraging, profitability, competition, social facilitation

INTRODUCTION

Spencer introduced the idea of "survival of the fittest" in "The Principles of Biology" (1864), and Darwin adopted this term in his 5th edition of "The Origin of Species" (1869). A naïve biological thinking might therefore be that only optimal individuals have been selected for survival. If it was, we might find a monochromatic world in which only a few fittest phenotypes predominate. Conversely, nature is full of diverse organisms with distinct behavioral phenotypes, even within a sympatric group of animals of the same species. We must consider some adaptive processes that make animals appropriately deviate from the optimality. Deviations due to social inter-individual interaction could be one of such critical processes. In this report, we focus on social foraging behavior as it allows us to theoretically and empirically make quantitative examinations on issues of optimality. Through introducing ecological theories of foraging, we argue that social facilitation and enhanced choice impulsiveness could be described as adaptive deviations.

OPTIMAL FORAGING, PROFITABILITY, AND INTERTEMPORAL CHOICES

Optimal Diet Menu Model and Impulsive Choices

Classical theories usually assume that optimal foragers maximize the long-term averaged gain rate. Foragers explore food (searching time, abbreviated as T_s) and exploit it (consuming or handling time, T_h), so that the gain rate R is given for E_f (net energy gain) as a substitution variable for fitness [1].

$$R = \frac{E_f}{T_s + T_h}$$

As initially formulated as Holling's disc equation [2] and subsequently by Charnov [3] in his diet menu model, optimal foragers must maximize R. Assuming the stochastic nature of food resources, Charnov reached a somewhat paradoxical conclusion that the R-maximizer must make decisions (i.e., action choice between attacking or passing-over) based on short-sighted estimation of profitability e/h, where e represents energetic gain and h handling time for the encountered food item. Namely, they attack the food if the expected profitability exceeds the lost opportunity, that is the potential food gained by passing over (or giving up) the encountered food item. We should consider (1) prospective (expected) profitability of the food and (2) highly uncertain lost-opportunity, the latter of which may be approximated by the average gain rate they have had (**Figure 1A** left).

As profitability is the product of gain (e) and proximity ($1/h$), the foraging choice may be isomorphic to the intertemporal choice (ITC) paradigm widely adopted to study impulsive/self-control issues (Ainslie-Rachalin theory [4]; **Figure 1A** right). Here, a set of two options $\{SS, LL\}$ is given as:

$$SS = \left(e_1,\ h_1^{-1}\right)\ LL = \left(e_2,\ h_2^{-1}\right), \text{where } (e_1 < e_2) \cap (h_1^{-1} > h_2^{-1}).$$

Based on behaviors in the ITC task [5–7], we formulated simple pico-economics for chicks in terms of profitability-based behaviors [8]. Briefly, as chicks feed on grain particles with small e, they must make h small by adopting highly impulsive choices, so that a certain level of profitability (e/h) is achieved. Choice impulsiveness does not necessarily represent internal anomalies in the machineries, but can be an adaptive trait with external (ecological) validity. An extreme case of such foraging economics is found in star-nosed mole rats [9], where the handling time was as short as 120 ms for the tiny food particles they eat. Hyperbolic discounting of future rewards is assumed to be a corollary to the profitability rule, simply because h includes the delay time for food. However, the zero-one rule (another principal prediction derived from the menu model [1]) is not met in many empirical studies including chicks. Instead, matching to the relative profitability has been the norm in most cases. Applicability of the menu model is thus limited.

Optimal Patch Use Model (Marginal Value Theorem) and Decision of Disengagements

Charnov proposed another important idea, the optimal patch-use model, which is based on marginal value theorem [10] (**Figure 1B** left). He assumes a single forager that sequentially visits a series of food patches. The model is characterized by (1) unevenly distributed food items in patches, and (2) resource depletion by the forager's consumption, as the convex curve of cumulative gain illustrates. The optimal forager must disengage from the patch at a point where gain rate is maximized (indicated by slope of the dashed red line in **Figure 1B** left) by action selection. If staying put in the patch, the forager will find the next food item in a short period of time. If disengaging, it must invest considerable time to find the new patch, which is more beneficial than the food item available in the old patch. Here again, the choice of actions might be translated to the intertemporal choice of options (**Figure 1B** right).

The possible commonality between the patch-use behavior and the ITC task has been challenged both theoretically and empirically [11–18]. So far, the foraging behavior cannot be translated to the ITC task, or these are simply not compatible. Actually, animals are less patient in the ITC task compared with their behaviors in natural foraging situations, and the temporal discounting measured in ITC task does not fit well with the decision to disengage. The underlying decision mechanism is also distinct, as pharmacological treatment using selective serotonin reuptake inhibitor (SSRI, fluvoxamine) suppressed impulsive choices but delayed the disengagement, contrary to the prediction based on the commonality [19]. Furthermore, the impulsivity measure can drastically change depending on how subjects are informed of the relevant parameters, such as the time after the food option was consumed (so-called post-reward delay) [7, 20]. Interspecies comparisons suggest a clear dependency on ecological factors such as diet preference [21, 22], but these factors do not uniquely characterize impulsivity of each species.

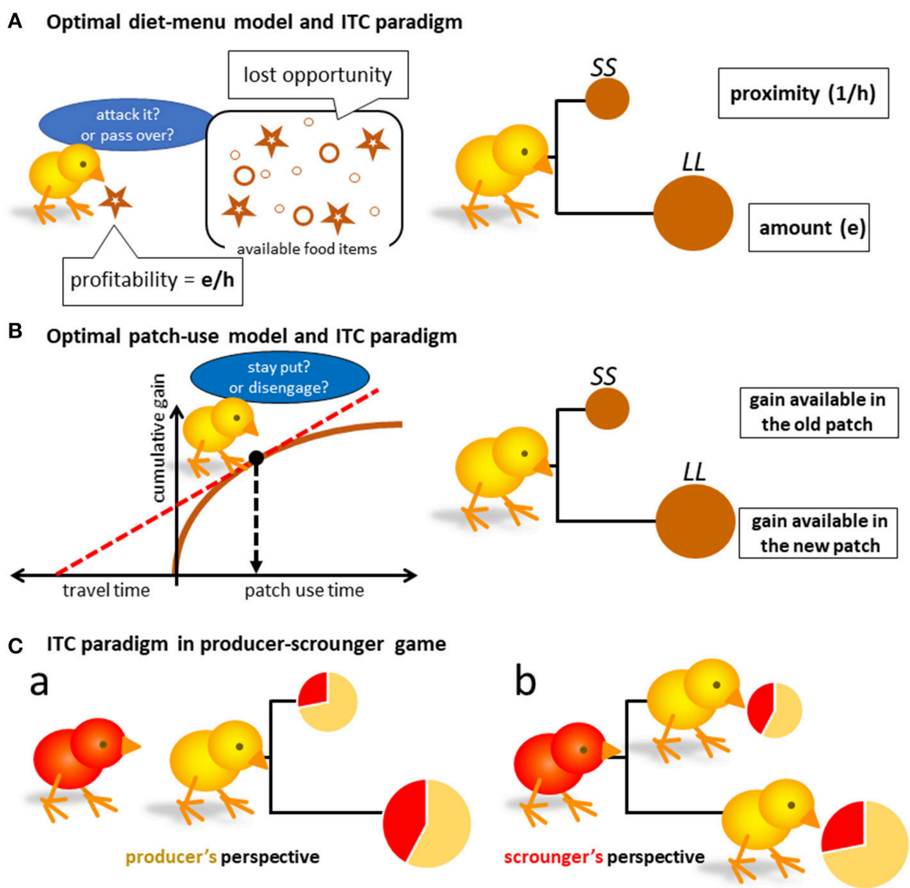

FIGURE 1 | Possible isomorphism between optimal foraging theories and the intertemporal choice paradigm **(A,B)**, which represent the optimal diet-menu model and the patch-use model, respectively. *SS* denotes a small and short delay reward, and *LL* a large and long delay alternative. Though the menu model **(A)** can be translated to the intertemporal choice (ITC) paradigm, the latter patch-use behavior cannot. In addition to the classical frameworks of optimal theories, we must address to the social foraging situation (**C**a,b) to understand what the results of ITC tests could mean in terms of ecology.

THE ITC PARADIGM IN SOCIAL FORAGING SITUATIONS

Producer-Scrounger Resource Conflict

Though these models may explain a single forager's behavior, we must consider that foraging is generally a social event, and animals compete and/or cooperate in both exploration and exploitation phases [23]. As the foragers' payoffs are mutually interdependent, individual decisions toward optimization inevitably result in sub-optimal outcome. To avoid starvation, animals should rather adopt a mixed strategy such as kleptoparasitism, which comprises producer and scrounger tactics [24]. Here, producers search and find food, and scroungers take free ride on the producer's discovery. If foragers freely change between the two alternative tactics, a stable equilibrium emerges at the point where the producer's gain is on par with the scrounger's. The validity of this framework has been shown in various animal studies [25–27].

What if the social foraging situation appears in the ITC paradigm? Intuitively, the producer chooses the more proximate food option, if the producer's share is higher for that proximate option (**Figures 1C**a). On the other hand, the scrounger chooses the more proximate producer by the same token (**Figures 1C**b). Under some circumstances, impulsive (or time-preference based) choices can be more beneficial. In the following sections, we will examine these possibilities by constructing analytical model and numerical calculation based on empirically-obtained parameters of behavior.

Rather than a mixed group composed of producers and scroungers, we presume a homogeneous group of opportunistic foragers that produce and scrounge simultaneously. In addition, we focus only on what the producing foragers gain. To produce food in nature, the foragers must pay a certain cost (such as traveling time, energy, and vigilance for food search) and make choices, whereas scroungers take free ride without paying the cost. We therefore assumed that only the producing foragers would update the food memory (value of food options), whereas the scroungers would not. We also disregard the cost and examine only the gain. Furthermore, foragers are assumed to exhaust all the food in a patch they find, rather than to disengage midway as the marginal value theorem assumes. As will be shown below, predictions based on these lines of simplification fit well with the behaviors of chicks that forage in densely packed flocks.

Assumptions

A pair of opportunistic foragers are assumed in a patchy food condition (**Figure 2**). When one forager encounters a patch comprising sharable food (F), it acts as producer (P), and rushes to F at a constant running speed v. Another forager immediately detects F discovered by P, acts as scrounger (S), and rushes to F at the same speed v. It might be possible to assume that S starts rushing to F only after P reaches F. We however assume that S rushes to P as soon as P rushes to F, because chicks run in response to the companion's (competitor's) running even before the companion starts to consume the food (social facilitation of running, [28]). Pecking food is also facilitated by companion's pecking (local enhancement), but the facilitated pecking is not causally linked with the facilitated running. Chicks run for other's run, and peck for other's peck, but these two types of facilitation are separate processes [29]. P is located closer to F (at distance $d > 0$) than S is, and S is at distance $\delta > 0$ from P. The time required for each player's decision is short and therefore ignored.

A: amount of food items in a patch
d: distance between F and P
δ: distance between P and S
v: running velocity of P and S

According to one of the representative frameworks developed in social foraging theory [24], we assume a non-ephemeral food patch F (**Figure 2A**). We assume that each forager has a fixed speed of food consumption (s, amount per time per individual). As soon as P reaches F after a delay (D), the patch F supplies a finite amount of food (A) at once, and P starts to consume F at the speed s. P monopolizes F for a period (T) until S arrives at F at the point denoted as Φ. No conflict occurs between P and S, and F is thereafter consumed at twice the speed ($2s$) until F is exhausted at Ω. P's gain rate therefore does not diminish by sharing food with S. We may otherwise assume an ephemeral food patch that supplies food by itself at a rapidly declining rate. In such a patch type, the finder's share would be higher in favor of P's gain. In the following, however, we focus only on the non-ephemeral type for simplicity.

The delay D and the finder's share (monopolizing time by P, denoted as T) are given as;

$$D = \frac{d}{v}$$
$$T = \frac{\delta}{v}$$

If Φ follows Ω, the food patch is exhausted by P before S arrives. This situation is met when

$$\frac{A}{s} \leq T = \frac{\delta}{v} \tag{1}$$

or

$$A \leq \frac{s\delta}{v} = sT \tag{1'}$$

If otherwise and Φ precedes Ω, the food patch is shared after Φ.

Profitability is illustrated as the slope of gain at Ω (dashed brown line with an arrow in **Figure 2A**) for each P (yellow arrow with a dashed line). We assume that foragers stay at F until Ω, when the food is exhausted. Depending on whether scrounging occurs, the profitability that P gains is given by the $prof_P$ functions as;

$$\text{If } \Phi \text{ follows } \Omega, \quad prof_P = \frac{A}{D + A/s} \tag{2}$$

$$\text{If } \Phi \text{precedes } \Omega, \quad prof_P = \frac{A + sT}{2D + T + A/s} \tag{3}$$

Analysis of the Model

We mathematically examine the conditions where SS has a higher profitability than LL. As the inequality (1′) predicts, following 3 cases are considered;

Case (1); no scrounging for both SS and LL

$$A_{SS} < A_{LL} \leq sT \tag{4}$$

Case (2); scrounging for LL but not for SS

$$A_{SS} \leq sT < A_{LL} \tag{5}$$

Case (3); scrounging for both LL and SS

$$sT < A_{SS} < A_{LL} \tag{6}$$

Here, A_{SS} and A_{LL} denote the amount of SS and LL, respectively. Similarly, the delay D for SS and LL is denoted as D_{SS} and D_{LL}. By definition,

$$A_{SS} < A_{LL} \text{ and } D_{SS} < D_{LL}$$

These cases are schematically illustrated in **Figure 2B** for different δ for levels of scrounging, Ba for case (1), Bb for case (2), and Bc for case (3). In the following, we will show conditions where

$$prof_P\ (SS) > prof_P\ (LL) \tag{7}$$

holds.

Case (1)

According to the formulae (2), the inequality formula (4) is given as;

$$\frac{A_{SS}}{D_{SS} + A_{SS}/s} > \frac{A_{LL}}{D_{LL} + A_{LL}/s} \tag{8}$$

which is equivalent for $\forall s > 0$ to a simpler form;

$$\frac{A_{SS}}{D_{SS}} > \frac{A_{LL}}{D_{LL}} \tag{8'}$$

SS is more profitable than LL for $\forall T > 0$ and $\forall s > 0$, when and only when the A/D ratio is higher for SS than for LL. The area of A/D ratios that satisfy (8′) is illustrated in dark brown in **Figure 3** left. Note that this area is identical to the area where A/D ratio of SS is higher than that of LL. The time required for consumption is thus disregarded for comparing the profitability between SS and LL. The finder's share (T) is also disregarded.

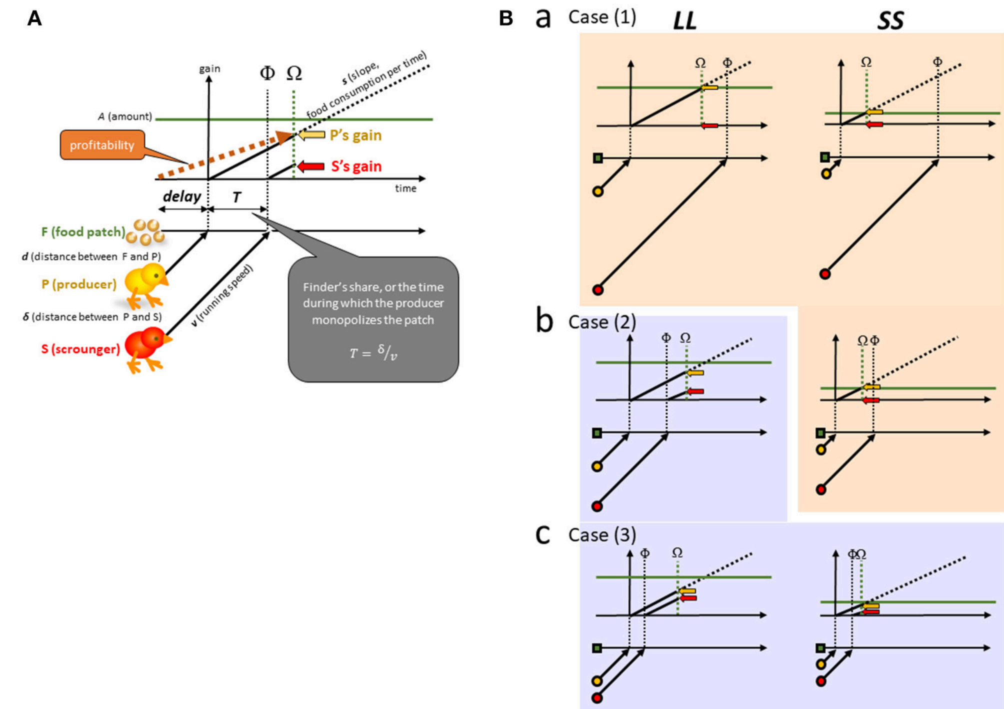

FIGURE 2 | (A) A group comprising two opportunistic foragers is assumed. Each of the two foragers searches for food, and if one finds food (F), it acts a producing individual (P). Another forager immediately detects the discovery and rushes to P as a scrounging individual (S). A food patch of finite amount (*A*) is placed at a long distance (*d*) from P, representing the *LL* option (left columns). A food patch of a smaller amount at a short distance represents the *SS* option (right columns). **(B)** We consider three levels of scrounging as long [**B**a; Case (1)], intermediate [**B**b; Case (2)] and short [**B**c; Case (3)] distances between P and S (δ). In Case (1), scrounging does not occur for both LL and SS. In Case (2), scrounging occurs only for LL. In Case (3), scrounging occur for both LL and SS. Scrounging inevitably reduces the P's finder's share, or the time during which P monopolizes the food (*T*).

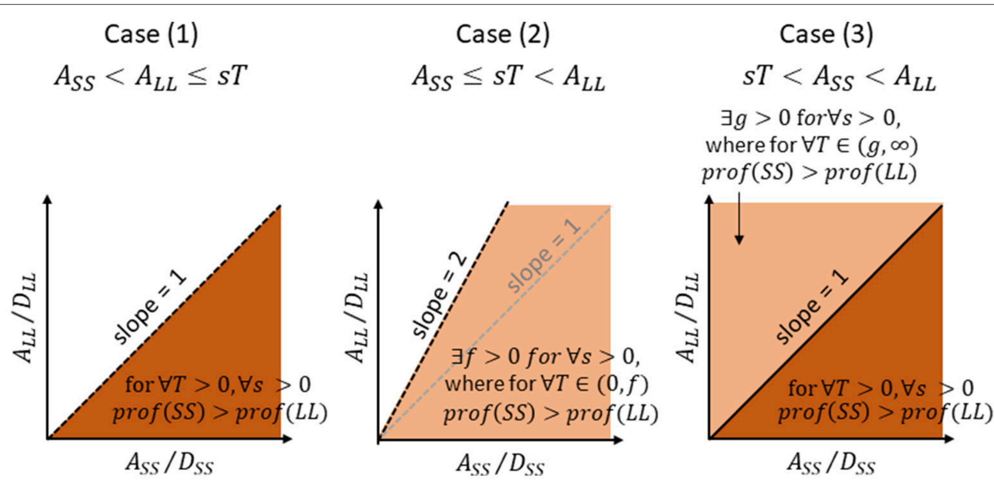

FIGURE 3 | Areas of parameters (amount-to-delay ratio, *A/D* ratio) for *SS* (abscissa) and *LL* (ordinate) where *SS* gives rise to a higher profitability than *LL*, for Case (1), Case (2), and Case (3), respectively. See text for further explanations.

Case (2)

Similarly based on (2) and (3), (4) is given as;

$$\frac{A_{SS}}{D_{SS} + A_{SS}/s} > \frac{A_{LL} + sT}{2D_{LL} + T + A_{LL}/s} \tag{9}$$

which is equivalent with;

$$f = \frac{2A_{SS}\left(D_{LL}/D_{SS}\right) - A_{LL}}{s} > T \tag{9'}$$

Considering (5), we have;

$$A_{SS}/s \leq T < \min\{A_{LL}/s,\ f\} \tag{10}$$

The upper limit of T exists for $\forall s > 0$ if the numerator of (9′) is positive, namely when

$$\frac{2A_{SS}}{D_{SS}} > \frac{A_{LL}}{D_{LL}} \tag{11}$$

holds. The area of A/D ratios that satisfy (11) is illustrated in light brown in **Figure 3** center. Note that this area is wider than the area where A/D ratio of *SS* is higher than that of *LL*. If otherwise and f is negative, (9) does not hold for $\forall T \geq 0$.

Case (3)

Based on (3), (7) is given as;

$$\frac{A_{SS} + sT}{2D_{SS} + T + A_{SS}/s} > \frac{A_{LL} + sT}{2D_{LL} + T + A_{LL}/s} \tag{12}$$

which is equivalent with;

$$g = \frac{A_{LL}D_{SS} - A_{SS}D_{LL}}{(D_{LL} - D_{SS})\,s} < T \tag{12′}$$

Considering (6), we have;

$$\max\{g,\ 0\} < T < A_{SS}/s \tag{13}$$

When the numerator of (12′) is 0 or negative, g is also negative, so that (12) holds for $\forall T \geq 0$. Namely when;

$$\frac{A_{SS}}{D_{SS}} \geq \frac{A_{LL}}{D_{LL}} \tag{14}$$

The A/D area that satisfy (14) is illustrated in dark brown in **Figure 3** right. If otherwise, the T has a non-zero lower limit given by (12′).

Predictions of the Analytical Model

From the analytical model, we obtain the following lines of prediction.

- P always gains more than S does.
- If not scrounged, the profitability that P gains from *SS* and *LL* options follows the respective amount-to-delay ratio (A/D ratio).
- If scrounged and the finder's share (T) is short, profitability of *SS* can be a higher than that of *LL*, even when A/D ratio of *SS* is lower than that of *LL*.
- For this reversal to occur, the finder's share (T) must be either shorter than an upper limit, or longer than a lower limit, depending on the A/D ratios of *SS* and *LL*, and the speed of food consumption (s).

To gain a higher profitability for a given d, S must reduce δ and/or increase v by forming a dense flock of rapidly running foragers. On the other hand, P also has to increase v to counteract S's scrounging. Interactions between S and P would result in an arms race, so v increases to its upper limit given by physiological constraints, and T consequently becomes shorter. The shorter the T, the lower the $prof_p$ would be. However, because *SS* is more resistant against scrounging than *LL*, P would choose *SS* more frequently than *LL*.

The food supply rate (s) needs a careful consideration. The present analytical model assumes that food is available at once as soon as foragers arrive at F, and the consumption rate is determined by the foragers behavior. Alternatively, the food resource may determine s, a constant that foragers cannot control. We analyzed this alternative situation (**Supplementary Text** and **Figures 1S**, **2S**) and found basically the same predictions. In either model, a higher s would make P to reduce the scrounging effect and increase $prof_p$ at the same time. The scrounger S also gains by increasing its consumption speed, so that an arms race would arise also for s, leading to its upper limit delineated by physiological constraints.

We may generalize the model to include more than 2 foragers, namely one P and two or more S that scrounge the P's discovery. A group of opportunistic foragers composed of n individuals may encounter food patches at a higher rate than the two foragers assumed here. In this study, however, we focus only on how much does P gain, and we do not consider what S gains. We ignored this because we supposed that only P forms memory associated with food of a certain profitability, whereas S does not update the food memory. This assumption is actually not correct, and chicks in the scrounging situation also learn the association between color cues and the food rewards [30]. Further theoretical and empirical considerations are needed on this point.

A serious consideration must be given for whether the analytical model is realistic. In the following, based on experimental behavioral data we have obtained so far in a series of chick studies, we will construct a numerical simulation further in search of the conditions where *SS* is more profitable than *LL*.

Numerical Simulation of Profitability in Social Foraging Situation

We construct numerical **simulation** to specify the conditions in which the $prof_p$ of *SS* is higher than that of *LL*. To do so, we modify some of the assumptions discussed so far. First, we now assume that the amount of food in patch (A) is a discrete value (integer) rather than continuous. Actually, foragers make discrete acts of attack for each piece of food in the patch, rather than smoothly sucking up fluid food. Second, each act of attack adds a certain unitary handling time, reducing $prof_p$ in a stepwise manner. Third, attacks often fail, further reducing $prof_p$. Finally, and most importantly, the amount (A) varies from a patch to another, even without scrounging. We take these ecologically realistic situations into consideration, together with behavioral parameters obtained in behavioral studies in domestic chicks [30].

Amount and Delay

We assume that the forager sequentially encounters food patches in a field comprising only *SS* options, or another field comprising only *LL* options (**Figures 4A,B**). The forager does not travel, and the time between patch encounters is disregarded. Instead, the

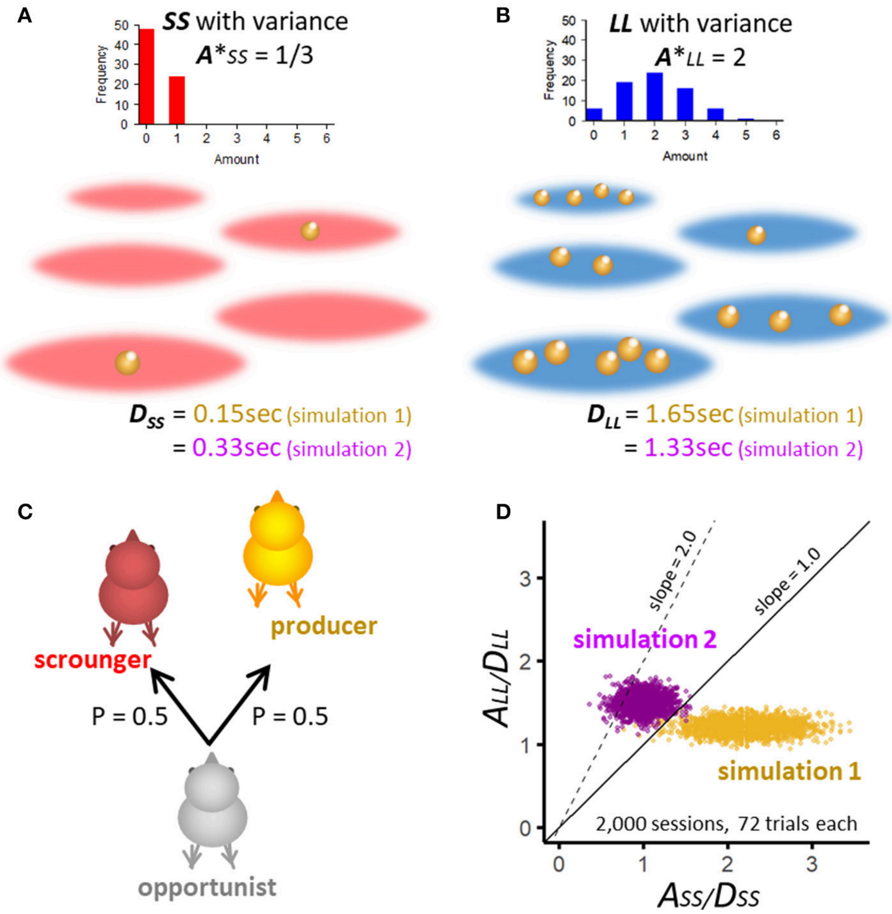

FIGURE 4 | Assumptions of numerical simulation constructed for realistic situations for *SS* **(A)** and *LL* patches **(B)**. For each patch, forager waits for a constant delay time, namely 0.15 s for *SS* and 1.65 s for *LL*. To mimic natural food patches, the amount of food items varies randomly according to binomial distribution. The expected amount of food was set as 1/3 for *SS* and 2 for *LL*, so that the latter is 6 times larger. **(C)** Opportunistic forager act producer or scrounger at equal probability = 0.5 in a manner independent from the food amount variations. **(D)** *A/D* ratio of randomly generated *SS* and *LL* pairs in 2,000 simulated sessions (each composed of 72 trials) are plotted for two sets of simulations. In simulation 1, *A/D* ratio is distributed mostly in the area where A_{SS}/D_{SS} is larger than A_{LL}/D_{LL}. In simulation 2, *A/D* ratio of *LL* is higher than *SS* and lies mainly below the line with slope = 2, corresponding to Case (2) of **Figure 3**.

forager evaluates each patch by its short-term profitability. In the following, we consider only the profitability gained by the producer ($prof_p$).

One patch has a fixed number ($A = 1$ or 6) of discrete food items, each of which independently becomes available at a probability of 1/3, so that natural variations in food amount in patches are mimicked. In the field comprising *SS* patches ($A = 1$, **Figure 4A**), the expected amount is ($A^* = 1/3$). Similarly, in the field comprising *LL* patches ($A = 6$, **Figure 4B**), the number of food items varies widely with its expected amount 6 times larger than that of *SS* patches ($A^* = 2$). For these patches, we assume a fixed *delay* of 0.15 s for *SS* and 1.65 s for *LL*. These *delay* values are taken from our behavioral study [30]. In the numerical simulation we assume;

A: total number of food items in a patch, varies according to binomial distribution; (integer, $0 \leq A \leq 6$ for *LL*, $0 \leq A \leq 1$ for *SS*).

The simulated opportunistic chick encounters food patches for 72 trials in one session, and the chick acts P or S at equal probability of 0.5 (**Figure 4C**). Notice that the amount (*A*) is determined independently of whether the chick acts P or S. **Figure 4D** shows two sets of simulation, in which A/D ratios of randomly generated *SS* and *LL* pairs are plotted in 2,000 sessions, each composed of 72 trials. In **simulation 1** (**Table 1**), delays are chosen so that *SS* is higher than *LL* in A^*/D ratio. In **simulation 2** (**Table 2**), A^*/D ratio of *SS* is lower than *LL* but it is distributed mostly below the line of slope = 2.0, corresponding to the parametric area shown in Case (2) of the analytical model (**Figure 3**).

Food Consumption Speed and Profitability Functions

The food consumption speed (*s*) is a constant determined by the foraging chicks, namely by how quickly and how accurately they attack the food. For one action of attack, the chick invests a unitary handling time (τ). Because attacks (pecks) sometimes fail, the chick repeats attack more times than the number of items, and the ratio (number of attacks per item) is given as $\kappa \geq 1$. The chick also pays an energetic cost for attacks, but it is ignored here. Based on our previous experiments in chicks, we estimate;

τ: handling time per attack: 0.25 (sec/peck)
κ: number of attacks invested per food item: 1.21 (pecks/grain)

Here, τ represents the time invested for one action of pecking, and κ the accuracy of pecking. The food consumption speed (s) is therefore given as;

$$s = 1/\tau\kappa = 3.31 \text{ (grain/sec)} \quad (15)$$

For A^*_{SS} and A^*_{LL} shown in **Table 1**, each of the 3 cases therefore corresponds to the following range of T values;

Case (1): $0.60 \leq T$
Case (2): $0.10 \leq T < 0.60$
Case (3): $0.00 \leq T < 0.10$

In the simulation, following the formula (1), profitability is given for each trial by (2) when;

$$A/s = A\tau\kappa \leq T$$

or by (3) when;

$$T < A/s = A\tau\kappa$$

holds, respectively.

Profitability When Scrounging Does Not Occur

Initially, we calculate $prof_P$ for *SS* and *LL* under the condition where scrounging does not occur (or T is set 999 sec for ∞) for **simulation 1** (**Table 3**) and **2** (**Table 4**). Average of $prof_P$ in 2,000 sessions is lower than the $prof_P$ computed for the averaged amount (A^*) for both *SS* and *LL*. It is because $prof_P$ is given by upward-convex functions in either (2) or (3), so that Jensen's inequality holds. As D_{SS} is shorter than D_{LL}, due to a higher degree of non-linearity for *SS*, a larger difference occurs in *SS* than in *LL*.

Profitability When Scrounging Occurs

For **simulation 1**, we calculated $prof_P$in two different conditions, namely when the food amount (A_{LL} and A_{SS}) does not vary (**Figure 5Aa**), and when the amount varies from trial to trial (**Figure 5Ab-d**), respectively. When the amount does not vary, *SS* exceeds *LL* for $\forall T \in [0, \infty)$ (**Figure 5Aa**). When the amount varies, Jensen's inequality occurs, and *LL* tends to be higher than *SS* for $T = 1.0sec$ (**Figure 5Ac**), but not for $T = 0.2sec$ (**Figure 5Ab**). Further systematic survey for $T \in [0, 1.0]$ reveals a reversal at around 0.4–0.6 s (**Figure 5Ad**); dashed horizontal lines (blue for *LL* and red for *SS*) indicate the average of simulated $prof_p$ obtained for $T = \infty$ *or* 999. Clearly *SS* is more resistant against scrounging than *LL*.

We also calculated $prof_P$ for **simulation 2** in two different conditions (**Figure 5B**). When the amount does not vary, *LL* exceeds *SS* except $T = 0.10sec$ (Ba). When the amount varies, as the effect of the Jensen's inequality is stronger for *SS* than for *LL*, *LL* exceeds *SS* for $\forall T \in [0, \infty)$. Still, as shown in **Figure 5Bd**, *SS* is more resistant against scrounging than *LL*.

Upper and lower limits for T [f and g, given by (9′) and (12′)] are calculated for the averaged amount (A^*) of *SS* and *LL* such as;

simulation 1:

$f = 1.21$ (sec), which is higher than 0.6 [the upper limit for Case (2)]
$g = -0.038$ (sec), which is lower than 0.0 [the lower limit for Case (3)]

simulation 2:

$f = 0.295$(sec), which is lower than 0.6 [the upper limit for Case (2)]
$g = 0.044$(sec), which is higher than 0.0 [the lower limit for Case(3)]

In simulation 1, f and g do not need to be considered as limitations, whereas in simulation 2 these values significantly limit the parametric space where *SS* has a higher profitability than *LL*.

TABLE 1 | Amount, delay, and *A/D* ratio (**simulation 1**).

	SS	***LL***
A^* (averaged amount, grain)	1/3	2
D (delay, sec)	0.15	1.65
A^*/D ratio (grain/sec)	2.22	1.21
Simulated *A/D* ratio (grain/sec); average of 2,000 sessions	2.224	1.213

TABLE 2 | Amount, delay, and *A/D* ratio (**simulation 2**).

	SS	***LL***
A^* (averaged amount, grain)	1/3	2
D (delay, sec)	1/3	4/3
A^*/D ratio (grain/sec)	1.00	1.50
Simulated *A/D* ratio (grain/sec); average of 2,000 sessions	0.995	1.502

TABLE 3 | Profitability (**simulation 1**).

	SS	***LL***
$prof_P$ for averaged amount A^*; no scrounging	1.33	0.89
Average of $prof_P$ in 2,000 sessions; no scrounging	0.74	0.83

TABLE 4 | Profitability (**simulation 2**).

	SS	***LL***
$prof_P$ for averaged amount A^*; no scrounging	0.77	1.03
Average of $prof_P$ in 2,000 sessions; no scrounging	0.52	0.96

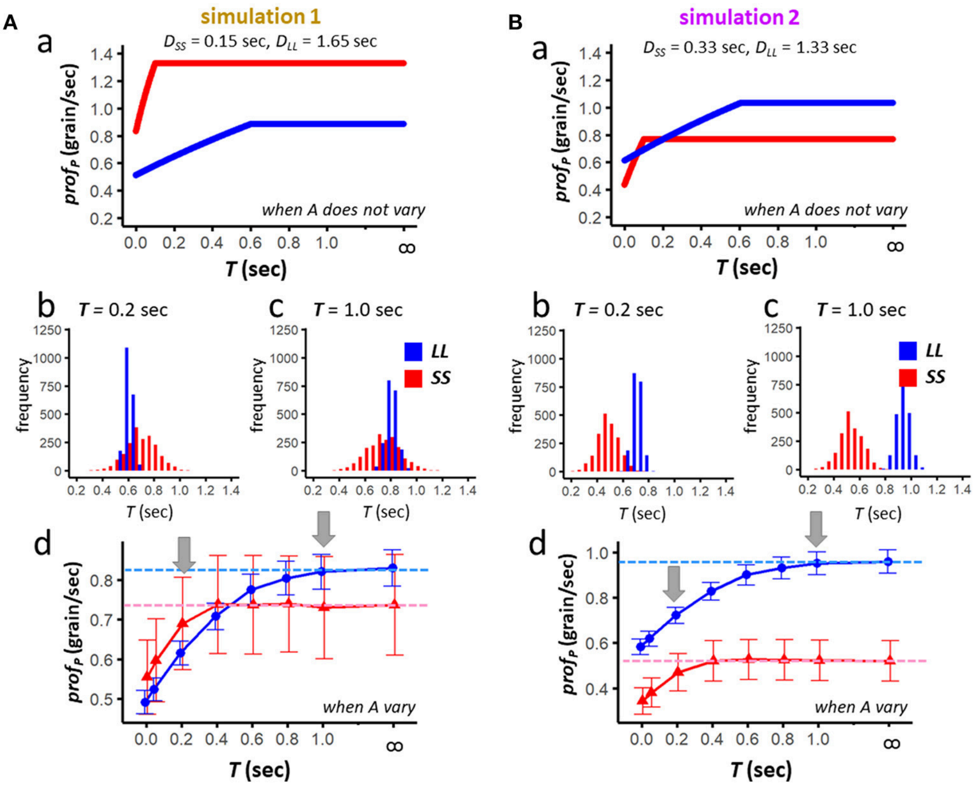

FIGURE 5 | Results of the numerical **simulation 1 (A)** and **simulation 2 (B)**. For each set of simulation parameters, averaged profitability of *LL* (blue) and *SS* (red) is examined in two conditions; namely, when the food amount does not vary (**A**a,**B**a), and when it varies from trial to trial (**A**b–d,**B**b–d). Results of 2,000 simulated sessions (each representing average of 72 trials) are shown for T (0.00, 0.05, 0.20, 0.40, 0.60, 0.80, 0.10, and 999 or ∞); mean and standard deviation are shown each. Dashed horizontal lines in (**A**d,**B**d) denote the profitability when scrounging does not occur.

Following lines of main conclusions are thus drawn;

- Results of the numerical simulations support a prediction of the analytical model that *SS* can be more profitable than *LL* when *T* is short.
- The non-linearity of $prof_P$ functions (upward-convex for amount) makes SS less profitable than *LL* due to the Jensen's inequality.
- Therefore, if *LL* has a higher *A*/*D* ratio than *SS*, *SS* has little chance to exceed *LL* in profitability under the realistic ecological situation.

EMPIRICAL SUPPORTS

Perceived Competition for Impulsiveness

When trained in group of three individuals for the conventional ITC task, chicks developed a strongly *SS*-biased preference (or impulsive choice) by ca. 2-fold compared with those trained alone [30] in good accordance with the theoretical examinations described above. A high level of impulsiveness appeared even when actual conflict of food did not occur. It must also be noticed that the social effects were conditional but not contextual, and the presence of a competitor at the binary choice test did not matter [31], suggesting that the profitability-linked values are stored as lasting reference memory after cumulative experiences of foraging.

However, the amount of food reward should vary at each trial, as those trained with fixed amount (no-risk condition) did not develop impulsive choices [32] in a manner contradictory to what we predict from the profitability variances. Risky food (i.e., larger profitability variance) may generally shift the choice toward indifference, rather than enhanced impulsiveness. The enhanced *SS* preference in the risk condition is also contradictory to another empirical finding that the chicks are risk averse [33]. When the delay varied, on the other hand, riskier option was given a paradoxically high value [33]. It remains to be intensively studied as to how the risk interacts with the profitability-based decision making.

Social Facilitation of Foraging Effort and Behavioral Synchronization

Patch-use behavior also drastically changes according to social foraging. When a chick was placed in an I-shaped maze equipped with a feeder at each terminal, the chick spontaneously started shuttling between the two feeders. The food (grain of millet) was supplied at variable intervals (1 grain at every 6.7–60 s) without any associated cues. Even in such an uncertain context, chicks allocated the residence time at the two feeders according to the relative gain rate as would be expected by the Herrnstein's matching law in psychology [34] and the ideal free distribution in ecology [35]. When two chicks were placed in the maze (social context), they immediately ran more than in the single context, and the runs were highly synchronized [28, 29]. The social facilitation [36] was accompanied by a precise matching to the reinforcement ratio [37]. See **Supplementary Materials** for example video clips of the socially-facilitated running and behavioral synchronization. One video (denoted as "single") shows a pair of chicks separated by opaque wall, so that chicks are invisible to each other. Another video ("pair") shows those chicks separated by transparent wall. As argued above, foraging pairs try to reduce δ and increase ν to their limits, so that the share of the producer (T_p) is minimized. Under such a high scrounging condition, *SS* could be a better option than *LL* is in terms of their short-sighted currency of the profitability.

Does the facilitated work-cost really pay? Running more means visiting feeders more frequently, and chicks could personally gain more information about food availability. However, paired chicks may gain public information from their companion. An experimental group of single foragers was confronted with a mirror in the maze, and they also showed socially-facilitated runs, but ended up with under-matching results as in the group of single chicks. Conversely, precise matching was achieved in chicks paired with a real conspecific [37]. These simple experiments gave an answer in favor of the latter public information hypothesis [38, 39]. Collective intelligence could emerge even in such a small group as a pair of chicks.

EMBEDDED SOCIALITY AND ECOLOGICAL VALIDITY

Apparently irrational impulsiveness and excessive work investment could be reasonably accounted for in terms of foraging ecology. In particular, adjustments associated with the producer-scrounger resource conflict proved to be pre-embedded in decisions mechanisms of newly-hatched domestic chicks, allowing them to change decisions flexibly in response to the social and economic circumstances. One is the social impulsivity that is conditionally induced by competition experienced in the past. Another is the social facilitation that appears contextually dependent on the present availability of public information on the food patch from flock companions.

In accord with the social impulsivity, visual perception of a competitor suppressed the neuronal representation of predicted rewards in the basal ganglia [6, 40]. The social facilitation also has neural substrates in the limbic pallium [29, 41]. Lesions to the nucleus accumbens made chicks impulsive [5] just as the competitive training did [30]. Lesions to the limbic pallium caused a perseveration [42] as exemplified by the socially-facilitated foraging [28]. Although the responsible internal processes are yet largely elusive (however, see [43–45] for recent advances in primates and humans), further quantitative analyses of behaviors based on foraging ecology would give us a valid understanding of the hidden processes, leading to the evolutionary basis of learning and cognition [46].

AUTHOR CONTRIBUTIONS

TM conceived the ideas. YO and HA constructed numeric simulation. YO, HA, and TM wrote the manuscript.

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SUPPLEMENTARY MATERIAL

The Supplementary Material for this article can be found online at: https://www.frontiersin.org/articles/10.3389/fams.2018.00049/full#supplementary-material

Videos S1 and S2 | Example video clips showing chicks shuttling between two feeders placed at both terminal ends of an I-shaped maze. Food (grains of millet) was delivered a low rate following variable interval schedule. The time of food delivery is indicated by flashing LEDs placed near the feeders. In one video ("**Video S1**"), chicks were separated by an opaque wall that separated two lanes. In another video ("**Video S2**"), separation was made by a transparent wall, so that chicks were visible with each other. In both cases, actual conflict of food did not occur, and the competition was fictitious. Distracting motor sounds were added to avoid possible association between the feeders' sound and the delivery of food.

Datasheet 1 | Supplementary analysis of an alternative model. This model is constructed with an assumption that the food patch F supplies food at a constant supply rate denoted by *s* (grain/sec). Basically, the same conclusions are obtained as in the model shown in the main text.

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Inconsistency and Subjective Time Dilation Perception in Intertemporal Decision Making

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A large number of studies have demonstrated that intertemporal decision making process usually results in preferences that reverse over time, or choices that are inconsistent over time. Inconsistency can be explained by different discount models by the effect of reward value perception at different moments. Otherwise, one can also understand inconsistency as the result of the time perception effect. Here, we address inconsistency as the result of a subjective time dilation perception effect. We use arguments inspired by the special theory of relativity and focused our study on a generalized model that encompasses psychophysical effects on time perception, where we look for a transformation of the time interval between the pay times of two rewards. Additionally, we present a generalized two-argument hyperbolic utility function for the Bernoulli (logarithmic) one, associating their difference to subjective time intervals.

Keywords: econophysics, psychophysics, intertemporal decision making, inconsistency, time perception, generalized models, utility functions

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1. INTRODUCTION

Individuals subjected to intertemporal decision making have to choose between two rewards: a smaller and more immediate and a greater and later one. In intertemporal decision making, the time interval between the present instant and the delivery time of the reward is called **delay**. Studies have led to a strong consensus that later rewards are discounted (or devalued) relative to more immediate ones [1]. The value of a reward, V, decreases as the delay increases. The undiscounted (real) value of a given reward is called **objective value**, V_0. The reward value to be received with a given delay, $V(t)$, is called **subjective value** and is equal to the subjective value V_0 discounted. Experiments with humans and animals have been carried out to determine the indifference points [2–8]. **Discount functions** model the behavior of a reward subjective value as a function of the delay, being monotonic decreasing and vanishing functions. Despite the difficulty of measuring $V(t)$ (by the indifference point determination), several phenomenological models have been addressed to establish discount functions that adequately describe the discount process as a function of the experimentally observed delay. At the outset, the exponential and hyperbolic functions are the main models, which can be retrieved as particular cases of more general ones [8–10].

Discount models can be elaborated taking into account time perception/distortion effects. In Physics, according to the special theory of relativity, **time dilation** is an effect characterized by the difference in the elapsed time measured by two observers. That difference may be due to the fact that observers are in different inertial systems moving uniformly and rectilinearly with respect to each other or because they are under the action of gravitational fields of different intensities [11]. In cases involving two inertial reference systems, an observer measures a shorter time interval ("proper time") between two **co-local events** (that happen at the same place in her/his system) than another observer, who measures the time interval between these same events from her/his system (for her/him, the events happen at different places). The expression for time dilation is $\Delta t = \gamma \Delta t_0$, where Δt_0 is the time interval between two co-local events for an observer in some inertial reference system (**proper time**), Δt is the time interval between those same events, but measured by an observer in a reference system moving with velocity v with respect to the first one. Here, $\gamma = 1/\sqrt{1-(v/c)^2}$ is the **Lorentz factor**, where v is the relative velocity of the inertial systems and c is the speed light.

Returning to intertemporal decision making, a dynamically inconsistent individual prefers smaller and more immediate rewards, but opts for greater and later ones in distant futures, as if the increase in the delay in receiving the rewards distorts her/his perception of $\Delta\tau$—the time interval between the pay times of two rewards. Here, we propose to deal with the dynamic inconsistency as the result of a subjective time dilation effect of the interval $\Delta\tau$ perceived by the decision maker. We obtain a generalized transformation equation for the effect of $\Delta\tau$ distortion, similar to that from the special theory of relativity. Our proposal is an important contribution to the characterization of subjective time in an individual basis, which is provided by the $\tilde{q}$ parameter. According to a study conducted in 2017 by Agostino et al. [12], characterizing subjective time in an individual basis is indispensable to study the deviations from average. Additionally, we present a generalized two-argument hyperbolic utility function for the Bernoulli (logarithmic) one, associating their difference to subjective time intervals. This issue relates two distinct subjetive perceptions: time and value.

2. MODELS

Here, we present the exponential and hyperbolic discount models and their first and second derivatives with respect to time as the impulsivity and degree of inconsistency, respectively. Takahashi et al. and Cajueiro discount models are similar and allow us to understand impulsivity and inconsistency as subjective time perception. This is suitably described mathematically using the generalized logarithm and exponential functions.

In standard economic theory, the present value of a future reward decreases with a fixed ratio per unit of delay, in the same way that a bank balance increases with a fixed interest rate over time. In this case, the discount of the real (objective) value of a reward is characterized by an exponential decay model [13]:

$$V^{(e)}(t) = V_0 e^{-kt}, \tag{1}$$

where the parameter k is the rate at which an individual discounts late rewards. High k values correspond to discount curves with more pronounced decay. In this model, the preference between two intertemporal rewards does not depend on how much the two rewards options are moved into the future with the same amount of time.

However, experimental results [14–19] show that the reward value discount as a function of the delay is best described by a hyperbolic function [5]:

$$V^{(h)}(t) = \frac{V_0}{1+kt}\,. \tag{2}$$

In intertemporal choices, **impulsivity** is defined as the preference for smaller and immediate rewards to greater and later ones [7]. Let the individual "A" chose the smaller and more immediate reward $V_1(t)$, and if individual "B" chooses the greater and later reward $V_2(t+\tau)$, we say "A" is more impulsive than "B." The relative variation of the discount function is used as a measure of impulsivity in the context of intertemporal decision making. The **discount rate** is the relative variation of the discount function [7]:

$$I = -\frac{\mathrm{d}(\ln V)}{\mathrm{d}t} = -\frac{1}{V}\frac{\mathrm{d}V}{\mathrm{d}t}. \tag{3}$$

The anti-impulsive behavior is defined as **self-control**.

Returning to the example, where the "A" is more impulsive than "B," if "A" changes her/his choice after a certain delay t (if she/he happens to prefer the greater and later reward), her/his intertemporal choice is said to be **dynamically inconsistent**. Experiments involving humans and animals [2, 20–25] have shown that individuals tend to prefer smaller and more immediate rewards, but opt for greater and later ones in distant futures. In decision making studies, this preference reversal over time is called **dynamic inconsistency** in intertemporal choices [7, 25]. The **degree of inconsistency** was defined by Prelec in 2004 [26] and interpreted by Takahashi in 2010 as the time variation of I:

$$\mathbb{I}(t) = \frac{\mathrm{d}I}{\mathrm{d}t}, \tag{4}$$

where I is given by Equation (3). Defining the quantity that measures the degree of inconsistency as the temporal variation of the so-called impulsivity (the preference for smaller and immediate rewards to greater and later rewards [7, 26]), several models attribute this behavior to the effects of psychophysical perception of delay [1, 8, 9, 27–31]. For the exponential discount model, which describes the behavior of the rational decision-makers from neoclassical economic theory, the discount rate $I^{(e)}(t) = k$ is constant and, therefore, the degree of inconsistency vanishes ($\mathbb{I}^{(e)} = 0$). Thus, the exponential model can not describe the inconsistency observed experimentally in intertemporal decision making. For the hyperbolic discount model, the discount rate $I^{(h)}(t) = kV^{(h)}(t)/V_0$ is a decreasing function of t. In this case, the value of a reward is strongly discounted on relatively small delays, but it is more moderately discounted as the delay increases. For this model, the degree of inconsistency does not vanish and is: $\mathbb{I}^{(h)}(t) = -[I^{(h)}(t)]^2$.

Recent studies [28–31] analyze the discount process from the perspective the time perception. Takahashi et al. [8] proposed to include the logarithmic perception of delay, according the **second**

law of psychophysics (or **Weber-Fechner's law**), on the temporal exponential discount, calling

$$t' = a\ln(1 + bt) \tag{5}$$

the subjective time interval, where a and b are psychophysical parameters with $g = ka$, one has [28]: $V^{(T)}(t) = V_0 e^{-kt'} = V_0 e^{-ka\ln(1+bt)} = V_0/(1 + bt)^g$. For this model, $I^{(T)}(t) = g^2/(1 + bt)$ and $\mathbb{I}^{(T)}(t) = -bI^{(T)}(t)/(1 + bt)$. It is interesting to point out that as $g \to 0$, this model retrieves the exponential behavior and when $g = 1$, the hyperbolic one.

Using the $\tilde{q}$-logarithm and $\tilde{q}$-exponential functions allows one the retrieve known models without taking limits, since these limits are implicit [10, 32–39]. The **$\tilde{q}$-logarithm function** $\ln_{\tilde{q}}(x)$ is defined as the value under the curve $f_{\tilde{q}}(w) = 1/w^{1-\tilde{q}}$ in the interval $w \in [1, x]$ [40]:

$$\ln_{\tilde{q}}(x) = \int_1^x \frac{\mathrm{d}w}{w^{1-\tilde{q}}} = \lim_{\tilde{q}'\to\tilde{q}} \frac{x^{\tilde{q}'} - 1}{\tilde{q}'} = \begin{cases} \frac{x^{\tilde{q}}-1}{\tilde{q}}, & \text{for } \tilde{q} \neq 0 \\ \ln(x), & \text{for } \tilde{q} = 0 \end{cases}. \tag{6}$$

For any value of $\tilde{q}$, the area is negative for $0 < x < 1$, null for $x = 1$ ($\ln_{\tilde{q}}(1) = 0$) and positive for $x > 1$. This function is not the logarithm function in base $\tilde{q}$ ($\log_{\tilde{q}}(x)$), but the generalization for the definition of natural logarithm with a parameter. For $\tilde{q} = 0$, $\ln_0(x) = \ln(x)$, the natural logarithm function. The point $x = 1$ is special because $\ln_{\tilde{q}}(1) = 0$. The **$\tilde{q}$-exponential function** $\exp_{\tilde{q}}(x)$ is defined as the value w, in such a way that the area under the curve $f_{\tilde{q}}(w) = 1/w^{1-\tilde{q}}$, in the interval $w \in [1, \exp_{\tilde{q}}(x)]$, is x. In other words, it is the inverse of the $\tilde{q}$-logarithm function $\exp_{\tilde{q}}[\ln_{\tilde{q}}(x)] = x = \ln_{\tilde{q}}[\exp_{\tilde{q}}(x)]$ and reads:

$$\exp_{\tilde{q}}(x) = \begin{cases} \lim_{\tilde{q}'\to\tilde{q}}(1 + \tilde{q}'x)^{1/\tilde{q}'}, & \text{if } \tilde{q}x \geq -1 \\ 0, & \text{otherwise} \end{cases}, \tag{7}$$

where $\exp_{\tilde{q}}(x)$ is not real valued if $\tilde{q}x < -1$. This is a nonnegative function $\exp_{\tilde{q}}(x) \geq 0$ and $x = 0$ is a special point because $\exp_{\tilde{q}}(0) = 1$, independently of the value of $\tilde{q}$. For $\tilde{q} = 0$, $\exp_0(x) = \exp(x)$, the exponential function.

Let us point out two properties that make the algebraic manipulations easier with these functions. Consider the following properties [40]:

$$\ln_{\tilde{q}}(ab) = \ln_{\tilde{q}}(a) \oplus_{\tilde{q}} \ln_{\tilde{q}}(b) \tag{8}$$

$$\ln_{\tilde{q}}(a/b) = \ln_{\tilde{q}}(a) \ominus_{\tilde{q}} \ln_{\tilde{q}}(b) \tag{9}$$

$$\exp_{\tilde{q}}(a \oplus_{\tilde{q}} b) = \exp_{\tilde{q}}(a)\exp_{\tilde{q}}(b) \tag{10}$$

$$\exp_{\tilde{q}}(a \ominus_{\tilde{q}} b) = \exp_{\tilde{q}}(a)/\exp_{\tilde{q}}(b); \tag{11}$$

with the sum and subtraction operators defined as:

$$a \oplus_{\tilde{q}} b = a + b + \tilde{q}ab \tag{12}$$

$$a \ominus_{\tilde{q}} b = \frac{a - b}{1 + \tilde{q}b}. \tag{13}$$

Generalized operator can be defined for multiplication and division, but this is out of the scope of this paper. Note that the result of the $\tilde{q}$-minus operation is a hyperbole on the b variable.

In 2006, Cajueiro [9] proposed a $\tilde{q}$-generalized discount function, given by:

$$V^{(C)}(t) = \frac{V_0}{\exp_{\tilde{q}}(k_{\tilde{q}}t)}, \tag{14}$$

where V_0 is the objective value of the reward and $k_{\tilde{q}}$ is an impulsivity parameter. For $\tilde{q} = 0$, Equation (14) retrieves the exponential discount function (Equation 1). For $\tilde{q} = 1$, it retrieves the hyperbolic discount function (Equation 2). In Equation (14), using $\tilde{q} = 1/(ka)$ and $k_{\tilde{q}} = kab$, this model is mathematically equivalent to the Weber-Fechner's exponential with psychophysical effects on time perception model. In this way, $\tilde{q}$ models the subjectivity of one individual. It is expected that different individuals have different $\tilde{q}$ values, as in Anteneodo et al. [32].

3. RESULTS

In this section, we define a proper time (reference time) and analytically calculate a subjective time perception transformation. Let us consider an intertemporal choice process involving two rewards, "1" and "2." The objective value of the reward "2" is greater than that of the reward "1," $V_1(0) < V_2(0)$. These rewards must be paid with different delays, $t = 0$ and $t = \Delta\tau$, respectively (see **Figure 1A**). One compares the values of these two rewards and prefers/chooses the one "perceived" as greater. Since individuals tend to prefer smaller and more immediate rewards, let us suppose $V_1(0) > V_2(\Delta\tau)$, which leads to the choice of the smaller and immediate reward $V_1(0)$ (the **objective value** of the reward "1") to the greater and later one $V_2(\Delta\tau)$ (the **subjective value** of the reward "2" in the delay $\Delta\tau$) (see **Figure 1B**). If these same two options are presented repeatedly, but gradually decreasing the value of $\Delta\tau$ each time, there is a delay ($\Delta\tau_0$) where $V_1(0) = V_2(\Delta\tau_0)$. For this delay, where the individual changes his choice and starts choosing $V_2(\Delta\tau)$ to $V_1(0)$ (see **Figure 1C**).

In this context, for models that predict the dynamic inconsistency, $\Delta\tau_0$ is analogous to the "proper time" from the special theory of relativity and it is defined as the maximum delay from which the individual prefers the greater and later reward, $V_2(\Delta\tau_0)$, to the smaller and immediate one, $V_1(0)$. For the Cajueiro's generalized model (Equation 14), $V_{02}/V_{01} = \exp_{\tilde{q}}(k_{\tilde{q}}\Delta\tau_0^{(C)})$, leading to (see derivation process in the **Supplementary Material**):

$$\Delta\tau_0^{(C)} = \frac{1}{k_{\tilde{q}}}\ln_{\tilde{q}}\left(\frac{V_{02}}{V_{01}}\right), \tag{15}$$

where $V_{01} = V_1(0)$ and $V_{02} = V_2(0)$ are the objective values of the rewards "1" and "2," respectively, and the superscript (C) is a reference to the generalized model.

For $\tilde{q} = 0$, the exponential model, $\Delta\tau_0^{(C)} = (\ln V_{02} - \ln V_{01})/k_0 = [u^{(0)}(V_{02}) - u^{(0)}(V_{01})]/k_0$, which is the difference of the Bernoulli's (logarithmic) utility functions [41] $u^{(0)}(V) = \ln V$ for the reference values in monetary unities. One can write a generalized utility function based on this analogy. Since from

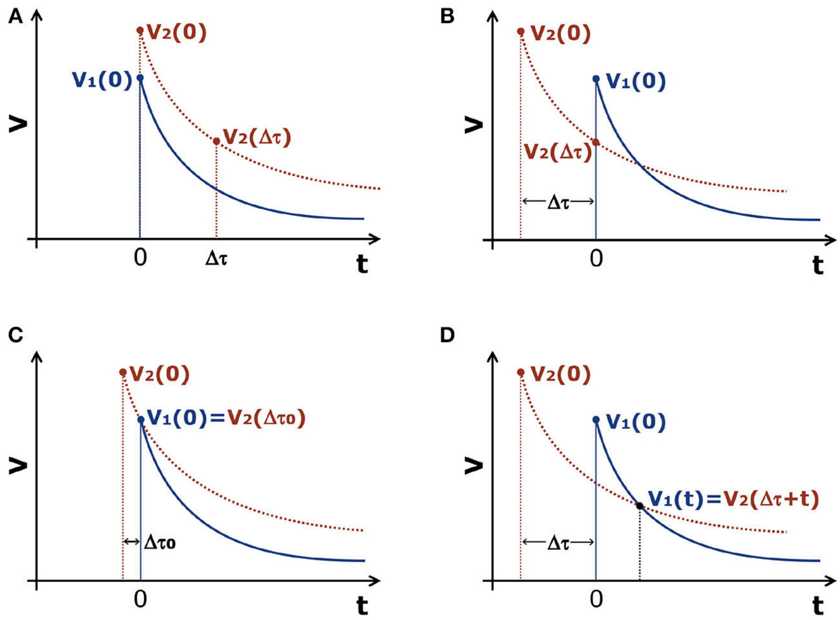

FIGURE 1 | Intertemporal choice process involving two rewards, "1" and "2," where $V_1(0) < V_2(0)$. The curves are from discount models that predict the dynamic inconsistency. The continuous curve refers to V_1 and the dashed one to V_2. **(A)** One should choose between these two rewards, to be paid with different delays: $V_1(0)$ (the objective value of the reward "1") or $V_2(\Delta\tau)$ (the subjective value of the reward "2" in the delay $\Delta\tau$). **(B)** To facilitate the comparison of these values, the curve of reward "2" was translated to the left to a "distance" of $\Delta\tau$, so that $V_1(0)$ and $V_2(\Delta\tau)$ were vertically aligned; one sees that $V_1(0) > V_2(\Delta\tau)$, which leads to the choice of reward "1." **(C)** Decreasing gradually the value of $\Delta\tau$, shifting the curve of reward "2" to the right, one finds a delay ($\Delta\tau_0$) where $V_1(0) = V_2(\Delta\tau_0)$. In this case, $\Delta\tau_0$ is analogous to the "proper time" of the special theory of relativity and it is defined as the maximum delay from which individuals prefer the greater and later reward to the smaller and immediate one. **(D)** There is a specify value of t where $V_1(t) = V_2(\Delta\tau + t)$. Here, There is an intertemporal preference reversal, because from this point individuals prefer the greater and later reward, $V_2(\Delta\tau + t)$, to the smaller and more immediate one, $V_1(t)$.

Equation (9), $\ln_{\tilde{q}}(V_{02}/V_{01}) = \ln_{\tilde{q}}(V_{02}) \ominus_{\tilde{q}} \ln_{\tilde{q}}(V_{01})$ with $\tilde{q}$-minus operator given by Equation (13), one can build a two-argument hyperbolic subjective utility function

$$u^{(\tilde{q})}(a, b) = \frac{a}{1 + \tilde{q}b}. \tag{16}$$

This leads to $\ln_{\tilde{q}}(V_{02}/V_{01}) = u^{(\tilde{q})}\left[\ln_{\tilde{q}}(V_{02}), \ln_{\tilde{q}}(V_{01})\right] - u^{(\tilde{q})}\left[\ln_{\tilde{q}}(V_{01}), \ln_{\tilde{q}}(V_{01})\right]$ and (see derivation process in the **Supplementary Material**)

$$\Delta\tau_0^{(C)} = \frac{u^{(\tilde{q})}\left[\ln_{\tilde{q}}(V_{02}), \ln_{\tilde{q}}(V_{01})\right] - u^{(\tilde{q})}\left[\ln_{\tilde{q}}(V_{01}), \ln_{\tilde{q}}(V_{01})\right]}{k_{\tilde{q}}}, \tag{17}$$

which shows that the time interval can be written as the difference of two utility functions.

Analogously, for models that predict the dynamic inconsistency, like the hyperbolic one (Equation 1) [1], we can present repeatedly the same previous two options, $V_1(0)$ and $V_2(\Delta\tau)$, but with equal and gradual increases in the delays for receiving the rewards. Thus, one expects that the choice also changes (intertemporal preference reversal), i.e., $V_1(t) = V_2(\Delta\tau + t)$ and one prefers the greater and later reward, $V_2(\Delta\tau + t)$, to the smaller and more immediate one, $V_1(t)$, from a certain time t (see **Figure 1D**). In the same way that the special theory of relativity presents a relation between the time intervals Δt and Δt_0, we propose expressions that relates $\Delta\tau$ and $\Delta\tau_0$. For the Cajueiro's generalized model (Equation 14) (see derivation process in the **Supplementary Material**):

$$\Delta\tau^{(C)}(t) = \left(1 + \tilde{q}k_{\tilde{q}}t\right)\Delta\tau_0^{(C)}, \tag{18}$$

where $1 + \tilde{q}k_{\tilde{q}}t$ is analogous to the Lorentz factor γ from the special theory of relativity. As expected for the exponential model, where $\tilde{q} = 0$, $\Delta\tau^{(e)} = \Delta\tau_0^{(e)}$, but subjective time dilation is expected for any $\tilde{q} > 0$.

If $t = 0$, one must choose between a smaller and immediate reward (an objective value) and a greater and later one (a subjective value). In this case, she/he agrees to wait a maximum time $\Delta\tau_0$ to choose the greater and later reward. But, as t increases, one starts to choose between a smaller and more immediate reward and a greater and later one (two subjective values). Here, she/he agrees to wait a maximum time $\Delta\tau$ [Equation (18)]—the temporal interval between the pay times of the rewards – to choose the greater and later reward, where $\Delta\tau > \Delta\tau_0$. In this paper, we assume that a gradual increase of t leads one to experience an increasing kind of subjective time dilation. Thus, the time t can make one feel the same "sensation" when she/he subjectively evaluates the "duration" of $\Delta\tau_0$ and $\Delta\tau$.

[1]and exponential with psychophysical effects on time perception (Equation 2) one

4. DISCUSSION

In this paper, we use arguments inspired by the special theory of relativity to deal with the dynamic inconsistency in intertemporal choices as the result of a subjective time dilation effect. We define the maximum time delay for which individuals prefer a greater and later reward to a smaller and immediate one, $\Delta\tau_0$, and relate it to the "proper time" from the special theory of relativity. In the same way, we define the maximum time delay for which individuals prefer a greater and later reward to a smaller and more immediate in a future time t, $\Delta\tau$. Focusing the study on a generalized model, which encompasses other ones that predict the dynamic inconsistence (for instance, the hyperbolic one), we find a factor, analogous to the Lorentz factor γ from the special theory of relativity, which relates $\Delta\tau$ and $\Delta\tau_0$: $\Delta\tau^{(C)} = (1+\tilde{q}k_{\tilde{q}}t)\Delta\tau_0^{(C)}$, where the superscript (C) is a reference to the generalized model.

We assume that the gradual increase of t leads one to experience an increasing kind of subjective time dilation, in a similar way to that performed by the increase of velocity in the special theory of relativity. Thus, the increase of time t makes the individual that subjectively evaluates the "duration" of $\Delta\tau$ feel the same time sensation caused by the "duration" of $\Delta\tau_0$, even $\Delta\tau > \Delta\tau_0$. It is important to point out that we assume individuals have the same value of k (discount rate) for the two rewards – the greater and later one an the smaller and more immediate one.

We stress that the time dilation effect of the special theory of relativity is a consequence of two hypothesis [11]: (1) The Principle of Relativity—there are an infinite number of inertial systems of reference in which all physical laws assume their simplest form; (2) The Principle of the Constancy of Light—in inertial system, the velocity of light has the same value when measured with length-measures and clocks of the same kind. Here, the subjective time dilation effect proposed in this paper is not the description of a physical effect, but a new interpretation for the dynamic inconsistency, and the consequent preference reversal over time in intertemporal choices. This interpretation is derived from models that predict the dynamic inconsistency, which are covered here by a generalized model, and permit a new way of facing this anomaly.

The use of generalized models provides a simple and practical way to include different psychophysical effects on time perception on the temporal discount functions. For instance, the logarithmic based Weber-Fechner and the power-law based **Stevens' law** [42] (**third law of psychophysics**) can be written in a unified way using the presented generalization of the logarithm function. Based on Equation (5), one writes the subjetive time as:

$$t'' = a\ln_s(1+bt)\ , \tag{19}$$

where the second law of phychophysics is retrieved for $s=0$. In 2011, Destefano and Martinez [33] proposed a very general and unified model for the discount process taking into account:

$$V^{(D)}(t) = \frac{V_0}{\exp_{\tilde{q}}\left[k_{\tilde{q}}a\ln_s(1+bt)\right]}\ . \tag{20}$$

In Destefano and Martinez [33], a complete study of possible values of $\tilde{q}$ and s has been performed. Also, the authors have shown that it is possible to dissociate the degree of inconsistency in two distinct parts of perception: one for value and other for time. The authors demonstrated that the direct analysis of the degree of inconsistency is the natural measure that favors the interpretation of the discount process. For the model of Equation 20,

$$\Delta\tau_0^{(D)} = \frac{1}{b}\left[\exp_s\left(\frac{\Delta\tau_0^{(C)}}{a}\right)-1\right] \tag{21}$$

and

$$\Delta\tau^{(D)}(t) = \frac{1}{b}\left\{\left(1+b\Delta\tau_0^{(D)}\right)\exp_s\left[\left(\frac{V_{02}}{V_{01}}\right)^{\tilde{q}}\frac{\ln_s(1+bt)}{1+b\Delta\tau_0^{(D)}}\right]-1\right\}-t\ . \tag{22}$$

This equation does not simply connect $\Delta\tau^{(D)}(t)$ with $\Delta\tau_0^{(D)}$ as in Eq. (18). This non-linear behavior may lead to some effect that will be studied in detail in a near future.

A study conducted in 2017 by Agostino et al. [12] have shown the importance of the individual differences to the average and individual psychophysical functions of long-range time representation. It suggests that the study of the deviations from exponential discount models in intertemporal choices to other ones that predict dynamic inconsistency must involve "... the characterization of subjective time in an individual-participant basis." That is exactly what our model provides, fitting individual data and account for the differences in discount rate, as the $\tilde{q}$ parameter can be individually adjusted an generate different discount functions. Our characterization of the subjective time in intertemporal choices procedures also covers cases where dynamic inconsistency is not involved, since the generalized discount model that we adopt encompasses other ones, like the exponential model, which do not deal with this anomaly.

To conclude, we have shown that dynamic inconsistency in intertemporal decision making can be seen as the result of a subjective time dilation perception effect. Based on a well-established theoretical framework, we have found a simple transformation equation for the time interval between the pay times of two rewards, showing that this subjective perception effect can be modeled by generalized models that encompasses particular cases that predict dynamic inconsistency. We also have found a broader transformation equation, derived from a very general and unified discount model, which will be further studied. Since the Bernoulli utility function is logarithmic, one can face it as a special case of a generalized hyperbolic one $u^{(\tilde{q})}(a,b) = a/(1+\tilde{q}b)$. In this way, our "proper time" can be written as $\Delta\tau_0^{(C)} = \left\{u^{(\tilde{q})}\left[\ln_{\tilde{q}}(V_{02}),\ln_{\tilde{q}}(V_{01})\right]-u^{(\tilde{q})}\left[\ln_{\tilde{q}}(V_{01}),\ln_{\tilde{q}}(V_{01})\right]\right\}/k_{\tilde{q}}$, which associate a time interval with the difference of two utility functions. Our proposals of a two-value utility function and a time interval transformation unveils the subtle connection between the subjectivity on value and time perceptions. Anomalies in intertemporal decision making can be translated and quantified in terms of value perceptions and shall be further explored.

AUTHOR CONTRIBUTIONS

Both authors contributed equally on the discussion of the ideas, development of the study and writing the manuscript.

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SUPPLEMENTARY MATERIAL

The Supplementary Material for this article can be found online at: https://www.frontiersin.org/articles/10.3389/fams.2018.00054/full#supplementary-material

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A Mathematical Analysis of the Improving Sequence Effect for Monetary Rewards

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In this paper, we mathematically formalize the concept of improving sequence effect, which is one of the main anomalies of the discounted utility model [1]. The improving sequence effect implies a preference for a given sequence of outcomes, which increase over time, and has been empirically demonstrated for both monetary and nonmonetary results (hedonic experiences and health-related outputs). Nevertheless, to date, there is no mathematical treatment of this anomaly in the context of intertemporal choice, which allows us to relate this paradox to other anomalies, such as the delay and magnitude effects. In this way, the present manuscript has filled this gap. More specifically, we have proved that the improving sequence effect for monetary rewards cannot be rationalized by using a separable discount function but only by considering a non-separable discount function. Moreover, under certain conditions, we have proved that the delay and magnitude effects are necessary conditions for the existence of the improving sequence effect.

Keywords: improving sequence effect, intertemporal choice, discounted utility model, delay effect, magnitude effect, discount function

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1. INTRODUCTION

Generally speaking, the *improving sequence effect* is an anomaly revealed in the ambit of intertemporal choice by which individuals prefer sequences of outcomes, which increase over time, rather than decrease, or flat sequences with equal mean [2]. In other words, people like improvement in such a way that they would prefer to leave the best outcomes for the last maturities of the sequence [3–5]. Thus, rather than experiencing the best outcome sooner, people usually prefer to postpone a good outcome. Obviously, this is a violation of the discounted utility model [1], since individuals show a negative time preference for sequences of outcomes rather than a positive time preference, as displayed for single outcomes.

Read and Powell [6] define the improving sequence effect for monetary sequences as follows: "Given a choice between two ways of distributing a fixed amount of money over time, either as a falling sequence going from large to small, or a rising sequence going from small to large, a rational decision maker will choose a falling sequence. This is because at a non-zero rate of interest the falling sequence dominates the rising one, since any unspent money can earn interest. Despite this, there is evidence that people often prefer rising to falling sequences."

More specifically, Loewenstein and Sicherman [7] and Loewenstein and Prelec [8] define this anomaly for income sequences in the following way: "Given a positive real rate of interest, a worker presented with alternative income sequences $X = (x_1, \ldots, x_n)$ and $Y = (y_1, \ldots, y_n)$ for otherwise identical jobs where

$\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} y_i$, $x_i > y_i$, for $i = 1, \ldots, j$, and $y_i > x_i$, for $i = j+1, \ldots, n$, should select X over Y. [. . .] Sequence X dominates Y; by selecting X and saving appropriately, workers could enjoy greater consumption in every period." However, empirical evidence shows that workers actually prefer increasing wage profiles over flat or decreasing wage profiles of greater monetary present value.

Despite this, many researchers have theoretically and empirically studied the improving sequence effect, most are cited in the following section, and none presented a mathematical analysis of this anomaly. In this way, the main contribution of this paper is the rationalization of the improving sequence effect as an anomaly, observed in the context of an intertemporal choice by using a non-separable discount function. Moreover, this manuscript finds an interesting result linking the improving sequence effect to the delay and magnitude effects.

Therefore, the main objective of this paper is to present an analysis of the improving sequence effect for monetary sequences. The organization of this paper is as follows. Section 2 provides three mathematical definitions of the improving sequence effect and the implications between them. On the other hand, section 3 includes a proof of the present value maximization principle for decreasing income sequences by using a separable discount function. Section 4 introduces the concept of a non-separable discount function to explain the improving sequence effect in the context of discrete and continuous distributions of capital. Finally, after demonstrating the relationship between the improving sequence effect and the delay and magnitude effects (section 5), section 6 summarizes and concludes the study.

2. MATHEMATICAL DEFINITIONS OF THE IMPROVING SEQUENCE EFFECT

Although there are several theoretical definitions of the improving sequence effect (seen in the Introduction section of this paper), to the best of our knowledge, no mathematical definitions have been proposed yet. For this reason, we provide three mathematical definitions, which support the ideas previously indicated in the Introduction section. But, before this, we have to highlight two important observations:

- Our definitions restrict the idea provided by Loewenstein and Sicherman [8] because we only consider income sequences variable in (increasing or decreasing) arithmetic progression and not general income sequences. Even a preference for a mixed response (increasing-decreasing-increasing) has been found in the case of nonmonetary outcomes [9].
- However, our third definition allows for a given income sequence to be preferred, not only over all decreasing income sequences but also over the remaining increasing income sequences.

Before discussing the three definitions of the improving sequence effect, let us introduce the following definitions from the past.

Definition 1. *Let R be a set of money amounts and T a set of time instants. The pair (R, T) is said to be a distribution of capital if there is a one-to-one correspondence between the elements of R and the elements of T.*

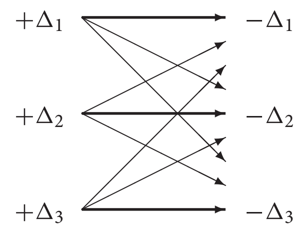

Chart 1 | Scheme corresponding to Definition 2.

In our definitions of the improving sequence effect, we will only consider a special set of money amounts, viz a sequence variable in an arithmetic progression.

Definition 2. *Given an amount S and a period of time n, an intertemporal choice is said to satisfy the improving sequence effect if, for every $\Delta > 0$, the sequence*

$$(a, 1), (a + \Delta, 2), (a + 2\Delta, 3), \ldots, (a + (n-1)\Delta, n), \quad (1)$$

where $a = \frac{S}{n} - \frac{n-1}{2}\Delta > 0$, is preferred over the rest of the decreasing sequences variable in an arithmetic progression whose positive terms mature at $1, 2, \ldots, n$, all terms summing up to S.

Definition 2 can be schematically viewed in **Chart 1**, where the arrow $+\Delta_i \longrightarrow -\Delta_j$ $(i, j = 1, 2, 3)$ means that the sequence

$$(a, 1), (a + \Delta_i, 2), (a + 2\Delta_i, 3), \ldots, (a + (n-1)\Delta_i, n),$$

is preferred to

$$(a, 1), (a - \Delta_j, 2), (a - 2\Delta_j, 3), \ldots, (a - (n-1)\Delta_j, n),$$

both satisfying the conditions displayed in definition 2.

Definition 2, which is called the *strong definition* of the improving sequence effect, does not completely fit the idea underlying this anomaly since, in the empirical studies on monetary sequences [2, 4, 6, 10–14], there are some decreasing sequences, which are preferred over other improving sequences, whose terms sum up to the same amount. Therefore, we provide a new definition, called the *semi-strong definition* of the improving sequence effect.

Definition 3. *Given an amount S and a period of time n, an intertemporal choice is said to satisfy the improving sequence effect if, for every $\Delta > 0$, the sequence (1) is preferred over the following decreasing sequence*

$$(a + (n-1)\Delta, 1), (a + (n-2)\Delta, 2), \ldots, (a + \Delta, n-1), (a, n),$$

all terms summing up to S.

Chart 2 schematizes definition 3.

Despite introducing the former two definitions, the following one fits the idea of the improving sequence effect provided by Loewenstein and Sicherman [8] better. In other words, the conditions imposed on definition 2 and 3, need to be relaxed, giving rise to the *weak definition* of the improving sequence effect.

$$\begin{tikzcd} +\Delta_1 \arrow[r] & -\Delta_1 \\ +\Delta_2 \arrow[r] & -\Delta_2 \\ +\Delta_3 \arrow[r] & -\Delta_3 \end{tikzcd}$$

Chart 2 | Scheme corresponding to Definition 3.

$$\begin{tikzcd} +\Delta_1 & -\Delta_1 \\ +\Delta_2 \arrow[u] \arrow[d] \arrow[r] \arrow[ur] \arrow[dr] & -\Delta_2 \\ +\Delta_3 & -\Delta_3 \end{tikzcd}$$

Chart 3 | Scheme corresponding to Definition 4.

DEFINITION 4. *Given an amount S and a period of time n, the intertemporal choice is said to satisfy the improving sequence effect if there is a $\Delta > 0$ such that sequence (1) is preferred over the rest of the sequences whose positive terms mature at $1, 2, \ldots, n$ and are constant or variable (increasing or decreasing) in an arithmetic progression, all terms summing up to S.*

Chart 3 schematizes definition 4.

These three definitions fit the idea of the improving sequence effect (and the negative time preference). However, as indicated, taking into account the results obtained from the empirical studies on this effect, definition 4 fits the empirically observed behavior better than the other behaviors and also allows a computational treatment, when analytically solving the equation, leading to the preferred sequence.

PROPOSITION 1. *Definition 2 implies definition 3, and definition 3 implies definition 4.*

Proof. See **Appendix** in Supplementary Material. □

Observe that definitions 2, 3, and 4 involve all values of the common difference Δ such that the terms of the corresponding sequences are positive. However, the set of differences could be restricted to a neighborhood of zero of the form $]-k, k[$, where $k > 0$, and incremental times to an interval $[0, h]$, where $h > 0$, giving rise to the respective "local" concepts of improving sequence effect. Finally, in the rest of this paper, we only considered definitions 3 and 4.

3. PRESENT VALUE MAXIMIZATION PRINCIPLE FOR INCOME SEQUENCES

Loewenstein and Sicherman [7] proved through a numerical example that the present value of a decreasing sequence is greater than the present value of the constant and the improving sequences, provided that their terms total the same amount ($150,000). This statement can be generalized to mean that the present value of a sequence variable in an arithmetic progression, whose terms total a fixed amount, is decreasing with respect to the difference of the progression. This result is consistent with the financial principles: the smaller the difference, the higher the amount of the first term, which consequently is discounted using a shorter period of time than the others. Moreover, it is relevant to note that Loewenstein and Sicherman [7] used the exponential discount, which is characterized by the use of a constant instantaneous discount rate. If the instantaneous discount rate [15] is constant (exponential discounting), it is intuitive to see that the discount function cannot explain the improving sequence effect. Now, we wonder if this result can be generalized to any separable discount function, regardless of whether its instantaneous discount rate is increasing or decreasing [16]. The answer to this question is affirmative, but before that, we will introduce some formal nomenclature and notation.

Let $F(t)$ be a (separable) discount function and let $a, a+\Delta, a+2\Delta, \ldots, a+(n-1)\Delta$ be an ordinary annuity variable in an arithmetic progression, where $a > 0$ is the first term and Δ is the common difference, such that the sum of all terms is constant and equal to S:

$$a + (a+\Delta) + (a+2\Delta) + \cdots + [a+(n-1)\Delta] = S.$$

Next, the following result can be stated.

PROPOSITION 2. *The present value of the ordinary annuity whose terms are variable in arithmetic progression:*

$$a, a+\Delta, a+2\Delta, \ldots, a+(n-1)\Delta$$

such that

$$a + (a+\Delta) + (a+2\Delta) + \cdots + [a+(n-1)\Delta] = S,$$

using any separable discount function, is decreasing with respect to Δ.

Proof. See **Appendix** in Supplementary Material. □

Proposition 2 can also be shown if $a, a+\Delta, a+2\Delta, \ldots, a+(n-1)\Delta$ is an annuity due variable in arithmetic progression.

COROLLARY 1. *The present value of the ordinary annuity variable in arithmetic progression*

$$a, a+\Delta, a+2\Delta, \ldots, a+(n-1)\Delta$$

such that

$$a + (a+\Delta) + (a+2\Delta) + \cdots + [a+(n-1)\Delta] = S,$$

using the exponential discounting $F(t) = (1+i)^{-t}$, with $i > 0$, is decreasing with respect to Δ. □

4. THE CASE OF NON-SEPARABLE DISCOUNT FUNCTIONS

Proposition 2 shows that, in order to explain the improving sequence effect, it is not possible to consider a separable discount function [17]. So, our first task was to define a non-separable discount function. See, for example, Lisei's paper [18].

DEFINITION 5. *A non-separable discount function is a real-valued function*

$$F : \mathbb{R} \times \mathbb{R}^+ \to \mathbb{R}$$

such that

$$(x, t) \mapsto F(x, t),$$

satisfying the following conditions:

1. *$F(x, 0) = x$, for every $x \in \mathbb{R}$.*
2. *$F(0, t) = 0$, for every $t \in \mathbb{R}^+$.*
3. *F is increasing with respect to x. If F is differentiable, $\frac{\partial F(x,t)}{\partial x} > 0$.*
4. *F is decreasing (respectively, increasing) with respect to t, if $x > 0$ (respectively, if $x < 0$). If F is differentiable, $\frac{\partial F(x,t)}{\partial t} < 0$ (respectively, $\frac{\partial F(x,t)}{\partial t} > 0$), if $x > 0$ (respectively, if $x < 0$).*

As an immediate consequence of conditions 1 and 3, it can be deduced that

$$F(x, t) < x, \text{ if } x > 0, \text{ and } F(x, t) > x, \text{ if } x < 0.$$

$F(x, t)$ represents the amount at time 0, which is indifferent to the amount x at time t.

4.1. Improving Sequence Effect With Discrete Distributions of Capital

In order to verify the existence of the improving sequence effect by using a given non-separable discount function, we had to solve the following problem: to maximize the present value

$$PV = \sum_{t=0}^{n-1} F(a + \Delta t, t + 1).$$

This optimization problem is equivalent to solving the following equation in Δ:

$$\frac{\mathrm{d}PV}{\mathrm{d}\Delta} = 0,$$

that is to say,

$$\sum_{t=0}^{n-1} \left(t - \frac{n-1}{2}\right) \left.\frac{\partial F(x, t+1)}{\partial x}\right|_{x=a+\Delta t} = 0, \tag{2}$$

where

$$a = \frac{S}{n} - \frac{n-1}{2}\Delta.$$

Among all solutions of equation (2), we had to dismiss those such that some $a + \Delta t$ ($t = 0, 1, \ldots, n-1$) was negative. Taking into account that all solutions are relative extremes, we had to choose the value of Δ corresponding to the greatest present value. To do this, if Δ_0 is a solution of equation (2), we had to verify that

$$\left.\frac{\mathrm{d}^2 PV}{\mathrm{d}\Delta^2}\right|_{x_0=a_0+\Delta_0 t} < 0, \tag{3}$$

where

$$a_0 = \frac{S}{n} - \frac{n-1}{2}\Delta_0,$$

that is to say

$$\sum_{t=0}^{n-1} \left(t - \frac{n-1}{2}\right)^2 \left.\frac{\partial^2 F(x, t+1)}{\partial x^2}\right|_{x_0=a_0+\Delta_0 t} < 0.$$

EXAMPLE 1. *The non-separable discount function $F(x, t) = x \exp\{-kt/x^\alpha\}$, with $k > 0$ and $\alpha > 1$ [19], satisfies the improving sequence effect for $k = 0.10$, $n = 3$, $\alpha = 2$, and $S = 6$. In effect, in this case,*

$$\frac{\partial F(x, t)}{\partial x} = \left(1 + \frac{kt\alpha}{x^\alpha}\right) \exp\{-kt/x^\alpha\}$$

and

$$\frac{\partial^2 F(x, t)}{\partial x^2} = \frac{kt\alpha}{x^{\alpha+1}} \left(1 + \frac{kt}{x^\alpha} - \alpha\right) \exp\{-kt/x^\alpha\}.$$

To solve equation (2), we obtained the value $\Delta_0 = 0.536$. In other words, this value of Δ maximizes the present value, as represented in **Figure 1**. Proposition 2 proved that if the discount function depends on time t and linearly on the reward x

$$F(x, t) = xF(t),$$

it is not rational that the improving sequence effect can hold. In other words, the improving sequence effect can occur when the discount function depends on time and is not linear with respect to the discounted outcome

$$F = F(x, t),$$

as confirmed by example 1.

Next, we wondered if alternatively this calculation could be easier if we worked with continuous distributions of capital. Before continuing with the development of the next section, we are going to introduce the following definition.

DEFINITION 6. *A distribution of capital $D = (R, T)$ is said to be continuous if T is a real interval and $R(t)$ is a continuous function [20], where $R(t)\mathrm{d}t$ is the elemental amount corresponding to time t.*

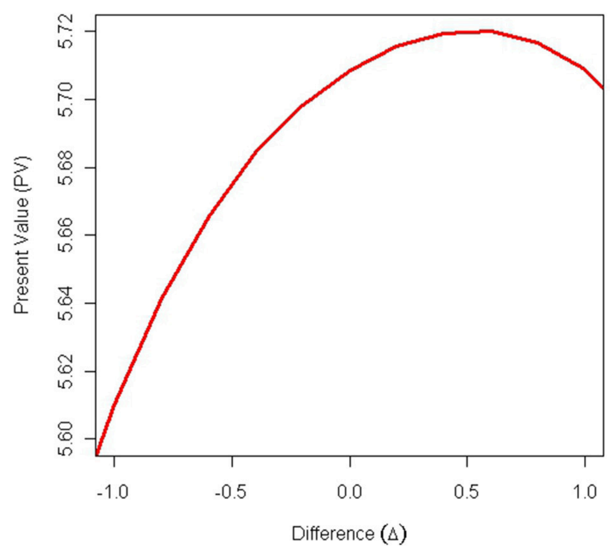

FIGURE 1 | Present value as a function of the common difference Δ (Example 1).

4.2. Improving Sequence Effect With Continuous Distributions of Capital

The objective of this subsection is to use continuous annuities for the analysis and description of the improving sequence effect. In this case, as indicated, we can not use separable discount functions. Therefore, we have to introduce the expression of the present value of a continuous annuity by using a non-separable discount function, $F(x,t)$:

$$PV = \lim_{n\to\infty} \sum_{k=1}^{n} F\left[R\left(\frac{k}{n}l\right), \frac{k}{n}l\right]\frac{l}{n}$$

or, equivalently,

$$\int_0^l F(R(t), t)\mathrm{d}t.$$

Obviously, in the treatment of the improving sequence effect, we will work with linear continuous annuities. More specifically, we will work with the family of continuous annuities defined by a straight line, $R(t) = mt + n$, such that $R(t) > 0$, for every t in $[0, l]$, and where the area defined by $R(t)$, the x-axis, and the vertical lines $t = 0$ and $t = l$ is constant and equal to S. From this, there are two important remarks:

1. The slope m can be positive or negative as long as the condition $R(t) > 0$, for every t in $[0, l]$, holds.
2. The values of l and S are given as constants.

In this context, the expression of the present value of the continuous annuity $R(t) = mt+n$, valued with the non-separable discount function $F(x, t)$, is

$$PV = \int_0^l F(mt + n, t)\mathrm{d}t. \tag{4}$$

Before going further, it is necessary to show the relationship between m and n. To do this, we took the following condition into account:

$$\int_0^l (mt + n)\mathrm{d}t = S,$$

from where

$$\left(m\frac{t^2}{2} + nt\right)\Big|_0^l = S,$$

or, equivalently,

$$m\frac{l^2}{2} + nl = S,$$

and finally,

$$n = \frac{S}{l} - \frac{ml}{2}.$$

Therefore,

$$R(t) = m\left(t - \frac{l}{2}\right) + \frac{S}{l}.$$

Now, we formulate the following question: Is there any value of m which maximizes the present value? If this value m_0 is positive, the improving sequence effect would hold. To do this, let us calculate the first derivative of the present value with respect to m:

$$\frac{\mathrm{d}PV}{\mathrm{d}m} = \int_0^l \frac{\partial F(x,t)}{\partial x}\Big|_{x=mt+n} \frac{\mathrm{d}R(t)}{\mathrm{d}m}\mathrm{d}t.$$

Hence,

$$\frac{\mathrm{d}R(t)}{\mathrm{d}m} = t - \frac{l}{2}$$

and consequently

$$\frac{\mathrm{d}PV}{\mathrm{d}m} = \int_0^l \frac{\partial F(x,t)}{\partial x}\Big|_{x=\left(t-\frac{l}{2}\right)m+\frac{S}{l}} \left(t - \frac{l}{2}\right)\mathrm{d}t. \tag{5}$$

Observe that, in equation (5), it is satisfied that

- $\frac{\partial F(x,t)}{\partial x}\Big|_{x=\left(t-\frac{l}{2}\right)m+\frac{S}{l}} > 0$, by the definition of $F(x, t)$.
- $t - \frac{l}{2} \leq 0$, for every t in $\left[0, \frac{l}{2}\right]$, and $t - \frac{l}{2} \geq 0$, for every t in $\left[\frac{l}{2}, l\right]$.

Therefore, the sign of the integral depends on the increase or the decrease of the factor $\frac{\partial F(x,t)}{\partial x}\Big|_{x=\left(t-\frac{l}{2}\right)m+\frac{S}{l}}$ with respect to t.

Finally, in order to prove that the obtained value of m, denoted by m_0, is a maximum, it is necessary that

$$\left.\frac{\mathrm{d}^2 PV}{\mathrm{d}m^2}\right|_{m=m_0} < 0.$$

Therefore, the use of continuous distributions of capital can facilitate the resolution of equation (2), which arises in the case of a discrete distribution of capital, and it is presented as an alternative of easy calculation.

5. IMPROVING SEQUENCE EFFECT AND THE DELAY AND MAGNITUDE EFFECTS

Before analyzing the relationship between the improving sequence effect and the delay and magnitude effects, the following statement can be enunciated:

PROPOSITION 3. *A non-separable discount function $F(x,t)$ describes the semi-strong improving sequence effect, for every period length $h \leq h_0$, where $h_0 > 0$, and every amount difference $-\Delta_0 \leq \Delta \leq \Delta_0$, with $\Delta_0 > 0$, if and only if $\frac{\partial^2 F(x,t)}{\partial x \partial t} \geq 0$ and, moreover, the set*

$$M := \left\{(x,t) \in \mathbb{R} \times \mathbb{R}^+ : \frac{\partial^2 F(x,t)}{\partial x \partial t} > 0\right\}$$

is dense in the set

$$N := \left\{(x,t) \in \mathbb{R} \times \mathbb{R}^+ : \frac{\partial^2 F(x,t)}{\partial x \partial t} \geq 0\right\}$$

in the usual topology of $\mathbb{R}^2$.

Proof. See **Appendix** in Supplementary Material. □

In what follows, our aim was to mathematically relate the increasing sequence effect to the delay and magnitude effects but, before this, we had to define them. *Delay effect* consists of using higher discount rates for short intervals than for large ones. This way, discount rates decrease as waiting time (to obtain the reward) increases.

In the experiment conducted by Benzion et al. [21], the inferred (mean) discount rates for postponing the receipt of a 200-dollar amount were 42.8, 25.5, 23.0, and 19.5% for delays of 6 months, 1, 2, and 4 years, respectively. In other words, they obtained higher average discount rates for shorter time intervals.

More specifically, Prelec and Loewenstein [22] formalized this effect in the following way: $(x,t_1) \sim (y,t_2)$ implies $(x,t_1+h) \prec (y,t_2+h)$, for $x < y$ and $h > 0$.

On the other hand, some researchers found that the rates used to discount large amounts were lower than the rates used to discount small ones, which was labeled as the *magnitude effect* [5]. Thaler [23] carried out an experiment where the participants were indifferent between receiving \$15 right away and \$60 in a year, \$250 now and \$350 in a year, and \$3,000 immediately and \$4,000 in a year, implying discount rates of 139, 34, and 29%, respectively.

This effect was expressed formally by Prelec and Loewenstein [22] as follows: $(x,t_1)\sim(y,t_2)$ implies $(\alpha x,t_1)\prec(\alpha y,t_2)$, for $\alpha > 1$ and $0 < x < y$ (which implies $t_1 < t_2$). Moreover, Gerber and Rohde [24] proposed this definition: preferences exhibit the magnitude effect at date t if the outcomes x, y, X, and Y are given with $0 < x < X$ and $0 < y < Y$, then $(x,0)\sim(y,t)$ and $(X,0)\sim(Y,t)$ implies $X/Y > x/y$. In the same way, Read [25] mathematically defined the magnitude effect: imagine that a small outcome (x_1^s) and a large outcome (x_1^l) are respectively equated with outcomes x_2^s and x_2^l available at different times in the future, such as $u(x_1^s) = u(x_2^s)$ and $u(x_1^l) = u(x_2^l)$, then $x_1^l/x_2^l > x_1^s/x_2^s$. However, we will use a stronger definition of the magnitude effect: $(x,t_1)\sim(y,t_2)$ implies $(x+k,t_1)\prec(y+k,t_2)$, for $k > 0$ and $0 < x < y$.

There are two behavioral explanations for this anomaly:

1. Individuals perceive and are influenced not only by relative differences but also by absolute differences in monetary amounts. The difference between \$100 now and \$150 in a year seems to be greater than the difference between \$10 now and \$15 in a year; for this reason, a lot of people are willing to wait for \$50 but not for \$5 [22].
2. Mental accounting [26] affects the amount entered into a mental checking. Large amounts are considered as savings and small amounts as consumption. Thus, the cost of waiting for a small reward may be perceived as a foregone consumption, whilst the opportunity cost of waiting for a large amount is perceived as a foregone interest [27].

Delay effect is equivalent to expecting that the instantaneous discount rate

$$\delta(x,t) = -\frac{\frac{\partial F(x,t)}{\partial t}}{x\frac{\partial F(x,t)}{\partial x}}$$

is decreasing with respect to the time t, that is to say,

$$\frac{\partial \delta(x,t)}{\partial t} < 0.$$

Analogously, magnitude effect is equivalent to expecting that the instantaneous discount rate is decreasing with respect to the amount x, that is to say,

$$\frac{\partial \delta(x,t)}{\partial x} < 0.$$

Once the delay and magnitude effects are characterized, the following two corollaries demonstrate that they are necessary conditions for the existence of the semi-strong improving sequence effect in terms of proposition 3.

COROLLARY 2. *If a non-separable discount function $F(x,t)$ describes the semi-strong improving sequence effect for every period length $h \leq h_0$ and every amount difference $-\Delta_0 \leq \Delta \leq \Delta_0$ and $\frac{\partial^2 F(x,t)}{\partial t^2} \geq 0$, then it satisfies the delay effect.*

Proof. See **Appendix** in Supplementary Material. □

EXAMPLE 2. *Let us consider the so-called hyperbolic discount function deformed by the amount [28], that is to say, the following* *non-separable discount function:*

$$F(x,t) = \frac{x}{1+it/x} = \frac{x^2}{x+it},$$

defined for every $x \in \mathbb{R}^+$, where $i > 0$. Thus, one has

- $\frac{\partial F(x,t)}{\partial t} = -\frac{ix^2}{(x+it)^2}$.
- $\frac{\partial F(x,t)}{\partial x} = \frac{x^2+2ixt}{(x+it)^2}$.

Therefore, as

$$\frac{\partial^2 F(x,t)}{\partial t^2} = \frac{2i^2x^2}{(x+it)^3} > 0$$

and

$$\delta(x,t) = \frac{ix^2}{x^3+2ix^2t} = \frac{i}{x+2it},$$

$F(x,t)$ satisfies the condition involved in the statement of corollary 2 and the delay effect (observe that $\delta(x,t)$ is strictly decreasing with respect to t).

However, as

$$\frac{\partial^2 F(x,t)}{\partial x \partial t} = -\frac{2i^2xt}{(x+it)^3} < 0,$$

$F(x,t)$ does not satisfy the local semi-strong definition of the improving sequence effect.

COROLLARY 3. *If a non-separable discount function $F(x,t)$ describes the semi-strong improving sequence effect for every period length $h \leq h_0$ and every amount difference $-\Delta_0 \leq \Delta \leq \Delta_0$ and $\frac{\partial^2 F(x,t)}{\partial x^2} \geq 0$, then it satisfies the magnitude effect.*

Proof. See **Appendix** in Supplementary Material. □

These results fit the experimental findings from Duffy and Smith [14] who found a positive relationship between the preference for increasing payments (sequence effect) and the size of those payments (magnitude effect). Nevertheless, the converse statement of corollary 3 is not true, as shown in example 3.

EXAMPLE 3. *With the same non-separable discount function of example 2, one has*

$$\frac{\partial^2 F(x,t)}{\partial x^2} = \frac{2i^2t^2}{(x+it)^3} > 0.$$

Thus, $F(x,t)$ satisfies the condition involved in the statement of corollary 3 and the magnitude effect (observe that $\delta(x,t)$ is strictly decreasing with respect to x). However, as indicated in example 2, $F(x,t)$ does not satisfy the local semi-strong definition of the improving sequence effect.

Summarizing, under the same hypothesis of corollaries 2 and 3 some additional conditions are required in the instantaneous discount rate so that the converse statements holds. This task will be left for further research.

6. CONCLUSION

This paper contributes to the existing literature on the improving sequence effect by presenting a mathematical analysis of this anomaly. First of all, in order to relate this effect with other effects, we proposed three mathematical definitions of this paradox. These three definitions fit the idea of the improving sequence effect (negative time preference); however, based on the results from the empirical studies, definition 4 fits the empirically observed behavior better than the others and allows a computational treatment to find the best sequence.

Moreover, it has been mathematically proven that the present value of a sequence variable in an arithmetic progression, whose terms total a fixed amount, decreases with respect to the difference of the progression either using the exponential discount function or any other discount function. Additionally, we introduced a new methodology to detect and explain the improving sequence effect with discrete distributions of capital by using non-separable discount functions (see Example 1).

Finally, a relationship between the improving sequence effect and the delay and magnitude effects was presented. More specifically, we proved that, under certain hypotheses, the semi-strong improving sequence effect is a sufficient (but not a necessary) condition for both the delay and magnitude effects.

AUTHOR CONTRIBUTIONS

All authors listed have made a substantial, direct and intellectual contribution to the work, and approved it for publication.

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SUPPLEMENTARY MATERIAL

The Supplementary Material for this article can be found online at: https://www.frontiersin.org/articles/10.3389/fams.2018.00055/full#supplementary-material

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